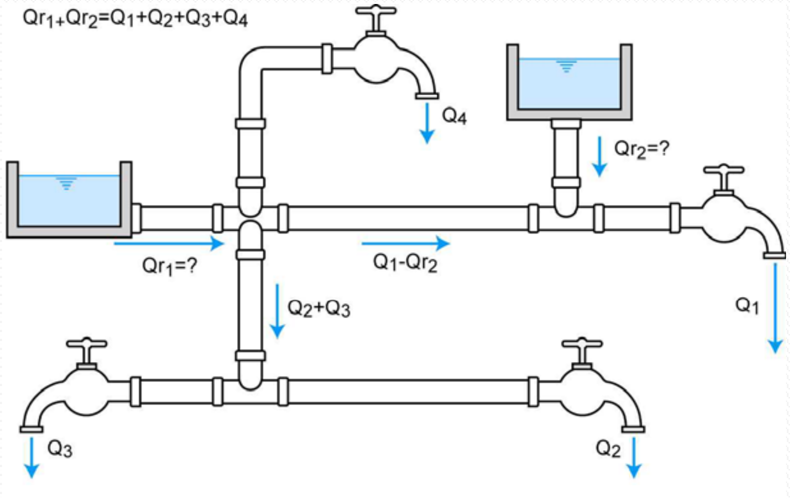
NETWORK HYDRAULICS
CEI 431

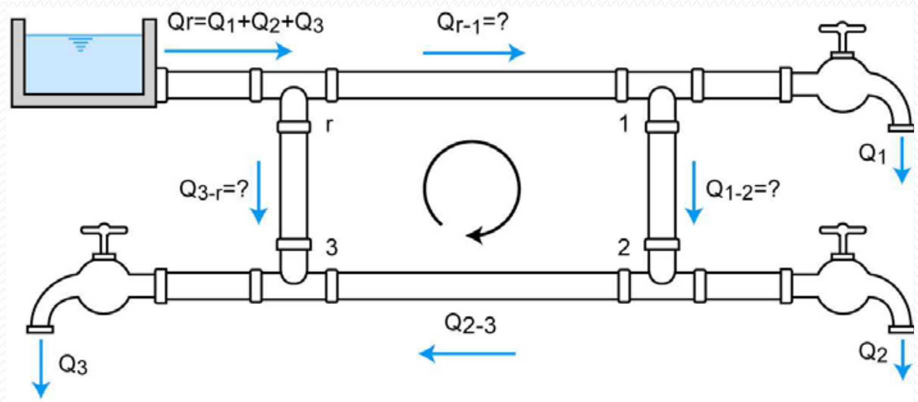
Network

- This chapter describes the analysis of steady flow in pipe systems.
- In any analysis problem all of the physical features of the network are known, and the solution process endeavours to determine the discharge in every pipe and the pressure, etc. at every node of the network.
- We believe it is important for an engineer to understand what is being accomplished in these computer solutions. To aid engineers in gaining this knowledge, we begin with the basic principles, and the equations that embody them, that interrelate the discharge in each pipe and the pressure at each node of the network.

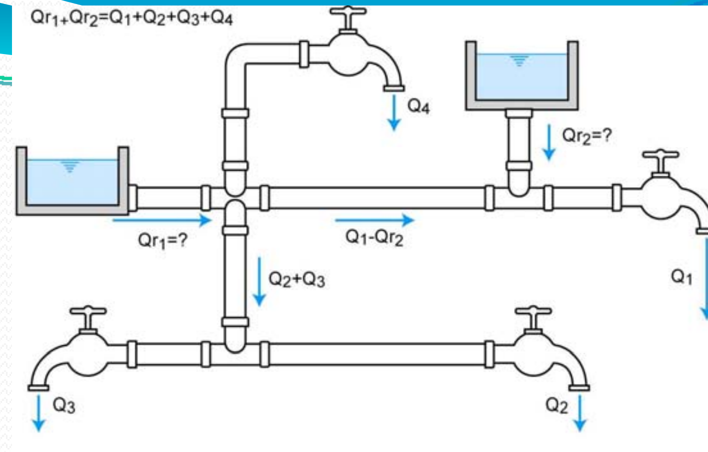


Branched System (Tree)

