لجنة الميكانيك
تقدم لكم..

[المكتبة التخصصية]

لجنة الميكانيك
Polytechnic

Mech.MuslimEngineer.Net

FB.com/Groups/Mid.Group

0789434018 MechFet

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Chapter 5

Two Degree of Freedom Systems

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.5 Equations of motion

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = 0$$
$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \qquad (E_1)$$

With $x_i(t) = X_i \cos(\omega t + \phi)$; $i = 1, 2$, Eqs. (E_1) give the frequency equation

$$\begin{vmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 \\ -k_2 & -\omega^2 m_2 + k_2 \end{vmatrix} = 0$$

or
$$\omega^4 - \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right) \omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0 \qquad (E_2)$$

Roots of Eq. (E_2) are

$$\omega_1^2, \omega_2^2 = \frac{k_1 + k_2}{2 m_1} + \frac{k_2}{2 m_2} \mp \sqrt{\frac{1}{4} \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right)^2 - \frac{k_1 k_2}{m_1 m_2}} \qquad (E_3)$$

If
$$\vec{X}^{(1)} = \begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} = r_1 X_1^{(1)} \end{Bmatrix} \quad \text{and} \quad \vec{X}^{(2)} = \begin{Bmatrix} X_1^{(2)} \\ X_2^{(2)} = r_2 X_1^{(2)} \end{Bmatrix},$$

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-m_1 \omega_1^2 + k_1 + k_2}{k_2} = \frac{k_2}{-m_2 \omega_1^2 + k_2} \qquad (E_4)$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-m_1 \omega_2^2 + k_1 + k_2}{k_2} = \frac{k_2}{-m_2 \omega_2^2 + k_2} \qquad (E_5)$$

General solution of (E_1) is

$$x_1(t) = X_1^{(1)} \cos(\omega_1 t + \phi_1) + X_1^{(2)} \cos(\omega_2 t + \phi_2)$$
$$x_2(t) = r_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2 X_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (E_6)$$

where $X_1^{(1)}$, $X_1^{(2)}$, ϕ_1 and ϕ_2 can be found using Eqs. (5.18).

For $m_1 = m$, $m_2 = 2m$, $k_1 = k$ and $k_2 = 2k$, (E_3) gives

$$\omega_1^2 = (2 - \sqrt{3}) \frac{k}{m} \quad , \quad \omega_2^2 = (2 + \sqrt{3}) \frac{k}{m} \qquad (E_7)$$

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

when $k = 1000$ N/m and $m = 20$ kg,

$\omega_1 = 3.6603$ rad/sec and $\omega_2 = 13.6603$ rad/sec

$$r_1 = \frac{k_2}{-m_2\omega_1^2 + k_2} = 1.36604, \quad r_2 = \frac{k_2}{-m_2\omega_2^2 + k_2} = -0.36602$$

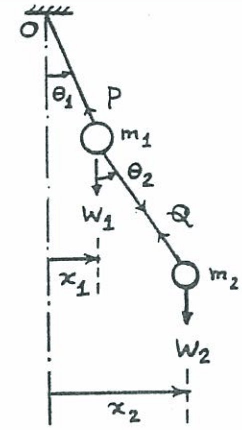
With $x_1(0) = 1$, $\dot{x}_1(0) = 0$, $x_2(0) = -1$ and $\dot{x}_2(0) = 0$, Eqs. (5.18) give $X_1^{(1)} = -0.36602$, $X_1^{(2)} = -1.36603$, $\phi_1 = 0$, $\phi_2 = 0$

Response of the system is

$$x_1(t) = -0.36602 \cos 3.6603\,t - 1.36603 \cos 13.6603\,t$$

$$x_2(t) = -0.5 \cos 3.6603\,t + 0.5 \cos 13.6603\,t$$

5.6 Taking moments about O and mass m_1,



$$m_1 l_1^2 \ddot{\theta}_1 = -W_1(l_1 \sin\theta_1) + Q \sin\theta_2 (l_1 \cos\theta_1) - Q\cos\theta_2 (l_1 \sin\theta_1)$$
$$= -W_1 l_1 \theta_1 + W_2 l_1 (\theta_2 - \theta_1) \qquad (E_1)$$

assuming $Q \simeq W_2$.

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_2 (l_1 \ddot{\theta}_1) = -W_2(l_2 \sin\theta_2)$$
$$= -W_2 l_2 \theta_2 \qquad (E_2)$$

Using the relations $\theta_1 = \frac{x_1}{l_1}$ and $\theta_2 = \frac{x_2 - x_1}{l_2}$, Eqs. (E_1) and (E_2) become

$$m_1 l_1 \ddot{x}_1 + \left[W_1 + W_2\left(\frac{l_1 + l_2}{l_2}\right)\right] x_1 - \frac{W_2 l_1}{l_2} x_2 = 0 \qquad (E_3)$$

$$m_2 l_2 \ddot{x}_2 - W_2 x_1 + W_2 x_2 = 0 \qquad (E_4)$$

When $m_1 = m_2 = m$, $l_1 = l_2 = l$ and $W_1 = W_2 = mg$, Eqs. (E_3) and (E_4) give

$$\left.\begin{aligned} ml\ddot{x}_1 + 3mg\,x_1 - mg\,x_2 &= 0 \\ ml\ddot{x}_2 - mg\,x_1 + mg\,x_2 &= 0 \end{aligned}\right\} \qquad (E_5)$$

For harmonic motion $x_i(t) = X_i \cos\omega t$; $i = 1,2$, Eqs. (E_5) become

$$\left.\begin{aligned} -\omega^2 ml\,X_1 + 3mg\,X_1 - mg\,X_2 &= 0 \\ -\omega^2 ml\,X_2 - mg\,X_1 + mg\,X_2 &= 0 \end{aligned}\right\} \qquad (E_6)$$

from which the frequency equation can be obtained as

$$\omega^4 m^2 l^2 - (4 m^2 l g)\omega^2 + 2 m^2 g^2 = 0$$

i.e. $\omega_1^2, \omega_2^2 = (2 \mp \sqrt{2}) \frac{g}{l}$

$\therefore \quad \omega_1 = 0.7654\sqrt{\frac{g}{l}}, \quad \omega_2 = 1.8478\sqrt{\frac{g}{l}}$

PRELIMINARY VERSION 12/14/2010

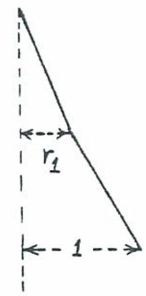
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Ratio of amplitudes is given by Eq. (E_6) as

$$\frac{X_1}{X_2} = \frac{m g}{-\omega^2 m l + 3 m g} = \frac{1}{\left(-\omega^2 \frac{l}{g} + 3\right)}$$

In mode 1, $\omega_1 = 0.7654 \sqrt{\frac{g}{l}}$, $r_1 = \left(\frac{X_1}{X_2}\right)^{(1)} = 0.4142$

No node.



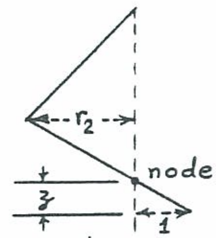
mode 1

In mode 2, $\omega_2 = 1.8478 \sqrt{\frac{g}{l}}$,

$$r_2 = \left(\frac{X_1}{X_2}\right)^{(2)} = -2.4133$$

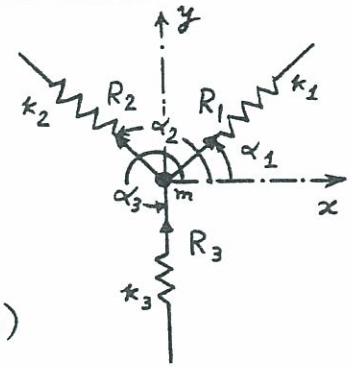
one node located at z:

$$\frac{z}{1} = \frac{1-z}{2.4133} \quad \text{or} \quad z = 0.2930$$



mode 2

(5.7) Let R_1, R_2 and R_3 be the restoring forces in springs. Equations of motion of mass m in x and y directions are



$$m\ddot{x} = \sum_{i=1}^{3} R_i \cos\alpha_i \qquad (E_1)$$

$$m\ddot{y} = \sum_{i=1}^{3} R_i \sin\alpha_i \qquad (E_2)$$

where $R_i = -k_i\,(x\cos\alpha_i + y\sin\alpha_i)$ $\quad(E_3)$

Eqs. (E_1) to (E_3) give

$$m\ddot{x} + \sum_{i=1}^{3} k_i\,(x\cos^2\alpha_i + y\sin\alpha_i\cos\alpha_i) = 0 \qquad (E_4)$$

$$m\ddot{y} + \sum_{i=1}^{3} k_i\,(x\sin\alpha_i\cos\alpha_i + y\sin^2\alpha_i) = 0 \qquad (E_5)$$

For $\alpha_1 = 45^\circ$, $\alpha_2 = 135^\circ$, $\alpha_3 = 270^\circ$ and $k_1 = k_2 = k_3 = k$, Eqs. (E_4) and (E_5) reduce to

$$m\ddot{x} + k x = 0 \qquad (E_6)$$

$$m\ddot{y} + 2k y = 0 \qquad (E_7)$$

These equations are uncoupled. For harmonic motion,

$x(t) = X\cos(\omega t + \phi)$, $\quad y(t) = Y\cos(\omega t + \phi)$, and hence

$\omega_1 = \sqrt{\frac{k}{m}}$ for motion in x direction

$\omega_2 = \sqrt{\frac{2k}{m}}$ for motion in y direction

Natural modes are given by

$$x(t) = X\cos\left(\sqrt{\frac{k}{m}}\,t + \phi_1\right)$$

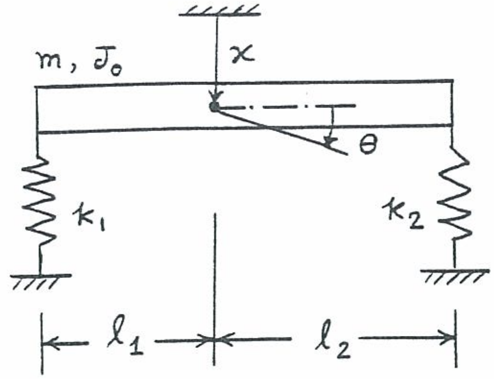
$$y(t) = Y\cos\left(\sqrt{\frac{2k}{m}}\,t + \phi_2\right)$$

where X, ϕ_1, Y and ϕ_2 can be determined from initial conditions.

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.8 Equations of motion in terms of x and θ:



$$m\ddot{x} + k_1(x - l_1\theta) + k_2(x + l_2\theta) = 0 \qquad (E_1)$$

$$J_0\ddot{\theta} - k_1 l_1 (x - l_1\theta) + k_2 l_2 (x + l_2\theta) = 0 \qquad (E_2)$$

For free vibration,

$$x(t) = X \cos(\omega t + \phi) \qquad (E_3)$$

$$\theta(t) = \Theta \cos(\omega t + \phi) \qquad (E_4)$$

and Eqs. (E_1) and (E_2) become

$$\begin{bmatrix} -m\omega^2 + k_1 + k_2 & -(k_1 l_1 - k_2 l_2) \\ -(k_1 l_1 - k_2 l_2) & -J_0\omega^2 + k_1 l_1^2 + k_2 l_2^2 \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (E_5)$$

Frequency equation is

$$\begin{vmatrix} -m\omega^2 + k_1 + k_2 & -(k_1 l_1 - k_2 l_2) \\ -(k_1 l_1 - k_2 l_2) & -J_0\omega^2 + k_1 l_1^2 + k_2 l_2^2 \end{vmatrix} = 0 \qquad (E_6)$$

i.e.,

$$\begin{vmatrix} -\omega^2 + 5000 & 100 \\ 100 & -0.3\,\omega^2 + 2030 \end{vmatrix} = 0$$

i.e.,

$$0.3\,\omega^4 - 3530\,\omega^2 + 10.14 \times 10^6 = 0$$

i.e.,

$$\omega^2 = 6785.3373, \quad 4981.3293$$

$$\therefore \quad \omega_1 = 70.5785 \text{ rad/sec}, \qquad \omega_2 = 82.3732 \text{ rad/sec}$$

Mode shapes:

$$(-1000\,\omega_1^2 + 5 \times 10^6)\,X + 0.1 \times 10^6\,\Theta = 0$$

or

$$\left.\frac{X}{\Theta}\right|_{\omega_1} = \frac{-0.1 \times 10^6}{-1000\,\omega_1^2 + 5 \times 10^6} = -5.3476$$

and

$$\left.\frac{X}{\Theta}\right|_{\omega_2} = \frac{-0.1 \times 10^6}{-1000\,\omega_2^2 + 5 \times 10^6} = 0.05601$$

5.9

$$k_g = \text{stiffness of girder} = \frac{48\,E\,I}{\ell^3} = \frac{48\,(6\,(10^{12}))}{(30\,(12))^3} = 6.1728\,(10^6)\ \text{lb/in}$$

$$k = \text{stiffness of rope} = \frac{A\,E}{\ell} = \frac{A\,(30\,(10^6))}{(20)\,(12)} = 12.5\,(10^4)\,A\ \text{lb/in}$$

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

m_1 = mass of trolley = 8000/386.4 = 20.7039 lb—sec^2/in
m_2 = mass of load = 2000/386.4 = 5.1760 lb—sec^2/in

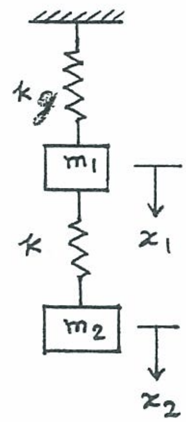
Desired frequency value: $\omega_1 > 20$ Hz. Let $\omega_1 = 25$ Hz $= 157.08$ rad/sec or $\omega_1^2 = 24674.1264$ (rad/sec)^2.

Fundamental natural frequency is given by (see Eq. (E3) in solution of Problem 5.5):

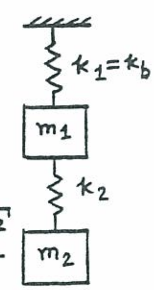
$$\omega_1^2 = \frac{k_g + k}{2\, m_1} + \frac{k}{2\, m_2} - \sqrt{\frac{1}{4}\left(\frac{k_g + k}{m_1} + \frac{k}{m_2}\right)^2 - \frac{k_g\, k}{m_1\, m_2}} \qquad (1)$$

Using the known values of k_g, m_1, and m_2, a series of trial values of A are given and Eq. (1) is evaluated to find the corresponding values of ω_1^2. The results are given in the table below. It can be seen that A = 1.1 in^2 yields a frequency that satisfies the specification.

A (in^2)	k (lb/in)	ω_1^2 (rad/sec)^2	ω_1 (rad/sec)
0.6	0.7500 (10^5)	0.1434 (10^5)	119.7
0.7	0.8750 (10^5)	0.1715 (10^5)	131.0
0.8	0.1000 (10^6)	0.1946 (10^5)	139.5
0.9	0.1125 (10^6)	0.2176 (10^5)	147.5
1.0	0.1250 (10^6)	0.2406 (10^5)	155.1
1.1	0.1375 (10^6)	0.2662 (10^5)	163.2
1.2	0.1500 (10^6)	0.2918 (10^5)	170.8
1.3	0.1625 (10^6)	0.3123 (10^5)	176.7
1.4	0.1750 (10^6)	0.3379 (10^5)	183.8
1.5	0.1875 (10^6)	0.3610 (10^5)	190.0



5.10 $k_1 = \dfrac{48\, E I}{l^3} = \dfrac{48\,(2.06 \times 10^{11})\,(0.02)}{(40)^3} = 3.09 \times 10^6$ N/m



$k_2 = 0.3 \times 10^6$ N/m, $m_1 = 1000$ kg, $m_2 = 5000$ kg

Eq. (E3) in the solution of problem 5.5 gives

$$\omega_1^2,\ \omega_2^2 = \frac{3.39 \times 10^6}{2000} + \frac{0.3 \times 10^6}{10\,000} \mp \sqrt{\frac{1}{4}\left(\frac{3.39 \times 10^6}{1000} + \frac{0.3 \times 10^6}{5000}\right)^2 - \frac{3.09 \times 0.3 \times 10^{12}}{5 \times 10^6}}$$

$$= (1.725 \mp 1.6704)\, 10^3$$

$\omega_1 = 7.3892$ rad/s, $\omega_2 = 58.2701$ rad/s

From Eqs. (E4) and (E5) of solution of problem 5.5,

$$r_1 = \frac{k_2}{-m_2\,\omega_1^2 + k_2} = \frac{0.3 \times 10^6}{-5000\,(54.6003) + 0.3 \times 10^6} = 11.1117$$

$$r_2 = \frac{k_2}{-m_2\,\omega_2^2 + k_2} = \frac{0.3 \times 10^6}{-5000\,(3395.4046) + 0.3 \times 10^6} = -0.01799$$

Mode shapes are $\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 11.1117 \end{Bmatrix} X_1^{(1)}$, $\vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ -0.01799 \end{Bmatrix} X_1^{(2)}$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.11 Frequency equation:

$$\left|\left[-\omega^2\,[m] + [k]\right]\right| = 0$$

or

$$\begin{vmatrix} (k_{11} - \omega^2\, m_1) & k_{12} \\ k_{21} & (k_{22} - \omega^2\, m_2) \end{vmatrix} = 0 \qquad (1)$$

Expansion of the determinantal equation (1) gives:

$$(m_1\, m_2)\,\omega^4 - (m_1\, k_{22} + m_2\, k_{11})\,\omega^2 + (k_{11}\, k_{22} - k_{12}^2) = 0 \qquad (2)$$

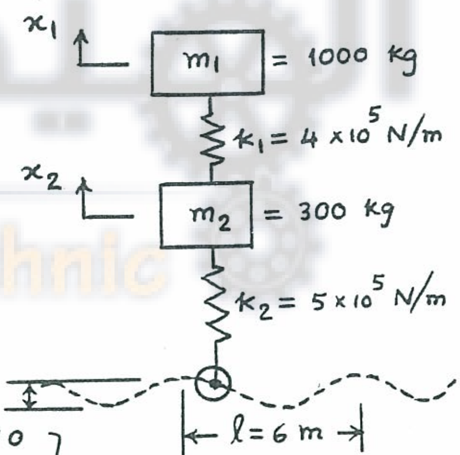
Roots of Eq. (2):

$$\omega_2^2,\ \omega_1^2 = \frac{(m_1\, k_{22} + m_2\, k_{11}) \pm \sqrt{\left\{(m_1\, k_{22} - m_2\, k_{11})^2 + 4\, m_1\, m_2\, k_{12}^2\right\}}}{2\, m_1\, m_2} \qquad (3)$$

Substitution of known expressions for k_{11}, k_{12}, and k_{22} into Eq. (3) yields:

$$\omega_2^2,\ \omega_1^2 = \frac{48}{7}\,\frac{E\,I}{m_1\, m_2}\left[(m_1 + 8\, m_2) \pm \sqrt{(m_1 - 8\, m_2)^2 + 25\, m_1\, m_2}\right] \qquad (4)$$

5.12 Equations of motion :



$$\left.\begin{aligned} m_1 \ddot{x}_1 + k_1 x_1 - k_1 x_2 &= 0 \\ m_2 \ddot{x}_2 + (k_1 + k_2) x_2 - k_1 x_1 &= 0 \end{aligned}\right\} \quad (E_1)$$

Let $x_i(t) = X_i \cos(\omega t + \phi)$;

$i = 1, 2$ $\qquad (E_2)$

Eq. (E_1) becomes

$$\begin{bmatrix} -m_1\omega^2 + k_1 & -k_1 \\ -k_1 & -m_2\omega^2 + k_1 + k_2 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is

$$\begin{vmatrix} -m_1\omega^2 + k_1 & -k_1 \\ -k_1 & -m_2\omega^2 + k_1 + k_2 \end{vmatrix} = 0$$

i.e.,

$$m_1 m_2\,\omega^4 - (m_1 k_1 + m_1 k_2)\,\omega^2 - k_1 m_2\,\omega^2 + k_1 k_2 = 0$$

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

i.e.,

$$\omega^2 = \Big[(m_1 k_1 + m_1 k_2 + m_2 k_1) \pm \big(m_1^2 k_1^2 + m_1^2 k_2^2 + m_2^2 k_1^2 + 2 m_1^2 k_1 k_2 - 2 m_1 m_2 k_1 k_2 + 2 m_1 m_2 k_1^2 \big)^{\frac{1}{2}} \Big] \Big/ 2 m_1 m_2 \qquad (E_3)$$

Since $m_1 = 1000$ kg, $m_2 = 300$ kg, $k_1 = 4 \times 10^5$ N/m and $k_2 = 5 \times 10^5$ N/m, Eq. (E_3) gives

$\omega_1 = 14.4539$ rad/sec, $\quad \omega_2 = 56.4897$ rad/sec

$$f_1 = \frac{14.4539}{2\pi} \text{ Hz} = \frac{s_1 (1000)}{3600} \left(\frac{1}{l} \right) = \frac{s_1}{21.6}$$

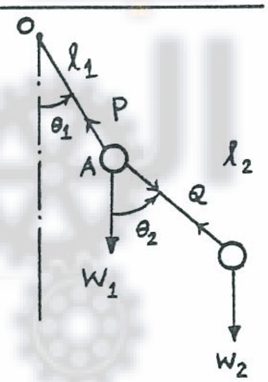
where $l = 6$ m and s_1 is in km/hr.

$$\therefore \; s_1 = \text{critical velocity \# 1} = \frac{14.4539}{2\pi} (21.6) = 49.6887 \text{ km/hr}$$

$$f_2 = \frac{56.4897}{2\pi} \text{ Hz} = s_2 \left(\frac{1000}{3600} \right) \left(\frac{1}{l} \right) = \frac{s_2}{21.6}$$

$$\therefore \; s_2 = \text{critical velocity \# 2} = \frac{56.4897}{2\pi} (21.6) = 194.1968 \text{ km/hr.}$$

5.13 Equations of motion for rotation about o and A give



$$m_1 l_1^2 \ddot{\theta}_1 = - W_1 l_1 \sin\theta_1 + Q \sin\theta_2 (l_1 \cos\theta_1) - Q \cos\theta_2 (l_1 \sin\theta_1)$$

$$= - W_1 l_1 \theta_1 + Q l_1 (\theta_2 - \theta_1)$$

$$= - W_1 l_1 \theta_1 + W_2 l_1 (\theta_2 - \theta_1) \quad \text{----} \quad (E_1)$$

since $Q \simeq W_2$.

$$m_2 l_2^2 \ddot{\theta}_2 + m_2 l_2 (l_1 \ddot{\theta}_1) = - W_2 l_2 \sin\theta_2$$

$$= - W_2 l_2 \theta_2 \quad \text{----} \quad (E_2)$$

For $m_1 = m_2 = m$ and $l_1 = l_2 = l$, Eqs. (E_1) and (E_2) become

$$m l \ddot{\theta}_1 + 2mg\,\theta_1 - mg\,\theta_2 = 0$$

$$m l \ddot{\theta}_1 + m l \ddot{\theta}_2 + mg\,\theta_2 = 0$$

Assuming $\theta_i(t) = \Theta_i \cos(\omega t + \phi)$; $i = 1, 2$, we get

$$\begin{bmatrix} -\omega^2 m l + 2mg & -mg \\ -\omega^2 m l & -\omega^2 m l + mg \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{----} (E_3)$$

Defining $\lambda = \dfrac{\omega^2 m l}{mg} = \dfrac{\omega^2 l}{g}$, frequency equation can be obtained as

$$\begin{vmatrix} -\lambda + 2 & -1 \\ -\lambda & -\lambda + 1 \end{vmatrix} = \lambda^2 - 4\lambda + 2 = 0$$

PRELIMINARY VERSION 12/14/2010

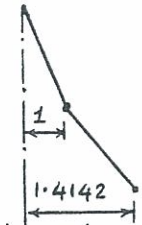
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$\lambda_1 = 2 - \sqrt{2} = 0.5858 , \qquad \lambda_2 = 2 + \sqrt{2} = 3.4142$

$\omega_1 = 0.7654 \sqrt{\frac{g}{l}} , \qquad \omega_2 = 1.8478 \sqrt{\frac{g}{l}}$

For ω_1, first equation in (E_3) gives for $\Theta_1 = 1$,

$$\Theta_2 = -\lambda_1 + 2 = \sqrt{2} = 1.4142$$



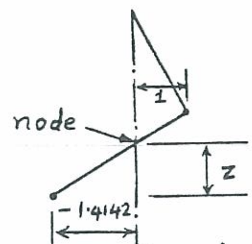
First mode

For ω_2, first equation in (E_3) gives for $\Theta_1 = 1$,

$$\Theta_2 = -\lambda_2 + 2 = -\sqrt{2} = -1.4142$$

Location of node:

$$\frac{z}{1.4142} = \frac{1-z}{1} \; ; \quad z = 0.4142$$



Second mode

(5.14) Eq. (E_3) in the solution of problem 5.5 gives

$$\omega_1^2, \omega_2^2 = \frac{k_1 + k_2}{2 m_1} + \frac{k_2}{2 m_2} \mp \sqrt{\frac{1}{4}\left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2}}$$

For $m_1 = m_2 = m$ and $k_1 = k_2 = k$, we get

$$\omega_1^2, \omega_2^2 = \frac{k}{m} + \frac{k}{2m} \mp \sqrt{\frac{1}{4}\left(\frac{3k}{m}\right)^2 - \frac{k^2}{m^2}} = \frac{3k}{2m} \mp \frac{k}{m}\sqrt{\frac{5}{4}}$$

$$\omega_1^2 = 0.382 \frac{k}{m} , \quad \omega_2^2 = 2.618 \frac{k}{m}$$

$$\omega_1 = 0.6181 \sqrt{\frac{k}{m}} , \quad \omega_2 = 1.6180 \sqrt{\frac{k}{m}}$$

Mode shapes are given by (see Eqs. (E_4) and (E_5) of problem 5.5)

$$r_1 = \frac{k_2}{-m_2 \omega_1^2 + k_2} = 1.6181 \; ; \quad \vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.6181 \end{Bmatrix} X_1^{(1)}$$

$$r_2 = \frac{k_2}{-m_2 \omega_2^2 + k_2} = -0.6180 \; ; \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -0.6180 \end{Bmatrix} X_1^{(2)}$$

(5.15) For the system of Fig. 5.5(a),

$$\vec{X}^{(1)T} [m] \vec{X}^{(2)} = \{ X_1^{(1)} \quad X_2^{(1)} \} \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} X_1^{(2)} \\ X_2^{(2)} \end{Bmatrix} = X_1^{(1)} X_1^{(2)} \{1 \quad r_1\} \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} 1 \\ r_2 \end{Bmatrix}$$

$$= X_1^{(1)} X_1^{(2)} (m_1 + m_2 r_1 r_2)$$

But

$$m_1 + m_2 r_1 r_2 = m_1 + m_2 \left(\frac{k_2}{-m_2 \omega_1^2 + k_2 + k_3}\right)\left(\frac{k_2}{-m_2 \omega_2^2 + k_2 + k_3}\right) \equiv \frac{N}{D}$$

where

$$N = m_1 (-m_2 \omega_1^2 + k_2 + k_3)(-m_2 \omega_2^2 + k_2 + k_3) + m_2 k_2^2 , \text{ and}$$

$$D = (-m_2 \omega_1^2 + k_2 + k_3)(-m_2 \omega_2^2 + k_2 + k_3)$$

PRELIMINARY VERSION 12/14/2010

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

By substituting

$$\omega_1^2, \omega_2^2 = \frac{(k_1+k_2)m_2 + (k_2+k_3)m_1}{2 m_1 m_2} \mp \frac{1}{2}\left\{\left[\frac{(k_1+k_2)m_2+(k_2+k_3)m_1}{m_1 m_2}\right]^2 - 4\,\frac{(k_1+k_2)(k_2+k_3)-k_2^2}{m_1 m_2}\right\}^{1/2}, \text{ we get}$$

$$N = m_1 m_2^2 \omega_1^2 \omega_2^2 - m_1 m_2 k_2 \omega_2^2 - m_1 m_2 k_3 \omega_2^2 - m_1 m_2 k_2 \omega_1^2 + m_1 k_2^2 + m_1 k_2 k_3 - m_1 m_2 k_3 \omega_1^2 + m_1 k_2 k_3 + m_1 k_3^2 + m_2 k_2^2$$

$$= m_1 m_2^2 \left\{ \left[\frac{(k_1+k_2)m_2+(k_2+k_3)m_1}{2 m_1 m_2}\right]^2 - \frac{1}{4}\left[\left(\frac{(k_1+k_2)m_2+(k_2+k_3)m_1}{m_1 m_2}\right)^2 - 4\left(\frac{(k_1+k_2)(k_2+k_3)-k_2^2}{m_1 m_2}\right)\right]\right\} - m_1 m_2 k_2 \left\{\frac{(k_1+k_2)m_2+(k_2+k_3)m_1}{m_1 m_2}\right\}$$

$$- m_1 m_2 k_3 \left\{\frac{(k_1+k_2)m_2+(k_2+k_3)m_1}{m_1 m_2}\right\} + m_1 k_2^2 + m_2 k_2^2 + 2 m_1 k_2 k_3 + m_1 k_3^2$$

$$= 0$$

5.16 Eq. (5.10) gives

$$\omega_1^2, \omega_2^2 = \frac{1}{2}\left\{\frac{(800)1+(700)2}{2}\right\} \mp \frac{1}{2}\left[\left\{\frac{(800)1+(700)2}{2}\right\}^2 - 4\left\{\frac{(800)(700)-(500)^2}{2}\right\}\right]^{1/2}$$

$$= 550 \mp 384.0573 = 165.9427,\ 934.0573$$

$\omega_1 = 12.8819$ rad/s, $\omega_2 = 30.5624$ rad/s

5.17 Eq. (E_3) in the solution of problem 5.5 gives

$$\omega_1^2, \omega_2^2 = \frac{8000}{2} + \frac{6000}{2} \mp \sqrt{\frac{1}{4}\left(\frac{8000}{1}+\frac{6000}{1}\right)^2 - \frac{12\times 10^6}{1}} = 917.2,\ 13082.8$$

$\omega_1 = 30.2853$ rad/sec, $\omega_2 = 114.3801$ rad/s

Eqs. (E_4) and (E_5) of solution of problem 5.5 give

$$r_1 = \frac{k_2}{-m_2\omega_1^2 + k_2} = \frac{6000}{-917.2+6000} = 1.1805; \quad \vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.1805 \end{Bmatrix} X_1^{(1)}$$

$$r_2 = \frac{k_2}{-m_2\omega_2^2 + k_2} = \frac{6000}{-13082.8+6000} = -0.8471; \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -0.8471 \end{Bmatrix} X_1^{(2)}$$

5.18 Same as example 5.1 with $m_1 = m_2 = 25$ kg and $k_1 = k_2 = k_3 = 50\,000$ N/m

$$\omega_1 = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\,000}{25}} = 44.7214 \text{ rad/s}$$

$$\omega_2 = \sqrt{\frac{3k}{m}} = \sqrt{\frac{150\,000}{25}} = 77.4597 \text{ rad/s}$$

General motion is given by Eq. (E_{10}) of Example 5.1:

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$x_1(t) = X_1^{(1)} \cos(44.7214\,t + \phi_1) + X_1^{(2)} \cos(77.4597\,t + \phi_2)$$
$$x_2(t) = X_1^{(1)} \cos(44.7214\,t + \phi_1) - X_1^{(2)} \cos(77.4597\,t + \phi_2)$$

5.19 Eq. (5.10) gives

$$\omega_1^2, \omega_2^2 = \frac{1}{2}\left\{\frac{3000(2) + 4000(1)}{2}\right\} \mp \frac{1}{2}\left[\left\{\frac{3000(2)+4000(1)}{2}\right\}^2 - 4\left\{\frac{3000(4000) - 1000^2}{2}\right\}\right]^{\frac{1}{2}}$$

$$= 1633.9746,\ 3366.0254$$

$\omega_1 = 40.4225$ rad/s, $\omega_2 = 58.0175$ rad/s

$$r_1 = \frac{k_2}{-m_2\omega_1^2 + k_2 + k_3} = \frac{1000}{-2(1633.9746) + 4000} = 1.3660$$

$$r_2 = \frac{k_2}{-m_2\omega_2^2 + k_2 + k_3} = \frac{1000}{-2(3366.0254) + 4000} = -0.3660$$

When $x_1(0) = x_2(0) = \dot{x}_2(0) = 0$ and $\dot{x}_1(0) = 20$ m/s,

$$X_1^{(1)} = \frac{1}{(-0.366 - 1.366)}\left[\frac{+0.366(20)}{40.4225}\right] = -0.1046$$

$$X_1^{(2)} = \frac{1}{-1.732}\left[\frac{1.366(20)}{58.0175}\right] = -0.2719$$

$$\phi_1 = \phi_2 = \tan^{-1}(\infty) = \frac{\pi}{2}$$

Motion of the two masses is given by Eq. (5.15):

$$x_1(t) = 0.1046 \sin 40.4225\,t + 0.2719 \sin 58.0175\,t$$
$$x_2(t) = +(1.3660)(0.1046) \sin 40.4225\,t - (-0.3660)(-0.2719) \sin 58.0175\,t$$
$$= 0.1429 \sin 40.4225\,t - 0.09952 \sin 58.0175\,t$$

5.20 (a) $\omega_1^2 = 917.2$, $\omega_2^2 = 13082.8$, $r_1 = 1.1805$, $r_2 = -0.8471$

$x_1(0) = 0.2$, $x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$

Eq. (5.18) gives

$$X_1^{(1)} = \frac{1}{(-0.8471 - 1.1805)}\left[(-0.8471)(0.2)\right] = 0.08356$$

$$X_1^{(2)} = \frac{1}{(-0.8471 - 1.1805)}\left[(-1.1805)(0.2)\right] = 0.11644$$

$$\phi_1 = \phi_2 = \tan^{-1}(0) = 0$$

$$x_1(t) = 0.08356 \cos 30.2853\,t + 0.11644 \cos 114.3801\,t$$
$$x_2(t) = (1.1805)(0.08356) \cos 30.2853\,t + (-0.8471)(0.11644) \cos 114.3801\,t$$
$$= 0.09864 \cos 30.2853\,t - 0.09864 \cos 114.3801\,t$$

(b) $x_1(0) = 0.2$, $x_2(0) = \dot{x}_1(0) = 0$, $\dot{x}_2(0) = 5.0$

Eq. (5.18) gives

$$X_1^{(1)} = \frac{1}{-2.0276}\left[\{-0.8471(0.2)\}^2 + \frac{(5)^2}{917.2}\right]^{1/2} = -0.1167$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$x_1^{(2)} = \frac{1}{-2.0276}\left[\{-1.1805\,(0.2)\}^2 + \frac{(-5)^2}{13082.8}\right]^{1/2} = -0.1184$$

$$\phi_1 = \tan^{-1}\left\{\frac{5.0}{30.2853\quad(-0.8471)(0.2)}\right\} = \tan^{-1}(-0.9745) = 135.7399^\circ$$

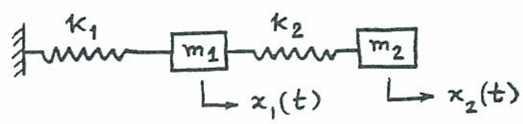
$$\phi_2 = \tan^{-1}\left\{\frac{-5.0}{114.3801\,(-1.1805)\,(0.2)}\right\} = \tan^{-1}(0.1851) = 10.4895^\circ$$

$$x_1(t) = -0.1167\,\cos(30.2853\,t + 135.7399^\circ) - 0.1184\,\cos(114.3801\,t + 10.4895^\circ)$$

$$x_2(t) = (1.1805)(-0.1167)\cos(30.2853\,t + 135.7399^\circ) - 0.8471\,(-0.1184)\,\cos(114.3801\,t + 10.4895^\circ)$$

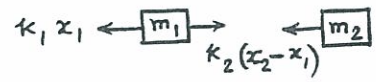
$$= -0.1378\,\cos(30.2853\,t + 135.7399^\circ) + 0.1003\,\cos(114.3801\,t + 10.4895^\circ)$$

5.21 Equivalent system is shown in figure:



$$k_i = 2\left(\frac{24\,E\,I_i}{h_i^3}\right) \quad ; \quad i = 1, 2$$

$$k_1 = k_2 = k = \frac{48\,E\,I}{h^3}\ ; \quad m_1 = 2m,\ m_2 = m$$



Equations of motion:
$$m_1\ddot{x}_1 + (k_1 + k_2)\,x_1 - k_2 x_2 = 0$$
$$m_2\ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0$$

For harmonic motion $x_i(t) = X_i\ \cos(\omega t + \phi)$; $i = 1, 2$, we get

$$\begin{bmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad ----(E_1)$$

Frequency equation is

$$\begin{vmatrix} (-\omega^2 m_1 + k_1 + k_2) & -k_2 \\ -k_2 & (-\omega^2 m_2 + k_2) \end{vmatrix} = 0$$

or $$\omega^4\, m_1 m_2 - \omega^2\,(m_2 k_1 + m_2 k_2 + m_1 k_2) + k_1 k_2 = 0$$

$$\omega^2 = \frac{(m_2 k_1 + m_2 k_2 + m_1 k_2) \pm \sqrt{(m_2 k_1 + m_2 k_2 + m_1 k_2)^2 - 4\,m_1 m_2 k_1 k_2}}{2\,m_1 m_2} \quad ---(E_2)$$

For given data,

$$\omega^2 = \frac{(mk + mk + 2mk) \pm \sqrt{(mk + mk + 2mk)^2 - 8\,m^2 k^2}}{4\,m^2} = \frac{k}{m}\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

$$\omega_1 = 0.5412\,\sqrt{\frac{k}{m}} = 3.7495\,\sqrt{\frac{E\,I}{m h^3}}\ ; \quad \omega_2 = 1.3066\,\sqrt{\frac{k}{m}} = 9.0524\,\sqrt{\frac{E\,I}{m h^3}}$$

From Eq. (E_1), we get

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

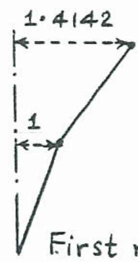
$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-\omega_1^2 m_1 + k_1 + k_2}{k_2} = \frac{-2m\,\omega_1^2 + 2k}{k} = \frac{-2(0.2929\,k) + 2k}{k}$$
$$= 1.4142$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-\omega_2^2 m_1 + k_1 + k_2}{k_2} = \frac{-2m\,\omega_2^2 + 2k}{k} = \frac{-2(1.7071\,k) + 2k}{k}$$
$$= -1.4142$$

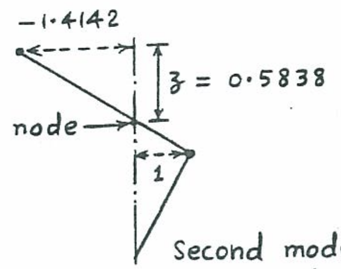
Mode shapes are:

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.4142 \end{Bmatrix} X_1^{(1)}$$

$$\vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ -1.4142 \end{Bmatrix} X_1^{(2)}$$



First mode (no node)

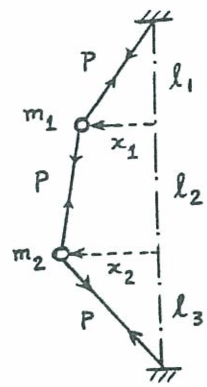


Second mode (one node)

Location of node:

$$\frac{z}{1.4142} = \frac{1-z}{1} \; ; \quad z = 0.5838$$

5.22 Let P be the tension in the string. Horizontal components of tension (along x_1 direction) in the string lying above and below m_1 are $-\frac{x_1 P}{l_1}$ and $-\frac{(x_1 - x_2)P}{l_2}$, respectively.



Newton's second law gives

$$m_1 \ddot{x}_1 = -\frac{x_1 P}{l_1} - \frac{(x_1 - x_2)P}{l_2} \quad \text{or} \quad m_1 \ddot{x}_1 + \frac{x_1}{l_1} P + \left(\frac{x_1 - x_2}{l_2}\right) P = 0$$

Similarly

$$m_2 \ddot{x}_2 + \frac{x_2}{l_3} P - \left(\frac{x_1 - x_2}{l_2}\right) P = 0$$

With $x_i(t) = X_i \cos(\omega t + \phi)$; $i = 1, 2$, and $l_1 = l_2 = l_3 = l$, $m_1 = m_2 = m$,

$$\begin{bmatrix} \left(-m\omega^2 + \frac{2P}{l}\right) & -\frac{P}{l} \\ -\frac{P}{l} & \left(-m\omega^2 + \frac{2P}{l}\right) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad ---- (E_1)$$

This gives the frequency equation

$$\left(-m\omega^2 + \frac{2P}{l}\right)^2 - \left(\frac{P}{l}\right)^2 = \left(-m\omega^2 + \frac{3P}{l}\right)\left(-m\omega^2 + \frac{P}{l}\right) = 0$$

$$\therefore \quad \omega_1 = \sqrt{\frac{P}{ml}} \, , \quad \omega_2 = \sqrt{\frac{3P}{ml}}$$

From first of Eqs. (E_1),

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-m\omega_1^2 + \frac{2P}{l}}{\left(\frac{P}{l}\right)} = 1 \; ; \quad r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-m\omega_2^2 + \frac{2P}{l}}{\left(\frac{P}{l}\right)} = -1$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

mode shapes are: $\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} X_1^{(1)}$, $\vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} X_1^{(2)}$

No node   one node at middle of the two masses

5.23 For $m_1 = 3m$, $m_2 = m$, $k_1 = 3k$ and $k_2 = k$, Eq. (E_2) in solution of problem 5.21 gives

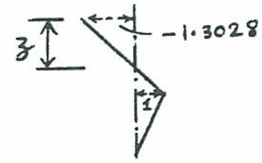
$$\omega^2 = \frac{(3mk + mk + 3mk) \pm \sqrt{(3mk + mk + 3mk)^2 - 36\,k^2 m^2}}{6\,m^2} = \frac{k}{6m}\left(7 \pm \sqrt{13}\right)$$

$$\omega^2 = 0.5657\,\frac{k}{m}\ ,\quad 1.7676\,\frac{k}{m}$$

$$\omega_1 = 0.7521\sqrt{\frac{k}{m}}\ ,\quad \omega_2 = 1.3295\sqrt{\frac{k}{m}}$$

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-\omega_1^2\, m_1 + k_1 + k_2}{k_2} = \frac{-0.5657(3) + 3 + 1}{1} = 2.3029$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-\omega_2^2\, m_1 + k_1 + k_2}{k_2} = \frac{-1.7676(3) + 3 + 1}{1} = -1.3028$$



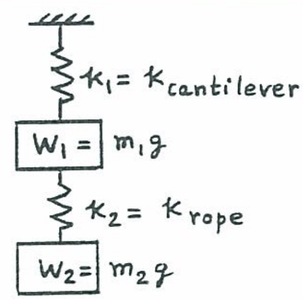
$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 2.3029 \end{Bmatrix} X_1^{(1)}\ ,\quad \vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ -1.3028 \end{Bmatrix} X_1^{(2)}$$

No node   one node

$$\frac{z}{1.3028} = \frac{1-z}{1}\ ;\quad z = 0.5657$$

5.24

$$k_1 = \frac{3EI}{b^3} = \frac{3E\left(\frac{1}{12}at^3\right)}{b^3} = \frac{E a t^3}{4 b^3}$$



$$k_2 = \frac{AE}{l} = \frac{\pi d^2 E}{4 l}$$

From problem 5.5,

$$\omega_{1,2}^2 = \frac{k_1 + k_2}{2 m_1} + \frac{k_2}{2 m_2} \mp \sqrt{\frac{1}{4}\left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2}}$$

$$= \left(\frac{E a t^3}{4 b^3} + \frac{\pi d^2 E}{4 l}\right)\frac{g}{2 W_1} + \frac{\pi d^2 E g}{8 l W_2}$$

$$\mp \sqrt{\left[\frac{1}{4}\left\{\left(\frac{E a t^3}{4 b^3} + \frac{\pi d^2 E}{4 l}\right)\frac{g}{W_1} + \frac{\pi d^2 E g}{4 l W_2}\right\}^2 - \frac{E^2 a t^3 \pi d^2\, g^2}{16\, l\, b^3\, W_1 W_2}\right]}$$

5.25 From solution of Problem 5.5, we find that for $m_1 = m$, $m_2 = 2\,m$, $k_1 = k$ and $k_2 = 2\,k$,

$$\omega_1^2 = (2 - \sqrt{3})\,\frac{k}{m}\ ;\ \omega_2^2 = (2 + \sqrt{3})\,\frac{k}{m}$$

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{k_2}{-m_2\,\omega_1^2 + k_2} = \frac{1}{-1 + \sqrt{3}}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{k_2}{-m_2\,\omega_2^2 + k_2} = \frac{1}{-1-\sqrt{3}}$$

First mode shape:

$$\begin{Bmatrix} X_1^{(1)} \\ X_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} X_1^{(1)} \cos(\omega_1 t + \phi_1) \\ r_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) \end{Bmatrix} = \begin{Bmatrix} X_1^{(1)} \cos(\omega_1 t + \phi_1) \\ \left(\dfrac{X_1^{(1)}}{\sqrt{3}-1}\right) \cos(\omega_1 t + \phi_1) \end{Bmatrix}$$

For the motion to be identical with the first normal mode, we need to have $X_1^{(2)} = 0$. This requires that (from Eq. (5.18)):

$$\frac{1}{r_2 - r_1}\left[\left\{-r_1\, x_1(0) + x_2(0)\right\}^2 + \frac{1}{\omega_2^2}\left\{r_1\, \dot{x}_1(0) - \dot{x}_2(0)\right\}^2\right]^{\frac{1}{2}} = 0$$

or

$$x_2(0) = r_1\, x_1(0) = \frac{x_1(0)}{\sqrt{3}-1}$$
$$\dot{x}_2(0) = r_1\, \dot{x}_1(0) = \frac{\dot{x}_1(0)}{\sqrt{3}-1}$$

5.26 Let $m_1 = m,\ m_2 = 2\,m,\ k_1 = k,\ k_2 = 2\,k$.
Initial conditions: $x_1(0) = 0,\ x_2(0) = 0.1$ m, $\dot{x}_1(0) = 0,\ \dot{x}_2(0) = 0$
Eqs. (5.18) yield:

$$X_1^{(1)} = \frac{1}{r_2 - r_1}\left[(0 - 0.1)^2\right]^{\frac{1}{2}} = \frac{0.1}{r_2 - r_1} = \frac{0.1}{\left(\dfrac{-1}{\sqrt{3}+1} - \dfrac{1}{\sqrt{3}-1}\right)} = -\frac{0.1}{\sqrt{3}}$$

$$X_1^{(2)} = \frac{1}{r_2 - r_1}\left[(0.1)^2\right]^{\frac{1}{2}} = \frac{0.1}{r_2 - r_1} = -\frac{0.1}{\sqrt{3}}$$
$$\phi_1 = \tan^{-1}(0) = 0$$
$$\phi_2 = \tan^{-1}(0) = 0$$

where ω_1 and ω_2 are given by Eq. (E3) of solution of Problem 5.5.