Looped Network



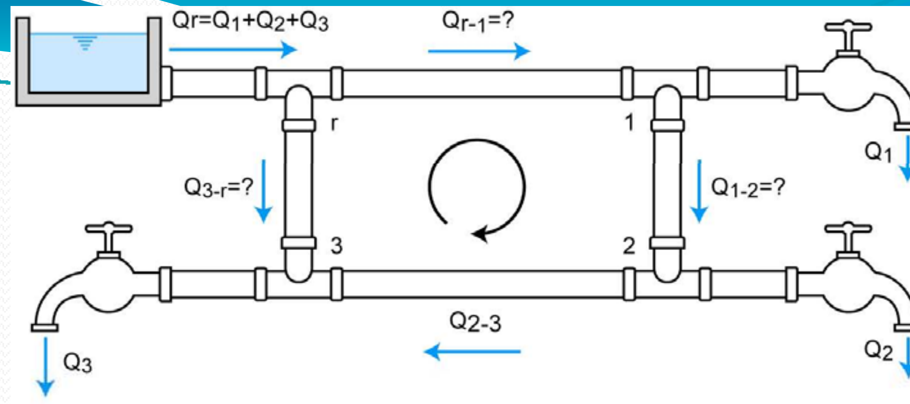
Branched Systems (Tree)



- **Pipe Flows**
  - **For known nodal demands, the rates can be partially determined.**
  - **Flow rates & directions in the pipe routes connecting the sources depend on the piezometric heads at the sources and the distribution of nodal demands.**
- **Velocities**
  - **Also partially known.**
- **Pressures**
  - **Conditions are the same as in case of the single source, once the flows and velocities have been determined.**
- **Hydraulic calculation**
  - **Single pipe calculation can only partially solve the system.**
  - **Additional condition is necessary.**

- **Tree distribution networks are used only in rural areas or for pipe irrigation.**

Looped Networks

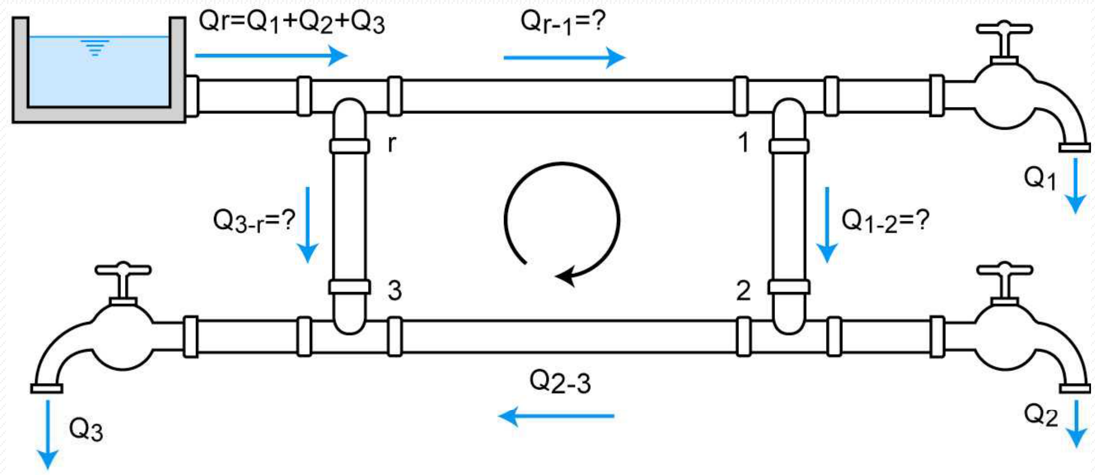


- Pipe Flows
  - Flow rates and directions are unknown.
- Velocities
  - The velocities and their directions are known only after the flows have been calculated.
- Pressures
  - Conditions are the same as in case of branched networks once the flows and hydraulic losses have been calculated for each pipe.
- Hydraulic calculation
  - The equations used for single pipe calculation are not sufficient.
  - Additional conditions have to be introduced.
  - Iterative calculation process is needed.

- Loops are needed for reliability purposes

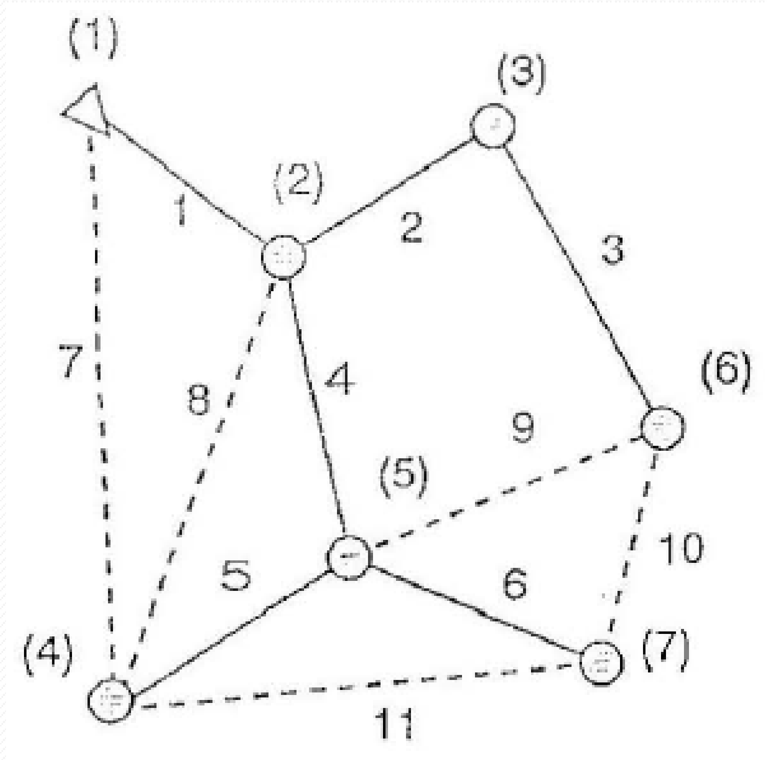
Network Elements

- **Nodes**
  - **Sink**
  - **Junction**
- **Pipes (Link)**
- **Loops**



Number of nodes	NN	
Number of pipes	NP	
Number of closed loops	NLP	= NP – (NN-1)

Network Elements



Number of nodes	**NN = 7**
Number of pipes	**NP = 11**
Number of closed loops	**NLP = NP – (NN-1) = 5**

Analysis, Design And Optimization

- The analysis problem - where the layout of the network, external flows and pipe characteristics are given - is to Calculate the flow through pipes. The solution of the analysis problem is unique irrespective of the technique used to obtain the solution.

- The design problem is the one where some of the network components, e.g. pipe diameters. pump lift or reservoir levels are required to obtain certain operation conditions. The design problem is commonly solved by trial and error using a series of analysis problems.

- The optimization problem is the one which satisfies the design constraints with the minimum cost of network . The cost includes both capital and running costs.

- Analysis is needed also to evaluate the different design alternatives and choose the optimal one.

Friction Head Loss Equation

$$H_f = K\,Q^n$$

Darcy-Weisbach Equation (1857)

$$\boxed{h_l = \frac{F\,L\,V^2}{2\,g\,D} = \frac{8\,F\,L\,Q^2}{\pi^2\,g\,D^5}}$$

Hazen – Williams Equation

$$\boxed{h_l = \frac{K_u\,L\,Q^{1.852}}{C_{HW}^{1.852}\quad D^{4.87}}}$$

K_u = 4.73 English
10.7 S.I.

Classification of Analysis Techniques

- Three different systems of equations can be developed for the solution of network analysis problems. These systems of equations are named after the variables that are regarded as the principal unknowns in that solution method.

  - *Q-equations*
  - *H-equations*
  - *ΔQ-equations*

Governing Equations

- Flow continuity at junction of pipes The sum of all ingoing and outgoing flows in each node equals zero ($\Sigma Q_i = 0.0$).
- Head loss continuity at loop of pipes, The sum of all head-losses along pipes that compose a complete loop equals zero ($\Sigma H_f = 0.0$).

Methods of Solution

- Hardy Cross Methods
  - Method of Balancing Heads
  - ***Method of Balancing Flows***
- Linear Theory
- Newton Raphson
- Gradient Algorithm