Resulting motion:

$$x_1(t) = X_1^{(1)} \cos(\omega_1 t + \phi_1) + X_1^{(2)} \cos(\omega_2 t + \phi_2) = -\frac{0.1}{\sqrt{3}}\left\{\cos \omega_1 t + \cos \omega_2 t\right\}$$

$$x_2(t) = r_1\, X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2\, X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

$$= \left(\frac{1}{\sqrt{3}-1}\right)\left(-\frac{0.1}{\sqrt{3}}\right)\cos \omega_1 t + \left(-\frac{1}{\sqrt{3}+1}\right)\left(-\frac{0.1}{\sqrt{3}}\right)\cos \omega_2 t$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$= -\frac{0.1}{\sqrt{3}}\left[\left(\frac{1}{\sqrt{3}-1}\right)\cos\omega_1 t - \left(\frac{1}{\sqrt{3}+1}\right)\cos\omega_2 t\right]$$

5.27 $\omega_1^2 \geq (2\pi \times 10)^2 = 3947.8602\ (\text{rad/sec})^2$

From solution of problem 5.24, this inequality can be expressed as:

$$\omega_1^2 = \left(\frac{E a t^3}{4 b^3} + \frac{\pi d^2 E}{4 l}\right)\frac{g}{2 W_1} + \frac{\pi d^2 E g}{8 l W_2}$$
$$- \sqrt{\left[\frac{1}{4}\left\{\left(\frac{E a t^3}{4 b^3} + \frac{\pi d^2 E}{4 l}\right)\frac{g}{W_1} + \frac{\pi d^2 E g}{4 l W_2}\right\}^2 - \frac{E^2 a t^3 \pi d^2 g^2}{16 l b^3 W_1 W_2}\right]}$$
$$\geq 3947.8602 \qquad (E_1)$$

Data: $E = 30 \times 10^6$ psi, $W_1 = 1000$ lb, $W_2 = 500$ lb, $g = 386.4$ in/s²,

$b = 30$ in, $l = 60$ in.

Unknowns: a, t, d.

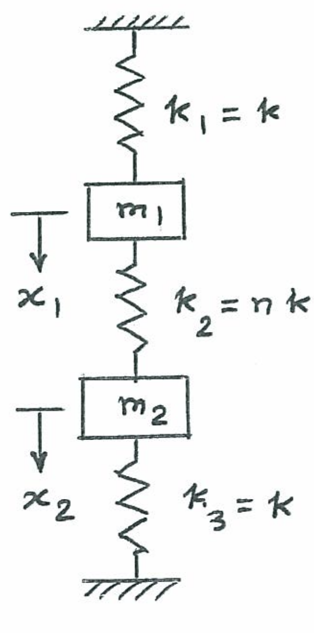
Let $a = 10\,t$ and $d = t$.

For this data, t is incremented from 0.1 in in increments of 0.01 in and the left hand side of the inequality (E_1) is evaluated. This gives a value of $t = 1.54$ in for satisfying (E_1).

$\therefore$ Design is $t = 1.54''$, $d = t = 1.54''$, $a = 10\,t = 15.4''$

5.28 Equations of motion:

$$\left.\begin{aligned} m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 &= 0 \\ m_2 \ddot{x}_2 - k_2 x_1 + (k_2 + k_3) x_2 &= 0 \end{aligned}\right\} \qquad (1)$$



Eigenvalue problem:

$$\begin{bmatrix} (-m_1\omega^2 + k_1 + k_2) & -k_2 \\ -k_2 & (-m_2\omega^2 + k_2 + k_3) \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (2)$$

For the data $k_1 = k_3 = 8$, $k_2 = n k = 8$; $m_1 = m_2 = m = 2$, Eq. (2) becomes

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\begin{bmatrix} -2\omega^2+16 & -8 \\ -8 & -2\omega^2+16 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3}$$

Frequency equation: $(-2\omega^2+16)^2 - 8^2 = 0$

or $\omega^2 = 4, 12$

or $\omega = 2,\ 3.4641$ (4)

For $\omega_1^2 = 4$; Eq.(3) gives

$[-2(4)+16]\, X_1 = 8\, X_2$ or $X_1 = X_2$

$$\therefore \vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} X_1^{(1)} \tag{5}$$

For $\omega_2^2 = 12$; Eq.(3) gives

$[-2(12)+16]\, X_1 = 8\, X_2$ or $X_2 = -X_1$

$$\therefore \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} X_1^{(2)} \tag{6}$$

Free vibration responses of masses m_1 and m_2 are given by

$$x_1(t) = X_1^{(1)} \cos(2t+\phi_1) + X_1^{(2)} \cos(3.4641\, t + \phi_2) \tag{7}$$

$$x_2(t) = X_1^{(1)} \cos(2t+\phi_1) - X_1^{(2)} \cos(3.4641\, t + \phi_2) \tag{8}$$

where $X_1^{(1)}$, $X_1^{(2)}$, ϕ_1 and ϕ_2 are constants to be determined from the initial conditions. Using the given initial conditions, Eqs. (7) and (8) yield

$$x_1(t=0) = 1 = X_1^{(1)} \cos\phi_1 + X_1^{(2)} \cos\phi_2 \tag{9}$$

$$x_2(t=0) = 0 = X_1^{(1)} \cos\phi_1 - X_1^{(2)} \cos\phi_2 \tag{10}$$

$$\dot{x}_1(t=0) = 0 = -2\, X_1^{(1)} \sin\phi_1 - 3.4641\, X_1^{(2)} \sin\phi_2 \tag{11}$$

$$\dot{x}_2(t=0) = 1 = -2\, X_1^{(1)} \sin\phi_1 + 3.4641\, X_1^{(2)} \sin\phi_2 \tag{12}$$

Eqs. (9) and (10) give:

$$X_1^{(1)} \cos\phi_1 = X_1^{(2)} \cos\phi_2 = \frac{1}{2} \tag{13}$$

Eqs. (11) and (12) yield:

$$X_1^{(1)} \sin\phi_1 = -0.25, \qquad X_1^{(2)} \sin\phi_2 = 0.1443 \tag{14}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

From Eqs. (13) and (14), we can find

$$X_1^{(1)} = \left\{ \left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{4}\right)^2 \right\}^{\frac{1}{2}} = 0.5590$$

$$\phi_1 = \tan^{-1}\left(\frac{X_1^{(1)} \sin \phi_1}{X_1^{(1)} \cos \phi_1} \right) = \tan^{-1}(-0.5) = -0.4636 \text{ rad}$$

$$X_1^{(2)} = \left\{ \left(\tfrac{1}{2}\right)^2 + (0.1443)^2 \right\}^{\frac{1}{2}} = 0.5204$$

$$\phi_2 = \tan^{-1}\left(\frac{X_1^{(2)} \sin \phi_2}{X_1^{(2)} \cos \phi_2} \right) = \tan^{-1}(0.2886) = 0.2810 \text{ rad}$$

Free vibration responses of m_1 and m_2:

$$x_1(t) = 0.5590 \cos(2t - 0.4636) + 0.5204 \cos(3.4641\, t + 0.2810)$$

$$x_2(t) = 0.5590 \cos(2t - 0.4636) - 0.5204 \cos(3.4641\, t + 0.2810)$$

5.29 From solution of problem 5.28, the free vibration responses of masses m_1 and m_2 are given by

$$x_1(t) = X_1^{(1)} \cos(2t + \phi_1) + X_1^{(2)} \cos(3.4641\, t + \phi_2) \tag{1}$$

$$x_2(t) = X_1^{(1)} \cos(2t + \phi_1) - X_1^{(2)} \cos(3.4641\, t + \phi_2) \tag{2}$$

where $X_1^{(1)}$, $X_1^{(2)}$, ϕ_1 and ϕ_2 are constants to be determined from the initial conditions. Using the given initial conditions, Eqs. (1) and (2) lead to

$$x_1(0) = 1 = X_1^{(1)} \cos \phi_1 + X_1^{(2)} \cos \phi_2 \tag{3}$$

$$x_2(0) = 0 = X_1^{(1)} \cos \phi_1 - X_1^{(2)} \cos \phi_2 \tag{4}$$

$$\dot{x}_1(0) = 0 = -2\, X_1^{(1)} \sin \phi_1 - 3.4641\, X_1^{(2)} \sin \phi_2 \tag{5}$$

$$\dot{x}_2(0) = 0 = -2\, X_1^{(1)} \sin \phi_1 + 3.4641\, X_1^{(2)} \sin \phi_2 \tag{6}$$

Eqs. (3) and (4) give:

$$X_1^{(1)} \cos \phi_1 = \tfrac{1}{2} \tag{7}$$

$$X_1^{(2)} \cos \phi_2 = \tfrac{1}{2} \tag{8}$$

Eqs. (5) and (6) give:

$$X_1^{(1)} \sin \phi_1 = 0 \tag{9}$$

$$X_1^{(2)} \sin \phi_2 = 0 \tag{10}$$

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Eqs. (7) and (9) give:

$$X_1^{(1)} = \left\{ \left(\tfrac{1}{2}\right)^2 + (0)^2 \right\}^{\frac{1}{2}} = \tfrac{1}{2} \quad ; \quad \phi_1 = \tan^{-1}(0) = 0$$

Eqs. (8) and (10) yield:

$$X_1^{(2)} = \left\{ \left(\tfrac{1}{2}\right)^2 + (0)^2 \right\}^{\frac{1}{2}} = \tfrac{1}{2} \quad ; \quad \phi_2 = \tan^{-1}(0) = 0$$

Hence, the free vibration responses of m_1 and m_2 are:

$$x_1(t) = \tfrac{1}{2} \cos 2t + \tfrac{1}{2} \cos 3.4641\, t$$

$$x_2(t) = \tfrac{1}{2} \cos 2t - \tfrac{1}{2} \cos 3.4641\, t$$

5.30 Results of Example 5.1:

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} X_1^{(1)} , \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} X_1^{(2)} , \quad \omega_1^2 = \frac{k}{m}, \quad \omega_2^2 = \frac{3k}{m}$$

$$[m] = m \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \quad [k] = k \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\vec{X}^{(1)T} \vec{X}^{(2)} = \{1 \quad 1\} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} X_1^{(1)} X_1^{(2)} = 0$$

$$\vec{X}^{(1)T} [m] \vec{X}^{(2)} = X_1^{(1)} X_1^{(2)} m \{1 \quad 1\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$= m X_1^{(1)} X_1^{(2)} \{1 \quad 1\} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = 0$$

$$\vec{X}^{(1)T} [m] \vec{X}^{(1)} = m \left(X_1^{(1)}\right)^2 \{1 \quad 1\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= 2m \left(X_1^{(1)}\right)^2 = c_1 = \text{constant}$$

$$\vec{X}_1^{(1)T} [k] \vec{X}^{(1)} = k \left(X_1^{(1)}\right)^2 \{1 \quad 1\} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$= k \left(X_1^{(1)}\right)^2 \{1 \quad 1\} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 2k \left(X_1^{(1)}\right)^2 = 2k \left(\frac{c_1}{2m}\right)$$

$$= c_1 \frac{k}{m} = c_1 \omega_1^2$$

$$\vec{X}^{(2)T} [m] \vec{X}^{(2)} = m \left(X_1^{(2)}\right)^2 \{1 \quad -1\} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$= 2m \left(X_1^{(2)}\right)^2 = c_2 = \text{constant}$$

$$\vec{X}^{(2)T} [k] \vec{X}^{(2)} = k \left(X_1^{(2)}\right)^2 \{1 \quad -1\} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

$$= k \left(X_1^{(2)}\right)^2 \{1 \quad -1\} \begin{Bmatrix} 3 \\ -3 \end{Bmatrix} = 6k \left(X_1^{(2)}\right)^2$$

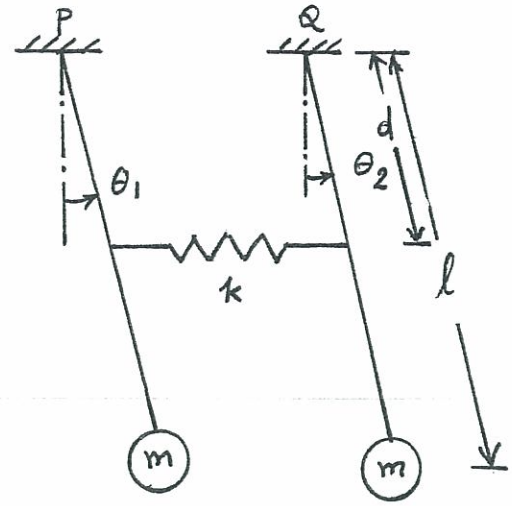
$$= 6k \left(\frac{c_2}{2m}\right) = c_2 \left(\frac{3k}{m}\right) = c_2 \omega_2^2$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.31 (a) Equations of motion:



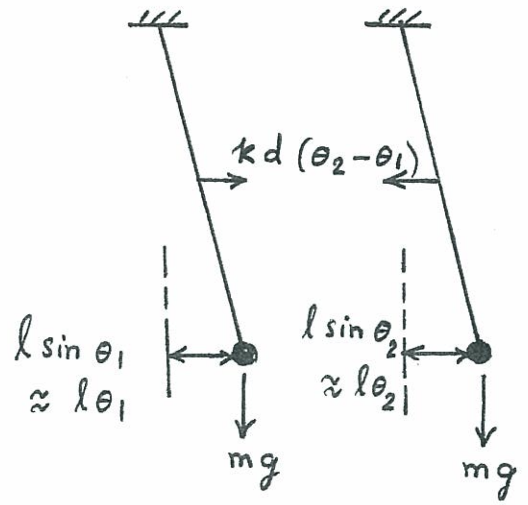
Assume: θ_1, θ_2 are small.

Moment equilibrium equations of the two masses about P and Q:

$$m l^2 \ddot{\theta}_1 + m g l \theta_1 + k d^2 (\theta_1 - \theta_2) = 0 \quad (1)$$

$$m l^2 \ddot{\theta}_2 + m g l \theta_2 - k d^2 (\theta_1 - \theta_2) = 0 \quad (2)$$

(b) Natural frequencies and mode shapes:



Free body diagram

Assume: Harmonic motion with

$$\theta_i(t) = \Theta_i \cos(\omega t - \phi); \; i = 1, 2 \quad (3)$$

where Θ_1 and Θ_2 are amplitudes of θ_1 and θ_2, respectively, ω is the natural frequency, and ϕ is the phase angle.

Using Eq. (3), Eqs. (1) and (2) can be expressed in matrix form as

$$-\omega^2 m l^2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} + \begin{bmatrix} mgl + kd^2 & -kd^2 \\ -kd^2 & mgl + kd^2 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (4)$$

Frequency equation:

$$\begin{vmatrix} -\omega^2 m l^2 + mgl + kd^2 & -kd^2 \\ -kd^2 & -\omega^2 m l^2 + mgl + kd^2 \end{vmatrix} = 0$$

or

$$\omega^4 - \omega^2 \left(\frac{2g}{l} + \frac{2kd^2}{ml^2} \right) + \left(\frac{g^2}{l^2} + \frac{2gkd^2}{ml^3} \right) = 0 \quad (5)$$

Solution of Eq. (5) gives

$$\omega_1^2 = \frac{g}{l}, \quad \omega_2^2 = \frac{g}{l} + \frac{2kd^2}{ml^2} \quad (6)$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

By substituting for ω_1^2 and ω_2^2 into Eq. (4), we obtain

$$\left(\frac{\Theta_2}{\Theta_1}\right)^{(1)} = 1 \quad \text{or} \quad \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \Theta_1^{(1)}$$

and

$$\left(\frac{\Theta_2}{\Theta_1}\right)^{(2)} = -1 \quad \text{or} \quad \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \Theta_1^{(2)}$$

Thus the motion of the masses in the two modes is given by

$$\vec{\theta}^{(1)}(t) = \begin{Bmatrix} \theta_1^{(1)}(t) \\ \theta_2^{(1)}(t) \end{Bmatrix} = \Theta_1^{(1)} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \cos(\omega_1 t + \phi_1) \qquad (7)$$

$$\vec{\theta}^{(2)}(t) = \begin{Bmatrix} \theta_1^{(2)}(t) \\ \theta_2^{(2)}(t) \end{Bmatrix} = \Theta_1^{(2)} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \cos(\omega_2 t + \phi_2) \qquad (8)$$

(c) Free vibration response:

Using linear superposition of natural modes, the free vibration response of the system is given by

$$\vec{\theta}(t) = c_1 \vec{\theta}^{(1)}(t) + c_2 \vec{\theta}^{(2)}(t) \qquad (9)$$

By choosing $c_1 = c_2 = 1$, with no loss of generality, Eqs. (7) to (9) lead to

$$\theta_1(t) = \Theta_1^{(1)} \cos(\omega_1 t + \phi_1) + \Theta_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (10)$$

$$\theta_2(t) = \Theta_1^{(1)} \cos(\omega_1 t + \phi_1) - \Theta_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (11)$$

where $\Theta_1^{(1)}$, ϕ_1, $\Theta_1^{(2)}$ and ϕ_2 are constants to be determined from the initial conditions. When $\theta_1(0) = a$, $\theta_2(0) = 0$, $\dot{\theta}_1(0) = 0$ and $\dot{\theta}_2(0) = 0$, Eqs. (10) and (11) yield

$$\left.\begin{aligned} a &= \Theta_1^{(1)} \cos\phi_1 + \Theta_1^{(2)} \cos\phi_2 \\ 0 &= \Theta_1^{(1)} \cos\phi_1 - \Theta_1^{(2)} \cos\phi_2 \\ 0 &= -\omega_1 \Theta_1^{(1)} \sin\phi_1 - \omega_2 \Theta_1^{(2)} \sin\phi_2 \\ 0 &= -\omega_1 \Theta_1^{(1)} \sin\phi_1 + \omega_2 \Theta_1^{(2)} \sin\phi_2 \end{aligned}\right\} \qquad (12)$$

Eqs. (12) can be solved for $\Theta_1^{(1)}$, ϕ_1, $\Theta_1^{(2)}$ and ϕ_2 to obtain

$$\left.\begin{aligned} \theta_1(t) &= a \cos\frac{\omega_2 - \omega_1}{2} t \cdot \cos\frac{\omega_2 + \omega_1}{2} t \\ \theta_2(t) &= a \sin\frac{\omega_2 - \omega_1}{2} t \cdot \sin\frac{\omega_2 + \omega_1}{2} t \end{aligned}\right\} \qquad (13)$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(d) conditions for beating:

When $\frac{2kd^2}{ml^2} << \frac{g}{l}$ or $k << \frac{mgl}{2d^2}$, (14)

the two frequency components in Eqs. (13), namely, $\frac{\omega_2-\omega_1}{2}$ and $\frac{\omega_2+\omega_1}{2}$, can be approximated as

$$\Omega_1 = \frac{\omega_2-\omega_1}{2} \simeq \frac{k}{2m}\frac{d^2}{\sqrt{gl^3}} \tag{15}$$

and

$$\Omega_2 = \frac{\omega_2+\omega_1}{2} \simeq \sqrt{\frac{g}{l}} + \frac{k}{2m}\frac{d^2}{\sqrt{gl^3}} \tag{16}$$

This implies that the motions of the pendulums are given by

$$\left.\begin{aligned} \theta_1(t) &\simeq a\cos\Omega_1 t\cdot\cos\Omega_2 t \\ \theta_2(t) &\simeq a\sin\Omega_1 t\cdot\sin\Omega_2 t \end{aligned}\right\} \tag{17}$$

This motion, Eqs. (17), denotes beating phenomenon.

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.36 With $k_{t1} = k_t$, $k_{t2} = 2\,k_t$, $J_1 = J_0$, $J_2 = 2\,J_0$, $k_{t3} = 0$ and $M_{t1} = M_{t2} = 0$, Eqs. (5.20) give

$$J_0 \ddot{\theta}_1 + 3\,k_t\,\theta_1 - 2\,k_t\,\theta_2 = 0$$

$$2\,J_0\,\ddot{\theta}_2 - 2\,k_t\,\theta_1 + 2\,k_t\,\theta_2 = 0$$

For harmonic solution, $\theta_i(t) = \Theta_i \cos(\omega t + \phi)$; $i = 1, 2$,

$$\begin{bmatrix} (-\omega^2 J_0 + 3\,k_t) & -2\,k_t \\ -2\,k_t & (-2\omega^2 J_0 + 2\,k_t) \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is

$$\begin{vmatrix} -\omega^2 J_0 + 3\,k_t & -2\,k_t \\ -2\,k_t & -2\,\omega^2 J_0 + 2\,k_t \end{vmatrix} = 2\,J_0^2\,\omega^4 - 8\,J_0\,k_t\,\omega^2 + 2\,k_t^2 = 0$$

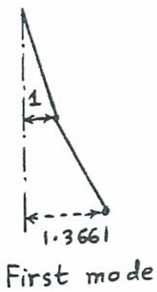
$$\omega^2 = (2 \mp \sqrt{3})\,\frac{k_t}{J_0} \quad ; \quad \omega_1 = 0.5176\,\sqrt{\frac{k_t}{J_0}}\,, \quad \omega_2 = 1.9319\,\sqrt{\frac{k_t}{J_0}}$$

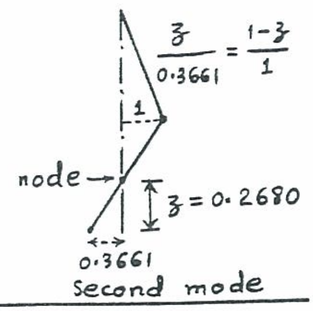
PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

$$r_1 = \frac{\Theta_2^{(1)}}{\Theta_1^{(1)}} = \frac{-J_0\,\omega_1^2 + 3\,k_t}{2\,k_t} = 1.3661$$

$$r_2 = \frac{\Theta_2^{(2)}}{\Theta_1^{(2)}} = \frac{-J_0\,\omega_2^2 + 3\,k_t}{2\,k_t} = -0.3661$$





5.37 Equation of motion of mass m: $m\ddot{x} = -k_2\,(x - r\theta)$ ---(E₁)

Equation of motion of cylinder of mass m_0 and mass moment of inertia $J_0 = \frac{1}{2} m_0 r^2$: $J_0\,\ddot{\theta} = -k_1 r^2 \theta - k_2\,(r\theta - x)\,r$ ---(E₂)

For $x(t) = X \cos(\omega t + \phi)$ and $\theta(t) = \Theta \cos(\omega t + \phi)$, Eqs. (E₁) and (E₂) give the frequency equation

$$\begin{vmatrix} -m\omega^2 + k_2 & -k_2 r \\ -k_2 r & -\frac{1}{2} m_0 r^2 \omega^2 + k_1 r^2 + k_2 r^2 \end{vmatrix} = 0$$

i.e. $$\omega^4 - \omega^2\left(\frac{k_2}{m} + \frac{2\{k_1 + k_2\}}{m_0}\right) + \frac{2\,k_1 k_2}{m_0\, m} = 0$$

$$\omega_1^2,\ \omega_2^2 = \frac{k_2}{2m} + \frac{(k_1 + k_2)}{m_0} \mp \sqrt{\frac{1}{4}\left(\frac{k_2}{m} + \frac{2\,k_1}{m_0} + \frac{2\,k_2}{m_0}\right)^2 - \frac{2\,k_1 k_2}{m\, m_0}}$$

5.38 For $J_1 = J_0$, $J_2 = 2\,J_0$, $k_{t_1} = k_{t2} = k_{t3} = k_t$, and $M_{t1} = M_{t2} = 0$, Eqs. (5.20) give

$$J_0\,\ddot{\theta}_1 + 2\,k_t\,\theta_1 - k_t\,\theta_2 = 0$$

$$2\,J_0\,\ddot{\theta}_2 - k_t\,\theta_1 + 2k_t\,\theta_2 = 0$$

For harmonic motion, these equations give

$$\begin{bmatrix} -\omega^2 J_0 + 2k_t & -k_t \\ -k_t & -2\omega^2 J_0 + 2\,k_t \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

from which the frequency equation can be obtained as

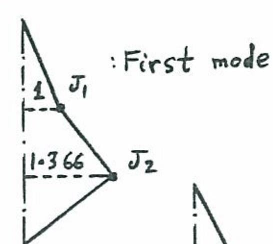
$$2\,J_0^2\,\omega^4 - 6\,J_0\,k_t\,\omega^2 + 3\,k_t^2 = 0$$

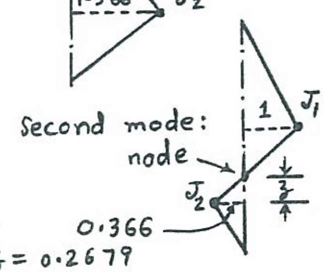
$$\omega_1^2,\ \omega_2^2 = \frac{1}{2}\left(3 \mp \sqrt{3}\right)\frac{k_t}{J_0}$$

$$\therefore\ \omega_1 = 0.79623\,\sqrt{\frac{k_t}{J_0}}\ ; \qquad \omega_2 = 1.53819\,\sqrt{\frac{k_t}{J_0}}$$

$$r_1 = \frac{\Theta_2^{(1)}}{\Theta_1^{(1)}} = \frac{-\omega_1^2 J_0 + 2k_t}{k_t} = 1.36603$$

$$r_2 = \frac{\Theta_2^{(2)}}{\Theta_1^{(2)}} = \frac{-\omega_2^2 J_0 + 2\,k_t}{k_t} = -0.36603$$





© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.39 Eqs. (5.20) give

$$J_0 \ddot{\theta}_1 + 6 k_t \theta_1 - 5 k_t \theta_2 = 0$$

$$5 J_0 \ddot{\theta}_2 - 5 k_t \theta_1 + 5 k_t \theta_2 = 0$$

These equations can be expressed as, for harmonic motion,

$$\begin{bmatrix} -\omega^2 J_0 + 6 k_t & -5 k_t \\ -5 k_t & -5 \omega^2 J_0 + 5 k_t \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is

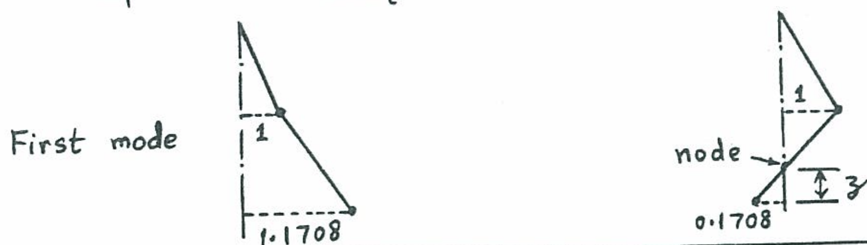
$$J_0^2 \omega^4 - 7 k_t J_0 \omega^2 + k_t^2 = 0$$

$$\omega_1^2, \omega_2^2 = \frac{k_t}{J_0} \left(\frac{7}{2} \mp \frac{1}{2} \sqrt{45} \right) = 0.1459 \frac{k_t}{J_0}, \quad 6.8541 \frac{k_t}{J_0}$$

$$\omega_1 = 0.38197 \sqrt{\frac{k_t}{J_0}}, \quad \omega_2 = 2.61803 \sqrt{\frac{k_t}{J_0}}$$

$$r_1 = \frac{\Theta_2^{(1)}}{\Theta_1^{(1)}} = \frac{-\omega_1^2 J_0 + 6 k_t}{5 k_t} = 1.1708$$

$$r_2 = \frac{\Theta_2^{(2)}}{\Theta_1^{(2)}} = \frac{-\omega_2^2 J_0 + 6 k_t}{5 k_t} = -0.1708$$

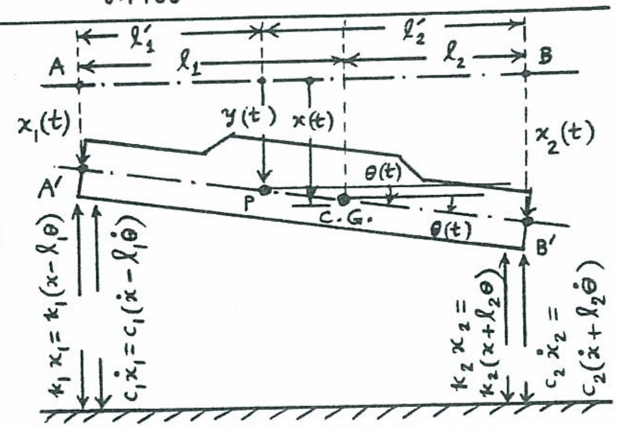


Second mode

$$\frac{z}{0.1708} = \frac{1-z}{1}$$

$$z = 0.1459$$

5.40 (i) Using $x(t)$ and $\theta(t)$:



For translatory motion:

$$m \ddot{x} = -k_1(x - l_1\theta) - c_1(\dot{x} - l_1\dot{\theta}) - k_2(x + l_2\theta) - c_2(\dot{x} + l_2\dot{\theta}) \quad \text{---} \quad (E_1)$$

For rotational motion about C.G.:

$$J_0 \ddot{\theta} = k_1(x - l_1\theta) l_1 + c_1(\dot{x} - l_1\dot{\theta}) l_1 - k_2(x + l_2\theta) l_2 - c_2(\dot{x} + l_2\dot{\theta}) l_2 \quad \text{---} \quad (E_2)$$

Eqs. (E_1) and (E_2) can be rewritten as

$$\begin{bmatrix} m & 0 \\ 0 & J_0 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_1 l_1 + c_2 l_2 \\ -c_1 l_1 + c_2 l_2 & c_1 l_1^2 + c_2 l_2^2 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_1 l_1 + k_2 l_2 \\ -k_1 l_1 + k_2 l_2 & k_1 l_1^2 + k_2 l_2^2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(ii) Using $y(t)$ and $\theta(t)$:

For translatory motion:

$$m\ddot{y} = -k_1(y - l_1'\theta) - c_1(\dot{y} - l_1'\dot{\theta}) - k_2(y + l_2'\theta) - c_2(\dot{y} + l_2'\dot{\theta}) - me\ddot{\theta} \quad \text{---- } (E_3)$$

For rotational motion:

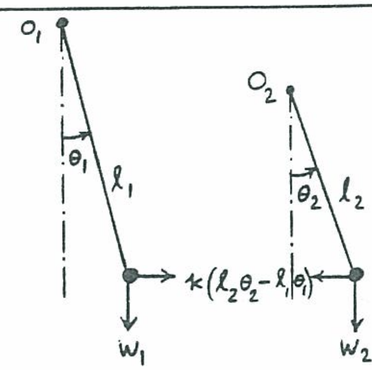
$$J_p\ddot{\theta} = k_1(y - l_1'\theta)\,l_1' + c_1(\dot{y} - l_1'\dot{\theta})\,l_1' - k_2(y + l_2'\theta)\,l_2' - c_2(\dot{y} + l_2'\dot{\theta})\,l_2' - me\ddot{y} \quad \text{---- } (E_4)$$

Eqs. (E_3) and (E_4) can be rewritten as

$$\begin{bmatrix} m & me \\ me & J_p \end{bmatrix}\begin{Bmatrix} \ddot{y} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c_1 + c_2 & -c_1 l_1' + c_2 l_2' \\ -c_1 l_1' + c_2 l_2' & c_1 l_1'^2 + c_2 l_2'^2 \end{bmatrix}\begin{Bmatrix} \dot{y} \\ \dot{\theta} \end{Bmatrix}$$

$$+ \begin{bmatrix} k_1 + k_2 & -k_1 l_1' + k_2 l_2' \\ -k_1 l_1' + k_2 l_2' & k_1 l_1'^2 + k_2 l_2'^2 \end{bmatrix}\begin{Bmatrix} y \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

5.41 For small angular deflections, equations of motion are



$$m_1 l_1^2\,\ddot{\theta}_1 = -W_1 l_1 \sin\theta_1 + k(l_2\theta_2 - l_1\theta_1)\,l_1\cos\theta_1$$

$$m_2 l_2^2\,\ddot{\theta}_2 = -W_2 l_2 \sin\theta_2 - k(l_2\theta_2 - l_1\theta_1)\,l_2\cos\theta_2$$

or

$$m_1 l_1^2\,\ddot{\theta}_1 + \theta_1(W_1 l_1 + k l_1^2) - k\,l_1 l_2\,\theta_2 = 0$$

$$m_2 l_2^2\,\ddot{\theta}_2 + \theta_2(W_2 l_2 + k l_2^2) - k\,l_1 l_2\,\theta_1 = 0$$

For harmonic motion, $\theta_i(t) = \Theta_i \cos(\omega t + \phi)$; $i = 1,2$, we get

$$\begin{bmatrix} -\omega^2 m_1 l_1^2 + W_1 l_1 + k\,l_1^2 & -k\,l_1 l_2 \\ -k\,l_1 l_2 & -\omega^2 m_2 l_2^2 + W_2 l_2 + k\,l_2^2 \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is

$$\omega^4(m_1 m_2\,l_1^2 l_2^2) - \omega^2\left[m_2 l_2^2(W_1 l_1 + k l_1^2) + m_1 l_1^2(W_2 l_2 + k l_2^2)\right]$$

$$+ \left[W_1 l_1 W_2 l_2 + W_2 l_2\,k\,l_1^2 + W_1 l_1\,k\,l_2^2\right] = 0 \quad \text{---- } (E_1)$$

Roots of this equation give the natural frequencies ω_1 and ω_2.

Amplitude ratios are given by

$$r_1 = \frac{\Theta_2^{(1)}}{\Theta_1^{(1)}} = \frac{-\omega_1^2\,m_1 l_1^2 + W_1 l_1 + k\,l_1^2}{k\,l_1 l_2}$$

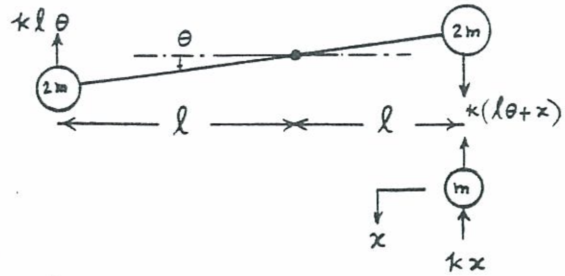
$$r_2 = \frac{\Theta_2^{(2)}}{\Theta_1^{(2)}} = \frac{-\omega_2^2\,m_1 l_1^2 + W_1 l_1 + k\,l_1^2}{k\,l_1 l_2}$$

where $W_1 = m_1 g$, $W_2 = m_2 g$.

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

5.42 Equations of motion:



$$4ml^2\ddot{\theta} = -kl\theta \cdot l - k(l\theta + x)l$$

$$m\ddot{x} = -kx - k(l\theta + x)$$

i.e.
$$4ml^2\ddot{\theta} + 2kl^2\theta + klx = 0$$
$$m\ddot{x} + 2kx + kl\theta = 0$$

For harmonic motion, these equations give

$$\begin{bmatrix} -4ml^2\omega^2 + 2kl^2 & kl \\ kl & -m\omega^2 + 2k \end{bmatrix} \begin{Bmatrix} \Theta \\ X \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is $4m^2\omega^4 - 10km\omega^2 + 3k^2 = 0$

$$\omega^2 = \frac{k}{m}\left(\frac{5}{4} \mp \frac{\sqrt{13}}{4}\right) = 0.3486\frac{k}{m}, \quad 2.1514\frac{k}{m}$$

$$\omega_1 = 0.5904\sqrt{\frac{k}{m}}, \quad \omega_2 = 1.4668\sqrt{\frac{k}{m}}$$

Amplitude ratios are

$$r_1 = \frac{X^{(1)}}{\Theta^{(1)}} = \frac{-4ml^2\omega_1^2 + 2kl^2}{-kl} = -0.6056\,l$$

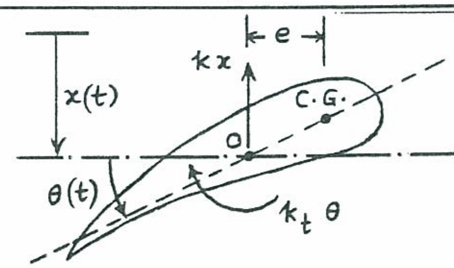
$$r_2 = \frac{X^{(2)}}{\Theta^{(2)}} = \frac{-4ml^2\omega_2^2 + 2kl^2}{-kl} = 6.6056\,l$$

Mode shapes are

$$\vec{X}^{(1)} = \begin{Bmatrix} \Theta^{(1)} \\ X^{(1)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -0.6056\,l \end{Bmatrix} \Theta^{(1)}$$

$$\vec{X}^{(2)} = \begin{Bmatrix} \Theta^{(2)} \\ X^{(2)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 6.6056\,l \end{Bmatrix} \Theta^{(2)}$$

5.43 Equations of motion:



$$m(\ddot{x} - e\ddot{\theta}) = -kx$$

$$J_{CG}\ddot{\theta} = -k_t\theta - kxe$$

i.e.
$$m\ddot{x} + kx - me\ddot{\theta} = 0$$
$$(J_0 - me^2)\ddot{\theta} + k_t\theta + kex = 0$$

For harmonic motion, we get the frequency equation as

$$\begin{vmatrix} -m\omega^2 + k & me\omega^2 \\ ke & -(J_0 - me^2)\omega^2 + k_t \end{vmatrix} = 0$$

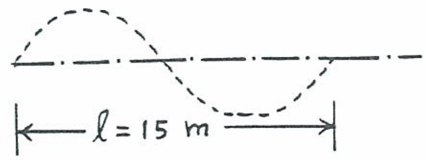
or
$$(J_0 - me^2)m\omega^4 - (J_0k + mk_t)\omega^2 + kk_t = 0$$

Roots of this equation give the natural frequencies of the system.

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الاتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.44 Speed becomes unfavorable when it is related to l as



$$v \tau_n = l$$

i.e., $v = \dfrac{l}{\tau_n} = l f_n = \dfrac{l \omega_n}{2\pi}$

Example 5.7 gives

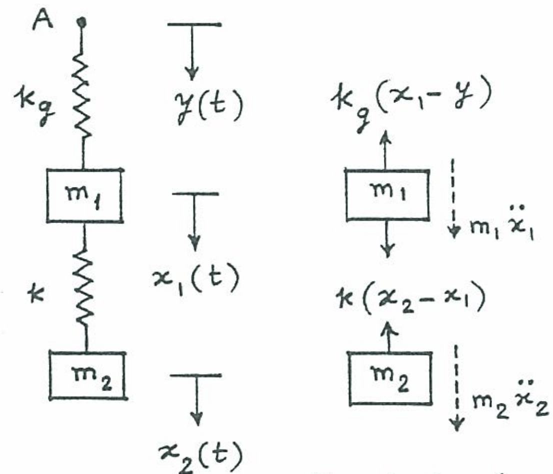
$\omega_1 = 5.8593$ rad/sec (bouncing)

$\omega_2 = 9.4341$ rad/sec (pitching)

$\therefore \; v_1 = \dfrac{l \omega_1}{2\pi} = \dfrac{15\,(5.8593)}{2\pi} = 13.9880$ m/s (bouncing)

$v_2 = \dfrac{l \omega_2}{2\pi} = \dfrac{15\,(9.4341)}{2\pi} = 22.5222$ m/s (pitching)

5.45



Free body diagrams of masses

Equations of motion:

$$m_1 \ddot{x}_1 + (k_g + k)\, x_1 - k\, x_2 = k_g\, y$$
$$m_2 \ddot{x}_2 - k\, x_1 + k\, x_2 = 0$$

Since velocity of crane in z-direction = 30 ft/min = 0.5 ft/sec, τ = time to complete one cycle = 10/0.5 = 20 sec, and $\omega = \dfrac{2\pi}{\tau} = \dfrac{2\pi}{20} = 0.31416$ rad/sec.

Base motion for m_1 (girder motion due to unevenness of rails):

$$y(t) = Y \sin \omega t$$

where Y = 2 in and $\omega = 0.31416$ rad/sec.

5.46 Road surface varies sinusoidally with amplitude, Y = 0.05 m and wavelength, d = 10 m. If v = velocity of automobile (m/sec), time to travel one wave length = $\tau = d/v$ sec. $\tau = 10/v$ sec, $\omega = \dfrac{2\pi}{\tau} = \dfrac{2\pi v}{10}$ rad/sec.

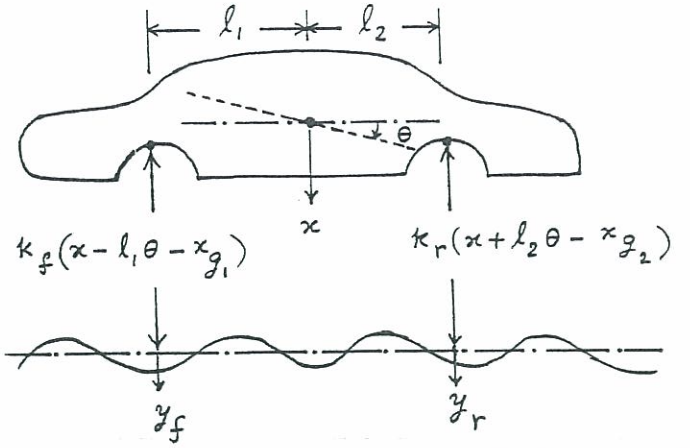
v = 50 km/hr = $(50\,(10^3))/(60\,(60))$ = 13.8889 m/sec,

$J_0 = m\, r_g^2 = 1000\,(0.9)^2 = 810$ kg–m^2.