Hardy Cross Solution Steps

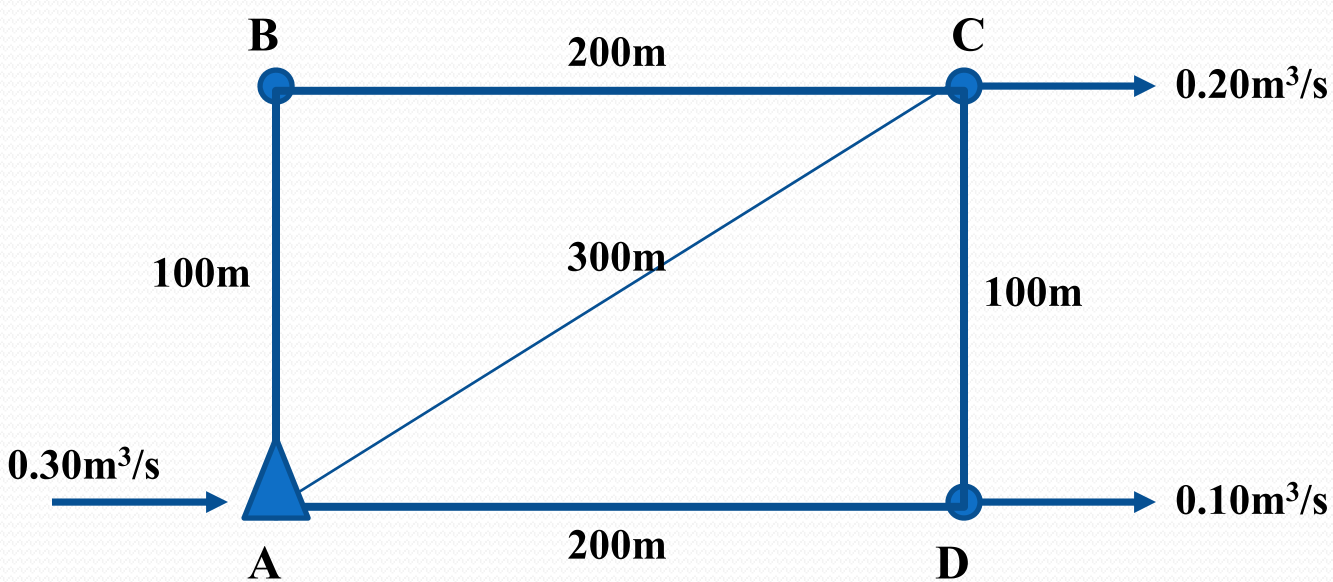
1. Arbitrary flows are assigned to each pipe; ($\Sigma Q_i = 0.0$).
2. Head-loss in each pipe is calculated.
3. The sum of the head-losses along each loop is checked.
4. If ΣH_f differs from the required accuracy, a flow correction δQ is introduced in loop 'i'.
5. Correction δQ is applied in each loop (clockwise or anti-clockwise). The iteration continues with Step 2

$$\delta Q_j = \frac{-\sum K\,Q_o^n}{n\sum\left|K\,Q_o^{n-1}\right|}$$

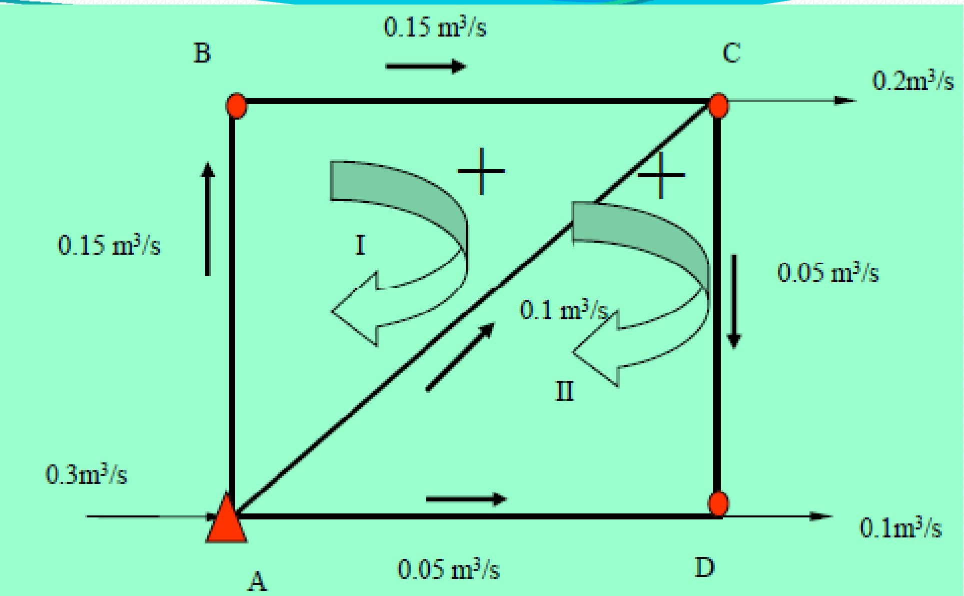
Hardy Cross Example

Using the Hardy-Cross method and hand calculation, estimate the flow rate in each of the pipes in the networks shown in Fig.