Equations of motion:

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.



y_f (y_r) = ground or base displacement of front (rear) wheels, downwards

For motion along x:

$$m\ddot{x} + x(k_f + k_r) + \theta(k_r\ell_2 - k_f\ell_1) = k_f y_f + k_r y_r \qquad (1)$$

For motion along θ:

$$J_0\ddot{\theta} + x(\ell_2 k_r - \ell_1 k_f) + \theta(k_r\ell_2^2 + k_f\ell_1^2) = k_r\ell_2 y_r - k_f\ell_1 y_f \qquad (2)$$

where the ground (base) motions can be expressed as

$$y_f(t) = Y\sin\omega t = 0.05\sin\frac{2\pi v}{10}t \ \ m \qquad (3)$$

$$y_r(t) = Y\sin(\omega t - \phi) = 0.05\sin\left(\frac{2\pi v}{10}t - \frac{2\pi(\ell_1+\ell_2)}{d}\right) \ m \qquad (4)$$

For given data, Eqs. (1) and (2) take the form:

$$1000\ddot{x} + 40(10^3)x + 15000\theta = 900\sin 8.7267t + 1100\sin(8.7267t - 1.5708) \qquad (5)$$

$$810\ddot{\theta} + 15000x + 67500\theta = 1650\sin(8.7267t - 1.5708) - 900\sin 8.7267t \qquad (6)$$

5.47 Natural frequencies are given by:

$$\left[-\omega^2\begin{bmatrix}m_1 & 0\\ 0 & m_2\end{bmatrix} + \begin{bmatrix}k_{11} & k_{12}\\ k_{21} & k_{22}\end{bmatrix}\right]\begin{Bmatrix}X_1\\ X_2\end{Bmatrix} = \begin{Bmatrix}0\\ 0\end{Bmatrix} \qquad (1)$$

where m_1 = mass of pulley $= \frac{200}{386.4}$ lb–sec²/in ; m_2 = mass of motor $= \frac{500}{386.4}$ lb–sec²/in

X_1 = amplitude of pulley, X_2 = amplitude of motor,

$$\frac{EI}{\ell^3} = \frac{(30(10^6))(\frac{\pi}{64}(2^4))}{(90^3)} = 32.3210 \text{ lb/in}$$

Frequency equation becomes:

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\begin{vmatrix} (-\omega^2\, m_1 + k_{11}) & k_{12} \\ k_{12} & (-\omega^2\, m_2 + k_{22}) \end{vmatrix} = 0$$

or

$$(m_1\, m_2)\, \omega^4 - (k_{11}\, m_2 + k_{22}\, m_1)\, \omega^2 + (k_{11}\, k_{22} - k_{12}^2) = 0 \tag{2}$$

From known data, Eq. (2) can be expressed as:

$$0.6698\, \omega^4 - 11563.2894\, \omega^2 + 7.3108\, (10^6) = 0 \tag{3}$$

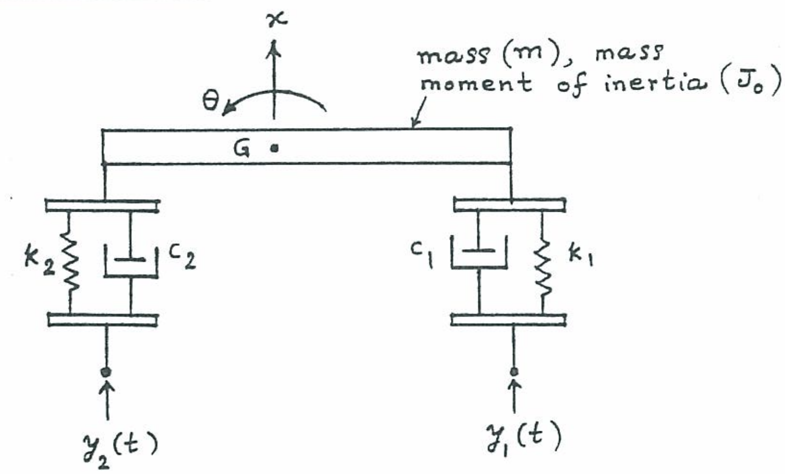
Roots of Eq. (3):

$$\omega^2 = 657.26642, \quad 16606.5300 \tag{4}$$

or

$$\omega_1 = 25.6372 \text{ rad/sec}, \quad \omega_2 = 128.8663 \text{ rad/sec}$$

5.48



1. Model the bicycle and the rider as a two d.o.f system as shown in figure.
2. Find the equivalent stiffness (k_1) and damping coefficient (c_1) of the front wheel in the vertical direction.
3. Find the equivalent stiffness (k_2) and damping coefficient (c_2), if applicable, of the rear wheel in the vertical direction.
4. Describe the road roughness under the wheels as $y_1(t)$ and $y_2(t)$.
5. Derive the equations of motion of the system subjected to base excitation.
6. Solve the resulting system of equations to find the steady state response.

5.49 (a)

Choose unknown coordinates as $x(t)$ and $\theta(t)$. Equations of motion:

$$m\, \ddot{x} = -k\, (x - \ell\, \theta/2) - 2\, k\, (x + \ell\, \theta/3) + F(t)$$

$$J_0\, \ddot{\theta} = k\, (x - \ell\, \theta/2)\, (\ell/2) - 2\, k\, (x + \ell\, \theta/3)\, (\ell/3) + F(t)\, (\ell/3)$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

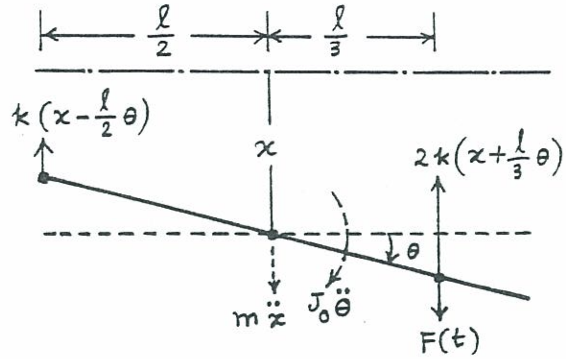
or

$$\begin{bmatrix} m & 0 \\ 0 & J_0 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 3\,k & k\,\ell/6 \\ k\,\ell/6 & 17\,k\,\ell^2/36 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ \ell\,F(t)/3 \end{Bmatrix}$$

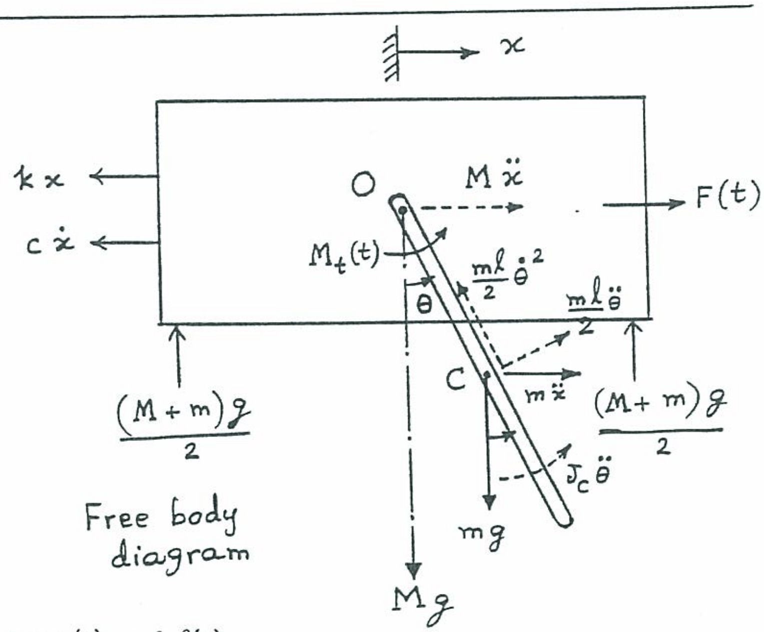
where $J_0 = \dfrac{m\,\ell^2}{12}$ and $F(t) = F_0 \sin \omega\, t$.

(b)

Elastic or static coupling.



5.50



Equations of motion with coordinates x(t) and θ(t):

For motion along x:

$$M\,\ddot{x} = -\,k\,x - c\,\dot{x} - (m\,\ell/2)\,\ddot{\theta}\cos\theta - m\,\ddot{x} + (m\,\ell/2)\,\dot{\theta}^2 \sin\theta + F(t) \qquad (1)$$

For rotation about O:

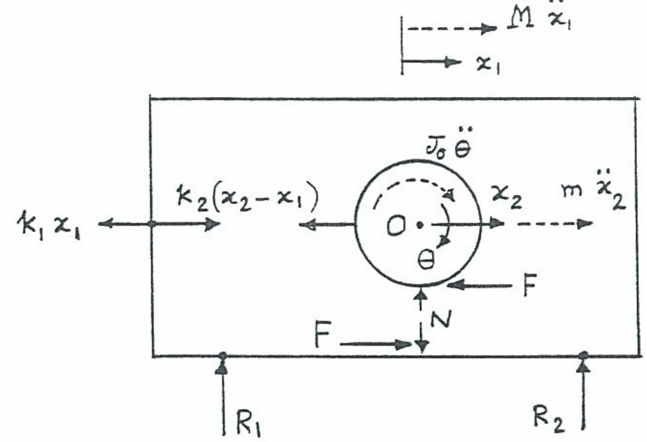
$$J_c\,\ddot{\theta} + (m\,\ell/2)\,\ddot{\theta}\,(\ell/2) + m\,\ddot{x}\,(\ell\cos\theta/2) = -\,m\,g\,(\ell/2)\sin\theta + M_t(t) \qquad (2)$$

Using $J_c = \dfrac{1}{12}\,m\,\ell^2$, $\cos\theta \approx 1$ and $\sin\theta \approx \theta$ and neglecting the nonlinear term involving $\dot{\theta}^2$ in Eq. (1), Eqs. (1) and (2) can be rewritten in matrix form as:

$$\begin{bmatrix} (M+m) & m\,\ell/2 \\ m\,\ell/2 & (J_c + m\,\ell^2/4) \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} c & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} k & 0 \\ 0 & m\,g\,\ell/2 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} F(t) \\ M_t(t) \end{Bmatrix}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

5.51



Free body diagram

N = normal reaction between cylinder and trailer, F = friction force, R_1, R_2 = reactions between trailer and ground.

Equation of motion for linear motion of cylinder:

$$\sum F = m\,\ddot{x}_2 \quad \text{or} \quad m\,\ddot{x}_2 = -F - k_2\,(x_2 - x_1) \tag{1}$$

Equation of motion for rotational motion of cylinder:

$$\sum M_0 = J_0\,\ddot{\theta} \quad \text{or} \quad J_0\,\ddot{\theta} = F\,r \tag{2}$$

where $J_0 = \frac{1}{2}\,m\,r^2$ and $\theta = \frac{x_2 - x_1}{r}$.

Equation of motion for linear motion of trailer:

$$\sum F = M\,\ddot{x}_1 \quad \text{or} \quad M\,\ddot{x}_1 = -k_1\,x_1 + k_2\,(x_2 - x_1) + F \tag{3}$$

Eq. (2) gives

$$F = \frac{J_0\,\ddot{\theta}}{r} = \frac{1}{r}\,(\frac{1}{2}\,m\,r^2)\left(\frac{\ddot{x}_2 - \ddot{x}_1}{r}\right) = \frac{m}{2}\,(\ddot{x}_2 - \ddot{x}_1) \tag{4}$$

Substitution of Eq. (4) into Eqs. (1) and (3) yields the equations of motion as:

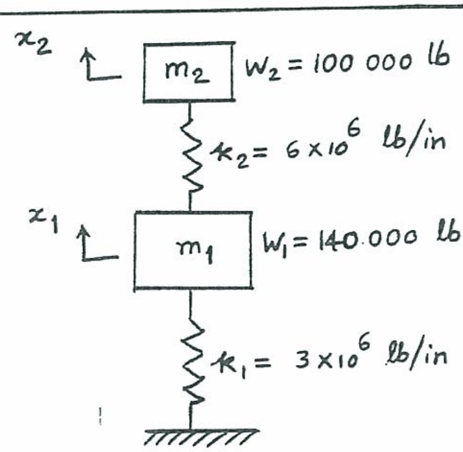
$$\frac{3\,m}{2}\,\ddot{x}_2 - \frac{1}{2}\,m\,\ddot{x}_1 - k_2\,x_1 + k_2\,x_2 = 0 \tag{5}$$

$$(M + \frac{m}{2})\,\ddot{x}_1 - \frac{m}{2}\,\ddot{x}_2 + x_1\,(k_1 + k_2) - k_2\,x_2 = 0 \tag{6}$$

5.52 (a) Natural frequencies of the system:

From problem 5.5,

$$\omega_{1,2}^2 = \frac{k_1 + k_2}{2m_1} + \frac{k_2}{2m_2} \mp \sqrt{\frac{1}{4}\left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2}}$$



© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech-MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\frac{\omega_{1,2}^2}{g} = \frac{9\times10^6}{28\times10^4} + \frac{6\times10^6}{20\times10^4} \mp \sqrt{\frac{1}{4}\left(\frac{9\times10^6}{14\times10^4} + \frac{6\times10^6}{10\times10^4}\right)^2 - \frac{18\times10^{12}}{140\times10^8}}$$

or

$$\omega_1 = 66.3408 \text{ rad/sec}$$

$$\omega_2 = 208.8557 \text{ rad/sec}$$

(b) Initial conditions of the system:

Let v_2 = initial velocity of anvil and frame just after the impact of tup

From conservation of momentum principle,

momentum of tup plus momentum of anvil just before impact = momentum of tup plus momentum of anvil just after impact

i.e.,

$$m_{tup}\, v_{tup} + m_{anvil}\,(0) = m_{tup}\, v_0 + m_{anvil}\, v_2 \qquad (E_1)$$

where v_0 = velocity of rebound of tup after impact

Also,

coefficient of restitution $(e) = \left(\dfrac{\text{relative velocity after impact}}{\text{relative velocity before impact}}\right)$

i.e.,

$$e = \frac{v_2 - v_0}{v_{tup}} \quad \text{or} \quad v_0 = v_2 - e\, v_{tup} \qquad (E_2)$$

From Eqs. (E_1) and (E_2),

$$v_2 = \frac{m_{tup}\, v_{tup}\,(1+e)}{m_{tup} + m_{anvil}} \qquad (E_3)$$

For given data,

$$v_2 = \frac{5000\,(180)\,(1+0.5)}{105\,000} = 12.8571 \text{ in/sec}$$

$\therefore$ Initial conditions are:

$$x_1(0) = 0, \quad \dot{x}_1(0) = 0$$

$$x_2(0) = 0, \quad \dot{x}_2(0) = 12.8571 \text{ in/sec}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(c) Displacements of anvil and foundation block:

We can use results of section 5.3 with $k_3 = 0$.

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{-m_1 \omega_1^2 + k_1 + k_2}{k_2} = \frac{-\frac{140\,000}{386.4}(4401.096) + 9\times 10^6}{6\times 10^6}$$

$$= 1.2342$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{-m_1 \omega_2^2 + k_1 + k_2}{k_2} = \frac{-\frac{140\,000}{386.4}(43620.696) + 9\times 10^6}{6\times 10^6}$$

$$= -1.1341$$

Response of the system can be computed using Eqs. (5.18):

$$X_1^{(1)} = \frac{1}{r_2 - r_1}\left(\frac{\dot{x}_2}{\omega_1}\right) = \frac{1}{-2.3683}\left(\frac{12.8571}{66.3408}\right) = -0.08183 \text{ in}$$

$$X_1^{(2)} = \frac{1}{r_2 - r_1}\left(\frac{\dot{x}_2}{\omega_2}\right) = \frac{1}{-2.3683}\left(\frac{12.8571}{208.8557}\right) = -0.02599 \text{ in}$$

$$\phi_1 = \tan^{-1}\left\{\frac{\dot{x}_2(0)}{0}\right\} = \frac{\pi}{2}$$

$$\phi_2 = \tan^{-1}\left\{-\frac{\dot{x}_2(0)}{0}\right\} = -\frac{\pi}{2}$$

Response is given by Eqs. (5.15):

$$x_1(t) = X_1^{(1)} \cos(\omega_1 t + \phi_1) + X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

$$= -0.08183 \cos\left(66.3408\, t + \frac{\pi}{2}\right) - 0.02599 \cos\left(208.8557\, t - \frac{\pi}{2}\right) \text{ in.}$$

$$x_2(t) = r_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2 X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

$$= -0.1010 \cos\left(66.3408\, t + \frac{\pi}{2}\right) + 0.02948 \cos\left(208.8557\, t - \frac{\pi}{2}\right) \text{ in.}$$

5.53 (a) Natural frequencies:

Equations of motion:

$$m_1 \ddot{x}_1 + k_1 x_1 - k_1 x_2 = F_1(t) \qquad (E_1)$$

$$m_2 \ddot{x}_2 + (k_1 + k_2) x_2 - k_1 x_1 = 0 \qquad (E_2)$$

Frequency equation:

$$\begin{vmatrix} -\omega^2 m_1 + k_1 & -k_1 \\ -k_1 & -m_2\omega^2 + k_1 + k_2 \end{vmatrix} = 0$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

or
$$\omega^4 - \left(\frac{k_1}{m_1} + \frac{k_1 + k_2}{m_2}\right)\omega^2 + \frac{k_1 k_2}{m_1 m_2} = 0$$

$$\therefore \omega_{1,2}^2 = \frac{k}{2m_1} + \frac{k_1 + k_2}{2m_2} \mp \sqrt{\frac{1}{4}\left(\frac{k_1}{m_1} + \frac{k_1+k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2}}$$

Here $m_1 = 2 \times 10^5$ kg, $m_2 = 2.5 \times 10^5$ kg, $k_1 = 150 \times 10^6$ N/m and $k_2 = 75 \times 10^6$ N/m.

$$\omega_{1,2}^2 = \frac{150 \times 10^6}{4 \times 10^5} + \frac{225 \times 10^6}{5 \times 10^5} \mp \sqrt{\frac{1}{4}\left(\frac{150 \times 10^6}{2 \times 10^5} + \frac{225 \times 10^6}{2.5 \times 10^5}\right)^2 - \frac{150 \times 75 \times 10^{12}}{5 \times 10^{10}}}$$

$$= 150,\ 1500 \quad (\text{rad/sec})^2$$

$$\therefore \omega_1 = 12.2474 \text{ rad/sec}, \quad \omega_2 = 38.7298 \text{ rad/sec}$$

(b) Response:

Assuming zero initial conditions, the Laplace transforms of (E_1) and (E_2) can be written as

$$m_1 s^2 \bar{x}_1(s) + k_1 \bar{x}_1(s) - k_1 \bar{x}_2(s) = \bar{F}_1(s)$$
$$m_2 s^2 \bar{x}_2(s) + (k_1 + k_2)\bar{x}_2(s) - k_1 \bar{x}_1(s) = 0$$

i.e.
$$(m_1 s^2 + k_1)\bar{x}_1(s) - k_1 \bar{x}_2(s) = \bar{F}_1(s)$$
$$-k_1 \bar{x}_1(s) + (m_2 s^2 + k_1 + k_2)\bar{x}_2(s) = 0$$

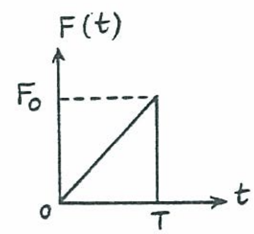
Solution of these equations gives

$$\bar{x}_1(s) = \left\{\frac{(m_2 s^2 + k_1 + k_2)}{m_1 m_2 s^4 + s^2(m_1 k_1 + m_1 k_2 + m_2 k_1) + k_1 k_2}\right\}\bar{F}_1(s) \quad \text{----}(E_3)$$

$$\bar{x}_2(s) = \left\{\frac{k_1}{m_1 m_2 s^4 + s^2(m_1 k_1 + m_1 k_2 + m_2 k_1) + k_1 k_2}\right\}\bar{F}_1(s) \quad \text{----}(E_4)$$

For the forcing function given,

$$\bar{F}_1(s) = \bar{F}(s) = \frac{F_0}{T}\left\{\frac{1}{s^2} - e^{-Ts}\left(\frac{T}{s} - \frac{1}{s^2}\right)\right\} \quad \text{----}(E_5)$$



For the given data, Eqs. (E_3) to (E_5) become

$$\bar{x}_1(s) = \frac{2.5 \times 10^5\, s^2 + 225 \times 10^6}{(5 \times 10^{10}\, s^4 + 825 \times 10^{11}\, s^2 + 11250 \times 10^{12})}\bar{F}_1(s)$$

$$= \frac{s^2 + 900}{2 \times 10^5\, s^4 + 330 \times 10^6\, s^2 + 45 \times 10^9}\bar{F}_1(s) \quad \text{----}(E_6)$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\bar{x}_2(s) = \frac{150 \times 10^6}{5\times10^{10}\, s^4 + 825\times10^{11}\, s^2 + 11250\times10^{12}}\ \bar{F}_1(s)$$

$$= \frac{30}{10^4\, s^4 + 165\times10^5\, s^2 + 225\times10^7}\ \bar{F}_1(s) \qquad ---- (E_7)$$

where $\bar{F}_1(s) = 2\times10^5 \left[\frac{1}{s^2} - e^{-0.5s}\left(\frac{1}{2s} - \frac{1}{s^2}\right)\right]$ ---- (E_8)

The inverse transforms of (E_6) and (E_7) yield $x_1(t)$ and $x_2(t)$.

5.54 Equations of motion for free vibration are (from Eqs. (5.1) and (5.2)):

$$\left.\begin{aligned} m_1\ddot{x}_1 + (c_1+c_2)\dot{x}_1 + (k_1+k_2)x_1 - c_2\dot{x}_2 - k_2 x_2 = 0 \\ m_2\ddot{x}_2 + (c_2+c_3)\dot{x}_2 + (k_2+k_3)x_2 - c_2\dot{x}_1 - k_2 x_1 = 0 \end{aligned}\right\} \qquad (E_1)$$

Assuming the solution as

$$x_i(t) = \tau_i\, e^{s_i t} \quad ; \quad i = 1,2$$

Eqs. (E_1) can be rewritten as

$$\begin{bmatrix} m_1 s^2 + (c_1+c_2)s + (k_1+k_2) & -(c_2 s + k_2) \\ -(c_2 s + k_2) & m_2 s^2 + (c_2+c_3)s + (k_2+k_3) \end{bmatrix} \begin{Bmatrix} \tau_1 \\ \tau_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad (E_2)$$

For a non-trivial solution of Eqs. (E_2),

$$\begin{vmatrix} m_1 s^2 + (c_1+c_2)s + k_1+k_2 & -(c_2 s + k_2) \\ -(c_2 s + k_2) & m_2 s^2 + (c_2+c_3)s + k_2+k_3 \end{vmatrix} = 0$$

i.e.,

$$s^4(m_1 m_2) + s^3\left[m_1(c_2+c_3) + m_2(c_1+c_2)\right] + s^2\left[m_1(k_2+k_3) + (c_1+c_2)(c_2+c_3) + m_2(k_1+k_2) - c_2^2\right] + s\left[(c_1+c_2)(k_2+k_3) + (c_2+c_3)(k_1+k_2) - 2c_2k_2\right] + \left[(k_1+k_2)(k_2+k_3) - k_2^2\right] = 0 \qquad (E_3)$$

This equation can be expressed as

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0 \qquad (E_4)$$

where $a_0, a_1, \ldots, a_4$ can be identified by comparing Eqs. (E_4) and (E_3).

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

Nature of possible solutions:

If s_1, s_2, s_3 and s_4 are the roots of Eq. (E_4), the general solution of the system can be expressed as

$$\left.\begin{aligned} x_1(t) &= \mathcal{C}_1^{(1)} e^{s_1 t} + \mathcal{C}_1^{(2)} e^{s_2 t} + \mathcal{C}_1^{(3)} e^{s_3 t} + \mathcal{C}_1^{(4)} e^{s_4 t} \\ x_2(t) &= \mathcal{C}_2^{(1)} e^{s_1 t} + \mathcal{C}_2^{(2)} e^{s_2 t} + \mathcal{C}_2^{(3)} e^{s_3 t} + \mathcal{C}_2^{(4)} e^{s_4 t} \end{aligned}\right\} \qquad (E_5)$$

where the constants $\mathcal{C}_1^{(i)}$, $i = 1$ to 4, can be found from the four initial conditions of the system, namely, $x_1(0)$, $x_2(0)$, $\dot{x}_1(0)$ and $\dot{x}_2(0)$. The ratios of amplitudes $\mathcal{C}_1^{(i)}/\mathcal{C}_2^{(i)}$ can be determined from Eqs. (E_2) as

$$\frac{\mathcal{C}_1^{(i)}}{\mathcal{C}_2^{(i)}} = \frac{c_2 s_i + k_2}{m_1 s_i^2 + (c_1 + c_2) s_i + k_1 + k_2} = \frac{m_2 s_i^2 + (c_2 + c_3) s_i + k_2 + k_3}{c_2 s_i + k_2};$$

$$i = 1, 2, 3, 4 \qquad (E_6)$$

If any root s_i has a positive real part, $x_1(t)$ and $x_2(t)$ will increase with time. If all s_i have negative real parts as

$$s_j = -r_j + i\, \omega_j$$

then the solution, $x_1(t)$, can be expressed as

$$x_1(t) = \sum_{j=1}^{4} \mathcal{C}_1^{(j)} e^{-r_j t} e^{i \omega_j t} = \sum_{j=1}^{4} \mathcal{C}_1^{(j)} e^{-r_j t} \left(\cos \omega_j t + i \sin \omega_j t \right)$$

If two roots s_1 and s_2 are complex conjugates as

$$s_1 = -(r_1 + i\, \omega_1) \quad \text{and} \quad s_2 = -(r_1 - i\, \omega_1),$$

$x_1(t)$ can be expressed as

$$\begin{aligned} x_1(t) = {} & e^{-r_1 t} \left\{ \mathcal{C}_1^{(1)} \cos \omega_1 t - i\, \mathcal{C}_1^{(1)} \sin \omega_1 t \right\} \\ & + e^{-r_1 t} \left\{ \mathcal{C}_1^{(2)} \cos \omega_1 t + i\, \mathcal{C}_1^{(2)} \sin \omega_1 t \right\} \\ & + \mathcal{C}_1^{(3)} e^{s_3 t} + \mathcal{C}_1^{(4)} e^{s_4 t} \end{aligned}$$

Similar expressions can be derived for $x_2(t)$.

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

5.55

Known data: $m_1 = m_2 = 10$ kg, $k_1 = k_2 = 2000$ N/m, $k_3 = 2$ N/m
$c_1 = 100$ N-s/m, $c_2 = c_3 = 1$ N-s/m
$x_1(0) = 0.2$ m, $x_2(0) = 0.1$ m, $\dot{x}_1(0) = \dot{x}_2(0) = 0$

Eqs. (E_3) and E_4 of the solution of Problem 5.54 give

$a_0 = m_1 m_2 = 100$

$a_1 = 10(2) + 10(101) = 1030$

$a_2 = 10(2002) + 101(2) + 10(4000) - 1 = 60221$

$a_3 = 101(2002) + 2(4000) - 2(1)(2000) = 206202$

$a_4 = 4000(2002) - 4 \times 10^6 = 4008000$

and

$$100 s^4 + 1030 s^3 + 60221 s^2 + 206202 s + 4008000 = 0 \quad (E_1)$$

Using PROGRAM 10, the roots of Eq. (E_1) can be found as

$$s_{1,2} = -1.4714 \pm i\, 8.8272 \quad (E_2)$$

$$s_{3,4} = -3.6786 \pm i\, 22.0668 \quad (E_3)$$

Thus the solution is given by

$$x_1(t) = \sum_{j=1}^{4} C_1^{(j)} e^{s_j t}, \quad x_2(t) = \sum_{j=1}^{4} C_2^{(j)} e^{s_j t} \quad (E_4, E_5)$$

where $C_1^{(j)}$, $j = 1,2,3,4$, can be found from the initial conditions, and the ratios of amplitudes $\left\{ \frac{C_1^{(j)}}{C_2^{(j)}} \right\}$ can be obtained from Eq. (E_6) in problem 5.46:

$$\frac{C_1^{(j)}}{C_2^{(j)}} = \frac{c_2 s_j + k_2}{m_1 s_j^2 + (c_1 + c_2) s_j + k_1 + k_2} = \frac{s_j + 2000}{10 s_j^2 + 101 s_j + 4000}; \quad j = 1,2,3,4 \quad (E_6)$$

For $j = 1$, $s_1 = -1.4714 + i\, 8.8272$ and (E_6) gives†

$$\alpha_1 = C_1^{(1)} / C_2^{(1)} = 0.6207 - i\, 0.1239 \quad (E_7)$$

For $j = 2$, $s_2 = -1.4714 - i\, 8.8272$ and (E_6) gives

$$\alpha_2 = C_1^{(2)} / C_2^{(2)} = 0.6207 + i\, 0.1239 \quad (E_8)$$

For $j = 3$, $s_3 = -3.6786 + i\, 22.0668$ and (E_6) yields

$$\alpha_3 = C_1^{(3)} / C_2^{(3)} = -1.3808 - i\, 0.7758 \quad (E_9)$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458. Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

For $j=4$, $s_4 = -3.6786 - i\,22.0668$ and (E_6) yields

$$\alpha_4 = C_1^{(4)}/C_2^{(4)} = -1.3808 + i\,0.7758 \qquad (E_{10})$$

Thus the solution of Eqs. (E_4) and (E_5) can be rewritten as

$$x_1(t) = \sum_{j=1}^{4} \alpha_j C_2^{(j)} e^{s_j t} \;, \quad x_2(t) = \sum_{j=1}^{4} \alpha_j e^{s_j t} \qquad (E_{11}, E_{12})$$

Since the pairs (α_1, α_2), (α_3, α_4), (s_1, s_2) and (s_3, s_4) are complex conjugates, we can express them as

$$\left.\begin{aligned} \alpha_1, \alpha_2 &= p_1 \pm i\,q_1 \;; \quad \alpha_3, \alpha_4 = p_2 \pm i\,q_2 \\ s_1, s_2 &= u_1 \pm i\,v_1 \;; \quad s_3, s_4 = u_2 \pm i\,v_2 \end{aligned}\right\} \qquad (E_{13})$$

and (E_{11}) and (E_{12}) can be simplified further. However, we proceed directly with (E_{11}) and (E_{12}) and use the initial conditions to evaluate the constants $C_2^{(j)}$; $j=1,2,3,4$:

$$\left.\begin{aligned} x_1(0) &= \alpha_1 C_2^{(1)} + \alpha_2 C_2^{(2)} + \alpha_3 C_2^{(3)} + \alpha_4 C_2^{(4)} = 0.2 \\ x_2(0) &= C_2^{(1)} + C_2^{(2)} + C_2^{(3)} + C_2^{(4)} = 0.1 \\ \dot{x}_1(0) &= s_1\alpha_1 C_2^{(1)} + s_2\alpha_2 C_2^{(2)} + s_3\alpha_3 C_2^{(3)} + s_4 s_4 C_2^{(4)} = 0 \\ \dot{x}_2(0) &= s_1 C_2^{(1)} + s_2 C_2^{(2)} + s_3 C_2^{(3)} + s_4 C_2^{(4)} = 0 \end{aligned}\right\} \qquad (E_{14})$$

Once $C_2^{(j)}$, $j=1,2,3,4$ are determined from Eqs. (E_{14}), the displacements of masses $x_1(t)$ and $x_2(t)$ can be obtained using Eqs. (E_{11}) and (E_{12}).

---

† If $s = a + ib$, $\quad s^2 = (a^2 - b^2) + i\,(2ab)$

If $x = \dfrac{a+bi}{c+di}$, it can be rewritten as

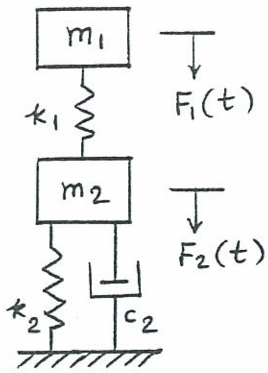
$$x = \frac{(a+bi)(c-di)}{(c^2+d^2)} = \left(\frac{ac+bd}{c^2+d^2}\right) + i\left(\frac{bc-ad}{c^2+d^2}\right)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.56



$$\omega = \frac{2\pi(1200)}{60} = 125.664 \text{ rad/sec}$$

$$F_1(t) = m e \omega^2 \cos \omega t$$

$$= \left(\frac{0.5}{386.4}\right)(6)(125.664)^2 \cos 125.664 t$$

$$= 122.6044 \cos 125.664 t \quad lb$$

$$m_1 = 800/386.4 = 2.0704 \ lb\text{-}s^2/in$$

$$m_2 = 2000/386.4 = 5.1760 \ lb\text{-}s^2/in$$

$k_1 = 2000$ lb/in, $k_2 = 1000$ lb/in, $c_2 = 200$ lb-s/in, $F_{10} = 122.6044$, $F_{20} = 0$.

Equations of motion are [substitute $k_1 = k_2$, $c_1 = c_2$, $m_1 = m_2$, $F_1 = 0$, $k_2 = k_1$, $c_2 = 0$, $m_2 = m_1$, $F_2 = F_1$, $k_3 = 0$, $c_3 = 0$ in Eq. (5.3)]:

$$\begin{bmatrix} m_2 & 0 \\ 0 & m_1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_2 \\ \ddot{x}_1 \end{Bmatrix} + \begin{bmatrix} c_2 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_2 \\ \dot{x}_1 \end{Bmatrix} + \begin{bmatrix} k_2 + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} x_2 \\ x_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ F_1(t) \end{Bmatrix} \quad \ldots (E_1)$$

or

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ 0 \end{Bmatrix} \quad \ldots (E_2)$$

Comparing (E_2) with Eq. (5.27), we find that

$m_{11} = m_1$, $m_{12} = 0$, $m_{22} = m_2$, $c_{11} = 0$, $c_{12} = 0$, $c_{22} = c_2$, $k_{11} = k_1$, $k_{12} = 0$ and $k_{22} = k_1 + k_2$.

Application of Eq. (5.31) leads to

$$Z_{11}(i\omega) = -\omega^2 m_{11} + i\omega c_{11} + k_{11} = -m_1 \omega^2 + k_1$$

$$Z_{12}(i\omega) = -\omega^2 m_{12} + i\omega c_{12} + k_{12} = -k_1$$

$$Z_{22}(i\omega) = -\omega^2 m_{22} + i\omega c_{22} + k_{22} = -m_2 \omega^2 + i\omega c_2 + k_1 + k_2$$

Response of the system can be expressed as

$$x_j(t) = X_j e^{i\omega t} = X_j \cos \omega t \quad (\text{real part})$$

with X_j given by Eq. (5.35):

$$X_1(i\omega) = \frac{Z_{22}(i\omega) \cdot F_{10} - Z_{12}(i\omega) \cdot F_{20}}{Z_{11}(i\omega) \cdot Z_{22}(i\omega) - Z_{12}^2(i\omega)}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$= \frac{(-m_2\omega^2 + i\omega c_2 + k_1 + k_2)\,F_{10}}{(-m_1\omega^2 + k_1)(-m_2\omega^2 + i\omega c_2 + k_1 + k_2) - k_1^2}$$

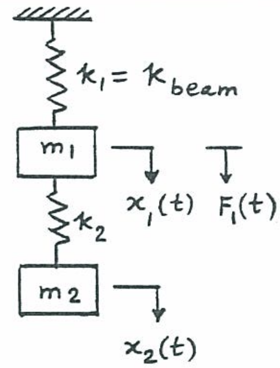
$$= \frac{\{-5.176\,(125.664)^2 + i(125.664)(200) + 3000\}\,122.6044}{\{[-2.0704\,(125.664)^2 + 2000][-5.176\,(125.664)^2 + i(125.664)(200) + 3000] - 4\times 10^6\}}$$

$$= (-40.0042 - 0.01919\,i)\times 10^{-4} \text{ in}$$

$$X_2(i\omega) = \frac{-Z_{12}(i\omega)\,F_{10} + Z_{11}(i\omega)\,F_{20}}{Z_{11}(i\omega)\,Z_{22}(i\omega) - Z_{12}^2(i\omega)} = \frac{k\;F_{10}}{Z_{11}(i\omega)\,Z_{22}(i\omega) - Z_{12}^2(i\omega)}$$

$$= \frac{2000\,(122.6044)}{(24.1272 - 7.7143\,i)\,10^8} = (0.9221 + 0.2948\,i)\times 10^{-4} \text{ in}$$

5.57 $k_1 = k_{beam} = \dfrac{192\,E\,(\frac{1}{12}a t^3)}{l^3} = \dfrac{16\,E\,a\,t^3}{l^3}$



Equations of motion:

$$\left.\begin{aligned} m_1\ddot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) &= F_1(t) = F_0 \cos\omega t \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) &= 0 \end{aligned}\right\}\ldots(E_1)$$

Assuming harmonic response

$$x_j(t) = X_j \cos\omega t \;;\; j = 1, 2$$

Eqs. (E_1) yield

$$X_1 = \frac{(k_2 - m_2\omega^2)\,F_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

$$X_2 = \frac{k_2\,F_0}{(k_1 + k_2 - m_1\omega^2)(k_2 - m_2\omega^2) - k_2^2}$$

For no steady state vibration of the beam, $X_1 = 0$ and hence the condition to be satisfied is

$$\frac{k_2}{m_2} = \omega^2$$

5.58 Equations of motion:

$$m_1\ddot{x}_1 + (k_1 + k_2)\,x_1 - k_2 x_2 = F_1(t) = F_0 \sin\omega t \quad \text{---- } (E_1)$$

$$m_2\ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \quad \text{---- } (E_2)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

We use $F_0 e^{i\omega t}$ (with $i = \sqrt{-1}$) for $F_1(t)$ and consider only the imaginary part at the end.

Let $x_j(t) = X_j e^{i\omega t}$; $j = 1, 2$

Eqs. (E_1) and (E_2) become

$$-m_1 \omega^2 X_1 e^{i\omega t} + (k_1 + k_2) X_1 e^{i\omega t} - k_2 X_2 e^{i\omega t} = F_0 e^{i\omega t}$$

$$-m_2 \omega^2 X_2 e^{i\omega t} + k_2 X_2 e^{i\omega t} - k_2 X_1 e^{i\omega t} = 0$$

i.e. $$[Z(i\omega)] \vec{X} = \vec{\underset{\sim}{F_0}} \qquad \text{---- } (E_3)$$

where $\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$, $\vec{\underset{\sim}{F_0}} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$,

$$[Z(i\omega)] = \begin{bmatrix} Z_{11}(i\omega) & Z_{12}(i\omega) \\ Z_{21}(i\omega) & Z_{22}(i\omega) \end{bmatrix},$$

$Z_{11}(i\omega) = -m_1\omega^2 + k_1 + k_2$, $Z_{12}(i\omega) = Z_{21}(i\omega) = -k_2$,

$Z_{22}(i\omega) = -m_2\omega^2 + k_2$.

Eqs. (5.35) give

$$X_1(i\omega) = \frac{(-m_2\omega^2 + k_2) F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2}$$

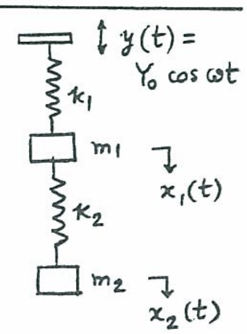
$$X_2(i\omega) = \frac{k_2 F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2}$$

Since $F_0 \sin \omega t = \mathrm{Im}\,(F_0 e^{i\omega t})$, $x_j(t) = \mathrm{Im}\,(X_j e^{i\omega t}) = X_j \sin \omega t$

$$\therefore \quad x_1(t) = \frac{(-m_2\omega^2 + k_2) F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \sin \omega t$$

$$x_2(t) = \frac{k_2 F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \sin \omega t$$

5.59 Equations of motion:



$$m_1 \ddot{x}_1 = -k_1(x_1 - y) - k_2(x_1 - x_2)$$

$$m_2 \ddot{x}_2 = -k_2(x_2 - x_1)$$

or

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = k_1 Y_0 \cos \omega t \qquad \text{---} (E_1)$$

$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0 \qquad \text{---} (E_2)$$

As there is no damping, the masses vibrate either in phase or 180° out of phase with respect to the base motion. Hence the response can be taken as

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$x_j(t) = X_j \cos \omega t \quad ; \quad j = 1,2 \qquad --- (E_3)$$

Eqs. (E_1) and (E_2) reduce to

$$(-\omega^2 m_1 + k_1 + k_2)\, X_1 - k_2\, X_2 = k_1 Y_0$$

$$-k_2\, X_1 + (-\omega^2 m_2 + k_2)\, X_2 = 0$$

i.e. $\left[Z(i\omega)\right] \vec{X} = \vec{F}$ where $\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}$, $\vec{F} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} = \begin{Bmatrix} k_1 Y_0 \\ 0 \end{Bmatrix}$,

$$Z_{11}(i\omega) = -\omega^2 m_1 + k_1 + k_2, \quad Z_{12}(i\omega) = Z_{21}(i\omega) = -k_2,$$

$$Z_{22}(i\omega) = -\omega^2 m_2 + k_2 .$$

Eqs. (5.35) and (E_3) give

$$x_1(t) = \frac{(-\omega^2 m_2 + k_2)\, k_1 Y_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \cos \omega t$$

$$x_2(t) = \frac{k_1 k_2 Y_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \cos \omega t$$

5.60 Equations of motion:

$$m_1 \ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2) x_1 - c_2 \dot{x}_2 - k_2 x_2 = F_1(t) = F_0 e^{i\omega t}$$

$$m_2 \ddot{x}_2 + c_2 \dot{x}_2 + k_2 x_2 - c_2 \dot{x}_1 - k_2 x_1 = 0$$

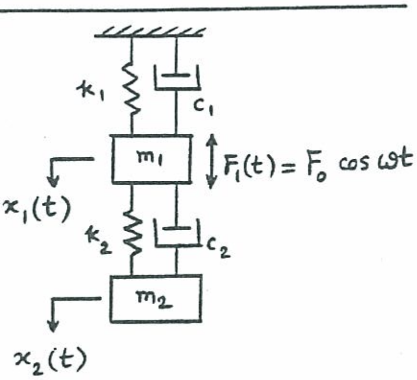
Let $x_j(t) = X_j e^{i\omega t}$; $j = 1,2$

$k_1 = k_2 = 500$ N/m

$m_1 = m_2 = 1$ kg

$c_1 = c_2 = c = 200$ N·s/m

$\omega = 1$ rad/s



Equations of motion become

$$\left[-\omega^2 m_1 + i\omega (c_1 + c_2) + k_1 + k_2\right] X_1 - (i\omega c_2 + k_2)\, X_2 = F_0 \qquad --- (E_1)$$

$$-(i\omega c_2 + k_2)\, X_1 + \left[-\omega^2 m_2 + i\omega c_2 + k_2\right] X_2 = 0 \qquad --- (E_2)$$

For given data, (E_1) and (E_2) become

$$\left[Z(i\omega)\right] \vec{X} = \vec{F}_0 \qquad --- (E_3)$$

where
$$Z_{11}(i\omega) = 400\, i + 999$$

$$Z_{12}(i\omega) = Z_{21}(i\omega) = -200\, i - 500$$

$$Z_{22}(i\omega) = 200\, i + 499$$

$$\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}, \quad \vec{F}_0 = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix} = \begin{Bmatrix} F_0 \\ 0 \end{Bmatrix}$$

Solution of (E_3) is, using Eqs. (5.35),

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$X_1 = \frac{(200\,i + 499)\,F_0}{(400i + 999)(200\,i + 499) - (-200i - 500)^2} = \frac{(200i + 499)\,F_0}{(199\,400\,i + 208501)}$$

$$= \frac{(200\,i + 499)(-199\,400\,i + 208501)\,F_0}{(199400\,i + 208\,501)(-199400\,i + 208501)}$$

$$= (17.2915 \times 10^{-4} - 6.9444 \times 10^{-4}\,i)\,F_0 \quad \text{---- } (E_4)$$

$$X_2 = \frac{(200\,i + 500)\,F_0}{(400\,i + 999)(200\,i + 499) - (-200i - 500)^2} = \frac{(200\,i + 500)\,F_0}{(199400\,i + 208501)}$$

$$= \frac{(200\,i + 500)(-199400\,i + 208501)\,F_0}{(199\,400\,i + 208\,501)(-199400\,i + 208501)}$$

$$= (17.3165 \times 10^{-4} - 6.9684 \times 10^{-4}\,i)\,F_0 \quad \text{---- } (E_5)$$

Final solution is given by the real parts as

$$x_1(t) = Re\left(X_1\,e^{i\omega t}\right) = Re\left(X_1 \cos\omega t + i\,X_1 \sin\omega t\right)$$

$$= Re\left[(17.2915 \times 10^{-4} - 6.9444 \times 10^{-4}\,i)\,F_0 \cos\omega t + (17.2915 \times 10^{-4}\,i + 6.9444 \times 10^{-4})\,F_0 \sin\omega t\right]$$

$$= 17.2915 \times 10^{-4}\,F_0 \cos\omega t + 6.9444 \times 10^{-4}\,F_0 \sin\omega t \quad \text{--- } (E_6)$$

$$x_2(t) = Re\left(X_2\,e^{i\omega t}\right) = Re\left(X_2 \cos\omega t + i\,X_2 \sin\omega t\right)$$

$$= Re\left[(17.3165 \times 10^{-4} - 6.9684 \times 10^{-4}\,i)\,F_0 \cos\omega t + (17.3165 \times 10^{-4}\,i + 6.9684 \times 10^{-4})\,F_0 \sin\omega t\right]$$

$$= 17.3165 \times 10^{-4}\,F_0 \cos\omega t + 6.9684 \times 10^{-4}\,F_0 \sin\omega t \quad \text{--- } (E_7)$$

5.61 Equations of motion:

$$m_1 \ddot{x}_1 + (k_1 + k_2)\,x_1 - k_2 x_2 = F_{10} \cos\omega t = Re\left(F_{10}\,e^{i\omega t}\right)$$

$$m_2 \ddot{x}_2 + (k_2 + k_3)\,x_2 - k_2 x_1 = F_{20} \cos\omega t = Re\left(F_{20}\,e^{i\omega t}\right)$$

Assuming $x_j(t) = X_j\,e^{i\omega t}$; $j = 1,2$ along with $F_j(t) = F_{j0}\,e^{i\omega t}$; $j = 1,2$, the equations of motion can be expressed as

$$(-\omega^2 m_1 + k_1 + k_2)\,X_1 - k_2 X_2 = F_{10}$$

$$-k_2 X_1 + (-\omega^2 m_2 + k_2 + k_3)\,X_2 = F_{20}$$

i.e. $[Z(i\omega)]\,\vec{X} = \vec{F_0}$ ---- (E_1)

where $Z_{11}(i\omega) = -\omega^2 m_1 + k_1 + k_2$, $Z_{12}(i\omega) = Z_{21}(i\omega) = -k_2$,

$Z_{22}(i\omega) = -\omega^2 m_2 + k_2 + k_3$,

$$\vec{X} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix}, \quad \vec{F_0} = \begin{Bmatrix} F_{10} \\ F_{20} \end{Bmatrix}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

solution of (E_1) can be expressed, using Eqs. (5.35), as

$$X_1 = \frac{(-\omega^2 m_2 + k_2 + k_3) F_{10} + k_2 F_{20}}{(-\omega^2 m_1 + k_1 + k_2)(-\omega^2 m_2 + k_2 + k_3) - k_2^2} \quad \text{---- } (E_2)$$

$$X_2 = \frac{k_2 F_{10} + (-\omega^2 m_1 + k_1 + k_2) F_{20}}{(-\omega^2 m_1 + k_1 + k_2)(-\omega^2 m_2 + k_2 + k_3) - k_2^2} \quad \text{---- } (E_3)$$

Since X_1 and X_2 are real (since there is no damping), the final solution is given by

$$x_1(t) = X_1 \cos \omega t$$
$$x_2(t) = X_2 \cos \omega t$$

where X_1 and X_2 are given by (E_2) and (E_3).