D for all pipes = 0.5m, F = 0.01



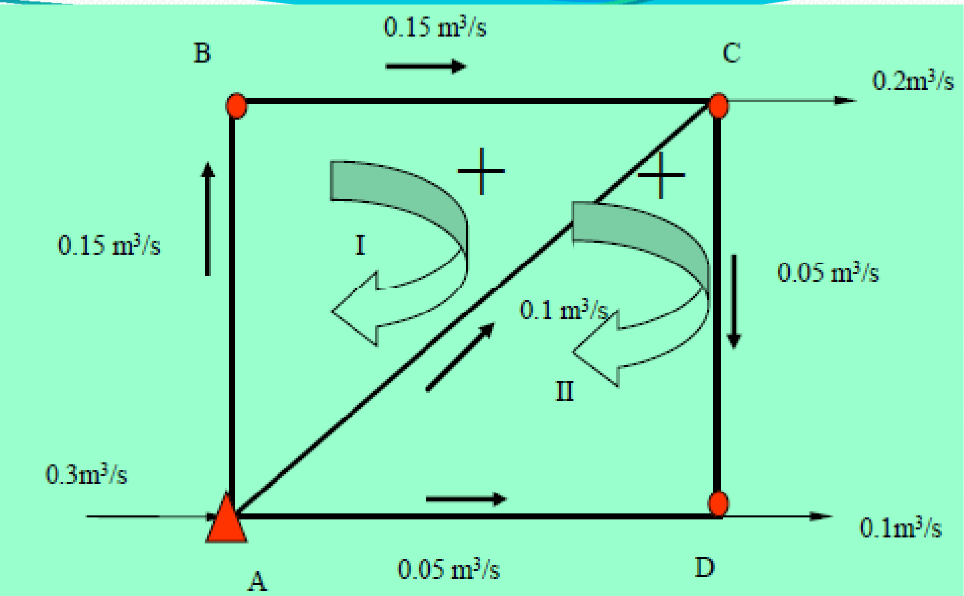
Hardy Cross – First Trial

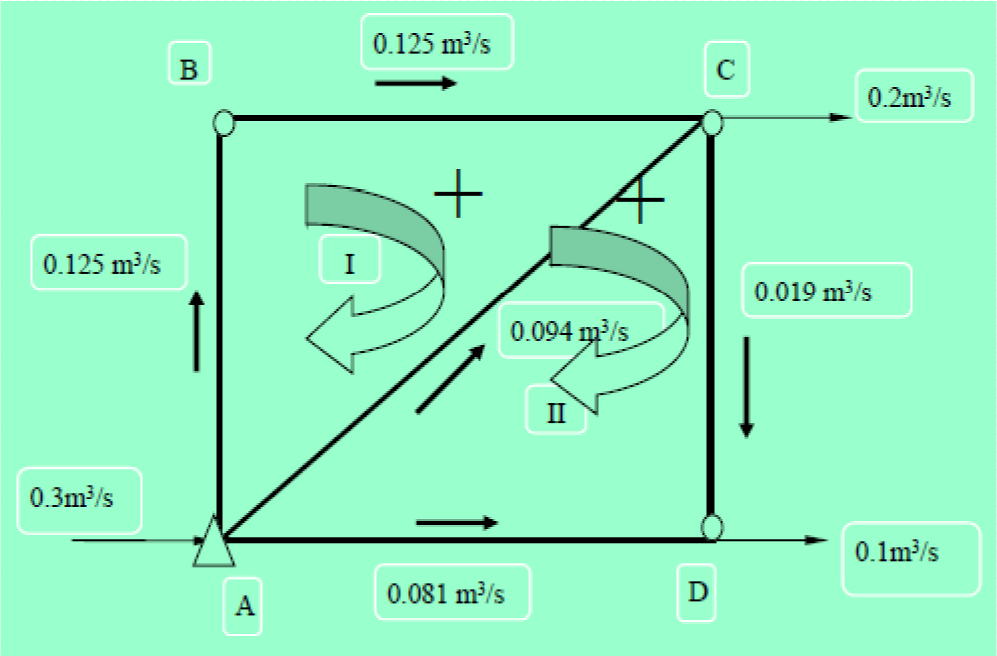


First Trial										
Loop	e	F	L	d	K	sign	Q	KQ^2	2k|Q|	Δ
I	AB	0.01	100	0.5	2.65	1.00	0.150	0.0596	0.79	
	BC	0.01	200	0.5	5.29	1.00	0.150	0.1191	1.59	
	CA	0.01	300	0.5	7.94	-1.00	0.100	-0.079	1.59	
							Sigma	0.0993	3.97	-0.025
Loop	e		L	d	K	sign	Q	KQ^2	2k|Q|	Δ
II	AC	0.01	300	0.5	7.94	1.00	0.10	0.0794	1.59	
	CD	0.01	100	0.5	2.65	1.00	0.05	0.0066	0.26	
	DA	0.01	200	0.5	5.29	-1.00	0.05	-0.013	0.53	
							Sigma	0.0728	2.38	-0.031

Hardy Cross – First Trial

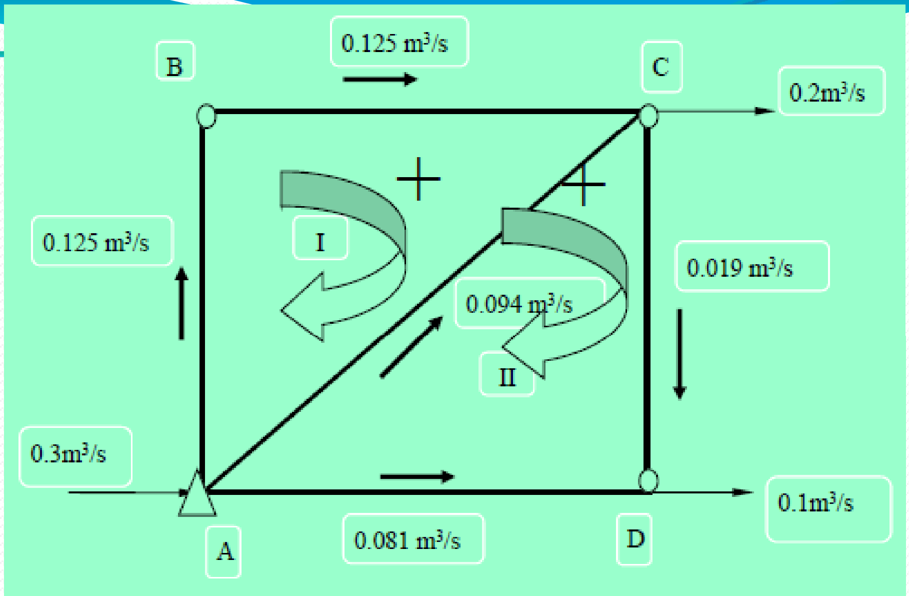
$$\delta Q_1 = -0.025$$





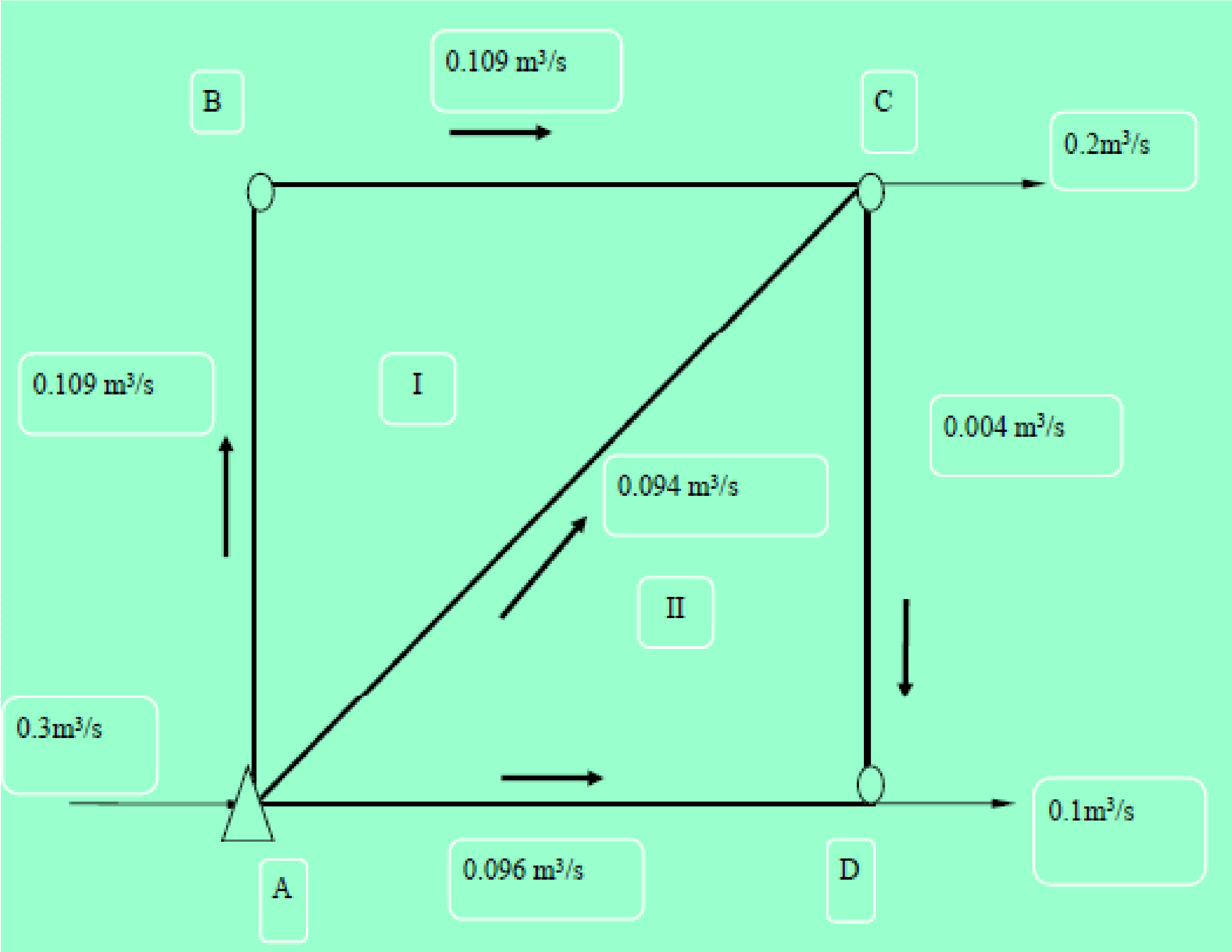
$$\delta Q2 = -0.031$$

Hardy Cross – Second Trial



Second Trial										
Loop	Pipe	F	L	d	K	sign	Q	KQ^2	2k|Q|	Δ
I	AB	0.01	100	0.50	2.65	1.00	0.13	0.04	0.66	
	BC	0.01	200	0.50	5.29	1.00	0.13	0.08	1.32	
	CA	0.01	300	0.50	7.94	-1.00	0.09	-0.07	1.49	
							Sigma	0.05	3.48	-0.016
Loop	Pipe		L	d	K	sign	Q	KQ^2	2k|Q|	Δ
II	AC	0.01	300	0.50	7.94	1.00	0.09	0.07	1.49	
	CD	0.01	100	0.50	2.65	1.00	0.02	0.00	0.10	
	DA	0.01	200	0.50	5.29	-1.00	0.08	-0.03	0.86	