5.62 From the solution of problem 5.58, we have

$$x_1(t) = \frac{(-m_2\omega^2 + k_2) F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \sin \omega t$$

$$x_2(t) = \frac{k_2 F_0}{(-m_1\omega^2 + k_1 + k_2)(-m_2\omega^2 + k_2) - k_2^2} \sin \omega t$$

For the data $F_1(t) = 50 \sin 4\pi t$, $F_0 = 50$ N, $\omega = 4\pi$ rad/s, $m_1 = 10$ kg, $m_2 = 5$ kg, $k_1 = 8000$ N/m and $k_2 = 2000$ N/m,

$$x_1(t) = \frac{(-5 \times 16\pi^2 + 2000)\, 50}{(-10 \times 16\pi^2 + 8000 + 2000)(-5 \times 16\pi^2 + 2000) - (2000)^2} \sin 4\pi t$$

$$= 0.009773 \sin 4\pi t \quad \text{meters}$$

$$x_2(t) = \frac{2000(50)}{(-10 \times 16\pi^2 + 8000 + 2000)(-5 \times 16\pi^2 + 2000) - (2000)^2} \sin 4\pi t$$

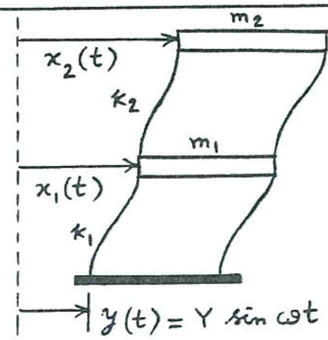
$$= 0.016148 \sin 4\pi t \quad \text{meters}$$

5.63 k_1 = total stiffness = 800 N/m

k_2 = total stiffness = 600 N/m

$m_1 = 50$ kg, $m_2 = 50$ kg

$Y = 0.2$ m, $\omega = \pi$ rad/s



Equations of motion:

$$m_1 \ddot{x}_1 = -k_1(x_1 - y) - k_2(x_1 - x_2)$$
$$m_2 \ddot{x}_2 = -k_2(x_2 - x_1)$$

i.e.

$$m_1 \ddot{x}_1 + (k_1 + k_2) x_1 - k_2 x_2 = k_1 y = k_1 Y \sin \omega t$$
$$m_2 \ddot{x}_2 + k_2 x_2 - k_2 x_1 = 0$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Assuming $x_i(t) = X_i \sin \omega t$; $i = 1, 2$, we get

$$(-m_1 \omega^2 + k_1 + k_2)\, X_1 - k_2 X_2 = k_1 Y$$

$$-k_2 X_1 + (-m_2 \omega^2 + k_2)\, X_2 = 0$$

For given data, these equations take the form

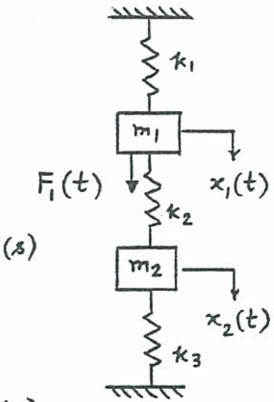
$$(-50\pi^2 + 1400)\, X_1 - 600\, X_2 = (800)(0.2)$$

$$-600\, X_1 + (-50\pi^2 + 600)\, X_2 = 0$$

Solution of these equations gives $X_1 = -0.06469$ m, $X_2 = -0.36439$ m

$\therefore\ x_1(t) = -0.06469 \sin \pi t$ m ; $x_2(t) = -0.36439 \sin \pi t$ m.

5.64 Equations of motion:



$$m_1 \ddot{x}_1 + (k_1 + k_2)\, x_1 - k_2 x_2 = F_1(t) \quad \text{--- } (E_1)$$

$$m_2 \ddot{x}_2 + (k_2 + k_3)\, x_2 - k_2 x_1 = 0 \quad \text{--- } (E_2)$$

Laplace transforms of (E_1) and (E_2) are

$$m_1[s^2 \bar{x}_1(s) - s\, x_1(0) - \dot{x}_1(0)] + (k_1 + k_2)\, \bar{x}_1(s) - k_2\, \bar{x}_2(s) = \bar{F}_1(s)$$

$$m_2[s^2 \bar{x}_2(s) - s\, x_2(0) - \dot{x}_2(0)] + (k_2 + k_3)\, \bar{x}_2(s) - k_2\, \bar{x}_1(s) = 0$$

Rearranging these equations, we get

$$(m_1 s^2 + k_1 + k_2)\, \bar{x}_1(s) - k_2\, \bar{x}_2(s) = \bar{F}_1(s) + s\, m_1 x_1(0) + m_1 \dot{x}_1(0) \quad \text{----} (E_3)$$

$$-k_2\, \bar{x}_1(s) + (m_2 s^2 + k_2 + k_3)\, \bar{x}_2(s) = s\, m_2 x_2(0) + m_2 \dot{x}_2(0) \quad \text{---} (E_4)$$

When $k_1 = k_2 = k_3 = k$ and $m_1 = m_2 = m$, (E_3) and (E_4) give

$$(m s^2 + 2k)\, \bar{x}_1(s) - k\, \bar{x}_2(s) = \bar{F}_1(s) + s\, m\, x_1(0) + m\, \dot{x}_1(0) \quad \text{---} (E_5)$$

$$-k\, \bar{x}_1(s) + (m s^2 + 2k)\, \bar{x}_2(s) = s\, m\, x_2(0) + m\, \dot{x}_2(0) \quad \text{---} (E_6)$$

Solution of Eqs. (E_5) and (E_6) gives

$$\bar{x}_1(s) = \frac{(m s^2 + 2k)\{\bar{F}_1(s) + m\, x_1(0) \cdot s + m\, \dot{x}_1(0)\} + k\{m\, x_2(0) \cdot s + m\, \dot{x}_2(0)\}}{(m s^2 + 2k)(m s^2 + 2k) - k^2}$$

$$\bar{x}_2(s) = \frac{k\{\bar{F}_1(s) + m\, x_1(0) \cdot s + \dot{x}_1(0)\, m\} + (m s^2 + 2k)\{m\, x_2(0) \cdot s + m\, \dot{x}_2(0)\}}{(m s^2 + 2k)(m s^2 + 2k) - k^2}$$

These equations become, for $x_1(0) = x_2(0) = \dot{x}_1(0) = \dot{x}_2(0) = 0$,

$F_1(t) = 5\, u(t)$, $\bar{F}_1(s) = \frac{5}{s}$, $m = 1$ and $k = 100$,

$$\bar{x}_1(s) = \frac{5\,(s^2 + 200)}{s\,[(s^2 + 200)^2 - 10000]} = \frac{5\,(s^2 + 200)}{s\,(s^2 + 300)\,(s^2 + 100)} \quad \text{---} (E_7)$$

$$\bar{x}_2(s) = 100 \left(\frac{5}{s}\right) \frac{1}{[(s^2 + 200)^2 - 10000]} = \frac{500}{s\,(s^2 + 300)\,(s^2 + 100)} \quad \text{---} (E_8)$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

By expressing

$$\bar{x}_1(s) = \frac{5(s^2+200)}{s(s^2+300)(s^2+100)} = \frac{A_1}{s} + \frac{A_2 s + A_3}{s^2+100} + \frac{A_4 s + A_5}{s^2+300}$$

$$\bar{x}_2(s) = \frac{500}{s(s^2+300)(s^2+100)} = \frac{B_1}{s} + \frac{B_2 s + B_3}{s^2+100} + \frac{B_4 s + B_5}{s^2+300}$$

we can find $A_1, A_2, \ldots, B_1, B_2, \ldots$ (partial fractions method).

This leads to

$$\bar{x}_1(s) = \frac{1}{30 s} - \frac{s}{40(s^2+100)} - \frac{s}{120(s^2+300)} \quad \text{---} (E_9)$$

$$\bar{x}_2(s) = \frac{1}{60 s} - \frac{s}{40(s^2+100)} + \frac{s}{120(s^2+300)} \quad \text{---} (E_{10})$$

Inverse Laplace transforms of (E_9) and (E_{10}) give

$$x_1(t) = \left(\frac{1}{30} - \frac{1}{40}\cos 10t - \frac{1}{120}\cos 10\sqrt{3}\,t\right) u(t)$$

$$x_2(t) = \left(\frac{1}{60} - \frac{1}{40}\cos 10t + \frac{1}{120}\cos 10\sqrt{3}\,t\right) u(t)$$

It is to be noted that $x_1(t) = \frac{1}{30}$ meter and $x_2(t) = \frac{1}{60}$ meter are the static deflections associated with 5 N static force applied to mass m_1. $\omega_1 = 10$ rad/s and $\omega_2 = 10\sqrt{3}$ rad/s are the two resonant frequencies associated with the two degree of freedom system.

5.65 Equivalent mass of cylinder with respect to $x_2 = (m_2)_{eq} = m_2 + \frac{J_0}{r^2}$

Equations of motion:

$$m_1 \ddot{x}_1 = -k(x_1 - x_2)$$

$$(m_2)_{eq}\, \ddot{x}_2 = -k(x_2 - x_1)$$

i.e.

$$m_1 \ddot{x}_1 + k x_1 - k x_2 = 0 \quad \text{----} (E_1)$$

$$\left(m_2 + \frac{J_0}{r^2}\right)\ddot{x}_2 + k x_2 - k x_1 = 0 \quad \text{----} (E_2)$$

Assuming $x_i(t) = X_i \cos(\omega t + \phi)$; $i = 1, 2$, Eqs. (E_1) and (E_2) can be written as

$$(-m_1\omega^2 + k) X_1 - k X_2 = 0$$

$$-k X_1 + \left(-\omega^2\left\{m_2 + \frac{J_0}{r^2}\right\} + k\right) X_2 = 0$$

Frequency equation is:

$$(-m_1\omega^2 + k)\left(-\omega^2 m_2 - \omega^2 \frac{J_0}{r^2} + k\right) - k^2 = 0$$

or

$$\omega^4\left(m_1 m_2 + \frac{m_1 J_0}{r^2}\right) - \omega^2\left(m_1 k + m_2 k + \frac{k J_0}{r^2}\right) = 0$$

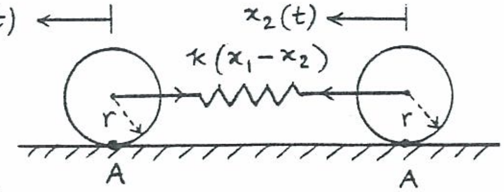
$$\omega_1 = 0, \qquad \omega_2 = \left\{\left(m_1 k + m_2 k + \frac{k J_0}{r^2}\right) \Big/ \left(m_1 m_2 + \frac{m_1 J_0}{r^2}\right)\right\}^{1/2}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.66 Equivalent mass of each cylinder for translatory motion, referred to point A, is



$$m_{eq} = \frac{J_A}{r^2} = \frac{\frac{mr^2}{2} + mr^2}{r^2} = \frac{3m}{2} = (m_1)_{eq} = (m_2)_{eq}$$

Equations of motion:

$$(m_1)_{eq}\,\ddot{x}_1 + k(x_1 - x_2) = 0$$
$$(m_2)_{eq}\,\ddot{x}_2 - k(x_1 - x_2) = 0$$

Frequency equation:

$$\begin{vmatrix} -(m_1)_{eq}\,\omega^2 + k & -k \\ -k & -(m_2)_{eq} + k \end{vmatrix} = 0$$

or $\quad (m_1)_{eq}(m_2)_{eq}\,\omega^4 - \omega^2\left[(m_1)_{eq}k + (m_2)_{eq}k\right] = 0$

$$\omega_1 = 0, \qquad \omega_2 = \sqrt{\frac{(m_1)_{eq}k + (m_2)_{eq}k}{(m_1)_{eq}(m_2)_{eq}}} = \sqrt{\frac{4k}{3m}}$$

5.67 For harmonic motion $x_i(t) = X_i \cos(\omega t + \phi)\,;\ i = 1, 2$ given equations lead to

$$(-\omega^2 a_1 + b_1)X_1 + c_1 X_2 = 0$$
$$b_2 X_1 + (-\omega^2 a_2 + c_2)X_2 = 0$$

Frequency equation is

$$\begin{vmatrix} -\omega^2 a_1 + b_1 & c_1 \\ b_2 & -\omega^2 a_2 + c_2 \end{vmatrix} = 0$$

or $\quad \omega^4(a_1 a_2) - \omega^2(a_1 c_2 + b_1 a_2) + (b_1 c_2 - c_1 b_2) = 0$

Condition for degeneracy is: $\quad b_1 c_2 - c_1 b_2 = 0$

5.68 Equations of motion:

$$J_1\ddot{\theta}_1 + k_t\theta_1 - k_t\theta_2 = 0$$
$$J_2\ddot{\theta}_2 + k_t\theta_2 - k_t\theta_1 = 0$$

For $\theta_i(t) = \Theta_i \cos(\omega t + \phi);\quad i = 1, 2,$

$$\begin{bmatrix} -\omega^2 J_1 + k_t & -k_t \\ -k_t & -\omega^2 J_2 + k_t \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Frequency equation is $\quad \omega^4(J_1 J_2) - \omega^2(J_1 k_t + J_2 k_t) = 0$

$$\omega_1 = 0, \qquad \omega_2 = \sqrt{\frac{k_t(J_1 + J_2)}{J_1 J_2}}$$

Amplitude ratios:

$$r_1 = \frac{\Theta_2^{(1)}}{\Theta_1^{(1)}} = \frac{-\omega_1^2 J_1 + k_t}{k_t} = 1$$

$$r_2 = \frac{\Theta_2^{(2)}}{\Theta_1^{(2)}} = \frac{-\omega_2^2 J_1 + k_t}{k_t} = -\frac{J_1}{J_2}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

General solution is given by equations similar to Eqs. (5.15). With $\dot{\theta}_1(0) = \dot{\theta}_2(0) = 0$, we obtain from equations similar to Eqs. (5.18):

$$\Theta_1^{(1)} = \frac{1}{r_2 - r_1}\left\{r_2\,\theta_1(0) - \theta_2(0)\right\} = -\left(\frac{J_2}{J_1+J_2}\right)\left[-\frac{J_1}{J_2}\,\theta_1(0) - \theta_2(0)\right]$$

$$= \left\{\frac{J_1\,\theta_1(0) + J_2\,\theta_2(0)}{J_1 + J_2}\right\}$$

$$\Theta_1^{(2)} = \frac{1}{r_2 - r_1}\left\{-r_1\,\theta_1(0) + \theta_2(0)\right\} = -\left(\frac{J_2}{J_1+J_2}\right)\left[\frac{J_1}{J_2}\,\theta_1(0) + \theta_2(0)\right]$$

$$= -\left\{\frac{J_1\,\theta_1(0) + J_2\,\theta_2(0)}{J_1 + J_2}\right\}$$

$$\phi_1 = \phi_2 = 0$$

$$\theta_1(t) = \Theta_1^{(1)} \cos 0t + \Theta_1^{(2)} \cos \omega_2 t = \Theta_1^{(1)} + \Theta_1^{(2)} \cos \omega_2 t$$

$$\theta_2(t) = \Theta_1^{(1)} - \frac{J_1}{J_2}\,\Theta_1^{(2)} \cos \omega_2 t$$

5.69 When $k_{t1} = 0$, the system becomes identical to the system of problem 5.68 with $k_t = k_{t1}$. Normal modes are given by

$$\vec{\Theta}^{(1)} = \begin{Bmatrix} \Theta_1^{(1)} \\ \Theta_2^{(1)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \Theta_1^{(1)} \quad ; \quad \vec{\Theta}^{(2)} = \begin{Bmatrix} \Theta_1^{(2)} \\ \Theta_2^{(2)} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -\left(\frac{J_1}{J_2}\right) \end{Bmatrix} \Theta_1^{(2)}$$

Equations of motion can be rewritten as

$$\ddot{\theta}_1 + \frac{k_t}{J_1}(\theta_1 - \theta_2) = 0 \qquad \text{---- } (E_1)$$

$$\ddot{\theta}_2 - \frac{k_t}{J_2}(\theta_1 - \theta_2) = 0 \qquad \text{---- } (E_2)$$

Subtracting (E_2) from (E_1) gives

$$(\ddot{\theta}_1 - \ddot{\theta}_2) + (\theta_1 - \theta_2)\left(\frac{k_t}{J_1} + \frac{k_t}{J_2}\right) = 0 \qquad \text{---- } (E_3)$$

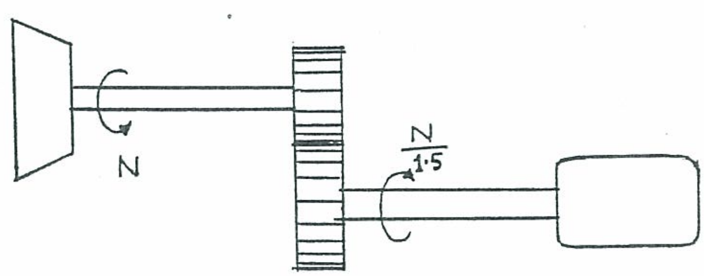
Defining $\alpha = \theta_1 - \theta_2$, (E_3) can be written as

$$\ddot{\alpha} + \left(\frac{k_t}{J_1} + \frac{k_t}{J_2}\right)\alpha = 0 \qquad \text{---- } (E_4)$$

This is a single equation for which the natural frequency is

$$\omega = \sqrt{\left(\frac{k_t}{J_1} + \frac{k_t}{J_2}\right)} = \sqrt{\frac{k_t(J_1 + J_2)}{J_1\,J_2}} \equiv \omega_2 \text{ of problem 5.45.}$$

5.70



PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Since the length of shaft 1 is small and its diameter large, it will be very rigid and hence the turbine and gear 1 are assumed to be rigidly connected. This helps in modeling the system as a two d.o.f. system.

$$J_{01} = J_{turbine} + J_{gear1} + \frac{J_{gear2}}{1.5^2} = 3000 + 500 + (1000/2.25) = 3944.4444 \text{ kg—m}^2$$

$$k_{t2} = \left(\frac{GJ}{\ell}\right)_{shaft2} = \frac{(80\,(10^9))\left(\frac{\pi}{32}\,(0.1^4)\right)}{1} = 7.854\,(10^5)\text{ N/m}$$

$$J_{02} = J_{generator} = 2000 \text{ kg—m}^2$$

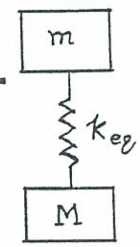
System is a semi-definite system. Its natural frequencies are given by (see Eq. (5.40)):

$$\omega_1 = 0$$

$$\omega_2 = \sqrt{\frac{k_{t2}\,(J_{01} + J_{02})}{J_{01}\,J_{02}}} = \sqrt{\frac{(78.54\,(10^4))\,(5944.4444)}{(3944.4444)\,(2000)}} = 24.3273 \text{ rad/sec}$$

5.71 Natural frequencies are given by Eq. (5.40):

$$\omega_1 = 0 \;;\quad \omega_2 = \sqrt{\frac{k\,(m_1 + m_2)}{m_1\,m_2}} = \sqrt{\frac{6\,k\,(m + M)}{m\,M}}$$



Assumption: Balloon is a point mass.

$$k_{eq} = 12\,k\,\cos^2 45^\circ = 6\,k$$

5.72 Speed of shaft = 6000 rpm = $\frac{6000\,(2\pi)}{60}$ = 628.32 rad/sec

Torsional stiffness (spring constant) of the hollow steel shaft:

$$k_t = \frac{\pi G\,(d_o^4 - d_i^4)}{32\,l} = \frac{\pi\,(11.5 \times 10^6)\,(2^4 - 1^4)}{32\,(15)}$$

$$= 1.1290125 \times 10^6 \text{ lb-in/rad}$$

Natural frequencies of the system, given by an equation similar to Eq. (5.40):

$$\omega_1 = 0, \quad \omega_2 = \left\{\frac{k_t\,(J_1 + J_2)}{J_1\,J_2}\right\}^{\frac{1}{2}} = \left\{\frac{1.1290125 \times 10^6\,(4+2)}{4(2)}\right\}^{\frac{1}{2}}$$

$$= 920.19529 \text{ rad/sec}$$

Second mode shape is given by

$$\alpha = \frac{\Theta_2}{\Theta_1} = \frac{k_{t2} - J_1\,\omega_2^2}{k_{t2}} = \frac{1.1290125 \times 10^6 - 4\,(920.19529)^2}{1.1290125 \times 10^6}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

or $\alpha = -2.0$ (E_1)

Torque transmitted before turbine is stopped (T):

$$T = \frac{63000\ (h.p.)}{\text{speed in rpm}} = \frac{63000\ (100)}{6000} = 1050.0 \text{ lb-in}$$

In view of the fact that $\omega_1 = 0$ corresponds to the same angular motions of the two mass moments of inertia, we have constant angular displacements and constant angular velocities so that the free vibration can be expressed as

$$\theta_1 = c_1 + c_2 t + c_3 \cos \omega_2 t + c_4 \sin \omega_2 t$$

$$\theta_2 = c_1 + c_2 t + c_3 \alpha \cos \omega_2 t + c_4 \alpha \sin \omega_2 t$$

where c_1, c_2, c_3 and c_4 are determined by the initial conditions.

The total initial angular displacement between turbine and generator is given by

$$\phi_0 = \frac{T}{k_t} = \frac{1050.0}{1.1290125 \times 10^6} = 0.00093 \text{ rad} = \Theta_1 - \Theta_2 \quad (E_2)$$

The initial angular displacements at the instant when turbine is suddenly stopped can be found by solving Eqs. (E_1) and (E_2):

$$\Theta_2 = -2\,\Theta_1$$

$$3\,\Theta_1 = 0.00093 \quad \text{or} \quad \Theta_1 = 0.00031 \text{ rad}$$

The initial angular velocities (speeds) at the instant when turbine is stopped are given by

$$\dot{\Theta}_1 = \dot{\Theta}_2 = 628.32 \text{ rad/sec}$$

Using the conditions

$$\theta_1(t=0) = \Theta_1 = 0.00031, \quad \theta_2(t=0) = \Theta_2 = -0.00062,$$

$$\dot{\theta}_1(t=0) = \dot{\Theta}_1 = 628.32, \quad \dot{\theta}_2(t=0) = \dot{\Theta}_2 = 628.32,$$

the constants c_1, c_2, c_3 and c_4 can be determined.

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.76 Equations of motion:

$$J_1 \ddot{\theta}_1 + c_{t1} \dot{\theta}_1 - c_{t2}(\dot{\theta}_2 - \dot{\theta}_1) + k_{t1}\theta_1 - k_{t2}(\theta_2 - \theta_1) = 0$$

$$J_2 \ddot{\theta}_2 + c_{t2}(\dot{\theta}_2 - \dot{\theta}_1) + k_{t2}(\theta_2 - \theta_1) = 0$$

i.e.,

$$\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} c_{t1}+c_{t2} & -c_{t2} \\ -c_{t2} & c_{t2} \end{bmatrix} \begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{Bmatrix} + \begin{bmatrix} k_{t1}+k_{t2} & -k_{t2} \\ -k_{t2} & k_{t2} \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{--- } (E_1)$$

Let $\theta_j(t) = X_j e^{st}$ ----- (E_2)

Eqs. (E_1) yield:

$$\left(\begin{bmatrix} J_1 s^2 & 0 \\ 0 & J_2 s^2 \end{bmatrix} + \begin{bmatrix} (c_{t1}+c_{t2})s & -c_{t2} s \\ -c_{t2} s & c_{t2} s \end{bmatrix} + \begin{bmatrix} (k_{t1}+k_{t2}) & -k_{t2} \\ -k_{t2} & k_{t2} \end{bmatrix} \right) \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} e^{st} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{---- } (E_3)$$

The characteristic equation becomes:

$$\begin{vmatrix} J_1 s^2 + (c_{t1}+c_{t2}) s + k_{t1} + k_{t2} & -(c_{t2} s + k_{t2}) \\ -(c_{t2} s + k_{t2}) & J_2 s^2 + c_{t2} s + k_{t2} \end{vmatrix} = 0$$

i.e.,

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0 \quad \text{--- } (E_4)$$

where

$$a_0 = J_1 J_2$$

$$a_1 = J_1 c_{t2} + J_2 (c_{t1} + c_{t2})$$

$$a_2 = J_1 k_{t2} + c_{t2}(c_{t1} + c_{t2}) + J_2 (k_{t1} + k_{t2}) - c_{t2}^2$$

$$a_3 = k_{t2}(c_{t1} + c_{t2}) + c_{t2}(k_{t1} + k_{t2}) - 2 c_{t2} k_{t2}$$

$$a_4 = k_{t2}(k_{t1} + k_{t2}) - k_{t2}^2$$

For the stability of the system, the conditions derived in section 5.8 are applicable:

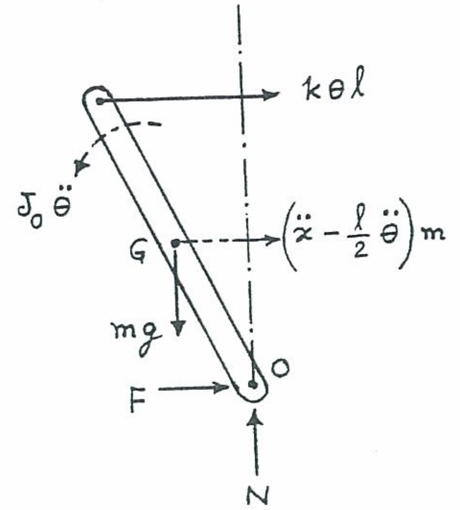
$$a_i > 0 \quad ; \quad i = 0, 1, 2, 3, 4$$

$$a_1 a_2 a_3 - a_1^2 a_4 - a_0 a_3^2 > 0$$

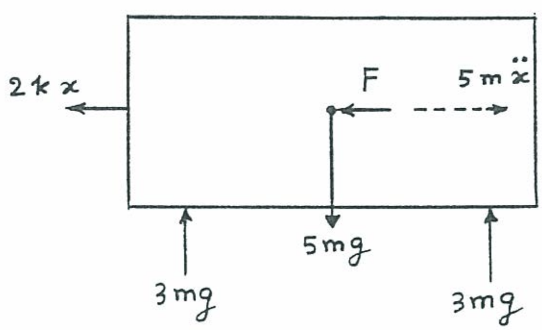
PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.77



Free body diagram of bar



Free body diagram of trailer

Equations of motion of bar:

$$m\,(\ddot{x} - \frac{\ell}{2}\,\ddot{\theta}) = k\,\theta\,\ell + F \tag{1}$$

$$J_0\,\ddot{\theta} - m\,(\ddot{x} - \frac{\ell}{2}\,\ddot{\theta})\,\frac{\ell}{2} = -\,k\,\theta\,\ell\,(\ell) + m\,g\,\frac{\ell}{2}\,\sin\theta \tag{2}$$

Equation of motion of trailer:

$$5\,m\,\ddot{x} = -\,F - 2\,k\,x \quad \text{or} \quad F = -\,2\,k\,x - 5\,m\,\ddot{x} \tag{3}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Equations (1) and (2) can be rewritten as:

$$m\left(\ddot{x} - \frac{\ell}{2}\ddot{\theta}\right) - k\,\theta\,\ell + 2\,k\,x + 5\,m\,\ddot{x} = 0 \tag{5}$$

$$J_0\,\ddot{\theta} - \frac{m\,\ell}{2}\left(\ddot{x} - \frac{\ell}{2}\ddot{\theta}\right) + k\,\theta\,\ell^2 - \frac{m\,g\,\ell\,\theta}{2} = 0 \tag{6}$$

$$\text{where} \quad J_0 = \frac{1}{3}\,m\,\ell^2 \tag{7}$$

Equations (5) and (6) can be expressed in matrix form as:

$$\begin{bmatrix} 6\,m & -\frac{m\,\ell}{2} \\ -\frac{m\,\ell}{2} & \frac{7}{12}\,m\,\ell^2 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 2\,k & -k\,\ell \\ 0 & \left(k\,\ell^2 - \frac{1}{2}\,m\,g\,\ell\right) \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{8}$$

Assuming a solution of the form:

$$x(t) = X\,e^{s\,t} \quad \text{and} \quad \theta(t) = \Theta\,e^{s\,t} \tag{9}$$

Eq. (8) can be expressed as:

$$\left[s^2 \begin{bmatrix} 6\,m & -\frac{m\,\ell}{2} \\ -\frac{m\,\ell}{2} & \frac{m\,\ell^2}{3} \end{bmatrix} + \begin{bmatrix} 2\,k & -k\,\ell \\ 0 & -\left(\frac{m\,g\,\ell}{2} - k\,\ell^2\right) \end{bmatrix} \right] \begin{Bmatrix} X \\ \Theta \end{Bmatrix} e^{s\,t} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{10}$$

By setting the determinant of the coefficient matrix in Eq. (10) equal to zero, we obtain:

$$\begin{vmatrix} (6\,m\,s^2 + 2\,k) & -\left(\frac{m\,\ell\,s^2}{2} + k\,\ell\right) \\ -\left(\frac{m\,\ell\,s^2}{2}\right) & \left(\frac{m\,\ell^2\,s^2}{3} - \frac{m\,g\,\ell}{2} + k\,\ell^2\right) \end{vmatrix} = 0 \tag{11}$$

which, upon expansion, gives:

$$\left(\frac{7}{4}\,m^2\,\ell^2\right)s^4 + \left(\frac{37}{6}\,m\,k\,\ell^2 - 3\,m^2\,g\,\ell\right)s^2 + \left(-\,m\,k\,g\,\ell + 2\,k^2\,\ell^2\right) = 0 \tag{12}$$

A comparison of Eq. (12) with Eq. (5.43) gives:

$$a_0 = \frac{7}{4}\,m^2\,\ell^2$$

$$a_1 = 0$$

$$a_2 = \frac{37}{6}\,m\,k\,\ell^2 - 3\,m^2\,g\,\ell$$

$$a_3 = 0$$

$$a_4 = 2\,k^2\,\ell^2 - m\,k\,g\,\ell$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Conditions for the stability of the system:

1. All coefficients a_i must be positive:

$$a_2 \geq 0 \quad \text{or} \quad k \geq \frac{18}{37}\,\frac{m\,g}{\ell}$$
$$a_4 \geq 0 \quad \text{or} \quad k \geq \frac{1}{2}\,\frac{m\,g}{\ell}$$

2.

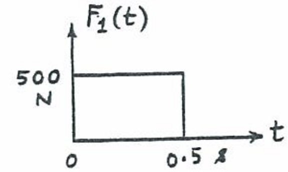
$$a_1\, a_2\, a_3 > a_0\, a_3^2 + a_4\, a_1^2$$

This is not applicable since both sides of the inequality are zero.

Thus the condition for stability is: $k \geq \frac{1}{2}\,\frac{m\,g}{\ell}$.

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.



5.90 Equations of motion are (Eqs. (5.1) and (5.2))

$$2m\ddot{x}_1 + 3k\,x_1 - 2k\,x_2 = F_1(t)$$

$$m\ddot{x}_2 - 2k\,x_1 + 3k\,x_2 = 0$$

Hence $[m] = \begin{bmatrix} 2m & 0 \\ 0 & m \end{bmatrix} = \begin{bmatrix} 20 & 0 \\ 0 & 10 \end{bmatrix}$, $[k] = \begin{bmatrix} 3k & -2k \\ -2k & 3k \end{bmatrix} = \begin{bmatrix} 6000 & -4000 \\ -4000 & 6000 \end{bmatrix}$,

$$\vec{F} = \begin{Bmatrix} F_1(t) \\ 0 \end{Bmatrix}$$

Frequency equation is $\left|-\omega^2[m] + [k]\right| = \omega^4 - 900\,\omega^2 + 100\,000 = 0$

Hence $\omega_1 = 11.3949$ rad/s, $\omega_2 = 27.7517$ rad/s

and $\tau_1 = 0.5514$ s, $\tau_2 = 0.2264$ s.

We select $\Delta t = 0.02$ s and use central difference method for numerical solution (see chapter 11 for details).

The main program which calls CDIFF, the subroutine EXTFUN and the output are given below [CDIFF is in Program 15.F]:

```
C ===================================================================
C
C PROGRAM
C MAIN PROGRAM WHICH CALLS CDIFF
C
C ===================================================================
C FOLLOWING 10 LINES CONTAIN PROBLEM-DEPENDENT DATA
      REAL M(2,2),K(2,2),MC(2,2),MK(2,2),MCI(2,2),MMC(2,2)
      DIMENSION C(2,2),XI(2),XDI(2),XDDI(2),XM1(2),F(2),R(2),RR(2),
     2 XMK(2),XMI(2),XM2(2),XP1(2),ZA(2),ZB(2),ZC(2),LA(2),LB(2,2),
     3 S(2),X(50,2),XD(50,2),XDD(50,2)
      DATA N,NSTEP,NSTEP1,DELT/2,49,50,0.02/
      DATA XI/0.0,0.0/
      DATA XDI/0.0,0.0/
      DATA M/20.0,0.0,0.0,10.0/
      DATA C/0.0,0.0,0.0,0.0/
      DATA K/6000.0,-4000.0,-4000.0,6000.0/
C END OF PROBLEM-DEPENDENT DATA
      CALL CDIFF (M,C,K,XI,XDI,XDDI,N,NSTEP,DELT,F,R,RR,XM1,XM2,XP1,
     2 MC,MK,MCI,XMK,MMC,XMI,ZA,ZB,ZC,LA,LB,S,X,XD,XDD,NSTEP1)
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
      WRITE (13,10)
10    FORMAT (//,38H SOLUTION BY CENTRAL DIFFERENCE METHOD,/)
      WRITE (13,20) N,NSTEP,DELT
20    FORMAT (12H GIVEN DATA:,/,3H N=,I5,4X,7H NSTEP=,I5,4X,6H DELT=,
     2 E15.8,/)
      WRITE (13,30)
30    FORMAT (10H SOLUTION:,//,5H STEP,3X,5H TIME,3X,7H X(I,1),3X,
     2 8H XD(I,1),2X,9H XDD(I,1),4X,7H X(I,2),3X,8H XD(I,2),2X,
     3 9H XDD(I,2),/)
      DO 40, I=1,NSTEP1
      TIME=REAL(I-1)*DELT
40    WRITE (13,50) I,TIME,X(I,1),XD(I,1),XDD(I,1),X(I,2),XD(I,2),XDD(
     2 I,2)
50    FORMAT (1X,I4,F8.4,6(1X,E10.4))
      STOP
      END
C ====================================================================
C
C SUBROUTINE EXTFUN
C THIS SUBROUTINE IS PROBLEM-DEPENDENT
C
C ====================================================================
      SUBROUTINE EXTFUN (F,TIME,N)
      DIMENSION F(N)
      F(1)=0.0
      F(2)=0.0
      IF (TIME .LE. 0.5) F(1)=500.0
      RETURN
      END
```

SOLUTION BY CENTRAL DIFFERENCE METHOD

GIVEN DATA:
N= 2 NSTEP= 49 DELT= 0.20000000E-01

SOLUTION:

STEP	TIME	X(I,1)	XD(I,1)	XDD(I,1)	X(I,2)	XD(I,2)	XDD(I,2)
1	0.0000	0.0000E+00	0.0000E+00	0.2500E+02	0.0000E+00	0.0000E+00	0.0000E+00
2	0.0200	0.5000E-02	0.0000E+00	0.2500E+02	0.0000E+00	0.0000E+00	0.0000E+00
3	0.0400	0.1940E-01	0.4850E+00	0.2350E+02	0.8000E-03	0.2000E-01	0.2000E+01
4	0.0600	0.4154E-01	0.9134E+00	0.1934E+02	0.4512E-02	0.1128E+00	0.7280E+01
5	0.0800	0.6905E-01	0.1241E+01	0.1344E+02	0.1379E-01	0.3247E+00	0.1391E+02
6	0.1000	0.9938E-01	0.1446E+01	0.7043E+01	0.3080E-01	0.6572E+00	0.1935E+02
7	0.1200	0.1302E+00	0.1530E+01	0.1347E+01	0.5632E-01	0.1063E+01	0.2127E+02
8	0.1400	0.1600E+00	0.1515E+01	-.2810E+01	0.8917E-01	0.1459E+01	0.1831E+02
9	0.1600	0.1877E+00	0.1436E+01	-.5164E+01	0.1262E+00	0.1747E+01	0.1050E+02
10	0.1800	0.2129E+00	0.1323E+01	-.6059E+01	0.1630E+00	0.1846E+01	-.6577E+00
⋮							
46	0.9000	0.2345E+00	-.1023E+01	-.8737E+01	0.1991E+00	-.5925E+00	-.2714E+02
47	0.9200	0.2101E+00	-.1165E+01	-.5533E+01	0.1715E+00	-.1120E+01	-.2565E+02
48	0.9400	0.1842E+00	-.1258E+01	-.3717E+01	0.1365E+00	-.1566E+01	-.1889E+02
49	0.9600	0.1571E+00	-.1325E+01	-.2966E+01	0.9808E-01	-.1837E+01	-.8195E+01
50	0.9800	0.1290E+00	-.1379E+01	-.2515E+01	0.6131E-01	-.1879E+01	0.3993E+01

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.91 (a) Frequency equation is

$$\left|-\omega^2[m]+[k]\right| = \left|-\omega^2\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix}+\begin{bmatrix}36 & -18\\ -18 & 18\end{bmatrix}\right| = \omega^4 - 270\,\omega^2 + 8100 = 0$$

```
C ==================================================================
C
C PROGRAM 6.F
C MAIN PROGRAM FOR CALLING THE SUBROUTINE QUART
C
C ==================================================================
C SOLUTION OF:  A(1)*(X**4)+A(2)*(X**3)+A(3)*(X**2)+A(4)*X+A(5)=0
      DIMENSION A(5),RR(4),RI(4)
C FOLLOWING LINE CONTAINS PROBLEM-DEPENDENT DATA
      DATA A/1.0,0.0,-270.0,0.0,8100.0/
C END OF PROBLEM-DEPENDENT DATA
      WRITE (26,10) (A(I),I=1,5)
 10   FORMAT (//,31H SOLUTION OF A QUARTIC EQUATION,//,6H DATA:,/,
     2 7H A(1) =,E15.6,/,7H A(2) =,E15.6,/,7H A(3) =,E15.6,/,
     3 7H A(4) =,E15.6,/,7H A(5) =,E15.6,/)
      CALL QUART (A,RR,RI)
      WRITE (26,20)
 20   FORMAT (/,7H ROOTS:,//,9H ROOT NO.,3X,10H REAL PART,5X,
     2 15H IMAGINARY PART,/)
      DO 30 I=1,4
 30   WRITE (26,40) I,RR(I),RI(I)
 40   FORMAT (I5,3X,E15.6,3X,E15.6)
      STOP
      END
```

```
SOLUTION OF A QUARTIC EQUATION

DATA:
A(1) =    0.100000E+01
A(2) =    0.000000E+00
A(3) =   -0.270000E+03
A(4) =    0.000000E+00
A(5) =    0.810000E+04

ROOTS:

ROOT NO.     REAL PART       IMAGINARY PART

    1      -0.153500E+02      0.000000E+00
    2      -0.586319E+01      0.000000E+00
    3       0.586319E+01      0.000000E+00
    4       0.153500E+02      0.000000E+00
```

(b) $\left(-0.2\,\omega_j^2+36\right) X_j^{(1)} - 18\, X_j^{(2)} = 0 \quad ; \quad j=1,2$

Writing $\quad X_j^{(2)} = \left(\dfrac{-0.2\,\omega_j^2+36}{18}\right) X_j^{(1)} \equiv r_j\, X_j^{(1)}$,

$$r_1 = \left\{-0.2\,(5.86319)^2+36\right\}/18 = 1.6180334$$

$$r_2 = \left\{-0.2\,(15.35)^2+36\right\}/18 = -0.6180278$$

Eqs. (5.18) give $\quad X_1^{(1)} = 1.44722$, $\quad X_1^{(2)} = 0.55279$, $\phi_1 = \phi_2 = 0$

Displacements of masses m_1 and m_2 are given by Eqs. (5.15):

$$x_1(t) = 1.44722 \cos(5.86319\,t) + 0.55279 \cos(15.35\,t)$$

$$x_2(t) = 2.34165 \cos(5.86319\,t) - 0.34164 \cos(15.35\,t)$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.92

```
C ======================================================================
C
C PROBLEM 5.92
C
C ======================================================================
      DIMENSION C(2,2),FZ(2)
      REAL K(2,2),M(2,2)
      COMPLEX Z(2,2),X(2),AA,BB,DEN
C INPUT DATA
      DATA M/0.1,0.0,0.0,0.1/
      DATA C/1.0,0.0,0.0,0.0/
      DATA K/40.0,-20.0,-20.0,20.0/
      DATA FZ/1.0,2.0/
      OMF=5.0
C END OF INPUT DATA
      DO 10 I=1,2
      DO 10 J=1,2
      A=-(OMF**2)*M(I,J)+K(I,J)
      B=OMF*C(I,J)
 10   Z(I,J)=CMPLX(A,B)
      DEN=Z(1,1)*Z(2,2)-Z(1,2)*Z(1,2)
      AA=Z(2,2)*CMPLX(FZ(1),0.0)
      BB=Z(1,2)*CMPLX(FZ(2),0.0)
      X(1)=(AA-BB)/DEN
      AA=Z(1,1)*CMPLX(FZ(2),0.0)
      BB=Z(1,2)*CMPLX(FZ(1),0.0)
      X(2)=(AA-BB)/DEN
      PRINT 20, X(1),X(2)
 20   FORMAT (//,2X,25H SOLUTION OF PROBLEM 5.49,//,2X,
     2 7H X(1) =,2E15.8,/,2X,7H X(2) =,2E15.8,/)
      PRINT 30, OMF
 30   FORMAT (2X,6H OMF =,E15.8)
      STOP
      END
```

```
 SOLUTION OF PROBLEM 5.92

 X(1) = 0.20095898E+00-0.68620138E-01
 X(2) = 0.34395310E+00-0.78423016E-01

 OMF = 0.50000000E+01
```


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.93 Free vibration response of the system shown in Fig. 5.24:

$k_1 = 1000,\ k_2 = 500,\ m_1 = 2,\ m_2 = 1,$

$x_1(0) = 1,\ x_2(0) = 0,\ \dot{x}_1(0) = -1,\ \dot{x}_2(0) = 0$

$$\omega_1^2,\ \omega_2^2 = \frac{k_1+k_2}{2m_1} + \frac{k_2}{2m_2} \mp \left\{ \frac{1}{4}\left(\frac{k_1+k_2}{m_1} + \frac{k_2}{m_2}\right)^2 - \frac{k_1 k_2}{m_1 m_2} \right\}^{\frac{1}{2}}$$

$$= \frac{1500}{4} + \frac{500}{2} \mp \left\{ \frac{1}{4}\left(\frac{1500}{2} + \frac{500}{1}\right)^2 - \frac{5\times10^5}{2} \right\}^{\frac{1}{2}}$$

$$= 250;\ 1000$$

$$\omega_1 = 15.8114 \text{ rad/s},\quad \omega_2 = 31.6228 \text{ rad/s} \qquad (E_1)$$

$$r_1 = \frac{X_2^{(1)}}{X_1^{(1)}} = \frac{k_2}{-m_2\omega_1^2 + k_2} = \frac{500}{-1(250)+500} = 2$$

$$r_2 = \frac{X_2^{(2)}}{X_1^{(2)}} = \frac{k_2}{-m_2\omega_2^2 + k_2} = \frac{500}{-1(1000)+500} = -1 \qquad (E_2)$$

$$x_1(t) = X_1^{(1)} \cos(\omega_1 t + \phi_1) + X_1^{(2)} \cos(\omega_2 t + \phi_2)$$

$$x_2(t) = r_1 X_1^{(1)} \cos(\omega_1 t + \phi_1) + r_2 X_1^{(2)} \cos(\omega_2 t + \phi_2) \qquad (E_3)$$

Initial conditions yield:

$$x_1(0) = 1 = X_1^{(1)} \cos(15.8114\, t + \phi_1) + X_1^{(2)} \cos(31.6228\, t + \phi_2)$$

$$x_2(0) = 0 = 2 X_1^{(1)} \cos(15.8114\, t + \phi_1) - X_1^{(2)} \cos(31.6228\, t + \phi_2)$$

$$\dot{x}_1(0) = -1 = -\omega_1 X_1^{(1)} \sin(15.8114\, t + \phi_1) - \omega_2 X_1^{(2)} \sin(31.6228\, t + \phi_2)$$

$$\dot{x}_2(0) = 0 = -r_1\omega_1 X_1^{(1)} \sin(15.8114\, t + \phi_1) - r_2\omega_2 X_1^{(2)} \sin(31.6228\, t + \phi_2) \qquad (E_4)$$

or

$$X_1^{(1)} \cos\phi_1 + X_1^{(2)} \cos\phi_2 = 1 \qquad (E_5)$$

$$2 X_1^{(1)} \cos\phi_1 - X_1^{(2)} \cos\phi_2 = 0 \qquad (E_6)$$

$$-15.8114\, X_1^{(1)} \sin\phi_1 - 31.6228\, X_1^{(2)} \sin\phi_2 = -1 \qquad (E_7)$$

$$-31.6228\, X_1^{(1)} \sin\phi_1 + 31.6228\, X_1^{(2)} \sin\phi_2 = 0 \qquad (E_8)$$

Solution of Eqs. (E_5) and (E_6):

$$X_1^{(1)} \cos\phi_1 = \frac{1}{3} \qquad (E_9)$$

$$X_1^{(2)} \cos\phi_2 = \frac{2}{3} \qquad (E_{10})$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Solution of Eqs. (E_7) and (E_8):

$$X_1^{(1)} \sin \phi_1 = 0.02108 \qquad (E_{11})$$

$$X_1^{(2)} \sin \phi_2 = 0.02108 \qquad (E_{12})$$

Eqs. (E_9) and (E_{11}) yield: $X_1^{(1)} = 0.334$, $\phi_1 = 0.06316$ rad

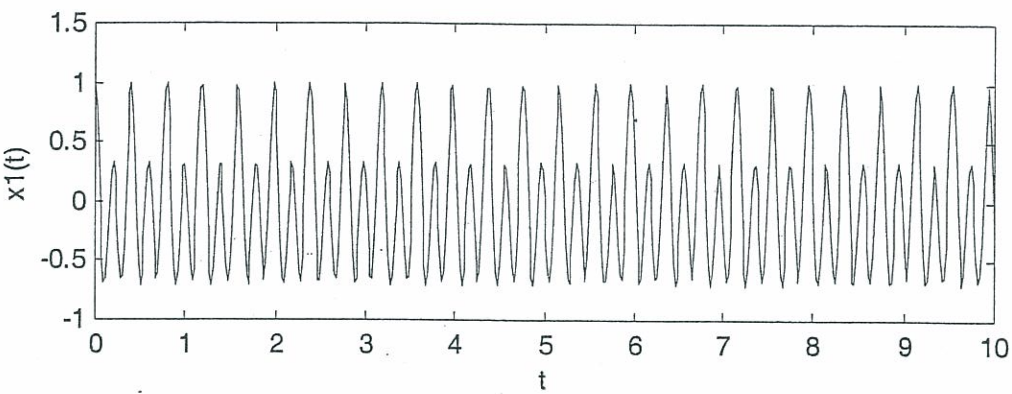
Eqs. (E_{10}) and (E_{12}) yield: $X_1^{(2)} = 0.667$, $\phi_2 = 0.03161$ rad

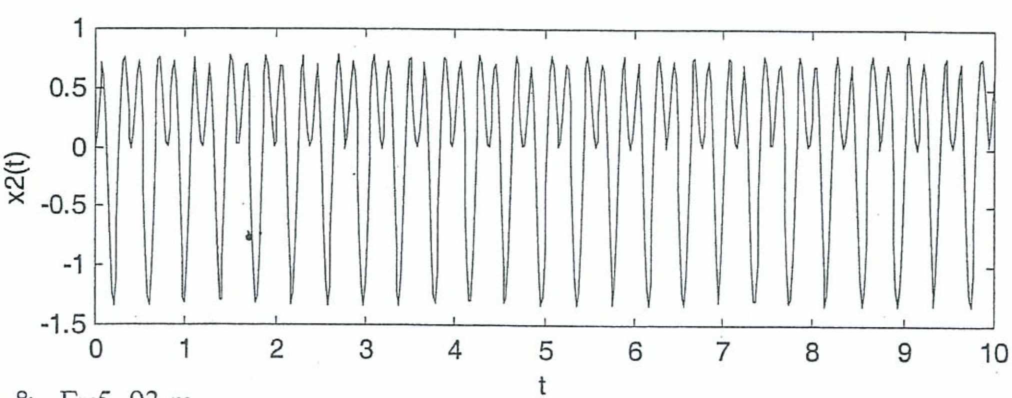
Response:

$$x_1(t) = 0.334 \cos(15.8114\, t + 0.06316) + 0.667 \cos(31.6228\, t + 0.03161) \qquad (E_{13})$$

$$x_2(t) = 0.668 \cos(15.8114\, t + 0.06316) - 0.667 \cos(31.6228\, t + 0.03161) \qquad (E_{14})$$

Plotting of Eqs. (E_{13}) and (E_{14}):






```
% Ex5_93.m
for i = 1: 501
   t(i) = 10 * (i-1)/500;
   x1(i) = 0.334 * cos(15.8114*t(i) + 0.06316)...
      + 0.667 * cos(31.6228*t(i) + 0.03161);
   x2(i) = 0.668 * cos(15.8114*t(i) + 0.06316)...
      - 0.667 * cos(31.6228*t(i) + 0.03161);
```


لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
end
subplot(211);
plot(t,x1);
xlabel('t');
ylabel('x1(t)')
subplot(212);
plot(t,x2);
xlabel('t');
ylabel('x2(t)')
```

5.94 For the initial conditions

$x_1(0) = 1$, $x_2(0) = 2$, $\dot{x}_1(0) = 1$ and $\dot{x}_2(0) = -2$,

Eqs. (E_3) of solution of Problem 5.93 yield

$$x_1(0) = 1 = X_1^{(1)} \cos\phi_1 + X_1^{(2)} \cos\phi_2 \qquad (E_1)$$

$$x_2(0) = 2 = 2\,X_1^{(1)} \cos\phi_1 - X_1^{(2)} \cos\phi_2 \qquad (E_2)$$

$$\dot{x}_1(0) = 1 = -15.8114\,X_1^{(1)} \sin\phi_1 - 31.6228\,X_1^{(2)} \sin\phi_2 \qquad (E_3)$$

$$\dot{x}_2(0) = -2 = -31.6228\,X_1^{(1)} \sin\phi_1 + 31.6228\,X_1^{(2)} \sin\phi_2 \qquad (E_4)$$

Eqs. (E_1) and (E_2) give:

$$X_1^{(1)} \cos\phi_1 = 1, \quad X_1^{(2)} \cos\phi_2 = 0 \qquad (E_5)$$

Eqs. (E_3) and (E_4) yield

$$X_1^{(1)} \sin\phi_1 = 0.02108, \quad X_1^{(2)} \sin\phi_2 = -0.04216 \qquad (E_6)$$

Equations (E_5) and (E_6) can be used to obtain

$$X_1^{(1)} = 1.000222, \quad \phi_1 = 0.02108 \text{ rad}$$

$$X_1^{(2)} = 0.04216, \quad \phi_2 = \frac{\pi}{2} \text{ rad}$$

Response of the system:

$$x_1(t) = 1.000222 \cos(15.8114\,t + 0.02108) + 0.04216 \cos\left(31.6228\,t + \frac{\pi}{2}\right) \qquad (E_7)$$

$$x_2(t) = 2.000444 \cos(15.8114\,t + 0.02108) - 0.04216 \cos\left(31.6228\,t + \frac{\pi}{2}\right) \qquad (E_8)$$

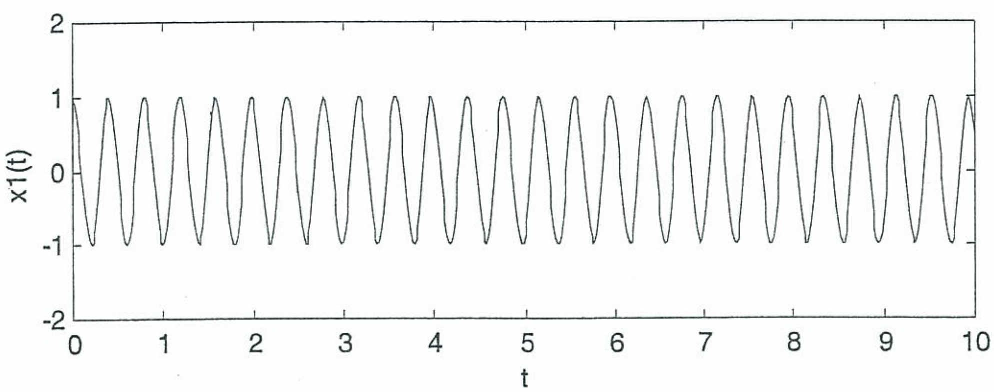
Plotting of Eqs. (E_7) and (E_8):

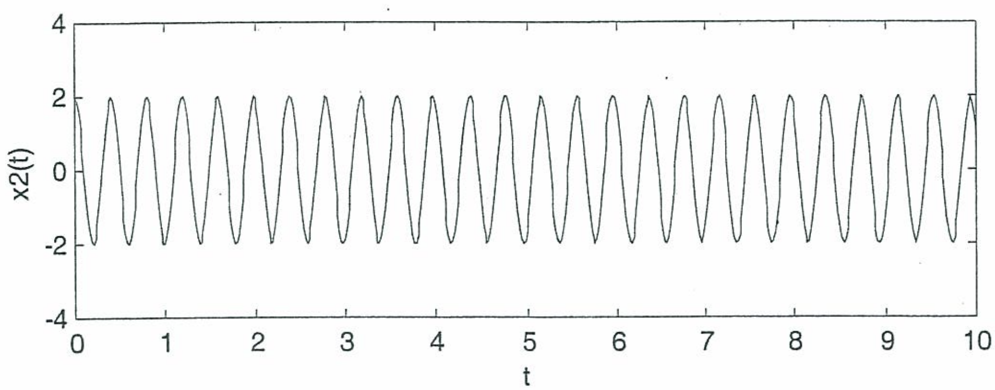
PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
% Ex5_94.m
for i = 1: 501
   t(i) = 10 * (i-1)/500;
   x1(i) = 1.000222 * cos(15.8114*t(i) + 0.02108)...
      + 0.04216 * cos(31.6228*t(i) + pi/2);
   x2(i) = 2.000444 * cos(15.8114*t(i) + 0.02108)...
      - 0.04216 * cos(31.6228*t(i) + pi/2);
end
subplot(211);
plot(t,x1);
xlabel('t');
ylabel('x1(t)')
subplot(212);
plot(t,x2);
xlabel('t');
ylabel('x2(t)')
```





PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.95

```
% Ex5_95.m
>>A = 1e6*[25 -5; -5 5]
A =
    25000000    -5000000
    -5000000     5000000
>>B = [10000 0; 0 5000]
B =
       10000           0
           0        5000
>>[V, D] = eig(A, B)
V =
    0.8719    0.2703
   -0.4896    0.9628
D =
  1.0e+003 *
    2.7808         0
         0    0.7192
```

5.96 Differential equations:

$$2\ddot{x}_1 + 20\dot{x}_1 - 5\dot{x}_2 + 50x_1 - 10x_2 = 2\sin 3t \qquad (E_1)$$

$$10\ddot{x}_2 - 5\dot{x}_1 + 5\dot{x}_2 - 10x_1 + 10x_2 = 5\cos 5t \qquad (E_2)$$

Let $y_1 = x_1$

$\dot{y}_1 = y_2 = \dot{x}_1$

$y_3 = x_2$

$\dot{y}_3 = y_4 = \dot{x}_2$

Equations (E_1) and (E_2) can be rewritten as

$$2\dot{y}_2 + 20y_2 - 5y_4 + 50y_1 - 10y_3 = 2\sin 3t$$

$$10\dot{y}_4 - 5y_2 + 5y_4 - 10y_1 + 10y_3 = 5\cos 5t$$

or

$$\frac{d}{dt}\begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} y_2 \\ -10y_2 + 2.5y_4 - 25y_1 + 5y_3 + \sin 3t \\ y_4 \\ 0.5y_2 - 0.5y_4 + y_1 - y_3 + 0.5\cos 5t \end{Bmatrix} \qquad (E_3)$$

or $$\dot{\vec{y}} = \vec{f} \qquad (E_4)$$

with $\vec{y} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix}$, $\vec{y}(0) = \begin{Bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{Bmatrix}$

PRELIMINARY VERSION 12/14/2010

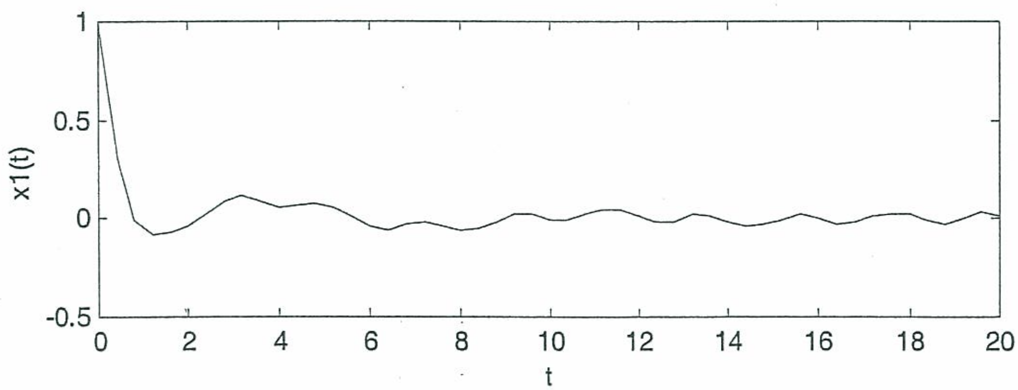
Mech.MuslimEngineer.Net

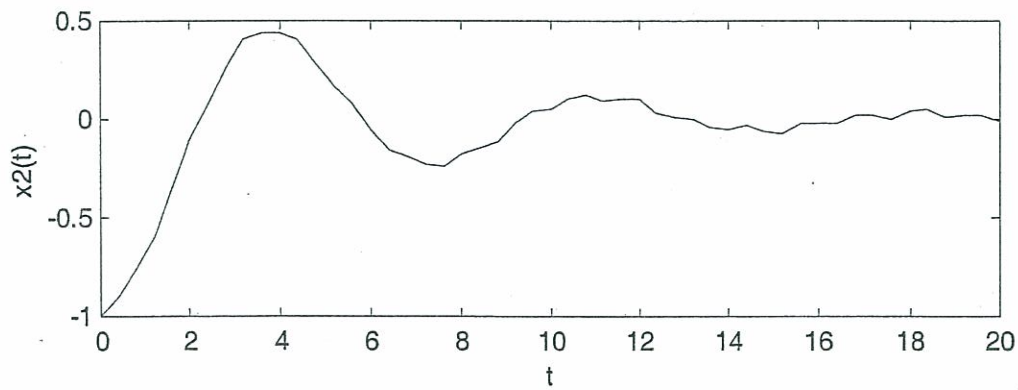
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

and $\vec{f}$ is given by the right hand side of Eq. (E_3).

Solution of Eq. (E_4) using MATLAB:

```
% Ex5_96.m
% This program will use the function dfun5_96.m , they should
% be in the same folder
tspan = [0: 0.4: 20];
y0 = [1; 0; -1; 0];
[t,y] = ode23('dfun5_96', tspan, y0);
disp('        t        x1(t)     xd1(t)     x2(t)      xd2(t)');
disp([t y]);
subplot(211);
plot(t,y(:,1));
xlabel('t');
ylabel('x1(t)');
subplot(212);
plot(t,y(:,3));
xlabel('t');
ylabel('x2(t)');

% dfun5_96.m
function f = dfun5_96 (t,y)
f = zeros(4,1);
f(1) = y(2);
f(2) = -10*y(2) + 2.5*y(4) - 25*y(1) + 5*y(3) + sin(3*t);
f(3) = y(4);
f(4) = 0.5*y(2) - 0.5*y(4) + y(1) - y(3) + 0.5*cos(5*t);
```





PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
Results of  Ex5_96
**********************************
>>Ex5_96
       t         x1(t)      xd1(t)      x2(t)       xd2(t)
       0         1.0000          0     -1.0000           0
  0.4000         0.3177    -1.4828     -0.8995      0.3577
  0.8000        -0.0076    -0.3482     -0.7604      0.3375
  1.2000        -0.0763    -0.0594     -0.5974      0.5230
  1.6000        -0.0741     0.0612     -0.3445      0.6835
  2.0000        -0.0356     0.1222     -0.1033      0.4929
  2.4000         0.0214     0.1625      0.0716      0.4371
  .
  .
  .
 18.8000        -0.0268     0.0072      0.0066     -0.0481
 19.2000         0.0010     0.1087      0.0196      0.0720
 19.6000         0.0331     0.0247      0.0233     -0.0721
 20.0000         0.0178    -0.0827     -0.0117     -0.0459
```

5.97 Equations:

$$2m\ddot{x}_1 + 3k\,x_1 - 2k\,x_2 = F_1(t)$$
$$m\ddot{x}_2 - 2k\,x_1 + 3k\,x_2 = 0$$

i.e.,

$$\ddot{x}_1 = -300\,x_1 + 200\,x_2 + \frac{1}{20}F_1(t)$$
$$\ddot{x}_2 = 400\,x_1 - 600\,x_2$$

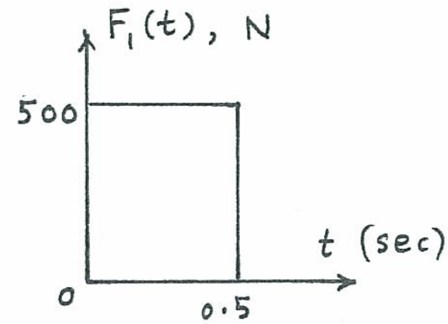
Let

$$\vec{y} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{Bmatrix} \quad \text{and} \quad \vec{y}(0) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \text{zero initial conditions assumed}$$

Then equations to be solved are:

$$\frac{d\vec{y}}{dt} = \begin{Bmatrix} y_2 \\ -300\,y_1 + 200\,y_3 + \frac{1}{20}F_1(t) \\ y_4 \\ 400\,y_1 - 600\,y_3 \end{Bmatrix} \qquad (E_1)$$

with $F_1(t)$ shown in the figure:



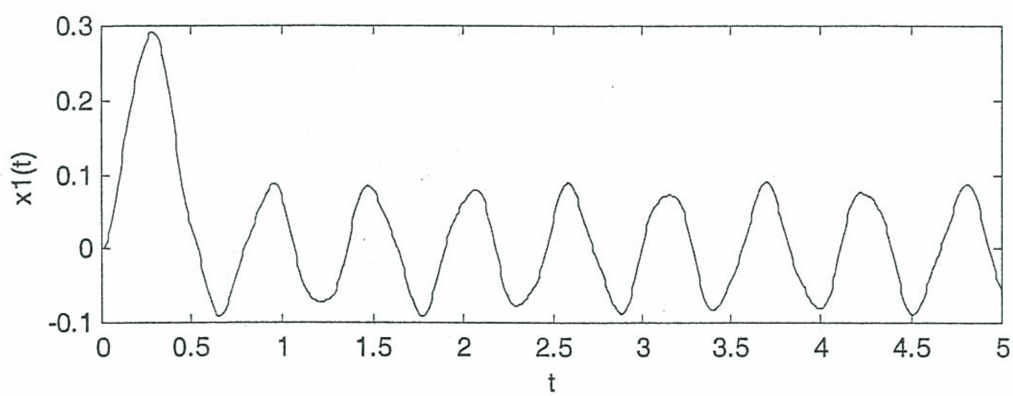
MATLAB solution of Eq. (E_1):

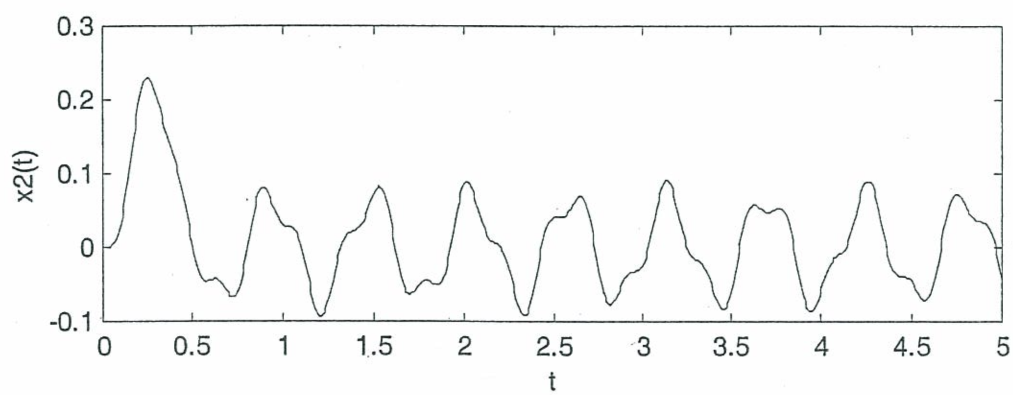
PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
% Ex5_97.m
% This program will use the function dfun5_97 .m, they should
% be in the same folder
tspan = [0: 0.01: 5];
y0 = [0; 0; 0; 0];
[t,y] = ode23('dfun5_97', tspan, y0);
subplot(211);
plot(t,y(:,1));
xlabel('t');
ylabel('x1(t)');
subplot(212);
plot(t,y(:,3));
xlabel('t');
ylabel('x2(t)');
```

```
% dfun5_97.m
function f = dfun5_97(t,y)
F1 = 500 * stepfun(t, 0.0) - 500 * stepfun(t, 0.5);
f = zeros(4,1);
f(1) = y(2);
f(2) = -300*y(1) + 200*y(3) + F1/20;
f(3) = y(4);
f(4) = 400*y(1) - 600*y(3);
```





PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.98 Frequency equation, Eq. (5.9):

$$m_1 m_2 \omega^4 - \{(k_1+k_2) m_2 + (k_2+k_3) m_1\} \omega^2 + \{(k_1+k_2)(k_2+k_3) - k_2^2\} = 0 \qquad (E_1)$$

With $m_1 = m_2 = 0.2$, $k_1 = k_2 = 18$ and $k_3 = 0$, Eq. (E_1) becomes

$$0.04\,\omega^4 - 10.8\,\omega^2 + 324 = 0$$

or $$\omega^4 - 270\,\omega^2 + 8100 = 0 \qquad (E_2)$$

Solution of Eq. (E_2) using MATLAB:

```
% Ex5_98.m
>>roots([1 0 -270 0 8100])
ans =
  15.35001820805078
 -15.35001820805078
   5.86318522754564
  -5.86318522754564
```

5.99
$$\begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{22} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} \\ k_{12} & k_{22} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (E_1)$$

$$F_j(t) = F_{j0}\, e^{i\omega t} \;;\; j = 1,2 \;;\; i = \sqrt{-1} \qquad (E_2)$$

$$x_j(t) = X_j\, e^{i\omega t} \;;\; j = 1,2 \qquad (E_3)$$

Eqs. (5.35):

$$X_1(i\omega) = \frac{Z_{22}(i\omega)\, F_{10} - Z_{12}(i\omega)\, F_{20}}{Z_{11}(i\omega)\, Z_{22}(i\omega) - Z_{12}^2(i\omega)} \qquad (E_4)$$

$$X_2(i\omega) = \frac{-Z_{12}(i\omega)\, F_{10} + Z_{11}(i\omega)\, F_{20}}{Z_{11}(i\omega)\, Z_{22}(i\omega) - Z_{12}^2(i\omega)} \qquad (E_5)$$

where
$$Z_{rs}(i\omega) = -\omega^2 m_{rs} + i\omega c_{rs} + k_{rs} \;;\; r, s = 1,2 \qquad (E_6)$$

Data: $m_{11} = m_{22} = 0.1$, $m_{12} = 0$, $c_{11} = 1.0$, $c_{12} = c_{22} = 0$,

$k_{11} = 40$, $k_{22} = 20$, $k_{12} = -20$, $F_{10} = 1$, $F_{20} = 2$,

$\omega = 5$

Hence $Z_{11}(i\omega) = 37.5 + 5i$, $Z_{12}(i\omega) = -20$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

and $Z_{22}(i\omega) = 17.5$

Solution using MATLAB:

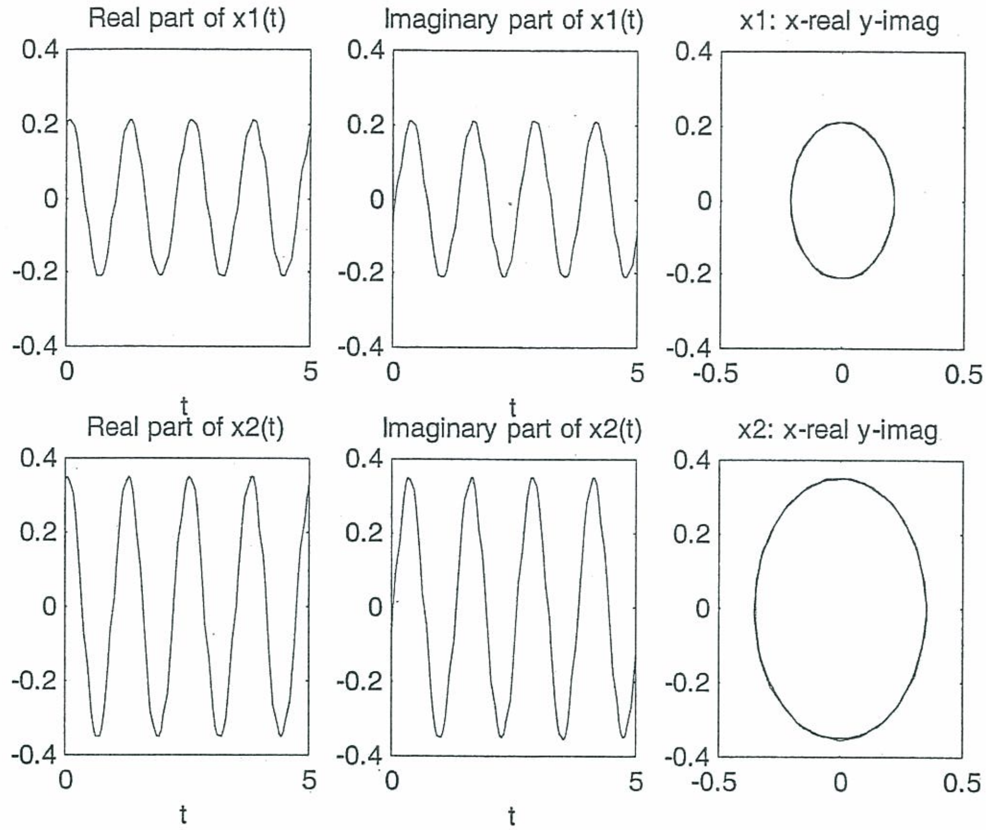
(Real and Imaginary parts of $x_1(t)$ and $x_2(t)$ given by Eq. (E_3))

```
% Ex5_99.m
m11 = 0.1;
m22 = 0.1;
m12 = 0;
c11 = 1.0;
c12 = 0;
c22 = 0;
k11 = 40;
k22 = 20;
k12 = -20;
F10 = 1;
F20 = 2;
w = 5;
z11 = complex((-w^2*m11 + k11), w*c11);
z12 = complex((-w^2*m12 + k12), w*c12);
z22 = complex((-w^2*m22 + k22), w*c22);
X1 = (z22*F10 - z12*F20)/(z11*z22 - z12*z12)
X2 = (-z12*F10 + z11*F20)/(z11*z22 - z12*z12)
for i = 1: 101
   t(i) = 5*(i-1)/100;
   x1(i) = X1 * exp(complex(0, w*t(i)));
   x2(i) = X2 * exp(complex(0, w*t(i)));
end
subplot(231);
plot(t, real(x1));
xlabel('t');
title('Real part of x1(t)');
subplot(232);
plot(t, imag(x1));
xlabel('t');
title('Imaginary part of x1(t)');
subplot(233);
plot(real(x1), imag(x1));
title('x1: x-real y-imag');
subplot(234);
plot(t, real(x2));
xlabel('t');
title('Real part of x2(t)');
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
subplot(235);
plot(t, imag(x2));
xlabel('t');
title('Imaginary part of x2(t)');
subplot(236);
plot(real(x2), imag(x2));
title('x2: x-real y-imag');
```