							Sigma	0.04	2.45	-0.015

Hardy Cross – Final Trial



The Linear Approximation

The linear approximation is a widely used method to solve the analysis problem with pipe flows as unknowns. It depends on approximating the non linear head loss equation in the following linear form,

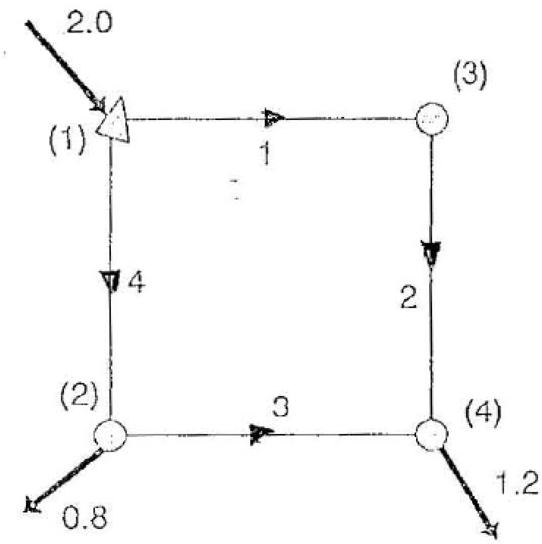
$$h_f = K' Q$$

$$\text{where } K' = K Q^{n-1}$$

The Linear Approximation

$$
\begin{aligned}
Q_1 \quad + Q_4 &= 2.0 \\
-Q_3 \quad + Q_4 &= 0.8 \\
Q_1 \quad - Q_2 &= 0.0 \\
2.0\, Q_1^{1.85} + 1.5\, Q_2^{1.85} - 3.0\, Q_3^{1.85} - 1.0\, Q_4^{1.85} &= 0.0
\end{aligned}
$$