5.100 Roots of the equation:

$$x^4 - 32x^3 + 244x^2 - 20x - 1200 = 0$$

Using MATLAB:

```
% Ex5_100.m
>>roots([1 -32 244 -20 -1200])
ans =
  20.00000000000001
  11.15980239097340
   2.77656274263302
  -1.93636513360642
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

5.101 The system shown in Fig. A can be drawn in equivalent form as shown in Fig. B where both pulleys have the same radius, r_1.

The equivalent mass moment of inertia of pulley 2 can be computed in different speed ratios as:

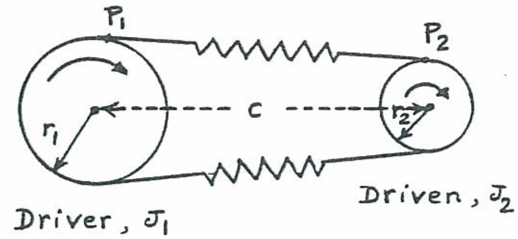


Fig. A

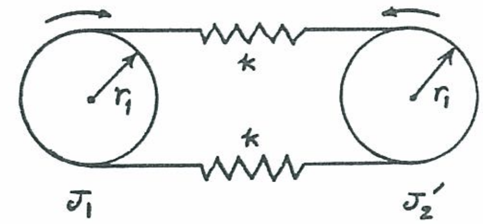


Fig. B

$$J_2' = J_2 \, (\text{speed ratio})^2$$

$$= J_2 \left(\frac{150}{350}\right)^2 ; \; J_2 \left(\frac{250}{350}\right)^2 ;$$

$$J_2 \left(\frac{450}{350}\right)^2 ; \; J_2 \left(\frac{750}{350}\right)^2$$

or

$$J_2' = 0.1837 \, J_2 \; ; \; 0.5102 \, J_2 \; ; \; 1.6531 \, J_2 \; ; \; 4.5918 \, J_2$$

Stiffness of the belt (on each side) is given by

$$k = \frac{A E}{l}$$

where A = cross-sectional area of belt, E = Young's modulus and l = length of the belt. Length of the belt (distance $P_1 P_2$ in Fig. A) is given by

$$l = \frac{1}{2} \left[4 c^2 - (D - d)^2 \right]^{\frac{1}{2}}$$

In this example, $c = 5$ m, $D = 1$ m, $d = 0.25$ m and hence

$$l = \frac{1}{2} \left[4 (5)^2 - (1 - 0.25)^2 \right]^{1/2} = 4.9859 \text{ m}$$

$$\therefore \; k = \frac{A \, (10^{10})}{4.9859} = 2.0057 \times 10^9 \, A \;\; \text{N/m}$$

Equation of motion:

$$J_1 \ddot{\theta}_1 + k_t (\theta_1 + \theta_2) = 0 \;\Rightarrow\; J_1 \ddot{\theta}_1 + k_t \theta_1 \left(1 + \frac{J_1}{J_2'}\right) = 0$$

$$J_2 \ddot{\theta}_2 + k_t (\theta_1 + \theta_2) = 0 \;\Rightarrow\; J_2 \ddot{\theta}_2 + k_t \theta_2 \left(\frac{J_2'}{J_1} + 1\right) = 0$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\therefore \quad \omega_n = \sqrt{k_t \left(\frac{J_1 + J_2'}{J_1 J_2'} \right)}$$

where $k_t = 2 k r_1^2$ (see solution of problem 5.68).

Here $J_1 = 0.1$ kg-m^2 and $J_2 = 0.2$ kg-m^2. In order for the natural frequency ω_n to be away from the speeds 150, 250, 350, 450 and 750 rpm {or, 15.708, 26.180, 36.652, 47.124 and 78.540 rad/sec},

$$\omega_n \le 15.708 \text{ rad/sec}$$

$$\omega_n \ge 78.540 \text{ rad/sec}$$

Since ω_n involves A (through k_t), it can be determined from the above inequalities.

5.102 Velocity of tup before impact is given by:

$$\frac{1}{2} m_{tup} v^2 = m_{tup} g h \quad \text{or} \quad v = \sqrt{2 g h} = \sqrt{2 (9.81)(2)} = 6.2642 \text{ m/sec}$$

(a) Impact is inelastic:

Conservation of momentum leads to:

$$m_{tup} v_{tup} + m_{anvil} (0) = (m_{tup} + m_{anvil}) v_0$$

$$\text{or} \quad v_0 = \frac{(1000)(6.2642)}{(1000 + 5000)} = 1.0440 \text{ m/sec}$$

(b) Natural frequencies:

$$\omega_{2,1}^2 = \frac{k_1 + k_2}{2 m_1} + \frac{k_2}{2 m_2} \pm \sqrt{\frac{1}{4} \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right)^2 - \frac{k_1 k_2}{m_1 m_2}}$$

Thus the natural frequency requirement can be stated as:

$$f_1^2 = \frac{\omega_1^2}{(2\pi)^2}$$

$$= \frac{1}{(2\pi)^2} \left\{ \frac{k_1 + k_2}{50000} + \frac{k_2}{10000} - \sqrt{\frac{1}{4} \left(\frac{k_1 + k_2}{25000} + \frac{k_2}{5000} \right)^2 - \frac{k_1 k_2}{125 (10^6)}} \right\} > (5^2) \tag{1}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(c) Free vibration response:

Initial conditions:

$$x_1(0) = x_2(0) = \dot{x}_1(0) = 0, \quad \dot{x}_2(0) = v_0 = 1.0440 \text{ m/sec}$$

Maximum forces in the springs:

$$F_1 = k_1 \, x_1 \,|_{max} \tag{2}$$

$$F_2 = k_2 \, (x_2 - x_1) \,|_{max} \tag{3}$$

For a helical spring, the shear stress (τ) under an axial force F is given by:

$$\tau = k_s \, \frac{8 \, F \, D}{\pi \, d^3} \tag{4}$$

where k_s = shear stress correction factor = $\frac{2D + d}{2\,D}$, D = mean coil diameter, and d = wire diameter.

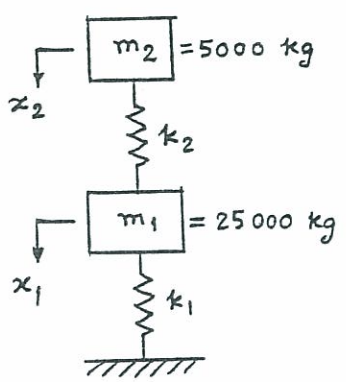
Ref: J. E. Shigley and C. R. Mischke, "Mechanical Engineering Design," 5th Ed., McGraw-Hill, New York, 1989.

Since stress is to be less than the yield stress witth a factor of safety of 1.5, we have

$$\tau_1 \le \frac{\tau_{yield}}{1.5} \tag{5}$$

$$\tau_2 \le \frac{\tau_{yield}}{1.5} \tag{6}$$

where τ_1 and τ_2 denote the shear stresses induced in the springs k_1 and k_2, respectively, and τ_{yield} is the shear stress corresponding to the yield stress of the material.



PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Chapter 6

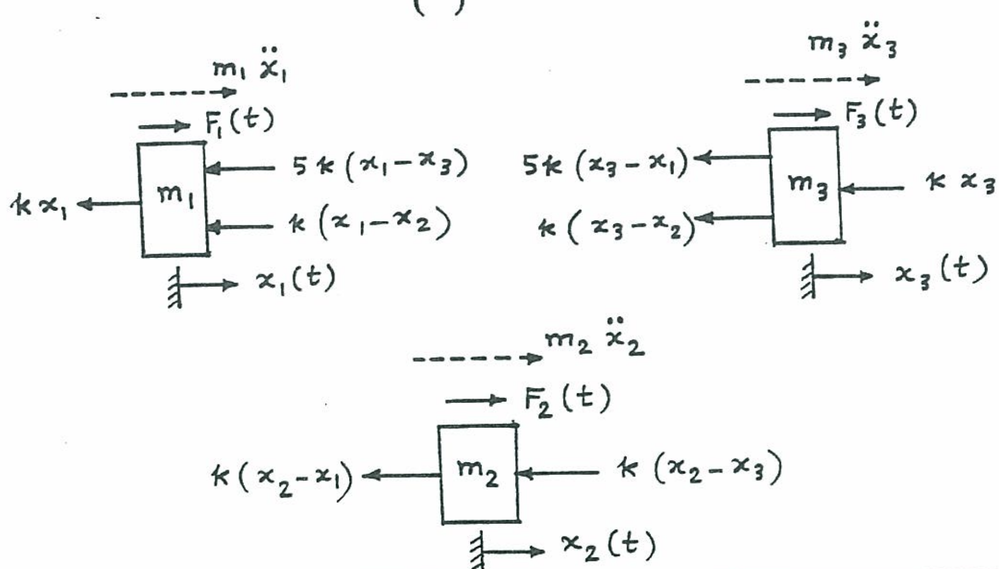
Multidegree of Freedom Systems

6.1 Equations of motion:

$$m_1\,\ddot{x}_1 = -k\,x_1 - 5\,k\,(x_1 - x_3) - k\,(x_1 - x_2) + F_1(t)$$
$$m_2\,\ddot{x}_2 = -k\,(x_2 - x_1) - k\,(x_2 - x_3) + F_2(t)$$
$$m_3\,\ddot{x}_3 = -5\,k\,(x_3 - x_1) - k\,(x_3 - x_2) - k\,x_3 + F_3(t)$$

or

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + k \begin{bmatrix} 7 & -1 & -5 \\ -1 & 2 & -1 \\ -5 & -1 & 7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} F_1(t) \\ F_2(t) \\ F_3(t) \end{Bmatrix}$$



6.2 Equations of motion:

$$J_0\,\ddot{\theta} = -2\,k\left(\frac{\ell}{4}\,\theta - x_1\right)\frac{\ell}{4} - c\left(\frac{\ell}{4}\,\dot{\theta} - \dot{x}_1\right)\frac{\ell}{4} - 3\,k\,(\theta\,\ell)\,\ell + M_t$$

$$2\,m\,\ddot{x}_1 = -2\,k\left(x_1 - \frac{\ell}{4}\,\theta\right) - c\left(\dot{x}_1 - \frac{\ell}{4}\,\dot{\theta}\right) - k\,(x_1 - x_2) + F_1$$

$$m\,\ddot{x}_2 = -k\,(x_2 - x_1) + F_2$$

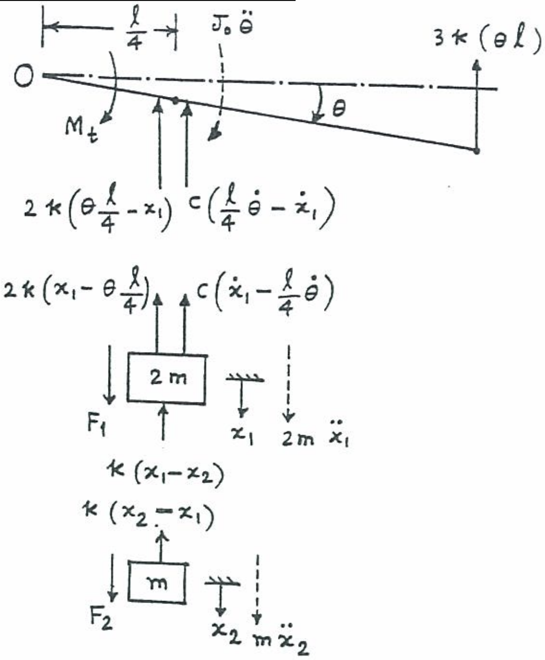
$$\text{where}\quad J_0 = \frac{1}{3}\,(2\,m)\,\ell^2 = \frac{2}{3}\,m\,\ell^2$$

These equations can be stated in matrix form as:

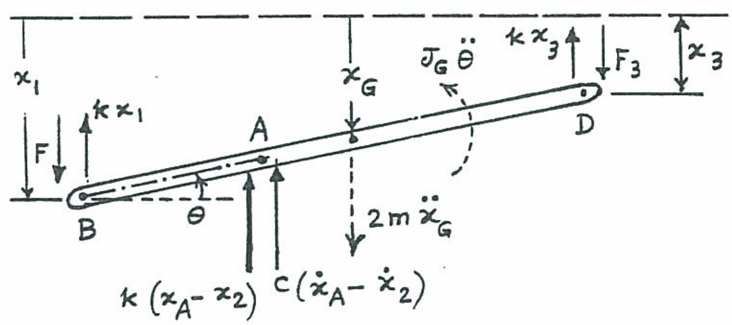
PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

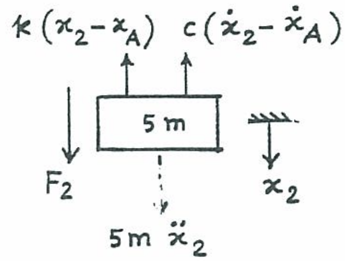
$$\begin{bmatrix} \frac{2}{3}\, m\, \ell^2 & 0 & 0 \\ 0 & 2\, m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} \frac{c\,\ell^2}{16} & -\frac{c\,\ell}{4} & 0 \\ -\frac{c\,\ell}{4} & c & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+ \begin{bmatrix} \frac{25\, k\, \ell^2}{8} & -\frac{k\,\ell}{2} & 0 \\ -\frac{k\,\ell}{2} & 3\, k & -k \\ 0 & -k & k \end{bmatrix} \begin{Bmatrix} \theta \\ x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} M_t \\ F_1 \\ F_2 \end{Bmatrix}$$



6.3





Equation of motion for rotation about B:

$$J_G\, \ddot{\theta} - 2\, m\, \ddot{x}_G \left(\frac{5\,\ell}{2}\right) = k\, x_3\, (5\,\ell) - F_3\, (5\,\ell) + k\,(x_A - x_2)\,(2\,\ell) + c\,(\dot{x}_A - \dot{x}_2)\,(2\,\ell) \tag{1}$$

Equation of motion for rotation about D:

$$J_G\, \ddot{\theta} + 2\, m\, \ddot{x}_G \left(\frac{5\,\ell}{2}\right) = -\, k\, x_1\, (5\,\ell) + F_1\, (5\,\ell) - k\,(x_A - x_2)\,(3\,\ell) - c\,(\dot{x}_A - \dot{x}_2)\,(3\,\ell) \tag{2}$$

Equation of motion of mass 5m in vertical direction:

$$5\, m\, \ddot{x}_2 = -\, k\,(x_2 - x_A) - c\,(\dot{x}_2 - \dot{x}_A) + F_2 \tag{3}$$

Noting that

$$J_G = \frac{1}{12}\,(2\,m)\,(5\,\ell)^2 = \frac{25}{6}\, m\, \ell^2 \quad ; \quad \theta = \frac{x_1 - x_3}{5\,\ell} \quad ; \quad x_G = \frac{x_1 + x_3}{2}$$

$$\text{and} \quad x_A = x_1 - 2\,\ell\,\theta = \frac{3}{5}\, x_1 + \frac{2}{5}\, x_3$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الاتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

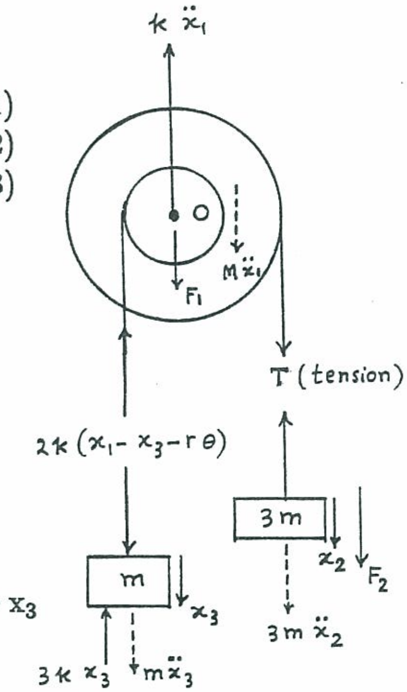
Eqs. (1) to (3) can be rewritten as:

$$\frac{m}{3}\,\ddot{x}_1 + \frac{2}{3}\,m\,\ddot{x}_3 + \frac{29}{25}\,k\,x_3 + \frac{6}{25}\,k\,x_1 - \frac{2}{5}\,k\,x_2 + \frac{6}{25}\,c\,\dot{x}_1 + \frac{4}{25}\,c\,\dot{x}_3 - \frac{2}{5}\,c\,\dot{x}_2 = F_3 \tag{4}$$

$$\frac{2}{3}\,m\,\ddot{x}_1 + \frac{m}{3}\,\ddot{x}_3 + \frac{34}{25}\,k\,x_1 - \frac{3}{5}\,k\,x_2 + \frac{6}{25}\,k\,x_3 + \frac{9}{25}\,c\,\dot{x}_1 - \frac{3}{5}\,c\,\dot{x}_2 + \frac{6}{25}\,c\,\dot{x}_3 = F_1 \tag{5}$$

$$5\,m\,\ddot{x}_2 - \frac{3}{5}\,k\,x_1 + k\,x_2 - \frac{2}{5}\,k\,x_3 - \frac{3}{5}\,c\,\dot{x}_1 + c\,\dot{x}_2 - \frac{2}{5}\,c\,\dot{x}_3 = F_2 \tag{6}$$

6.4 Equations of motion:



Mass M: $M\,\ddot{x}_1 = -\,k\,x_1 + T - 2\,k\,(x_1 - x_3 - r\,\theta) + F_1 \quad (1)$

Mass m: $m\,\ddot{x}_3 = -\,3\,k\,x_3 + 2\,k\,(x_1 - x_3 - r\,\theta) + F_3 \quad (2)$

Mass 3m: $3\,m\,\ddot{x}_2 = F_2 - T \quad (3)$

Rotation of pulley:

$$J_0\,\ddot{\theta} = T\,(3\,r) + r\,(2\,k)\,(x_1 - x_3 - r\,\theta) \tag{4}$$

Noting that

$$\theta = \frac{x_2 - x_1}{3\,r}$$

and

$$x_1 - x_3 - r\,\theta = x_1 - x_3 - r\left(\frac{x_2 - x_1}{3\,r}\right) = \frac{4}{3}\,x_1 - \frac{1}{3}\,x_2 - x_3$$

Eq. (4) can be used to find the tension T as:

$$T = \left(\frac{J_0}{9\,r^2}\right)(\ddot{x}_2 - \ddot{x}_1) - \frac{8}{9}\,k\,x_1 + \frac{2}{9}\,k\,x_2 + \frac{2}{3}\,k\,x_3 \tag{5}$$

Using the expression of T, Eqs. (1) to (3) can be rewritten as

$$\left(M + \frac{J_0}{9\,r^2}\right)\ddot{x}_1 - \frac{J_0}{9\,r^2}\,\ddot{x}_2 + \frac{41}{9}\,k\,x_1 - \frac{8}{9}\,k\,x_2 - \frac{8}{3}\,k\,x_3 = F_1(t) \tag{6}$$

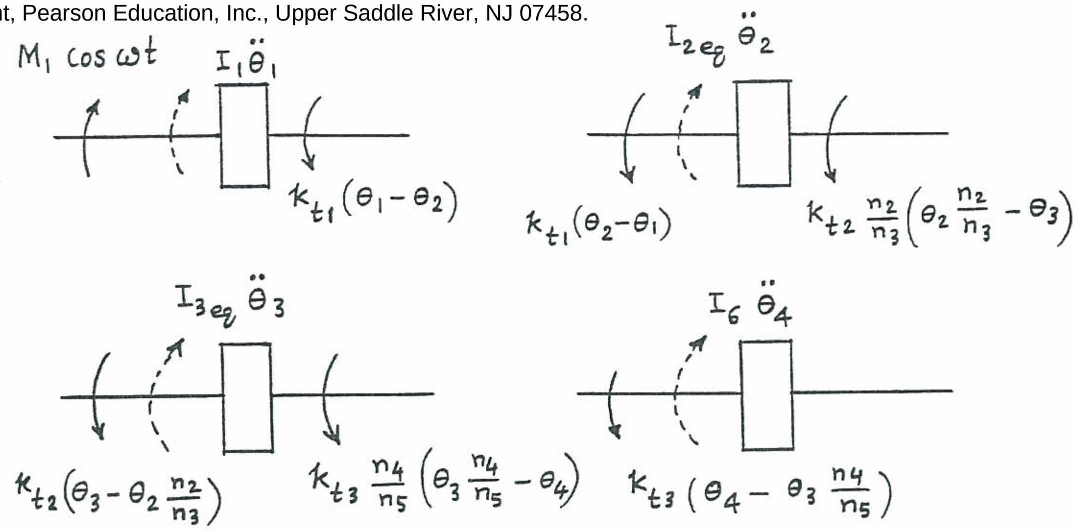
$$-\frac{J_0}{9\,r^2}\,\ddot{x}_1 + \left(3\,m + \frac{J_0}{9\,r^2}\right)\ddot{x}_2 - \frac{8}{9}\,k\,x_1 + \frac{2}{9}\,k\,x_2 + \frac{2}{3}\,k\,x_3 K = F_2(t) \tag{7}$$

$$m\,\ddot{x}_3 - \frac{8}{3}\,k\,x_1 + \frac{2}{3}\,k\,x_2 + 5\,k\,x_3 = F_3(t) \tag{8}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.5



$$I_2 eq = I_2 + I_3 \left(\frac{n_2}{n_3}\right)^2 \;;\; I_3 eq = I_4 + I_5 \left(\frac{n_4}{n_5}\right)^2$$

Equations of motion:

$$I_1 \ddot{\theta}_1 + k_{t1} (\theta_1 - \theta_2) = M_1 \cos \omega t$$

$$\left(I_2 + I_3 \frac{n_2^2}{n_3^2}\right) \ddot{\theta}_2 + k_{t1} (\theta_2 - \theta_1) + k_{t2} \frac{n_2}{n_3} (\theta_2 \frac{n_2}{n_3} - \theta_3) = 0$$

$$\left(I_4 + I_5 \frac{n_4^2}{n_5^2}\right) \ddot{\theta}_3 + k_{t2} (\theta_3 - \theta_2 \frac{n_2}{n_3}) + k_{t3} \frac{n_4}{n_5} (\theta_3 \frac{n_4}{n_5} - \theta_4) = 0$$

$$I_6 \ddot{\theta}_4 + k_{t3} (\theta_4 - \theta_3 \frac{n_4}{n_5}) = 0$$

6.6

$$M \ddot{x}_3 = - c_2 (\dot{x}_3 - \ell_1 \dot{\theta} - \dot{x}_1) - k_2 (x_3 - \ell_1 \theta - x_1)$$
$$- c_2 (\dot{x}_3 + \ell_1 \dot{\theta} - \dot{x}_2) - k_2 (x_3 + \ell_2 \theta - x_2)$$

$$J_G \ddot{\theta} = c_2 (\dot{x}_3 - \ell_1 \dot{\theta} - \dot{x}_1) \ell_1 + k_2 (x_3 - \ell_1 \theta - x_1) \ell_1$$
$$- c_2 (\dot{x}_3 + \ell_2 \dot{\theta} - \dot{x}_2) \ell_2 - k_2 (x_3 + \ell_2 \theta - x_2) \ell_2 \quad (2)$$

$$m_1 \ddot{x}_1 = - c_2 (\dot{x}_1 - \dot{x}_3 + \ell_1 \dot{\theta}) - k_2 (x_1 - x_3 + \ell_1 \theta)$$
$$- c_1 \dot{x}_1 - k_1 x_1 + F_1 \quad (3)$$

$$m_2 \ddot{x}_2 = - c_2 (\dot{x}_2 - \dot{x}_3 - \ell_2 \dot{\theta}) - k_2 (x_2 - x_3 - \ell_2 \theta)$$
$$- k_1 x_2 - c_1 \dot{x}_2 + F_2 \quad (4)$$

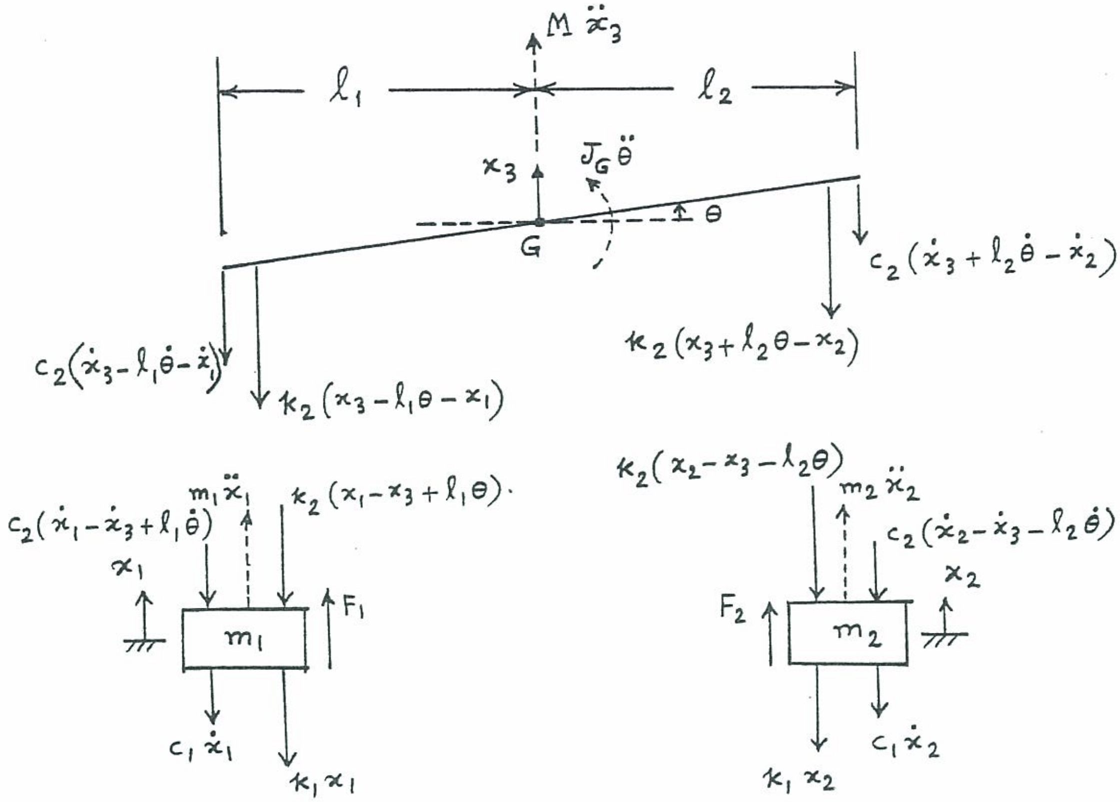
Eqs. (1) to (4) can be rewritten as

$$M_3 \dot{x}_3 + 2 c_2 \dot{x}_3 - c_2 \dot{x}_1 - c_2 \dot{x}_2 + 2 k_2 x_3$$
$$+ \theta (k_2 \ell_2 - k_2 \ell_1) - k_2 x_1 - k_2 x_2 = 0 \quad (5)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.



$$J_G\,\ddot{\theta} + (c_2\,\ell_1^2 + c_2\,\ell_2^2)\,\dot{\theta} + (-\,c_2\,\ell_1 + c_2\,\ell_2)\,\dot{x}_3 + c_2\,\ell_1\,\dot{x}_1 - c_2\,\ell_2\,\dot{x}_2 + (-\,k_2\,\ell_1 + k_2\,\ell_2)\,x_3 + k_2\,\ell_1\,x_1 - k_2\,\ell_2\,x_2 + (k_2\,\ell_1^2 + k_2\,\ell_2^2)\,\theta = 0 \qquad (6)$$

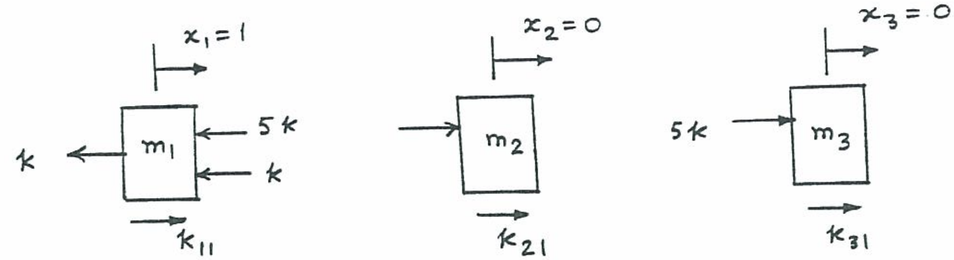
$$m_1\,\ddot{x}_1 + (c_1 + c_2)\,\dot{x}_1 - c_2\,\dot{x}_3 + c_2\,\ell_1\,\dot{\theta} + (k_1 + k_2)\,x_1 - k_2\,x_3 + k_2\,\ell_1\,\theta = F_1 \qquad (7)$$

$$m_2\,\ddot{x}_2 + (c_1 + c_2)\,\dot{x}_2 - c_2\,\dot{x}_3 - c_2\,\ell_2\,\dot{\theta} + (k_1 + k_2)\,x_2 - k_2\,x_3 - k_2\,\ell_2\,\theta = F_2 \qquad (8)$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

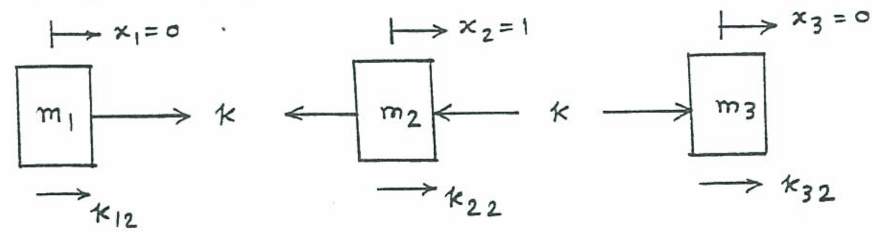
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.12



(i) Set $x_1 = 1$, $x_2 = x_3 = 0$:

Equilibrium of forces:
$k_{11} - k - 5k - k = 0$ or $k_{11} = 7k$; $k_{21} = -k$; $k_{31} = -5k$

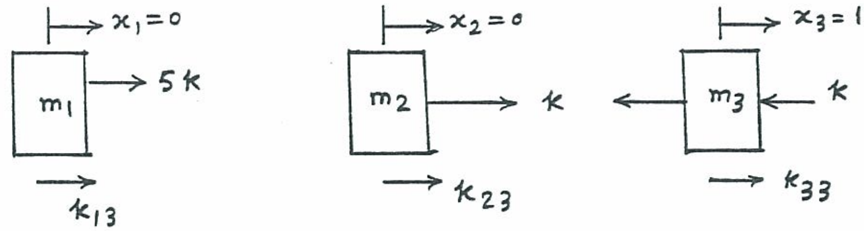


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

(ii) Set $x_2 = 1$, $x_1 = x_3 = 0$:

Equilibrium of forces:
$k_{12} = -k$; $k_{22} - k - k = 0$ or $k_{22} = 2k$; $k_{32} = -k$

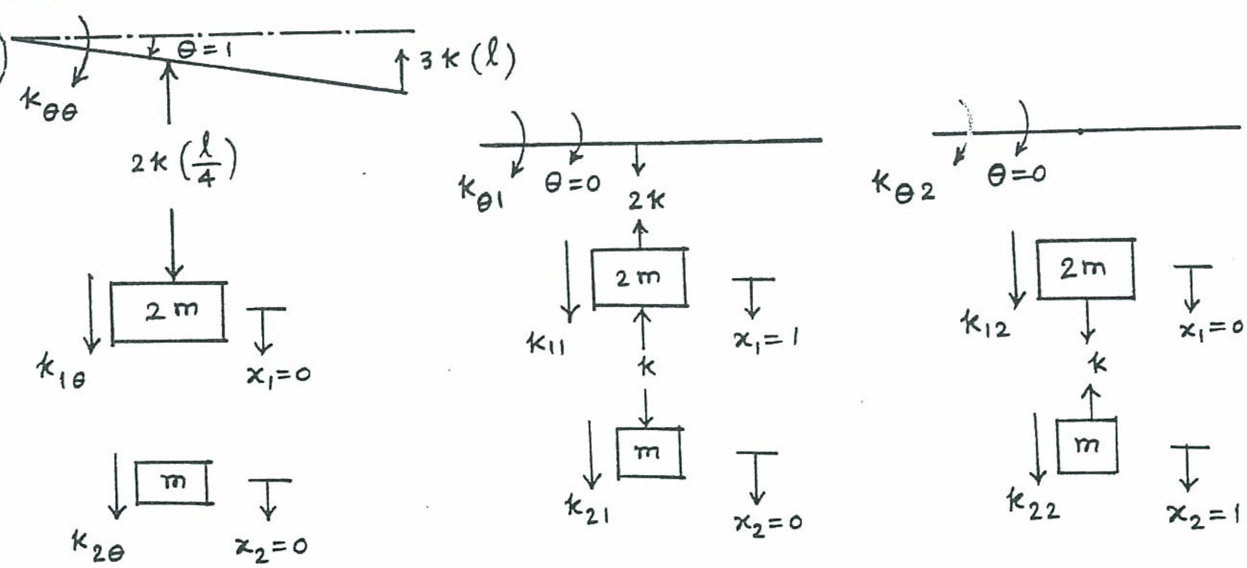


(iii) Set $x_3 = 1$, $x_1 = x_2 = 0$:

Equilibrium of forces:
$k_{13} = -5k$; $k_{23} = -k$; $k_{33} = 7k$

$$\therefore \quad [k] = \begin{bmatrix} 7 & -1 & -5 \\ -1 & 2 & -1 \\ -5 & -1 & 7 \end{bmatrix}$$

6.13



(i) Give $\theta = 1$, $x_1 = x_2 = 0$:

Equilibrium equations:

$$k_{\theta\theta} = 3\,k\,\ell^2 + 2\,k\,(\ell/4)^2 = \frac{25}{8}\,k\,\ell^2 \quad ; \quad k_{1\theta} = -\frac{k\,\ell}{2} \quad ; \quad k_{2\theta} = 0$$

(ii) Give $x_1 = 1$, $\theta = x_2 = 0$:

Equilibrium equations:

$$k_{\theta 1} = -2\,k\,(\ell/4) = -\frac{k\,\ell}{2} \quad ; \quad k_{11} = 2\,k + k = 3\,k \quad ; \quad k_{21} = -k$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

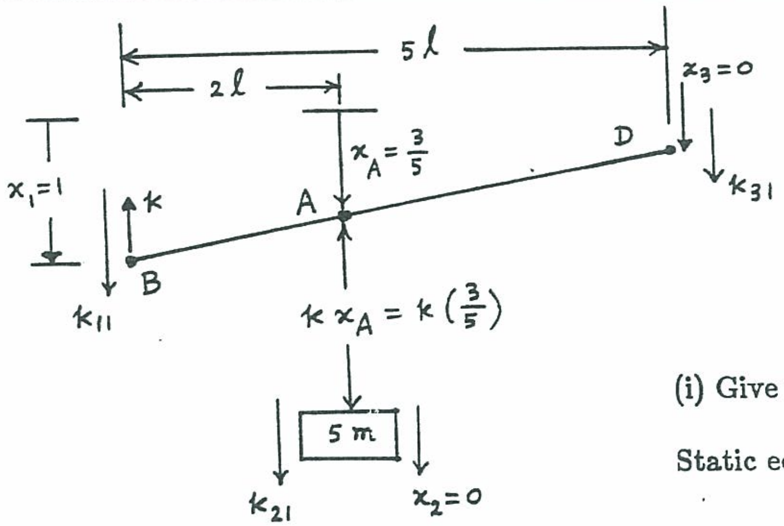
(iii) Give $x_2 = 1,\ \theta = x_1 = 0$:

Equilibrium equations:

$$k_{\theta 2} = 0\ ;\ k_{12} = -k\ ;\ k_{22} = k$$

$$\therefore\ [k] = \begin{bmatrix} \dfrac{25\,k\,\ell^2}{8} & -\dfrac{k\,\ell}{2} & 0 \\ -\dfrac{k\,\ell}{2} & 3\,k & -k \\ 0 & -k & k \end{bmatrix}$$

6.14



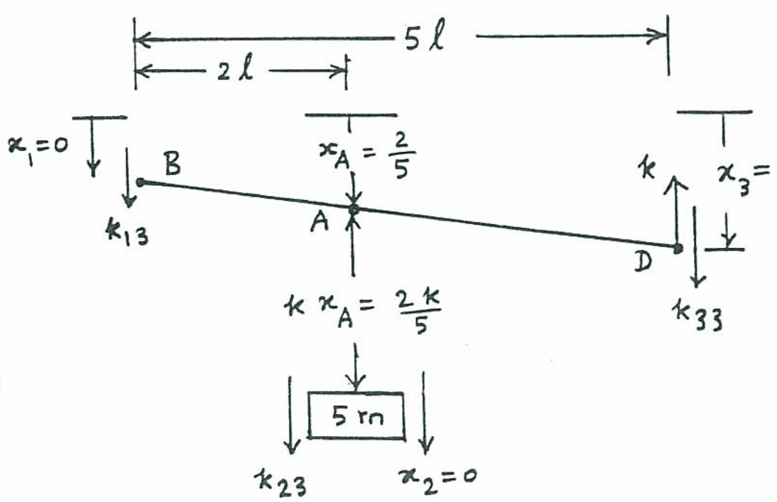
(i) Give $x_1 = 1,\ x_2 = x_3 = 0$:

Static equilibrium equations:

$$\sum M_B = 0 \ \text{ or }\ k_{31}\,(5\,\ell) - \frac{3}{5}\,k\,(2\,\ell) = 0 \ \text{ or }\ k_{31} = \frac{6}{25}\,k$$

$$\sum M_D = 0 \ \text{ or }\ k_{11}\,(5\,\ell) - k\,(5\,\ell) - \frac{3\,k}{5}\,(3\,\ell) = 0 \ \text{ or }\ k_{11} = \frac{34}{25}\,k$$

$$\sum F = 0 \text{ at mass 5m } \ \text{ or }\ k_{21} + \frac{3}{5}\,k = 0 \ \text{ or }\ k_{21} = -\frac{3}{5}\,k$$



(ii) Give $x_3 = 1,\ x_1 = x_2 = 0$:

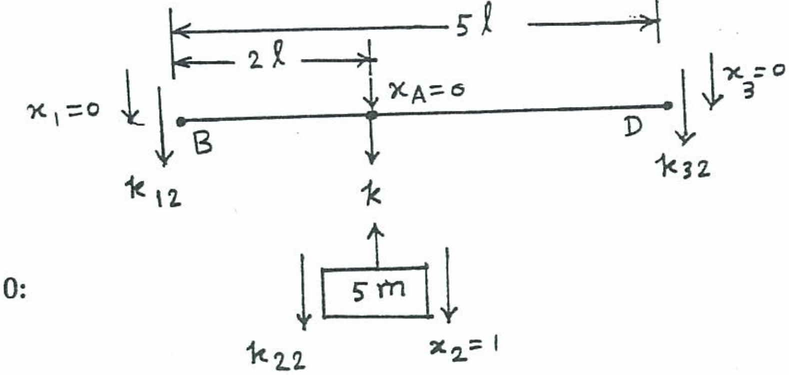
Static equilibrium equations:

$$\sum M_B = 0 \ \text{ or }\ k_{33}\,(5\,\ell) - \frac{2\,k}{5}\,(2\,\ell) - k\,(5\,\ell) = 0 \ \text{ or }\ k_{33} = \frac{29}{25}\,k$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

$$\sum M_D = 0 \text{ or } k_{13}\,(5\,\ell) - \frac{2\,k}{5}\,(3\,\ell) = 0 \text{ or } k_{13} = \frac{6}{25}\,k$$

$$\sum F = 0 \text{ at mass 5m or } k_{23} + \frac{2}{5}\,k = 0 \text{ or } k_{23} = -\frac{2}{5}\,k$$



(iii) Give $x_3 = 1$, $x_1 = x_2 = 0$:

Static equilibrium equations:

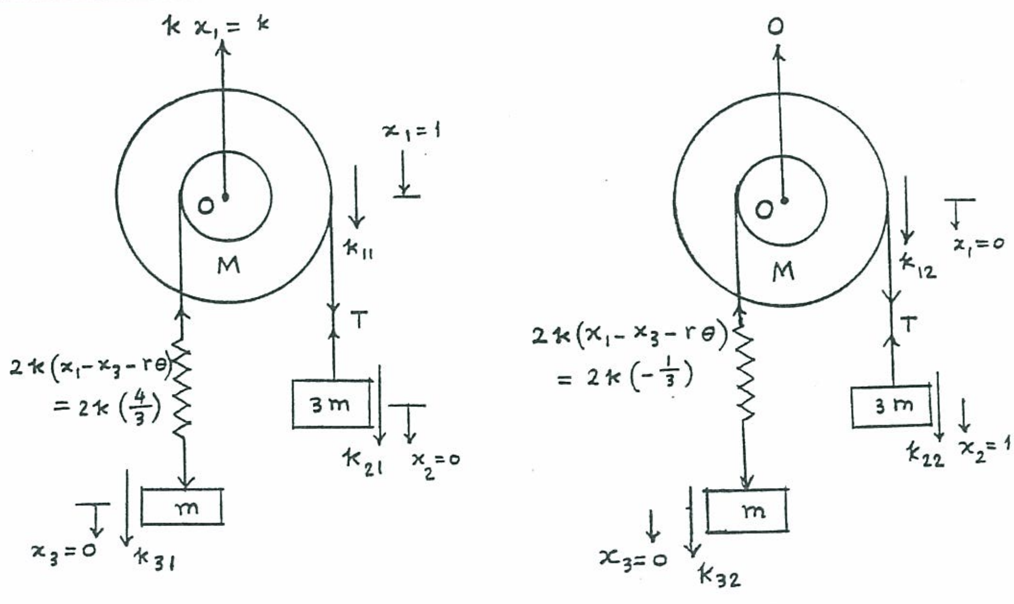
$$\sum M_B = 0 \text{ or } k_{32}\,(5\,\ell) + k\,(2\,\ell) = 0 \text{ or } k_{32} = -\frac{2}{5}\,k$$

$$\sum M_D = 0 \text{ or } k_{12}\,(5\,\ell) + k\,(3\,\ell) = 0 \text{ or } k_{12} = -\frac{3}{5}\,k$$

$$\sum F = 0 \text{ at mass 5m or } k_{22} - k = 0 \text{ or } k_{22} = k$$

$$\therefore\quad [k] = k\begin{bmatrix} \frac{34}{25} & -\frac{3}{5} & \frac{6}{25} \\ -\frac{3}{5} & 1 & -\frac{2}{5} \\ \frac{6}{25} & -\frac{2}{5} & \frac{29}{25} \end{bmatrix}$$

6.15



Here $\theta = \dfrac{x_2 - x_1}{3\,r}$; $x_1 - x_3 - r\,\theta = \dfrac{4}{3}\,x_1 - \dfrac{1}{3}\,x_2 - x_3$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(i) Give $x_1 = 1$, $x_2 = x_3 = 0$:

$$\sum F = 0 \text{ for mass M:} \quad k_{11} - k + T - \frac{8\,k}{3} = 0 \tag{1}$$

$$\sum F = 0 \text{ for mass m:} \quad k_{31} + \frac{8\,k}{3} = 0 \tag{2}$$

$$\sum F = 0 \text{ for mass 3m:} \quad k_{21} = T \tag{3}$$

$$\sum M_0 = 0 \text{ for pulley:} \quad T\,(3\,r) + 2\,k\left(\frac{4}{3}\right) r = 0 \;\text{ or }\; T = -\frac{8\,k}{9} \tag{4}$$

Eqs. (1) to (4) yield:

$$k_{21} = -\frac{8\,k}{9}, \quad k_{31} = -\frac{8\,k}{3}, \quad k_{11} = \frac{41}{9}\,k$$

(ii) Give $x_2 = 1$, $x_1 = x_3 = 0$:

$$\sum F = 0 \text{ for mass M:} \quad k_{12} + T + \frac{2\,k}{3} = 0 \tag{5}$$

$$\sum F = 0 \text{ for mass 3m:} \quad k_{22} - T = 0 \tag{6}$$

$$\sum F = 0 \text{ for mass m:} \quad k_{32} - \frac{2\,k}{3} = 0 \tag{7}$$

$$\sum M_0 = 0 \text{ for pulley:} \quad T\,(3\,r) + r\,(-\frac{2\,k}{3}) = 0 \tag{8}$$

Solution of Eqs. (5) to (8) yields:

$$k_{22} = T = \frac{2\,k}{9}, \quad k_{32} = \frac{2\,k}{3}, \quad k_{12} = -\frac{8\,k}{9}$$

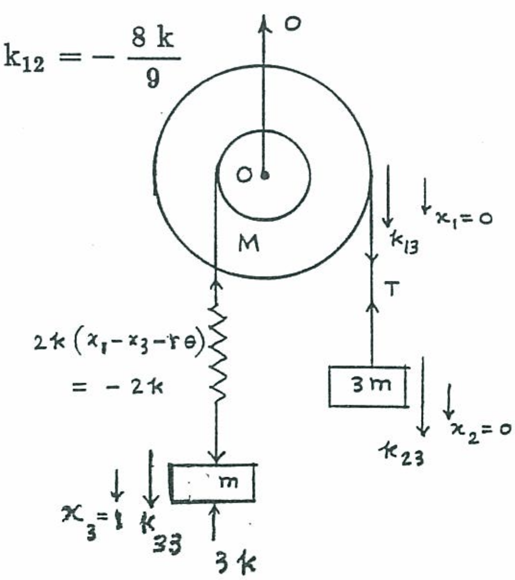
(iii) Give $x_3 = 1$, $x_1 = x_2 = 0$:

$$\sum F = 0 \text{ for mass M:} \quad k_{13} + T - (-2\,k) = 0 \tag{9}$$

$$\sum F = 0 \text{ for mass 3m:} \quad k_{23} - T = 0 \tag{10}$$

$$\sum F = 0 \text{ for mass m:} \quad k_{33} - 2\,k - 3\,k = 0 \tag{11}$$

$$\sum M_0 = 0 \text{ for pulley:} \quad T\,(3\,r) + r\,(-2\,k) = 0 \tag{12}$$



Solution of Eqs. (9) to (12) gives:

$$k_{33} = 5\,k, \quad k_{23} = \frac{2\,k}{3}, \quad k_{13} = -\frac{8\,k}{3}$$

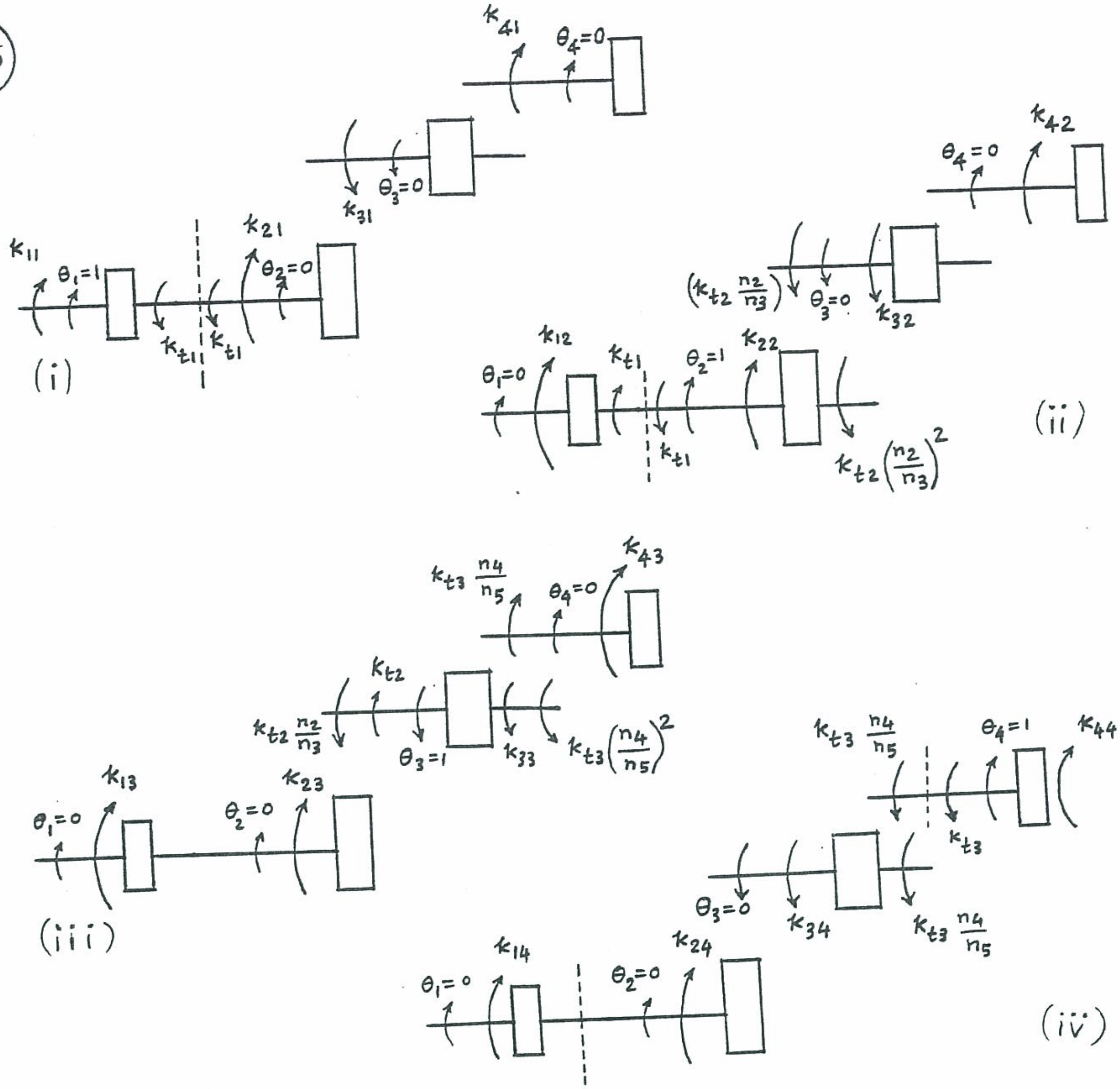
$$\therefore \quad [k] = k \begin{bmatrix} \frac{41}{9} & -\frac{8}{9} & -\frac{8}{3} \\ -\frac{8}{9} & \frac{2}{9} & \frac{2}{3} \\ -\frac{8}{3} & \frac{2}{3} & 5 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.16



(i) Set $\theta_1 = 1$, $\theta_2 = \theta_3 = \theta_4 = 0$:

Equilibrium equations give:

$$k_{11} = k_{t1}, \;\; k_{21} = -\, k_{t1}, \;\; k_{31} = k_{41} = 0$$

(ii) Set $\theta_2 = 1$, $\theta_1 = \theta_3 = \theta_4 = 0$:

Equilibrium equations yield:

$$k_{12} = -\, k_{t1}, \;\; k_{22} = k_{t1} + k_{t2}\left(\frac{n_2}{n_3}\right)^2 \;\;, \;\; k_{32} = -\, k_{t2}\left(\frac{n_2}{n_3}\right), \;\; k_{42} = 0$$

(iii) Set $\theta_3 = 1$, $\theta_1 = \theta_2 = \theta_4 = 0$:

Equilibrium equations provide:

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

$$k_{13} = 0,\ \ k_{23} = -\,k_{t2}\left(\frac{n_2}{n_3}\right)\ ,\ \ k_{33} = k_{t2} + k_{t3}\left(\frac{n_4}{n_5}\right)^2,\ \ k_{43} = -\,k_{t3}\left(\frac{n_4}{n_5}\right)$$

(iv) Set $\theta_4 = 1$, $\theta_1 = \theta_2 = \theta_3 = 0$:

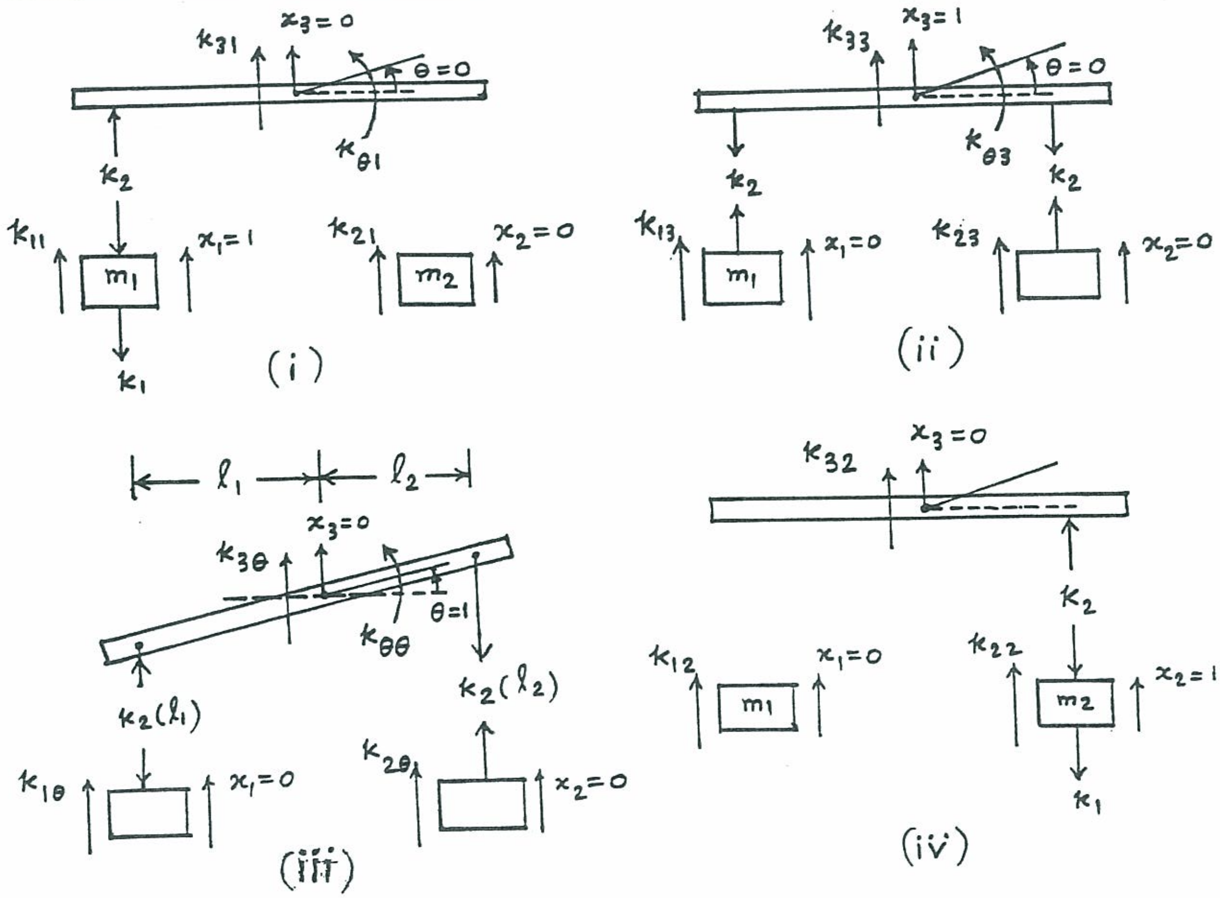
Equilibrium equations give:

$$k_{14} = k_{24} = 0,\ \ k_{34} = -\,k_{t3}\left(\frac{n_4}{n_5}\right),\ \ k_{44} = k_{t3}$$

Thus the stiffness matrix is:

$$[k] = \begin{bmatrix} k_{t1} & -\,k_{t1} & 0 & 0 \\ -\,k_{t1} & k_{t1} + k_{t2}\left(\frac{n_2}{n_3}\right)^2 & -\,k_{t2}\left(\frac{n_2}{n_3}\right) & 0 \\ 0 & -\,k_{t2}\left(\frac{n_2}{n_3}\right) & k_{t2} + k_{t3}\left(\frac{n_4}{n_5}\right)^2 & -\,k_{t3}\left(\frac{n_4}{n_5}\right) \\ 0 & 0 & -\,k_{t3}\left(\frac{n_4}{n_5}\right) & k_{t3} \end{bmatrix}$$

6.17



© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

(i) Set $x_1 = 1$, $x_2 = x_3 = \theta = 0$:

Equilibrium equations:

$$k_{11} = k_1 + k_2, \;\; k_{31} = -k_2, \;\; k_{\theta 1} = k_2\, \ell_1, \;\; k_{21} = 0$$

(ii) Set $x_3 = 1$, $x_1 = x_2 = \theta = 0$:

Equilibrium equations:

$$k_{33} = 2\, k_2, \;\; k_{13} = -k_2, \;\; k_{23} = -k_2, \;\; k_{\theta 3} = -k_2\, \ell_1 + k_2\, \ell_2$$

(iii) Set $\theta = 1$, $x_1 = x_2 = x_3 = 0$:

Equilibrium equations:

$$k_{\theta\theta} = k_2\, (\ell_1^2 + \ell_2^2), \;\; k_{\theta 3} = k_2\, (\ell_2 - \ell_1) \;\; , \;\; k_{1\theta} = k_2\, \ell_1, \;\; k_{2\theta} = -k_2\, \ell_2$$

(iv) Set $x_2 = 1$, $x_1 = x_3 = \theta = 0$:

Equilibrium equations:

$$k_{12} = 0, \;\; k_{32} = -k_2, \;\; k_{\theta 2} = -k_2\, \ell_2, \;\; k_{22} = k_1 + k_2$$

$$\therefore \;\; [k] = \begin{bmatrix} (k_1 + k_2) & 0 & -k_2 & k_2\, \ell_1 \\ 0 & (k_1 + k_2) & -k_2 & -k_2\, \ell_2 \\ -k_2 & -k_2 & 2\, k_2 & k_2\, (\ell_2 - \ell_1) \\ k_2\, \ell_1 & -k_2\, \ell_2 & k_2\, (\ell_2 - \ell_1) & k_2(\ell_1^2 + \ell_2^2) \end{bmatrix}$$

6.18 (i) Give $F_x = 1$, $M_\theta = 0$:

Same force of $F_x = 1$ is induced everywhere along the rope. Since k_1 and k_2 are in series,

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

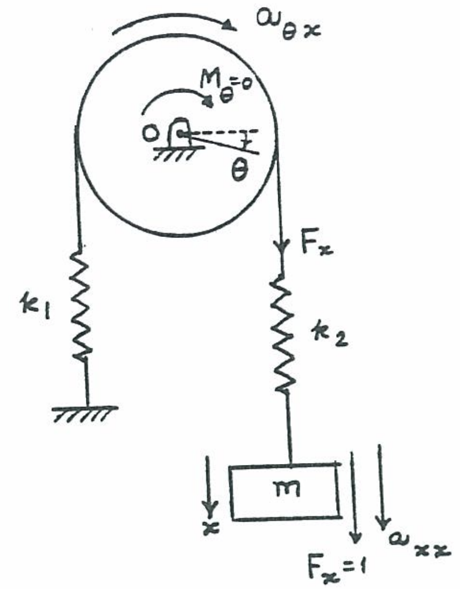
$$a_{xx} = \frac{\text{force}}{k_{eq}} = \text{deflection at m due } F_x \text{ of } 1 = \frac{k_1 + k_2}{k_1\, k_2}$$

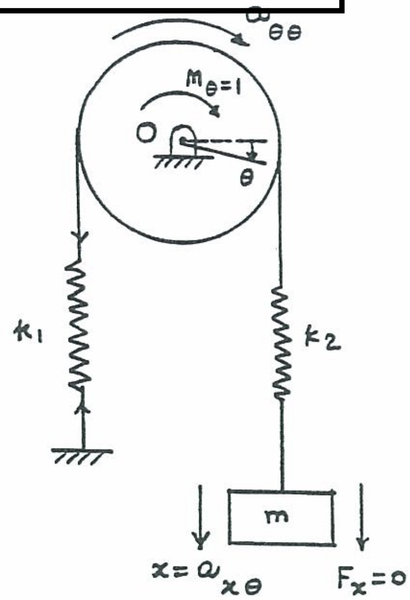
Linear deflection of k_1 under $F_x = 1$ is $\frac{1}{k_1}$, angular deflection θ due to linear displacement of $\frac{1}{k_1} = a_{\theta x}$.

$$a_{\theta x} = \text{linear deflection of } k_1 = a_{\theta x}\, r = \frac{1}{k_1} \;\; ; \;\; a_{\theta x} = \frac{1}{k_1\, r}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

PRELIMINARY VERSION 12/14/2010





(ii) Give $M_\theta = 1$, $F_x = 0$:

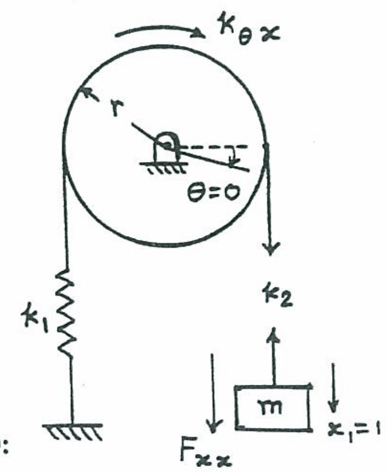
Extension of $k_1 = a_{\theta\theta}\ r$, spring force $= k_1\ (a_{\theta\theta}\ r)$.

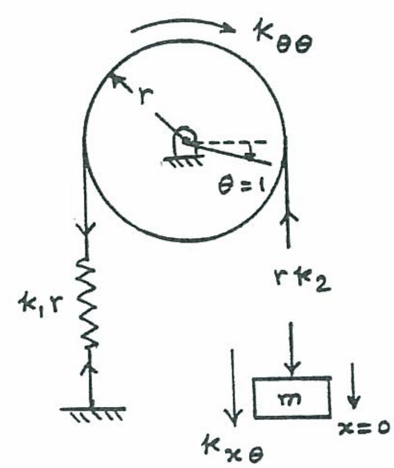
$$\sum M_\theta = 0 \ \text{ or } \ M_\theta = \left(k_1\ a_{\theta\theta}\ r\right) r = 1 \ \text{ or } \ a_{\theta\theta} = \frac{1}{k_1\ r^2}.$$

Displacement of $m = a_{x\theta} = a_{\theta\theta}\ r = \dfrac{1}{k_1\ r}$

$$\therefore \quad [a] = \begin{bmatrix} \dfrac{k_1 + k_2}{k_1\ k_2} & \dfrac{1}{k_1\ r} \\ \dfrac{1}{k_1\ r} & \dfrac{1}{k_1\ r^2} \end{bmatrix}$$

6.19





(i) Give $x = 1$, $\theta = 0$:

Equilibrium equations give:

$$k_{xx} - k_2 = 0 \ \text{ or } \ k_{xx} = k_2 \quad ; \quad k_{\theta x} + k_2\ r = 0 \ \text{ or } \ k_{\theta x} = -\, k_2\ r$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

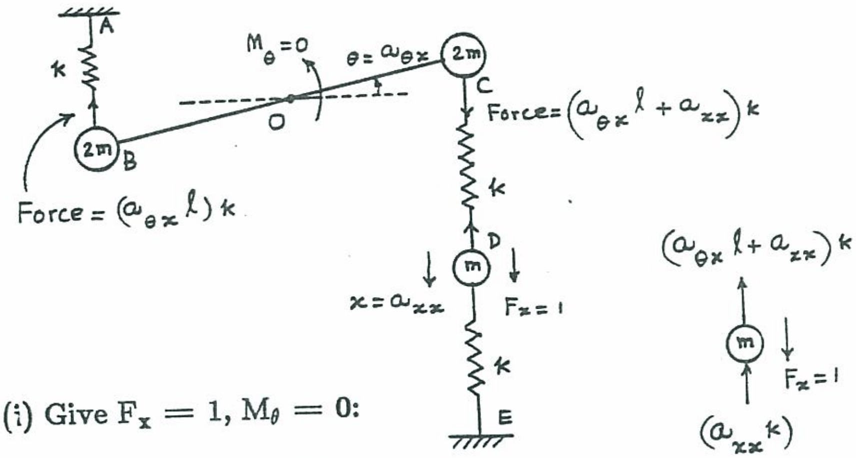
(ii) Give $\theta = 1$, $x = 0$:

Equilibrium equations yield:

$$k_{x\theta} + r\,k_2 = 0 \quad \text{or} \quad k_{x\theta} = -\,k_2\,r$$
$$k_{\theta\theta} - r^2\,k_2 - r^2\,k_1 = 0 \quad \text{or} \quad k_{\theta\theta} = r^2\,(k_1 + k_2)$$

$$\therefore \quad [k] = \begin{bmatrix} k_2 & -\,k_2\,r \\ -\,k_2\,r & (k_1 + k_2)\,r^2 \end{bmatrix}$$

6.20



(i) Give $F_x = 1$, $M_\theta = 0$:

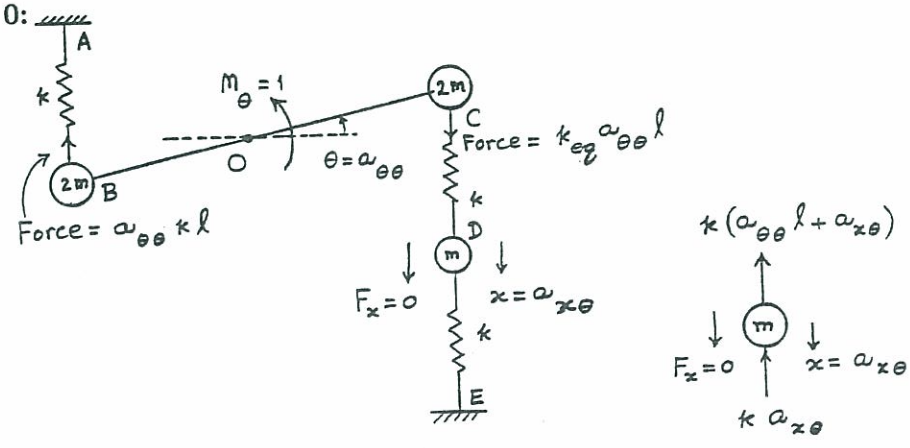
Extension of spring AB $= a_{\theta x}\,\ell$.
Total extension of spring CD $= (a_{\theta x}\,\ell + a_{xx})$.
Compression of spring DE $= a_{xx}$.

$$\sum F = 0 \quad \text{or} \quad a_{xx}\,k + a_{\theta x}\,\ell\,k + a_{xx}\,k = 1 \quad \text{or} \quad 2\,a_{xx}\,k + a_{\theta x}\,k\,\ell = 1 \qquad (1)$$
$$\sum M_0 = 0 \quad \text{or} \quad a_{\theta x}\,k\,\ell + (a_{\theta x}\,k\,\ell + a_{xx}\,k) = 0 \quad \text{or} \quad 2\,a_{\theta x}\,k\,\ell + a_{xx}\,k = 0$$

Solution of Eqs. (1) and (2):

$$a_{xx} = \frac{2}{3\,k} \;;\; a_{\theta x} = -\,\frac{1}{3\,k\,\ell}$$

(ii) Give $M_\theta = 1$, $F_x = 0$:



PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

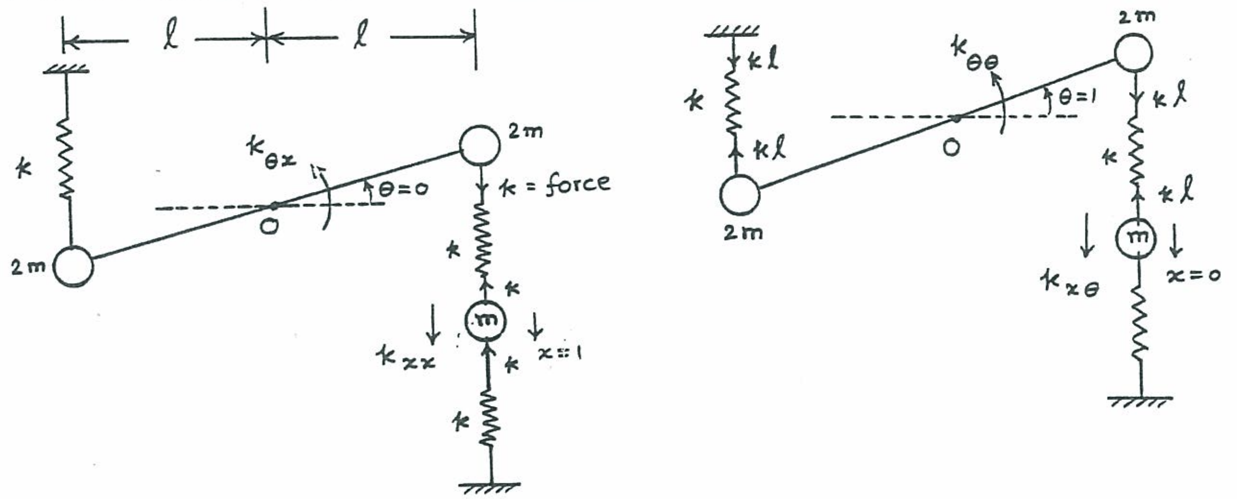
Extension of spring AB $= a_{\theta\theta}\,\ell$.
Total extension of springs CD and DE $= a_{\theta\theta}\,\ell$.
k_{eq} of series springs CD and DE $= \dfrac{k}{2}$.

$$\sum M_0 = 0 \text{ or } (a_{\theta\theta}\,k\,\ell)\,\ell + (k_{eq}\,a_{\theta\theta}\,\ell)\,\ell = 1 \text{ or } a_{\theta\theta} = \frac{2}{3\,k\,\ell^2} \tag{3}$$

$$\sum F = 0 \text{ or } k\,a_{x\theta} + k\,a_{\theta\theta}\,\ell + k\,a_{x\theta} = 0 \text{ or } a_{x\theta} = -\frac{1}{3\,k\,\ell} \tag{4}$$

$$\therefore\ [a] = \begin{bmatrix} \dfrac{2}{3\,k} & -\dfrac{1}{3\,k\,\ell} \\ -\dfrac{1}{3\,k\,\ell} & \dfrac{2}{3\,k\,\ell^2} \end{bmatrix}$$

6.21



(i) Give $x = 1,\ \theta = 0$:

$$\sum F = 0 \text{ or } k_{xx} - k - k = 0 \text{ or } k_{xx} = 2\,k$$
$$\sum M_0 = 0 \text{ or } k_{\theta x} - k\,\ell = 0 \text{ or } k_{\theta x} = k\,\ell$$

(ii) Give $\theta = 1,\ x = 0$:

$$\sum F = 0 \text{ or } k_{x\theta} - k\,\ell = 0 \text{ or } k_{x\theta} = k\,\ell$$
$$\sum M_0 = 0 \text{ or } k_{\theta\theta} - k\,\ell\,(\ell) - k\,\ell\,(\ell) = 0 \text{ or } k_{\theta\theta} = 2\,k\,\ell^2$$

$$\therefore\ [k] = \begin{bmatrix} 2\,k & k\,\ell \\ k\,\ell & 2\,k\,\ell^2 \end{bmatrix}$$

6.22 Kinetic energy of the system can be expressed as:

$$T = \frac{1}{2}\,(2m)\,(\ell\,\dot{\theta})^2 + \frac{1}{2}\,(2m)\,(\ell\,\dot{\theta}^2) + \frac{1}{2}\,m\,\dot{x}^2 = \frac{1}{2}\,(4m)\,(\ell\,\dot{\theta})^2 + \frac{1}{2}\,m\,\dot{x}^2$$

which can be expressed in matrix form as

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$T = \frac{1}{2}\,(\dot{x} \quad \dot{\theta})\,[m] \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} \qquad \text{where} \quad [m] = \begin{bmatrix} m & 0 \\ 0 & 4\, m\, \ell^2 \end{bmatrix}$$

6.23 Flexibility influence coefficients:

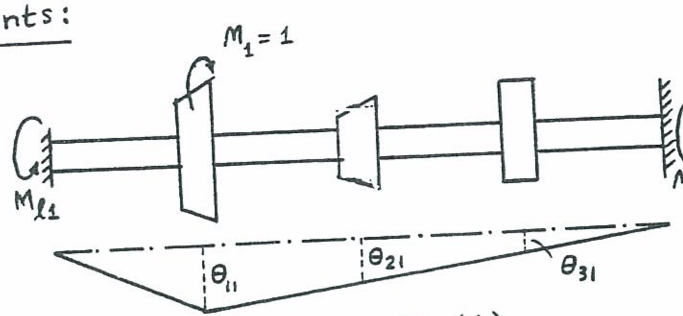


Fig.(A)

Spring constants of different sections of the shaft (k_i) are

$$k_i = \frac{(GJ)_i}{\ell_i}\ ; \quad i = 1,2,3,4$$

where $(GJ)_i$ = torsional rigidity, J_i = polar moment of inertia, and ℓ_i = length of section i of shaft.

Consider disc 1. Shaft to the left of disc 1 has a spring constant of k_1 while the shaft to the right side of disc 1 has an equivalent spring constant of $\quad k_{e1} = \dfrac{1}{\sum_{i=2}^{4}\left(\frac{1}{k_i}\right)}$

If we apply unit torque to disc 1 ($M_1 = 1$) as shown in Fig.(A), reactive torques at left and right ends of the shaft are

$$M_{r1} = k_{e1}\,\theta_{11}\,, \quad M_{\ell 1} = k_1\,\theta_{11}$$

Since $\quad M_{\ell 1} + M_{r1} = M_1 = 1$, we get

$$\theta_{11} = a_{11} = \left\{\sum_{i=2}^{4}\left(\frac{1}{k_i}\right)\right\} \Big/ \left\{k_1 \cdot \sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}$$

$$M_{\ell 1} = k_1\,\theta_{11} = \sum_{i=2}^{4}\left(\frac{1}{k_i}\right) \Big/ \left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}\ ; \quad M_{r1} = \frac{\theta_{11}}{\sum_{i=2}^{4}\left(\frac{1}{k_i}\right)} = \frac{1}{k_1 \cdot \sum_{i=1}^{4}\left(\frac{1}{k_i}\right)}$$

Also,

$$\theta_{31} = a_{31} = \frac{M_{r1}}{k_4} = \frac{1}{k_1 k_4 \left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}}$$