$$2.0\, Q_1 + 1.5\, Q_2 - 3.0\, Q_3 - 1.0\, Q_4 = 0.0$$



A

B

$A X = B \rightarrow X = A^{-1} B$

	A	B	C	D	E	F	G	H	I
1		Coefficient Matrix				RHS	sumhf		
2									
3	1	0	0	1		2			
4	0	0	-1	1		0.8			
5	1	-1	0	0		0			
6	2.000	1.500	-3.000	-1.000		0	-0.5		
7									
8		Inverted Coefficient Matrix				Q	Qupd	Qavg	
9									
10	0.533	-0.400	0.200	0.133		1.000	0.747	0.873	
11	0.533	-0.400	-0.800	0.133		1.000	0.747	0.873	
12	0.467	-0.600	-0.200	-0.133		1.000	0.453	0.727	
13	0.467	0.400	-0.200	-0.133		1.000	1.253	1.127	
14									

The Newton - Raphson Method

The nonlinear system of equations resulting from using any of the unknowns mentioned above can be solved by the Newton-Raphson method. The value of the unknowns or variables at iteration i + 1 is calculated as

$$\underline{X}_{i+1} = \underline{X}_i - D^{-1}\underline{F}(X_i)$$

where D^{-1} is the Jacobean matrix, which is the first derivative of the function F with respect to the variables X.

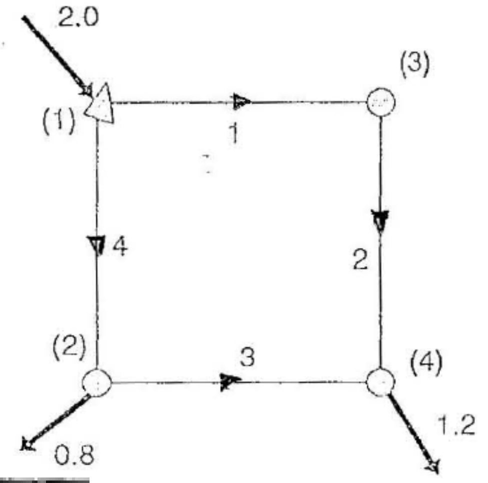
The Newton - Raphson Method

$$
\begin{aligned}
Q_1 \quad &+Q_4 = 2.0 \\
-Q_3 \quad &+Q_4 = 0.8 \\
Q_1 \quad &-Q_2 = 0.0
\end{aligned}
$$

$$\left[\frac{100-H_3}{2.0}\right]^{\frac{1}{1.85}} + \left[\frac{100-H_2}{1.0}\right]^{\frac{1}{1.85}} = 2.0$$

$$\left[\frac{100-H_2}{1.0}\right]^{\frac{1}{1.85}} - \left[\frac{H_2-H_4}{3.0}\right]^{\frac{1}{1.85}} = 0.8$$

$$\left[\frac{100-H_3}{2.0}\right]^{\frac{1}{1.85}} - \left[\frac{H_3-H_4}{1.5}\right]^{\frac{1}{1.85}} = 0.0$$



	A	B	C	D	E	F	G	H	I
1		H		F		Jacobian Matrix (D)			
2									
3	H2	97.000	F1	0.812	DF1	-0.327	-0.271	0.000	
4	H3	98.000	F2	0.209	DF2	-0.544	0.000	0.217	
5	H4	95.000	F3	-0.455	DF3	0.000	-0.533	0.262	
6									
7		Hupd		D-1*F		Inverse of Jacobian (D-1)			
8									
9		98.773		-1.773		-1.514	-0.929	0.769	
10		98.859		-0.859		-1.868	1.122	-0.929	
11		98.479		-3.479		-3.793	2.278	1.925	
12									
13	Q1	1.000							
14	Q2	1.455							
15	Q3	0.803							
16	Q4	1.811							

$$\underline{X}_{i+1} = \underline{X}_i - D^{-1}\underline{F}(\underline{X}_i)$$

- *Thank You*