$$\theta_{21} = a_{21} = M_{r1} \Big/ \left\{\sum_{i=3}^{4}\left(\frac{1}{k_i}\right)\right\} = \frac{1}{k_1 \cdot \left\{\sum_{i=3}^{4}\left(\frac{1}{k_i}\right)\right\} \cdot \left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}}$$

Consider disc 2. Shaft to the left side of disc 2 has an equivalent spring constant of k_{e2} and the shaft to its right side has an

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

equivalent spring constant of k_{e3} with

$$k_{e2} = \frac{1}{\sum_{i=1}^{2}\left(\frac{1}{k_i}\right)} , \qquad k_{e3} = \frac{1}{\sum_{i=3}^{4}\left(\frac{1}{k_i}\right)}$$

If we apply a unit torque to disc 2 ($M_2 = 1$), reactive torques at the left and right ends of shaft are

$$M_{\ell 2} = \theta_{22}\, k_{e2} , \qquad M_{r2} = \theta_{22}\, k_{e3} \qquad \text{with} \qquad M_2 = M_{\ell 2} + M_{r2} = 1$$

Hence

$$\theta_{22} = a_{22} = \left\{\sum_{i=1}^{2}\left(\frac{1}{k_i}\right)\right\}\cdot\left\{\sum_{i=3}^{4}\left(\frac{1}{k_i}\right)\right\}\Big/\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}$$

$$\theta_{32} = a_{32} = \frac{M_{r2}}{k_4} = \frac{\theta_{22}}{k_{e3}\cdot k_4} = \left\{\sum_{i=1}^{2}\left(\frac{1}{k_i}\right)\right\}\Big/\left[k_4\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}\right]$$

$$\theta_{12} = a_{12} = \frac{M_{\ell 2}}{k_1} = \frac{\theta_{22}}{k_{e2}\cdot k_1} = \left\{\sum_{i=3}^{4}\left(\frac{1}{k_i}\right)\right\}\Big/\left[k_1\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}\right]$$

Consider disc 3. Apply unit torque to disc 3 ($M_3 = 1$) to obtain

$$M_{\ell 3} = k_{e4}\,\theta_{33} , \qquad M_{r3} = k_4\,\theta_{33} , \qquad M_{r3} + M_{\ell 3} = M_3 = 1$$

where $k_{e4} = \dfrac{1}{\sum_{i=1}^{3}\left(\frac{1}{k_i}\right)}$

Hence

$$\theta_{33} = a_{33} = \left\{\sum_{i=1}^{3}\left(\frac{1}{k_i}\right)\right\}\Big/\left[k_4\cdot\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}\right]$$

$$\theta_{13} = a_{13} = \frac{M_{\ell 3}}{k_1} = \frac{1}{k_1 k_4\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}}$$

$$\theta_{23} = a_{23} = \frac{M_{\ell 3}}{\sum_{i=1}^{2}\left(\frac{1}{k_i}\right)} = \frac{1}{k_4\left\{\sum_{i=1}^{2}\left(\frac{1}{k_i}\right)\right\}\left\{\sum_{i=1}^{4}\left(\frac{1}{k_i}\right)\right\}}$$

Flexibility matrix is $[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Note: In this problem, it is much easier to derive the stiffness influence coefficients, k_{ij}, compared to a_{ij}. Hence it is advisable to find [k] first and then find [a] by inverting the matrix [k].

Stiffness influence coefficients:

Let angular displacement of disc 1 be unity ($\theta_1 = 1$) and of discs 2 and 3 be zero as shown in Fig. (B). If the torque applied to disc 1 is M_{11} and the reactions are $M_{\ell 1}$ and M_{21}, we have

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$M_{\ell 1} = k_1 \theta_{11} = k_1$, $M_{21} = k_2 \theta_{11} = k_2$

$M_{11} = M_{\ell 1} + M_{21} \equiv k_{11} = k_1 + k_2$

$k_{21} = -M_{21} = -k_2$ ($\because$ reactive torque is opposite to M_{11} in direction)

$k_{31} = M_{31} = 0$ ($\because$ disc 2 is fixed, no reactive torque is felt at disc 3)

Let displacement of disc 2 = 1 and displacements of discs 1 and 3 be zero.

$k_{22} = k_2 + k_3$, $k_{12} = -k_2$, $k_{32} = -k_3$

Similarly we can obtain

$k_{33} = k_3 + k_4$, $k_{13} = 0$, $k_{23} = -k_3$

stiffness matrix is $[k] = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix}$

Equations of motion of the system:

$$[m]\,\ddot{\vec{\theta}} + [k]\,\vec{\theta} = \vec{M}_t$$

where $[m] = \begin{bmatrix} J_{d1} & 0 & 0 \\ 0 & J_{d2} & 0 \\ 0 & 0 & J_{d3} \end{bmatrix}$ = matrix of mass moments of inertia of the discs

$\vec{\theta} = \begin{Bmatrix} \theta_1(t) \\ \theta_2(t) \\ \theta_3(t) \end{Bmatrix}$ and $\vec{M}_t = \begin{Bmatrix} M_{t1}(t) \\ M_{t2}(t) \\ M_{t3}(t) \end{Bmatrix}$ = vector of external torques applied to discs $= \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$

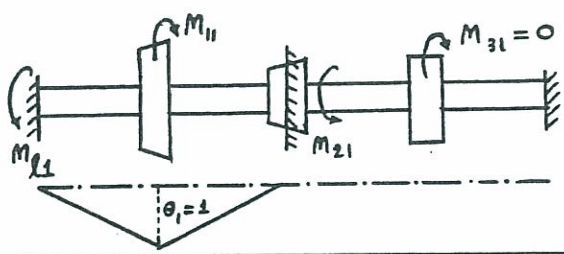
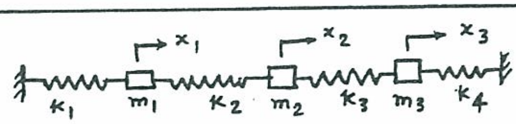


Fig. (B)

6.24 Stiffness influence coefficients:



Let $x_1 = 1$, $x_2 = x_3 = 0$.

Forces required at 1, 2, 3 are

$F_1 = k_1 + k_2 = k_{11}$; $F_2 = -k_2 = k_{21}$; $F_3 = 0 = k_{31}$

Let $x_2 = 1$, $x_1 = x_3 = 0$.

Forces required at 1, 2, 3 are

$F_1 = -k_2 = k_{12}$; $F_2 = k_2 + k_3 = k_{22}$; $F_3 = -k_3 = k_{32}$

Let $x_3 = 1$, $x_1 = x_2 = 0$.

Forces required at 1, 2, 3 are

$F_1 = 0 = k_{13}$; $F_2 = -k_3 = k_{23}$; $F_3 = k_3 + k_4 = k_{33}$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

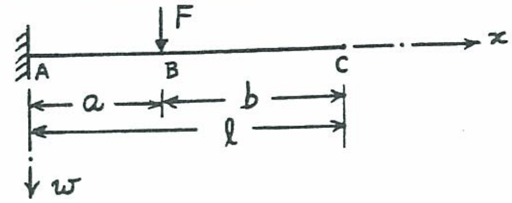
$$\therefore \quad [k] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 \\ -k_2 & (k_2+k_3) & -k_3 \\ 0 & -k_3 & (k_3+k_4) \end{bmatrix}$$

Flexibility influence coefficients:

Procedure and results are similar to those of problem 6.1.

Equations of motion: $[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{F}$

6.25 From strength of materials, the deflection of the cantilever beam shown is given by



$$w(x)\Big|_{in\ AB} = \frac{Fx^2}{6EI}(-x+3a) \quad --- (E_1)$$

$$w(x)\Big|_{in\ BC} = \frac{Fa^2}{6EI}(-a+3x) \quad --- (E_2)$$

Apply $F_1 = 1$, $F_2 = F_3 = 0$: $\quad a_{11} = (F=1,\ x=l,\ a=l \text{ in } (E_1)) = l^3/(3EI)$

$a_{21} = (F=1,\ x=2l,\ a=l \text{ in } (E_2)) = 5l^3/(6EI)$

$a_{31} = (F=1,\ x=3l,\ a=l \text{ in } (E_2)) = 4l^3/(3EI)$

Similarly apply $F_2 = 1$, $F_1 = F_3 = 0$ to get a_{22}, a_{32}, a_{12} and $F_3 = 1$, $F_1 = F_2 = 0$ to get a_{33}, a_{13}, a_{23}. Result is

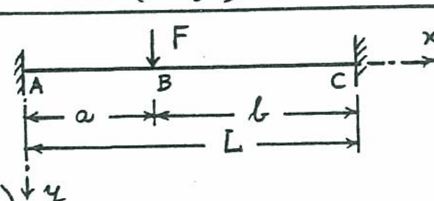
$$[a] = \frac{l^3}{EI}\begin{bmatrix} 1/3 & 5/6 & 4/3 \\ 5/6 & 8/3 & 14/3 \\ 4/3 & 14/3 & 9 \end{bmatrix}$$

Equations of motion:

$$[m]\ddot{\vec{w}} + [k]\vec{w} = \vec{0} \quad \text{or} \quad [a][m]\ddot{\vec{w}} + \vec{w} = \vec{0}$$

with $[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$ and $\vec{w} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}$.

6.26 Deflection of a fixed-fixed beam is



$$y(x)\Big|_{in\ AB} = \frac{Fb^2x^2}{6EIL^3}\{-x(3a+b)+3aL\} \quad --- (E_1)$$

$$y(x)\Big|_{in\ BC} = \frac{Fa^2(L-x)^2}{6EIL^3}\{-(L-x)(3b+a)+3bL\} \quad --- (E_2)$$

Apply $F_1 = 1$, $F_2 = F_3 = 0$:

$a_{11} = (F=1,\ a=l,\ b=3l,\ x=l,\ L=4l \text{ in } (E_1)) = 9l^3/(64EI)$

$a_{21} = (F=1,\ a=l,\ b=3l,\ x=2l,\ L=4l \text{ in } (E_2)) = l^3/(6EI)$

$a_{31} = (F=1,\ a=l,\ b=3l,\ x=3l,\ L=4l \text{ in } (E_2)) = 13l^3/(192EI)$

Similarly apply $F_2 = 1$, $F_1 = F_3 = 0$ to get a_{22}, a_{32}, a_{12} and

PRELIMINARY VERSION 12/14/2010

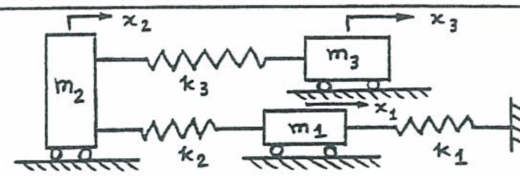
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$F_3 = 1$, $F_1 = F_2 = 0$ to get a_{33}, a_{13}, a_{23}. Result is

$$[a] = \frac{l^3}{EI}\begin{bmatrix} 9/64 & 1/6 & 13/192 \\ 1/6 & 1/3 & 1/6 \\ 13/192 & 1/6 & 9/64 \end{bmatrix}$$

6.27 Flexibility matrix:



a_{11} = deflection of m_1 for a unit load on $m_1 = 1/k_1$

m_2 and m_3 get same displacement (as rigid body motion) as there are no other forces or constraints.

$a_{21} = a_{31} = \frac{1}{k_1}$. If we apply unit load to m_2, equivalent stiffness is given by $\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$. $\quad a_{22} = \frac{1}{k_{eq}} = \frac{k_1 + k_2}{k_1 k_2}$.

Mass m_3 follows deflection of m_2. $\quad a_{32} = a_{22}$.

If we apply unit load to m_3, equivalent stiffness of springs is given by $\frac{1}{k_{eq}} = \frac{1}{k_3} + \frac{1}{k_1} + \frac{1}{k_2}$. $\quad a_{33} = \frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$.

Stiffness matrix:

Let $x_1 = 1$, $x_2 = x_3 = 0$. Forces required at 1, 2, 3 are

$F_1 = k_1 + k_2 = k_{11}$, $\quad F_2 = -k_2 = k_{21}$, $\quad F_3 = 0 = k_{31}$.

Let $x_2 = 1$, $x_1 = x_3 = 0$. Forces required at 1, 2, 3 are

$F_1 = -k_2 = k_{12}$, $\quad F_2 = k_2 + k_3 = k_{22}$, $\quad F_3 = -k_3 = k_{23}$.

Let $x_3 = 1$, $x_1 = x_2 = 0$. Forces required at 1, 2, 3 are

$F_1 = 0 = k_{13}$, $\quad F_2 = -k_3 = k_{23}$, $\quad F_3 = k_3 = k_{33}$.

$$[k] = \begin{bmatrix} (k_1 + k_2) & -k_2 & 0 \\ -k_2 & (k_2 + k_3) & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}; \quad [a] = \begin{bmatrix} 1/k_1 & 1/k_1 & 1/k_1 \\ 1/k_1 & \left(\frac{1}{k_1} + \frac{1}{k_2}\right) & \left(\frac{1}{k_1} + \frac{1}{k_2}\right) \\ 1/k_1 & \left(\frac{1}{k_1} + \frac{1}{k_2}\right) & \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right) \end{bmatrix}$$

6.28 Assume small deflections; hence tension in spring (P) remains constant.

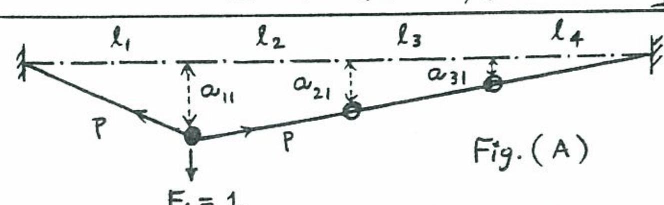


Fig. (A)

Let $F_1 = 1$, $F_2 = F_3 = 0$ as shown in Fig. (A).

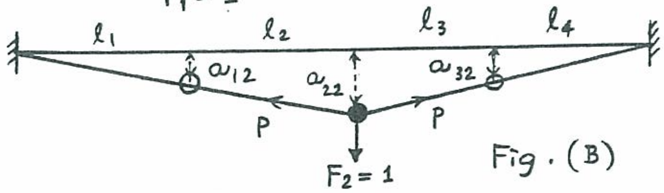


Fig. (B)

Vertical force balance gives

$$F_1 = 1 = \left(\frac{a_{11}}{l_1}\right) P + \left(\frac{a_{11}}{l_2 + l_3 + l_4}\right) P$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$a_{11} = \frac{1}{P\left(\frac{1}{l_1} + \frac{1}{l_2 + l_3 + l_4}\right)}$$

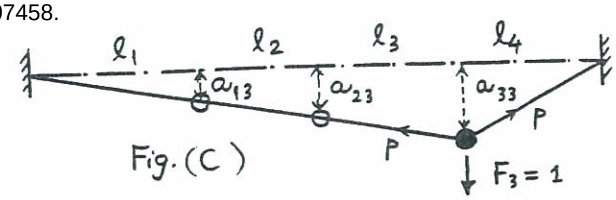


Fig. (C)

From relations of triangles,

$$\frac{a_{11}}{l_2+l_3+l_4} = \frac{a_{21}}{l_3+l_4} \quad \text{and} \quad \frac{a_{11}}{l_2+l_3+l_4} = \frac{a_{31}}{l_4}$$

$$a_{21} = \left(\frac{l_3+l_4}{l_2+l_3+l_4}\right) a_{11}, \quad a_{31} = \left(\frac{l_4}{l_2+l_3+l_4}\right) a_{11}$$

When $F_2 = 1$, $F_1 = F_3 = 0$, vertical force balance gives (Fig. (B))

$$F_2 = 1 = \left(\frac{a_{22}}{l_1+l_2}\right) P + \left(\frac{a_{22}}{l_3+l_4}\right) P \quad \Rightarrow \quad a_{22} = \frac{1}{P\left\{\frac{1}{l_1+l_2} + \frac{1}{l_3+l_4}\right\}}$$

From triangle relations $\quad a_{12} = \left(\frac{l_1}{l_1+l_2}\right) a_{22}, \quad a_{32} = \left(\frac{l_4}{l_3+l_4}\right) a_{22}$

When $F_3 = 1$, $F_1 = F_2 = 0$ (Fig. (C)), vertical force balance gives

$$F_3 = 1 = \left(\frac{a_{33}}{l_1+l_2+l_3}\right) P + \left(\frac{a_{33}}{l_4}\right) P \ ; \quad a_{33} = \frac{1}{P\left(\frac{1}{l_1+l_2+l_3} + \frac{1}{l_4}\right)}$$

From triangle relations, $\quad a_{23} = \left(\frac{l_1+l_2}{l_1+l_2+l_3}\right) a_{33}, \quad a_{13} = \left(\frac{l_1}{l_1+l_2+l_3}\right) a_{33}$

Equations of motion: $\quad [a][m]\ddot{\vec{w}} + \vec{w} = \vec{0}$

with $\quad [m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad \vec{w} = \begin{Bmatrix} w_1 \\ w_2 \\ w_3 \end{Bmatrix}.$

6.29 Let x_1, x_2 and x_3 denote the displacements of top, middle and bottom masses. Equations of motion are

$$m\ddot{x}_1 = -k x_1 - k(x_1 - x_2) - 3k(x_1 - x_3)$$

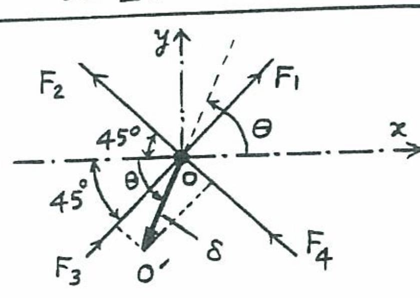
$$2m\ddot{x}_2 = -2k x_2 - k(x_2 - x_1) - k(x_2 - x_3)$$

$$m\ddot{x}_3 = -k x_3 - 3k(x_3 - x_1) - k(x_3 - x_2)$$

i.e.

$$\begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 5k & -k & -3k \\ -k & 4k & -k \\ -3k & -k & 5k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

6.30 Let o move to o' with δ small.



Then

$$F_1 = k\delta\cos(\theta - 45^\circ)$$

$$F_2 = k\delta\cos(135^\circ - \theta)$$

$$F_3 = k\delta\cos(\theta - 45^\circ)$$

$$F_4 = k\delta\cos(135^\circ - \theta)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Force along δ is:

$$F = F_1 \cos(\theta - 45^\circ) + F_2 \cos(135^\circ - \theta) + F_3 \cos(\theta - 45^\circ) + F_4 \cos(135^\circ - \theta)$$

$$= 2k\delta \left[\cos^2(\theta - 45^\circ) + \cos^2(135^\circ - \theta) \right]$$

$$= 2k\delta$$

$\therefore$ Stiffness influence coefficient of junction point in arbitrary direction $= F/\delta = 2k$

6.31 Stiffness matrix is given by Eq. (6.6):

$$[k] = \begin{bmatrix} (k_1+k_2) & -k_2 & 0 & 0 & \cdots & 0 \\ -k_2 & (k_2+k_3) & -k_3 & 0 & \cdots & 0 \\ 0 & -k_3 & (k_3+k_4) & -k_4 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & 0 & \cdots & (k_n+k_{n+1}) \end{bmatrix}$$

All elements except those on the three diagonals are zero. Hence $[k]$ is a band matrix. In fact, it is a tri-diagonal matrix.

6.32 We use the expression of kinetic energy to derive the mass matrix. Let the generalized coordinates be x_1, x_2 and x_3. The kinetic energy of the system is.

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

This can be expressed in matrix form as

$$T = \frac{1}{2} (\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3) \, [m] \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix}$$

with the mass matrix given by

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

6.33 We use the expression of kinetic energy to derive the mass matrix. Using the generalized coordinates θ, x_1 and x_2, the kinetic energy of the system can be expressed as:

$$T = \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2} (2m) \dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2$$

where $J_0 = J_G + (2m) \left(\frac{\ell}{2}\right)^2 = \frac{2}{3} m \ell^2$. T can be expressed in matrix form as:

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$T = \frac{1}{2} (\dot{\theta} \quad \dot{x}_1 \quad \dot{x}_2) [m] \begin{Bmatrix} \dot{\theta} \\ \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} \quad \text{with the mass matrix} \quad [m] = \begin{bmatrix} \frac{2}{3} m \ell^2 & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix}$$

6.34 We derive the mass matrix from the kinetic energy expression. Using the coordinates x_1, x_2 and x_3, the kinetic energy of the system can be expressed as (see figure in the solution of Problem 6.3):

$$T = \frac{1}{2} J_G \dot{\theta}^2 + \frac{1}{2} (2m) \dot{x}_G^2 + \frac{1}{2} (5m) \dot{x}_2^2 \tag{1}$$

$$\text{Using} \quad J_G = \frac{1}{12} (2m) (5\ell)^2 = \frac{25}{6} m \ell^2 \tag{2}$$

$$x_G = \frac{x_1 + x_3}{2}, \quad \theta = \frac{x_1 - x_3}{5\ell} \tag{3}$$

Eq. (1) can be rewritten as:

$$T = \frac{1}{2} \left(\frac{25}{6} m \ell^2 \right) \left(\frac{\dot{x}_1 - \dot{x}_3}{5\ell} \right)^2 + \frac{1}{2} (2m) \left(\frac{\dot{x}_1 + \dot{x}_3}{2} \right)^2 + \frac{1}{2} (5m) \dot{x}_2^2$$

$$= \frac{1}{2} \frac{2m}{3} \dot{x}_1^2 + \frac{1}{2} \frac{2m}{3} \dot{x}_3^2 + \frac{1}{2} \frac{1}{3} m (2 \dot{x}_1 \dot{x}_3) + \frac{1}{2} (5m) \dot{x}_2^2 \tag{4}$$

Equation (4) can be expressed in matrix form as

$$T = \frac{1}{2} (\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3) [m] \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} \quad \text{with the mass matrix} \quad [m] = m \begin{bmatrix} \frac{2}{3} & 0 & \frac{1}{3} \\ 0 & 5 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} \end{bmatrix}$$

6.35 The kinetic energy of the system can be expressed, in terms of the coordinates x_1, x_2 and x_3 as:

$$T = \frac{1}{2} M \dot{x}_1^2 + \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2} (3m) \dot{x}_2^2 + \frac{1}{2} m \dot{x}_3^2 \tag{1}$$

Using the relation $\theta = \dfrac{x_2 - x_1}{3r}$, Eq. (1) can be rewritten as

$$T = \frac{1}{2} \left(M + \frac{J_0}{9r^2} \right) \dot{x}_1^2 + \frac{1}{2} \left(\frac{J_0}{9r^2} + 3m \right) \dot{x}_2^2 - \frac{1}{2} \frac{J_0}{9r^2} (2 \dot{x}_1 \dot{x}_2) + \frac{1}{2} m \dot{x}_3^2 \tag{2}$$

By expressing T in matrix form as

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

$$T = \frac{1}{2}\,(\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3)\,[m]\begin{Bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{Bmatrix}$$

the mass matrix

$$[m] = \begin{bmatrix} \left(M + \frac{J_0}{9\,r^2}\right) & -\frac{J_0}{9\,r^2} & 0 \\ -\frac{J_0}{9\,r^2} & \left(\frac{J_0}{9\,r^2} + 3\,m\right) & 0 \\ 0 & 0 & m \end{bmatrix}$$

6.36 The kinetic energy of the system can be expressed as

$$T = \frac{1}{2}\,I_1\,\dot{\theta}_1^2 + \frac{1}{2}\,I_2\,\dot{\theta}_2^2 + \frac{1}{2}\,I_3\left(\dot{\theta}_2\,\frac{n_2}{n_3}\right)^2 + \frac{1}{2}\,I_4\,\dot{\theta}_3^2 + \frac{1}{2}\,I_5\left(\dot{\theta}_3\,\frac{n_4}{n_5}\right)^2 + \frac{1}{2}\,I_6\,\dot{\theta}_4^2$$

This can be expressed in matrix form as

$$T = \frac{1}{2}\,(\dot{\theta}_1 \quad \dot{\theta}_2 \quad \dot{\theta}_3 \quad \dot{\theta}_4)\,[m]\begin{Bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\end{Bmatrix}$$

and the mass matrix can be identified as

$$[m] = \begin{bmatrix} I_1 & 0 & 0 & 0 \\ 0 & \left(I_2 + I_3\,\frac{n_2^2}{n_3^2}\right) & 0 & 0 \\ 0 & 0 & \left(I_4 + I_5\,\frac{n_4^2}{n_5^2}\right) & 0 \\ 0 & 0 & 0 & I_6 \end{bmatrix}$$

6.37

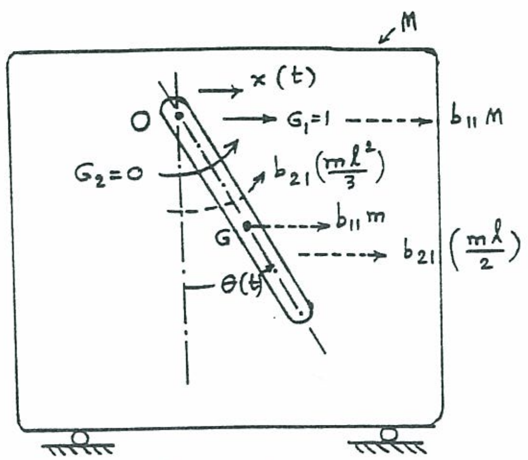


Fig. 1

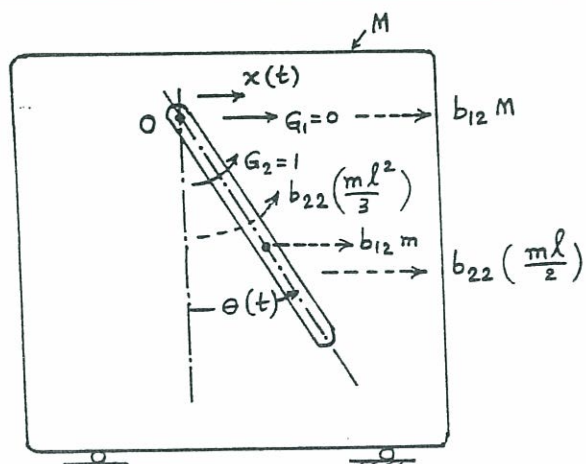


Fig. 2

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

Let x(t) and θ(t) indicate the coordinates to define the linear and angular positions of the trailer (M) and the pendulum (m), respectively. To derive the inverse inertia influence coefficients, first we apply a unit linear impulse at point O (along x(t)) and write the impulse-momentum relations as (see Fig. 1):

Linear impulse-momentum relation along x:

$$\dot{x} = 1 = b_{11}\,(M + m) + b_{21}\,(\frac{m\,\ell}{2}) \tag{1}$$

Angular impulse-momentum relation along θ:

$$\dot{\theta} = 0 = b_{11}\,(\frac{m\,\ell}{2}) + b_{21}\,(\frac{m\,\ell^2}{3}) \tag{2}$$

Solution of Eqs. (1) and (2) gives:

$$b_{11} = \frac{4}{4\,M + m}\ ;\ b_{21} = -\frac{6}{4\,M\,\ell + m\,\ell} \tag{3}$$

Next we apply a unit angular impulse at point O (along θ(t)) and write the impulse-momentum relations as (see Fig. 2):

Linear impulse-momentum relation along x:

$$\dot{x} = 0 = b_{12}\,(M + m) + b_{22}\,(\frac{m\,\ell}{2}) \tag{4}$$

Angular impulse-momentum about O along θ:

$$\dot{\theta} = 1 = b_{12}\,(\frac{m\,\ell}{2}) + b_{22}\,(\frac{m\,\ell^2}{3}) \tag{5}$$

Solution of Eqs. (4) and (5) gives:

$$b_{12} = -\frac{6}{4\,M\,\ell + m\,\ell}\ ;\ b_{22} = \frac{12\,(M + m)}{4\,M\,m\,\ell^2 + m^2\,\ell^2} \tag{6}$$

Thus the inverse mass matrix is given by

$$[b] = [m]^{-1} = \begin{bmatrix} \left(\frac{4}{4\,M + m}\right) & -\left(\frac{6}{4\,M\,\ell + m\,\ell}\right) \\ -\left(\frac{6}{4\,M\,\ell + m\,\ell}\right) & \left(\frac{12\,(M + m)}{4\,M\,m\,\ell^2 + m^2\,\ell^2}\right) \end{bmatrix} \tag{7}$$

6.38 The shear building can be modeled as shown below:

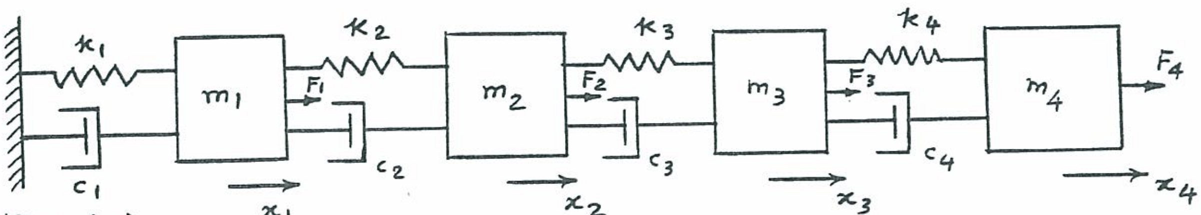


FIG. (A)

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

لجنة الميكانيك - الإتجاه الإسلامي

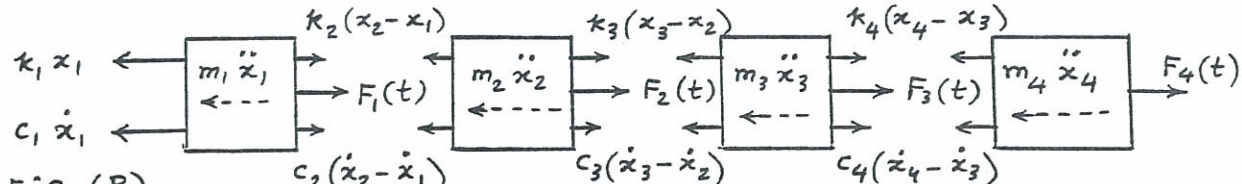


FIG. (B)

(a)
Equations of motion (from free body diagrams in Fig. B):

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1 x_1 - c_2(\dot{x}_2-\dot{x}_1) - k_2(x_2-x_1) = F_1(t)$$
$$m_2\ddot{x}_2 + c_2(\dot{x}_2-\dot{x}_1) + k_2(x_2-x_1) - c_3(\dot{x}_3-\dot{x}_2) - k_3(x_3-x_2) = F_2(t)$$
$$m_3\ddot{x}_3 + c_3(\dot{x}_3-\dot{x}_2) + k_3(x_3-x_2) - c_4(\dot{x}_4-\dot{x}_3) - k_4(x_4-x_3) = F_3(t)$$
$$m_4\ddot{x}_4 + c_4(\dot{x}_4-\dot{x}_3) + k_4(x_4-x_3) = F_4(t) \qquad \text{-----}(E_1)$$

(b) Lagrange's equations are

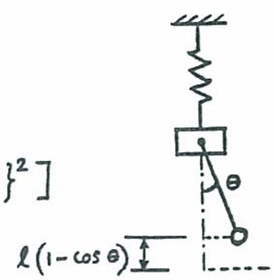
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = Q_j \qquad \text{-----}(E_2)$$

where T = kinetic energy, V = potential energy, R = Rayleigh's dissipation function, Q_j = j^{th} generalized force and q_j = j^{th} generalized coordinate:

$$T = \frac{1}{2}\left\{ m_1\dot{x}_1^2 + m_2\dot{x}_2^2 + m_3\dot{x}_3^2 + m_4\dot{x}_4^2 \right\}$$
$$V = \frac{1}{2}\left\{ k_1 x_1^2 + k_2(x_2-x_1)^2 + k_3(x_3-x_2)^2 + k_4(x_4-x_3)^2 \right\}$$
$$R = \frac{1}{2}\left\{ c_1\dot{x}_1^2 + c_2(\dot{x}_2-\dot{x}_1)^2 + c_3(\dot{x}_3-\dot{x}_2)^2 + c_4(\dot{x}_4-\dot{x}_3)^2 \right\}$$
$$Q_j = F_j \quad ; \quad j = 1,2,3,4$$

Using $q_j = x_j$; $j = 1,2,3,4$, the application of Eqs. (E_2) yields the equations of motion given in (E_1).

6.39 Coordinates of the bob are $(x + l\cos\theta,\ l\sin\theta)$



T = kinetic energy = kinetic energy of slider + kinetic energy of bob

$$= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}m\left[\left\{\frac{d}{dt}(x + l\cos\theta)\right\}^2 + \left\{\frac{d}{dt}(l\sin\theta)\right\}^2\right]$$
$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2(\sin^2\theta + \cos^2\theta) - \frac{1}{2}m(2\dot{x}\,l\sin\theta\ \dot{\theta})$$
$$= m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 - m\dot{x}\dot{\theta}\,l\sin\theta \simeq m\dot{x}^2 + \frac{1}{2}ml^2\dot{\theta}^2 \text{ for small } \theta.$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

V = potential energy = potential energy of spring + potential energy of bob

$$= \frac{1}{2} k x^2 + m g l \, (1 - \cos\theta)$$

(Note: Potential energy of slider need not be considered if $x=0$ corresponds to static equilibrium position)

Since $\cos\theta \simeq 1 - \frac{1}{2}\theta^2$, $\quad V = \frac{1}{2} k x^2 + \frac{1}{2} m g l \theta^2$

As there are no external forces, Lagrange's equations become

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial V}{\partial q_j} = 0 \; ; \quad j = 1, 2$$

Here $q_1 = x$ and $q_2 = \theta$

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial \dot{x}} = 2m\dot{x}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = 2m\ddot{x}, \quad \frac{\partial V}{\partial x} = kx$$

$$\frac{\partial T}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \dot{\theta}} = m l^2 \dot{\theta}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = m l^2 \ddot{\theta}, \quad \frac{\partial V}{\partial \theta} = m g l \theta$$

Lagrange's equations become

$$2m\ddot{x} + kx = 0 \; ; \quad m l^2 \ddot{\theta} + m g l \theta = 0 \text{ or } l\ddot{\theta} + g\theta = 0$$

6.40 (1) With x_1 and x_2 as generalized coordinates:

Since $\quad x_1 = x - l_1 \theta$ and $x_2 = x + l_2 \theta$,

$$x = \left(\frac{x_1 l_2 + x_2 l_1}{l_1 + l_2}\right) \quad \text{and} \quad \theta = \left(\frac{x_2 - x_1}{l_1 + l_2}\right)$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2 = \frac{1}{2} m \left(\frac{\dot{x}_1 l_2 + \dot{x}_2 l_1}{l_1 + l_2}\right)^2 + \frac{1}{2} J_0 \left(\frac{\dot{x}_2 - \dot{x}_1}{l_1 + l_2}\right)^2$$

$$\frac{\partial T}{\partial x_1} = 0, \quad \frac{\partial T}{\partial \dot{x}_1} = \frac{m}{2(l_1+l_2)^2}\left(2 l_2^2 \dot{x}_1 + 2 l_1 l_2 \dot{x}_2\right) + \frac{J_0}{2(l_1+l_2)^2}\left(2\dot{x}_1 - 2\dot{x}_2\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = \frac{m}{(l_1+l_2)^2}\left(l_2^2 \ddot{x}_1 + l_1 l_2 \ddot{x}_2\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_1 - \ddot{x}_2\right)$$

$$\frac{\partial T}{\partial x_2} = 0, \quad \frac{\partial T}{\partial \dot{x}_2} = \frac{m}{2(l_1+l_2)^2}\left(2 l_1^2 \dot{x}_2 + 2 l_1 l_2 \dot{x}_1\right) + \frac{J_0}{2(l_1+l_2)^2}\left(2\dot{x}_2 - 2\dot{x}_1\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = \frac{m}{(l_1+l_2)^2}\left(l_1^2 \ddot{x}_2 + l_1 l_2 \ddot{x}_1\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_2 - \ddot{x}_1\right)$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k x_2^2$$

$$\frac{\partial V}{\partial x_1} = k_1 x_1, \quad \frac{\partial V}{\partial x_2} = k_2 x_2$$

Lagrange's equations, Eq. (6.41), give

$$\frac{m}{(l_1+l_2)^2}\left(l_2^2 \ddot{x}_1 + l_1 l_2 \ddot{x}_2\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_1 - \ddot{x}_2\right) + k_1 x_1 = 0$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\frac{m}{(l_1+l_2)^2}\left(l_1^2 \ddot{x}_2 + l_1 l_2 \ddot{x}_1\right) + \frac{J_0}{(l_1+l_2)^2}\left(\ddot{x}_2 - \ddot{x}_1\right) + k_2 x_2 = 0$$

i.e.

$$\ddot{x}_1\left(m l_2^2 + J_0\right) + \ddot{x}_2\left(m l_1 l_2 - J_0\right) + x_1 (l_1+l_2)^2 k_1 = 0$$

$$\ddot{x}_1\left(m l_1 l_2 - J_0\right) + \ddot{x}_2\left(m l_1^2 + J_0\right) + x_2 (l_1+l_2)^2 k_2 = 0$$

(2) With x and θ as generalized coordinates:

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J_0 \dot{\theta}^2$$

$$V = \frac{1}{2} k_1 (x - l_1\theta)^2 + \frac{1}{2} k_2 (x + l_2\theta)^2$$

$$\frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial \dot{x}} = m\dot{x}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}}\right) = m\ddot{x}, \quad \frac{\partial V}{\partial x} = k_1(x - l_1\theta) + k_2(x + l_2\theta)$$

$$\frac{\partial T}{\partial \theta} = 0, \quad \frac{\partial T}{\partial \dot{\theta}} = J_0\dot{\theta}, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = J_0\ddot{\theta}, \quad \frac{\partial V}{\partial \theta} = -k_1 l_1 (x - l_1\theta) + k_2 l_2 (x + l_2\theta)$$

Lagrange's equations give

$$m\ddot{x} + k_1(x - l_1\theta) + k_2(x + l_2\theta) = 0$$

$$J_0\ddot{\theta} - k_1 l_1 (x - l_1\theta) + k_2 l_2 (x + l_2\theta) = 0$$

6.41

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_2 - x_1)^2 + \frac{1}{2} k_3 (x_3 - x_2)^2 + \frac{1}{2} k_4 x_3^2$$

$$\frac{\partial T}{\partial x_1} = 0, \quad \frac{\partial T}{\partial \dot{x}_1} = m_1\dot{x}_1, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = m_1\ddot{x}_1, \quad \frac{\partial V}{\partial x_1} = k_1 x_1 - k_2(x_2 - x_1)$$

$$\frac{\partial T}{\partial x_2} = 0, \quad \frac{\partial T}{\partial \dot{x}_2} = m_2\dot{x}_2, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m_2\ddot{x}_2, \quad \frac{\partial V}{\partial x_2} = k_2(x_2 - x_1) - k_3(x_3 - x_2)$$

$$\frac{\partial T}{\partial x_3} = 0, \quad \frac{\partial T}{\partial \dot{x}_3} = m_3\dot{x}_3, \quad \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_3}\right) = m_3\ddot{x}_3, \quad \frac{\partial V}{\partial x_3} = k_3(x_3 - x_2) + k_4 x_3$$

Lagrange's equations give

$$m_1\ddot{x}_1 + x_1(k_1 + k_2) - k_2 x_2 = 0$$

$$m_2\ddot{x}_2 - k_2 x_1 + x_2(k_2 + k_3) - k_3 x_3 = 0$$

$$m_3\ddot{x}_3 - k_3 x_2 + x_3(k_3 + k_4) = 0$$

6.42

Using θ_1, θ_2 and θ_3 as generalized coordinates, we find

$$T \simeq \frac{1}{2} m_1 (l_1\dot{\theta}_1)^2 + \frac{1}{2} m_2 (l_1\dot{\theta}_1 + l_2\dot{\theta}_2)^2 + \frac{1}{3} m_3 (l_1\dot{\theta}_1 + l_2\dot{\theta}_2 + l_3\dot{\theta}_3)^2 \quad \text{---} (E_1)$$

Let reference position correspond to $\theta_1 = \theta_2 = \theta_3 = 0$.

Vertical movement of m_1 is

$$l_1(1 - \cos\theta_1) \simeq l_1\left\{1 - \left(1 - \frac{\theta_1^2}{2}\right)\right\} = \frac{1}{2} l_1 \theta_1^2$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

vertical movement of $m_2 = \frac{1}{2} l_1 \theta_1^2 + \frac{1}{2} l_2 \theta_2^2$

vertical movement of $m_3 = \frac{1}{2} l_1 \theta_1^2 + \frac{1}{2} l_2 \theta_2^2 + \frac{1}{2} l_3 \theta_3^2$

$$V = \frac{m_1 g\, l_1 \theta_1^2}{2} + \frac{m_2 g}{2}\left(l_1\theta_1^2 + l_2\theta_2^2\right) + \frac{m_3 g}{2}\left(l_1\theta_1^2 + l_2\theta_2^2 + l_3\theta_3^2\right) \text{---} (E_2)$$

$$\frac{\partial T}{\partial \theta_1} = 0, \quad \frac{\partial T}{\partial \dot{\theta}_1} = m_1 l_1^2 \dot{\theta}_1 + m_2 l_1 \left(l_1\dot{\theta}_1 + l_2\dot{\theta}_2\right) + m_3 l_1 \left(l_1\dot{\theta}_1 + l_2\dot{\theta}_2 + l_3\dot{\theta}_3\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = (m_1+m_2+m_3)\, l_1^2 \ddot{\theta}_1 + (m_2+m_3)\, l_1 l_2 \ddot{\theta}_2 + m_3 l_1 l_3 \ddot{\theta}_3,$$

$$\frac{\partial V}{\partial \theta_1} = (m_1+m_2+m_3)\, g\, l_1 \theta_1$$

$$\frac{\partial T}{\partial \theta_2} = 0, \quad \frac{\partial T}{\partial \dot{\theta}_2} = m_2 l_2 \left(l_1\dot{\theta}_1 + l_2\dot{\theta}_2\right) + m_3 l_2 \left(l_1\dot{\theta}_1 + l_2\dot{\theta}_2 + l_3\dot{\theta}_3\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = (m_2+m_3)\, l_1 l_2 \ddot{\theta}_1 + (m_2+m_3)\, l_2^2 \ddot{\theta}_2 + m_3 l_2 l_3 \ddot{\theta}_3,$$

$$\frac{\partial V}{\partial \theta_2} = m_2 g\, l_2 \theta_2 + m_3 g\, l_2 \theta_2 = (m_2+m_3)\, g\, l_2 \theta_2$$

$$\frac{\partial T}{\partial \theta_3} = 0, \quad \frac{\partial T}{\partial \dot{\theta}_3} = m_3 l_3 \left(l_1\dot{\theta}_1 + l_2\dot{\theta}_2 + l_3\dot{\theta}_3\right),$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right) = m_3 l_3 \left(l_1\ddot{\theta}_1 + l_2\ddot{\theta}_2 + l_3\ddot{\theta}_3\right), \quad \frac{\partial V}{\partial \theta_3} = m_3 g\, l_3 \theta_3$$

Lagrange's equations give the equations of motion

$$\begin{bmatrix} (m_1+m_2+m_3)\, l_1^2 & (m_2+m_3)\, l_1 l_2 & m_3 l_1 l_3 \\ (m_2+l_3)\, l_1 l_2 & (m_2+m_3)\, l_2^2 & m_3 l_2 l_3 \\ m_3 l_1 l_3 & m_3 l_2 l_3 & m_3 l_3^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix}$$

$$+ \begin{bmatrix} (m_1+m_2+m_3)\, g\, l_1 & 0 & 0 \\ 0 & (m_2+m_3)\, g\, l_2 & 0 \\ 0 & 0 & m_3 g\, l_3 \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \text{---} (E_3)$$

6.43 (a) Let generalized coordinates be $q_1 = x$ and $q_2 = \theta$

Displacement of mass M is $x + l\theta$.

Kinetic energy of airplane is $T = \frac{1}{2} M_0 \dot{x}^2 + 2\left\{\frac{1}{2} M \left(\dot{x} + l\dot{\theta}\right)^2\right\}$

Potential energy is $V = 2\left(\frac{1}{2} k_t\, \theta^2\right)$

Lagrange's equations are $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i\,; \quad i = 1, 2$

Here $\frac{\partial T}{\partial x} = 0, \quad \frac{\partial T}{\partial \dot{x}} = M_0 \dot{x} + 2M\left(\dot{x} + l\dot{\theta}\right), \quad \frac{\partial V}{\partial x} = 0$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$\frac{\partial T}{\partial \theta} = 0$, $\frac{\partial T}{\partial \dot{\theta}} = 2 M \ell (\dot{x} + \ell \dot{\theta})$, $\frac{\partial V}{\partial \theta} = 2 k_t \theta$, $Q_1 = Q_2 = 0$

Hence Lagrange's equations become

$$\left.\begin{aligned} M_0 \ddot{x} + 2M(\ddot{x} + \ell \ddot{\theta}) &= 0 \\ 2M\ell(\ddot{x} + \ell\ddot{\theta}) + 2k_t\theta &= 0 \end{aligned}\right\} \qquad (E_1)$$

(b) If $x(t) = X \cos(\omega t + \phi)$ and $\theta(t) = \Theta \cos(\omega t + \phi)$, the equations of motion become

$$\left.\begin{aligned} (-M_0\omega^2 - 2M\omega^2) X - 2M\ell\omega^2 \Theta &= 0 \\ -2M\ell\omega^2 X - 2M\ell^2\omega^2\Theta + 2k_t\Theta &= 0 \end{aligned}\right\} \qquad (E_2)$$

which yields the frequency equation:

$$\begin{vmatrix} M_0\omega^2 + 2M\omega^2 & 2M\ell\omega^2 \\ 2M\ell\omega^2 & 2M\ell^2\omega^2 - 2k_t \end{vmatrix} = 0$$

i.e., $\omega^2 \left[2 M_0 M \ell^2 \omega^2 - 2 M_0 k_t - 4 M k_t \right] = 0$

i.e., $\omega^2 = 0$; $\omega^2 = \left(\frac{2 M_0 k_t + 4 M k_t}{2 M_0 M \ell^2} \right)$

Mode shapes: Eq. (E_2) gives

$$\frac{\Theta}{X} = \frac{2M\ell\omega^2}{-2M\ell^2\omega^2 + 2k_t}$$

For $\omega_1 = 0$; $\left.\frac{\Theta}{X}\right|_{\omega_1} = \frac{0}{2k_t} = 0 \Rightarrow \Theta = 0$

This corresponds to rigid body translation in x (vertical) direction.

For ω_2 ; $$\left.\frac{\Theta}{X}\right|_{\omega_2} = \frac{2M\ell\left(\frac{2M_0 k_t + 4M k_t}{2 M_0 M \ell^2}\right)}{-2M\ell^2\left(\frac{2M_0 k_t + 4Mk_t}{2M_0 M\ell^2}\right) + 2k_t}$$

$$= -\left(\frac{M_0}{2M\ell} + \frac{1}{\ell}\right)$$

(c) For $\omega_2 > 4\pi$ rad/sec,

$$\left(\frac{2M_0 k_t + 4Mk_t}{2MM_0\ell^2}\right) > 16\pi^2 \qquad (E_3)$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

When $M_0 = 1000$ kg, $M = 500$ kg and $l = 6$ m, inequality (E_3) becomes

$$\frac{2000\ k_t + 2000\ k_t}{(1 \times 10^6)(36)} > 16\pi^2$$

i.e., $k_t > 1.4212 \times 10^6$ N-m/rad.

6.44 Generalized coordinates: x_1, x_2, x_3:

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 + \frac{1}{2} m_3 \dot{x}_3^2$$

$$V = \frac{1}{2} k x_1^2 + \frac{1}{2} k (x_2 - x_1)^2 + \frac{1}{2} k (x_3 - x_2)^2 + \frac{1}{2} k x_3^2 + \frac{1}{2} (5k)(x_3 - x_1)^2$$

$$Q_i = F_i \ ; \ i = 1, 2, 3$$

$$\frac{\partial T}{\partial \dot{x}_i} = m_i \dot{x}_i \ ; \ i = 1, 2, 3 \qquad \frac{\partial V}{\partial x_1} = 7 k x_1 - k x_2 - 5 k x_3$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_i}\right) = m_i \ddot{x}_i \ ; \ i = 1, 2, 3 \qquad \frac{\partial V}{\partial x_2} = - k x_1 + 2 k x_2 - k x_3$$

$$\frac{\partial T}{\partial x_i} = 0 \ ; \ i = 1, 2, 3 \qquad \frac{\partial V}{\partial x_3} = - 5 k x_1 - k x_2 + 7 k x_3$$

Lagrange's equations yield the equations of motion:

$$m_1 \ddot{x}_1 + 7 k x_1 - k x_2 - 5 k x_3 = F_1(t)$$
$$m_2 \ddot{x}_2 - k x_1 + 2 k x_2 - k x_3 = F_2(t)$$
$$m_3 \ddot{x}_3 - 5 k x_1 - k x_2 + 7 k x_3 = F_3(t)$$

6.45 Generalized coordinates: θ, x_1, x_2:

$$T = \frac{1}{2} J_0 \dot{\theta}^2 + \frac{1}{2}(2m)\dot{x}_1^2 + \frac{1}{2} m \dot{x}_2^2$$

where $J_0 = \frac{1}{3}(2m)\ell^2 = \frac{2}{3} m \ell^2$.

$$V = \frac{1}{2}(2k)(x_1 - \frac{\ell}{4}\theta)^2 + \frac{1}{2} k (x_2 - x_1)^2 + \frac{1}{2}(3k)(\theta \ell)^2$$

$$R = \frac{1}{2} c (\dot{x}_1 - \frac{\ell}{4}\dot{\theta})^2$$

$$Q_\theta = M_t(t) \ ; \ Q_1 = F_1(t) \ ; \ Q_2 = F_2(t)$$

$$\frac{\partial T}{\partial \dot{\theta}} = J_0 \dot{\theta} \ ; \ \frac{\partial T}{\partial \dot{x}_1} = 2 m \dot{x}_1 \ ; \ \frac{\partial T}{\partial \dot{x}_2} = m \dot{x}_2$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = J_0\,\ddot{\theta}\ ;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = 2\,m\,\ddot{x}_1\ ;\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = m\,\ddot{x}_2$$

$$\frac{\partial T}{\partial \theta} = 0\ ;\ \frac{\partial T}{\partial x_i} = 0\ ;\ i = 1,\ 2$$

$$\frac{\partial R}{\partial \dot{\theta}} = -\,c\,\frac{\ell}{4}\,(\dot{x}_1 - \dot{\theta}\,\frac{\ell}{4})\ ;\ \frac{\partial R}{\partial \dot{x}_1} = c\,(\dot{x}_1 - \dot{\theta}\,\frac{\ell}{4})\ ;\ \frac{\partial R}{\partial \dot{x}_2} = 0$$

$$\frac{\partial V}{\partial \theta} = \left(\frac{25}{8}\,k\,\ell^2\right)\theta - \frac{1}{2}\,k\,\ell\,x_1$$

$$\frac{\partial V}{\partial x_1} = -\,\frac{1}{2}\,k\,\ell\,\theta + 3\,k\,x_1 - k\,x_2$$

$$\frac{\partial V}{\partial x_2} = -\,k\,x_1 + k\,x_2$$

Lagrange's equations, Eqs. (6.118), yield the equations of motion as:

$$\frac{2}{3}\,m\,\ell^2\,\ddot{\theta} + \frac{1}{16}\,c\,\ell^2\,\dot{\theta} - \frac{1}{4}\,c\,\ell\,\dot{x}_1 + \frac{25}{8}\,k\,\ell^2\,\theta - \frac{1}{2}\,k\,\ell\,x_1 = M_t(t)$$
$$2\,m\,\ddot{x}_1 - \frac{1}{4}\,c\,\ell\,\dot{\theta} + c\,\dot{x}_1 - \frac{1}{2}\,k\,\ell\,\theta + 3\,k\,x_1 - k\,x_2 = F_2(t)$$
$$m\,\ddot{x}_2 - k\,x_1 + k\,x_2 = F_2(t)$$

6.46 Generalized coordinates: x_i ; $i = 1,\ 2,\ 3$:

$$T = \frac{1}{2}\,J_G\,\dot{\theta}^2 + \frac{1}{2}\,(2m)\,\dot{x}_G^2 + \frac{1}{2}\,(5m)\,\dot{x}_2^2$$

$$= \frac{1}{2}\left(\frac{25}{6}\,m\,\ell^2\right)\left(\frac{\dot{x}_1 - \dot{x}_3}{5\,\ell}\right)^2 + \frac{1}{2}\,(2m)\left(\frac{\dot{x}_1 + \dot{x}_3}{2}\right)^2 + \frac{1}{2}\,(5m)\,\dot{x}_2^2$$

where the subscript G denotes the mass center of the bar with

$$J_G = \frac{1}{12}\,(2m)\,(5\,\ell)^2 = \frac{25}{6}\,m\,\ell^2\ ;\ \theta = \frac{x_1 - x_3}{5\,\ell}\ ;\ x_G = \frac{x_1 + x_3}{2}$$

$$V = \frac{1}{2}\,k\,x_1^2 + \frac{1}{2}\,k\,x_3^2 + \frac{1}{2}\,k\,(x_A - x_2)^2$$
$$= \frac{1}{2}\,k\,x_1^2 + \frac{1}{2}\,k\,x_3^2 + \frac{1}{2}\,k\,(\frac{3\,x_1 + 2\,x_3}{5} - x_2)^2$$

$$\text{since}\quad x_A = x_1 - \frac{2}{5}\,(x_1 - x_3) = \frac{3\,x_1 + 2\,x_3}{5}$$

$$R = \frac{1}{2}\,c\,(\dot{x}_A - \dot{x}_2)^2 = \frac{1}{2}\,c\left(\frac{3\,\dot{x}_1 + 2\,\dot{x}_3}{5} - \dot{x}_2\right)^2$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$Q_i(t) = F_i(t) \;\; ; \;\; i = 1,\, 2,\, 3$$

$$\frac{\partial T}{\partial \dot{x}_1} = \left(\frac{25\, m\, \ell^2}{6}\right)\left(\frac{1}{5\, \ell}\right)\left(\frac{\dot{x}_1 - \dot{x}_3}{5\, \ell}\right) + (2m)\, \frac{1}{2}\left(\frac{\dot{x}_1 + \dot{x}_3}{2}\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = \left(\frac{25\, m\, \ell^2}{6}\right)\left(\frac{1}{25\, \ell^2}\right)(\ddot{x}_1 - \ddot{x}_3) + \frac{m}{2}\,(\ddot{x}_1 + \ddot{x}_3)$$

$$\frac{\partial T}{\partial \dot{x}_2} = (5m)\, \dot{x}_2 \;\; ; \;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = (5m)\, \ddot{x}_2$$

$$\frac{\partial T}{\partial \dot{x}_3} = -\left(\frac{25\, m\, \ell^2}{6}\right)\left(\frac{1}{5\, \ell}\right)\left(\frac{\dot{x}_1 - \dot{x}_3}{5\, \ell}\right) + (2m)\, \frac{1}{2}\left(\frac{\dot{x}_1 + \dot{x}_3}{2}\right)$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_3}\right) = -\left(\frac{25\, m\, \ell^2}{6}\right)\left(\frac{1}{25\, \ell^2}\right)(\ddot{x}_1 - \ddot{x}_3) + \frac{m}{2}\,(\ddot{x}_1 + \ddot{x}_3)$$

$$\frac{\partial T}{\partial x_i} = 0 \;\; ; \;\; i = 1,\, 2,\, 3$$

$$\frac{\partial R}{\partial \dot{x}_1} = \frac{3}{5}\, c\left(\frac{3\, \dot{x}_1 + 2\, \dot{x}_3}{5} - \dot{x}_2\right) \qquad \frac{\partial V}{\partial x_1} = k\, x_1 + \frac{3}{5}\, k\left(\frac{3\, x_1 + 2\, x_3}{5} - x_2\right)$$

$$\frac{\partial R}{\partial \dot{x}_3} = \frac{2}{5}\, c\left(\frac{3\, \dot{x}_1 + 2\, \dot{x}_3}{5} - \dot{x}_2\right) \qquad \frac{\partial V}{\partial x_2} = -\, k\left(\frac{3\, x_1 + 2\, x_3}{5} - x_2\right)$$

$$\frac{\partial R}{\partial \dot{x}_2} = -\, c\left(\frac{3\, \dot{x}_1 + 2\, \dot{x}_3}{5} - \dot{x}_2\right) \qquad \frac{\partial V}{\partial x_3} = k\, x_3 + \frac{2}{5}\, k\left(\frac{3\, x_1 + 2\, x_3}{5} - x_2\right)$$

Application of Lagrange's equations gives the equations of motion as:

$$\frac{2}{3}\, m\, \ddot{x}_1 + \frac{1}{3}\, m\, \ddot{x}_3 + \frac{9}{25}\, c\, \dot{x}_1 - \frac{3}{5}\, c\, \dot{x}_2 + \frac{6}{25}\, c\, \dot{x}_3 + \frac{34}{25}\, k\, x_1 - \frac{3}{5}\, k\, x_2 + \frac{6}{25}\, k\, x_3 = F_1(t)$$

$$5\, m\, \ddot{x}_2 - \frac{3}{5}\, c\, \dot{x}_1 + c\, \dot{x}_2 - \frac{2}{5}\, c\, \dot{x}_3 - \frac{3}{5}\, k\, x_1 + k\, x_2 - \frac{2}{5}\, k\, x_3 = F_2(t)$$

$$\frac{1}{3}\, m\, \ddot{x}_1 + \frac{2}{3}\, m\, \ddot{x}_3 + \frac{6}{25}\, c\, \dot{x}_1 - \frac{2}{5}\, c\, \dot{x}_2 + \frac{4}{25}\, c\, \dot{x}_3 + \frac{6}{25}\, k\, x_1 - \frac{2}{5}\, k\, x_2 + \frac{29}{25}\, k\, x_3 = F_3(t)$$

6.47

$$T = \frac{1}{2}\, M\, \dot{x}_1^{\,2} + \frac{1}{2}\, J_0\, \dot{\theta}^2 + \frac{1}{2}\,(3m)\, \dot{x}_2^{\,2} + \frac{1}{2}\, m\, \dot{x}_3^{\,2}$$

$$V = \frac{1}{2}\, k\, x_1^2 + \frac{1}{2}\,(2k)\,(x_1 - x_3 - r\, \theta)^2 + \frac{1}{2}\,(3k)\, x_3^2$$

$$Q_i = F_i \;\; ; \;\; i = 1,\, 2,\, 3$$

Noting that $\theta = \left(\dfrac{x_2 - x_1}{3\, r}\right)$, we can express

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\frac{\partial T}{\partial \dot{x}_1} = M\,\dot{x}_1 + J_0 \left(-\frac{1}{3\,r}\right)\left(\frac{\dot{x}_2 - \dot{x}_1}{3\,r}\right) \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_1}\right) = M\,\ddot{x}_1 - \frac{J_0}{3\,r}\left(\frac{\ddot{x}_2 - \ddot{x}_1}{3\,r}\right)$$

$$\frac{\partial T}{\partial \dot{x}_2} = J_0 \left(\frac{1}{3\,r}\right)\left(\frac{\dot{x}_2 - \dot{x}_1}{3\,r}\right) + 3\,m\,\dot{x}_2 \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_2}\right) = \frac{J_0}{9\,r^2}\,(\ddot{x}_2 - \ddot{x}_1) + 3\,m\,\ddot{x}_2$$

$$\frac{\partial T}{\partial \dot{x}_3} = m\,\dot{x}_3 \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{x}_3}\right) = m\,\ddot{x}_3$$

$$\frac{\partial V}{\partial x_1} = k\,x_1 + (2k)\,\frac{4}{3}\,(\frac{4}{3}\,x_1 - \frac{1}{3}\,x_2 - x_3)$$

$$\frac{\partial V}{\partial x_2} = -\,\frac{1}{3}\,(2k)\,(\frac{4}{3}\,x_1 - \frac{1}{3}\,x_2 - x_3)$$

$$\frac{\partial V}{\partial x_3} = -\,(2k)\,(\frac{4}{3}\,x_1 - \frac{1}{3}\,x_2 - x_3) + (3k)\,x_3$$

Application of Lagrange's equations give the equations of motion:

$$\left(M + \frac{J_0}{9\,r^2}\right)\ddot{x}_1 - \frac{J_0}{9\,r^2}\,\ddot{x}_2 + \frac{41}{9}\,k\,x_1 - \frac{8}{9}\,k\,x_2 - \frac{8}{3}\,k\,x_3 = F_1(t)$$

$$-\,\frac{J_0}{9\,r^2}\,\ddot{x}_1 + \left(3\,m + \frac{J_0}{9\,r^2}\right)\ddot{x}_2 - \frac{8}{9}\,k\,x_1 + \frac{2}{9}\,k\,x_2 + \frac{2}{3}\,k\,x_3 = F_2(t)$$

$$m\,\ddot{x}_3 - \frac{8}{3}\,k\,x_1 + \frac{2}{3}\,k\,x_2 + 5\,k\,x_3 = F_3(t)$$

6.48 Generalized coordinates: θ_i ; $i = 1, 2, 3, 4$:

$$T = \frac{1}{2}\,I_1\,\dot{\theta}_1^2 + \frac{1}{2}\left(I_2 + I_3\,\frac{n_2^2}{n_3^2}\right)\dot{\theta}_2^2 + \frac{1}{2}\left(I_4 + I_5\,\frac{n_4^2}{n_5^2}\right)\dot{\theta}_3^2 + \frac{1}{2}\,I_6\,\dot{\theta}_4^2$$

$$V = \frac{1}{2}\,k_{t1}\,(\theta_2 - \theta_1)^2 + \frac{1}{2}\,k_{t2}\left(\theta_3 - \theta_2\,\frac{n_2}{n_3}\right)^2 + \frac{1}{2}\,k_{t3}\left(\theta_4 - \theta_3\,\frac{n_4}{n_5}\right)^2$$

$$Q_1 = M_1\,\cos\omega\,t$$

$$\frac{\partial T}{\partial \dot{\theta}_1} = I_1\,\dot{\theta}_1 \;;\; \frac{\partial T}{\partial \dot{\theta}_2} = \left(I_2 + I_3\,\frac{n_2^2}{n_3^2}\right)\dot{\theta}_2$$

$$\frac{\partial T}{\partial \dot{\theta}_3} = \left(I_4 + I_5\,\frac{n_4^2}{n_5^2}\right)\dot{\theta}_3 \;;\; \frac{\partial T}{\partial \dot{\theta}_4} = I_6\,\dot{\theta}_4$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_1}\right) = I_1\,\ddot{\theta}_1 \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_2}\right) = \left(I_2 + I_3\,\frac{n_2^2}{n_3^2}\right)\ddot{\theta}_2$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_3}\right) = \left(I_4 + I_5\,\frac{n_4^2}{n_5^2}\right)\ddot{\theta}_3 \;;\; \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}_4}\right) = I_6\,\ddot{\theta}_4$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\frac{\partial V}{\partial \theta_1} = -k_{t1}(\theta_2 - \theta_1)$$

$$\frac{\partial V}{\partial \theta_2} = k_{t1}(\theta_2 - \theta_1) - k_{t2}\left(\theta_3 - \theta_2\frac{n_2}{n_3}\right)\frac{n_2}{n_3}$$

$$\frac{\partial V}{\partial \theta_3} = k_{t2}\left(\theta_3 - \theta_2\frac{n_2}{n_3}\right) - k_{t3}\left(\theta_4 - \theta_3\frac{n_4}{n_5}\right)\frac{n_4}{n_5}$$

$$\frac{\partial V}{\partial \theta_4} = k_{t3}\left(\theta_4 - \theta_3\frac{n_4}{n_5}\right)$$

Equations of motion (from Lagrange's equations):

$$I_1\ddot{\theta}_1 - k_{t1}(\theta_2 - \theta_1) = M_1\cos\omega t$$

$$\left(I_2 + I_3\frac{n_2^2}{n_3^2}\right)\ddot{\theta}_2 + k_{t1}(\theta_2 - \theta_1) - k_{t2}\left(\theta_3 - \theta_2\frac{n_2}{n_3}\right)\frac{n_2}{n_3} = 0$$

$$\left(I_4 + I_5\frac{n_4^2}{n_5^2}\right)\ddot{\theta}_3 + k_{t2}\left(\theta_3 - \theta_2\frac{n_2}{n_3}\right) - k_{t3}\left(\theta_4 - \theta_3\frac{n_4}{n_5}\right)\frac{n_4}{n_5} = 0$$

$$I_6\ddot{\theta}_4 + k_{t3}\left(\theta_4 - \theta_3\frac{n_4}{n_5}\right) = 0$$

6.49 Equations of motion for the system of Fig. 6.8(a) are:

$$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = 0 \qquad \text{--- } (E_1)$$

$$m_2\ddot{x}_2 - k_2(x_1 - x_2) + k_3(x_2 - x_3) = 0 \qquad \text{--- } (E_2)$$

$$m_3\ddot{x}_3 - k_3(x_2 - x_3) = 0 \qquad \text{--- } (E_3)$$

Let $q_1 = x_1$, $q_2 = x_2 - x_1$ and $q_3 = x_3 - x_2$. Eqs. (E_1) to (E_3) become

$$m_1\ddot{x}_1 + k_1q_1 - k_2q_2 = 0 \Rightarrow \ddot{x}_1 + \frac{k_1}{m_1}q_1 - \frac{k_2}{m_1}q_2 = 0 \qquad \text{--- } (E_4)$$

$$m_2\ddot{x}_2 + k_2q_2 - k_3q_3 = 0 \Rightarrow \ddot{x}_2 + \frac{k_2}{m_2}q_2 - \frac{k_3}{m_2}q_3 = 0 \qquad \text{--- } (E_5)$$

$$m_3\ddot{x}_3 + k_3q_3 = 0 \Rightarrow \ddot{x}_3 + \frac{k_3}{m_3}q_3 = 0 \qquad \text{--- } (E_6)$$

(E_4) minus (E_5) gives $\quad (\ddot{x}_2 - \ddot{x}_1) - \frac{k_1}{m_1}q_1 + \left(\frac{k_2}{m_2} + \frac{k_2}{m_1}\right)q_2 - \frac{k_3}{m_2}q_3 = 0 \quad \text{--- } (E_7)$

(E_5) minus (E_6) gives $\quad (\ddot{x}_3 - \ddot{x}_2) - \frac{k_2}{m_2}q_2 + q_3\left(\frac{k_3}{m_3} + \frac{k_3}{m_2}\right) = 0 \quad \text{--- } (E_8)$

(E_4), (E_7) and (E_8) can be expressed as

$$\left.\begin{aligned}
&\ddot{q}_1 + \frac{k_1}{m_1}q_1 - \frac{k_2}{m_1}q_2 = 0\\
&\ddot{q}_2 - \frac{k_1}{m_1}q_1 + \left(\frac{k_2}{m_2} + \frac{k_2}{m_1}\right)q_2 - \frac{k_3}{m_2}q_3 = 0\\
&\ddot{q}_3 - \frac{k_2}{m_2}q_2 + \left(\frac{k_3}{m_3} + \frac{k_3}{m_2}\right)q_3 = 0
\end{aligned}\right\} \qquad \text{--- } (E_9)$$

For $k_i = k$ and $m_i = m$ $(i = 1, 2, 3)$, Eqs. (E_9) reduce to

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\left.\begin{array}{l} \ddot{q}_1 + \frac{k}{m} q_1 - \frac{k}{m} q_2 = 0 \\ \ddot{q}_2 - \frac{k}{m} q_1 + 2\frac{k}{m} q_2 - \frac{k}{m} q_3 = 0 \\ \ddot{q}_3 - \frac{k}{m} q_1 + 2\frac{k}{m} q_3 = 0 \end{array}\right\} \quad ---(E_{10})$$

For $q_i(t) = Q_i \cos(\omega t + \phi)$; $i = 1, 2, 3$, (E_{10}) give

$$\begin{bmatrix} \left(-\omega^2 + \frac{k}{m}\right) & -\frac{k}{m} & 0 \\ -\frac{k}{m} & \left(-\omega^2 + 2\frac{k}{m}\right) & -\frac{k}{m} \\ 0 & -\frac{k}{m} & \left(-\omega^2 + 2\frac{k}{m}\right) \end{bmatrix} \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad ---(E_{11})$$

Frequency equation is

$$\begin{vmatrix} -\omega^2 + \frac{k}{m} & -\frac{k}{m} & 0 \\ -\frac{k}{m} & -\omega^2 + 2\frac{k}{m} & -\frac{k}{m} \\ 0 & -\frac{k}{m} & -\omega^2 + 2\frac{k}{m} \end{vmatrix} = \omega^6 - 5\omega^4 \frac{k}{m} + 6\omega^2 \frac{k^2}{m^2} - \frac{k^3}{m^3} = 0$$

i.e. $\alpha^3 - 5\alpha^2 + 6\alpha - 1 = 0$ where $\alpha = \frac{\omega^2 m}{k}$.

Roots of this equation give

$\alpha_1 = 0.19806$, $\omega_1 = 0.44504 \sqrt{\frac{k}{m}}$

$\alpha_2 = 1.5553$, $\omega_2 = 1.2471 \sqrt{\frac{k}{m}}$

$\alpha_3 = 3.2490$, $\omega_3 = 1.8025 \sqrt{\frac{k}{m}}$

It can be seen that the eigenvalues are same in both problems.

6.50 Equations of motion (from problem 6.24):

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

For harmonic motion $x_i(t) = X_i \cos(\omega t + \phi)$; $i = 1, 2, 3$, we get

$$\begin{bmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 & 0 \\ -k_2 & -\omega^2 m_2 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & -\omega^2 m_3 + k_3 + k_4 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Frequency equation is

$$\begin{vmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 & 0 \\ -k_2 & -\omega^2 m_2 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & -\omega^2 m_3 + k_3 + k_4 \end{vmatrix} = 0$$

i.e. $$\left(-\omega^2 m_1 + k_1 + k_2\right)\left\{\left(-\omega^2 m_2 + k_2 + k_3\right)\left(-\omega^2 m_3 + k_3 + k_4\right) - k_3^2\right\}$$
$$+ k_2\left\{-k_2\left(-\omega^2 m_3 + k_3 + k_4\right)\right\} = 0$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

i.e. $$\omega^6 (m_1 m_2 m_3) - \omega^4 \left[m_1 m_2 (k_3 + k_4) + m_2 m_3 (k_1 + k_2) + m_1 m_3 (k_2 + k_3) \right] + \omega^2 \left[m_1 (k_2 + k_3)(k_3 + k_4) + m_2 (k_1 + k_2)(k_3 + k_4) + m_3 (k_1 + k_2)(k_2 + k_3) - m_1 k_3^2 - m_3 k_2^2 \right] - \left[(k_1 + k_2)(k_2 + k_3)(k_3 + k_4) + (k_1 + k_2) k_3^2 + (k_3 + k_4) k_2^2 \right] = 0$$

6.51 Equations of motion:

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad ---(E_1)$$

For harmonic motion, we get

$$\begin{bmatrix} -\omega^2 m_1 + k_1 + k_2 & -k_2 & 0 \\ -k_2 & -\omega^2 m_2 + k_2 + k_3 & -k_3 \\ 0 & -k_3 & -\omega^2 m_3 + k_3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad --(E_2)$$

This gives the frequency equation (for $k_1 = k$, $k_2 = 2k$, $k_3 = 3k$, $m_1 = m$, $m_2 = 2m$ and $m_3 = 3m$)

$$\begin{vmatrix} -\omega^2 m + 3k & -2k & 0 \\ -2k & -2\omega^2 m + 5k & -3k \\ 0 & -3k & -3\omega^2 m + 3k \end{vmatrix} = 0 \quad ---(E_3)$$

i.e. $$(-\omega^2 m + 3k)\left[(-2\omega^2 m + 5k)(-3\omega^2 m + 3k) - 9k^2 \right] + 2k \left[-2k(-3\omega^2 m + 3k) \right] = 0$$

i.e. $$2\alpha^3 - 13\alpha^2 + 19\alpha - 2 = 0 \quad \text{where } \alpha = \frac{\omega^2 m}{k} \quad ---(E_4)$$

This gives the roots

$\alpha_1 = 0.113992$, $\quad \alpha_2 = 2.00002$, $\quad \alpha_3 = 4.38600$

$\omega_1 = 0.337627 \sqrt{\frac{k}{m}}$, $\quad \omega_2 = 1.414221 \sqrt{\frac{k}{m}}$, $\quad \omega_3 = 2.094278 \sqrt{\frac{k}{m}}$

Mode shape in j^{th} mode:

Eq. (E_2) gives $$\frac{X_2^{(j)}}{X_1^{(j)}} = \frac{-\omega_j^2 m_1 + k_1 + k_2}{k_2} = \frac{-\omega_j^2 m + 3k}{2k}$$

$$\frac{X_3^{(j)}}{X_2^{(j)}} = \frac{k_3}{-\omega_j^2 m_3 + k_3} = \frac{3k}{-3\omega_j^2 m + 3k}$$

$$\frac{X_3^{(j)}}{X_1^{(j)}} = \frac{X_3^{(j)}}{X_2^{(j)}} \cdot \frac{X_2^{(j)}}{X_1^{(j)}} = \frac{3(-\omega_j^2 m + 3k)}{2(-3\omega_j^2 m + 3k)}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

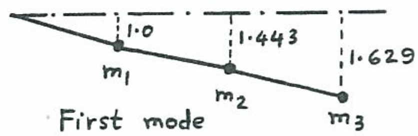
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\vec{X}^{(j)} = \begin{Bmatrix} X_1^{(j)} \\ X_2^{(j)} \\ X_3^{(j)} \end{Bmatrix} = X_1^{(j)} \begin{Bmatrix} 1 \\ (-\omega_j^2 m + 3k)/(2k) \\ 3(-\omega_j^2 m + 3k)/[2(-3\omega_j^2 m + 3k)] \end{Bmatrix} \quad \text{--- } (E_5)$$

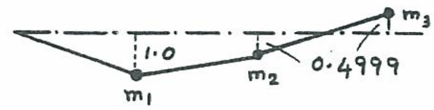
Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 1.443004 \\ 1.628659 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.49999 \\ -0.49998 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -0.693 \\ 0.204666 \end{Bmatrix}$$

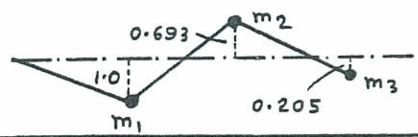
Mode shapes:



First mode



Second mode



Third mode

6.52 When $k_1 = 3k$, $k_2 = k_3 = k$, $m_1 = 3m$ and $m_2 = m_3 = m$, Eq. (E_2) of problem 6.46 gives the frequency equation

$$\begin{vmatrix} -3m\omega^2 + 4k & -k & 0 \\ -k & -m\omega^2 + 2k & -k \\ 0 & -k & -m\omega^2 + k \end{vmatrix} = 0$$

i.e. $(-3m\omega^2 + 4k)\left[(-m\omega^2 + 2k)(-m\omega^2 + k) - k^2\right] + k\left[-k(-m\omega^2 + k)\right] = 0$

i.e. $$3\alpha^3 - 13\alpha^2 + 14\alpha - 3 = 0 \quad \text{--- } (E_1)$$

where $\alpha = m\omega^2/k$. Roots of (E_1) are

$\alpha_1 = 0.284515$, $\quad \alpha_2 = 1.26053$, $\quad \alpha_3 = 2.78829$

$\omega_1 = 0.533399\sqrt{\frac{k}{m}}$, $\quad \omega_2 = 1.122733\sqrt{\frac{k}{m}}$, $\quad \omega_3 = 1.669817\sqrt{\frac{k}{m}}$

Eq. (E_5) of problem 6.51 gives the j^{th} mode shape as

$$\vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ (-3m\omega_j^2 + 4k)/k \\ (-3m\omega_j^2 + 4k)/(-m\omega_j^2 + k) \end{Bmatrix}$$

Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 3.146455 \\ 4.397653 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.21841 \\ -0.83833 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -4.36487 \\ 2.44081 \end{Bmatrix}$$

<u>Orthogonality of normal modes:</u>

$$[m] = m \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

$$\vec{X}^{(1)T}[m]\vec{X}^{(1)} = 1 \Rightarrow (1.0 \quad 3.146455 \quad 4.397653)\begin{Bmatrix}3.0\\3.146455\\4.397653\end{Bmatrix} m\, X_1^{(1)^2} = 1$$

$$\Rightarrow 32.239531\, m\, X_1^{(1)^2} = 1 \quad \text{or} \quad X_1^{(1)} = \frac{1}{\sqrt{m}}(0.176119)$$

$$\vec{X}^{(1)} = \frac{1}{\sqrt{m}}\begin{Bmatrix}0.176119\\0.554151\\0.774510\end{Bmatrix}$$

$$\vec{X}^{(2)T}[m]\vec{X}^{(2)} = 1 \Rightarrow (1.0 \quad 0.21841 \quad -0.83833)\begin{Bmatrix}3.0\\0.21841\\-0.83833\end{Bmatrix} m\, X_1^{(2)^2} = 1$$

$$\Rightarrow 3.7505\, m\, X_1^{(2)^2} = 1 \quad \text{or} \quad X_1^{(2)} = \frac{1}{\sqrt{m}}(0.516363)$$

$$\vec{X}^{(2)} = \frac{1}{\sqrt{m}}\begin{Bmatrix}0.516363\\0.112779\\-0.432883\end{Bmatrix}$$

$$\vec{X}^{(3)}[m]\vec{X}^{(3)} = 1 \Rightarrow (1.0 \quad -4.36487 \quad 2.44081)\begin{Bmatrix}3.0\\-4.36487\\2.44081\end{Bmatrix} m\, X_1^{(3)^2} = 1$$

$$\Rightarrow 28.00964\, m\, X_1^{(3)^2} = 1 \quad \text{or} \quad X_1^{(3)} = \frac{1}{\sqrt{m}}(0.18895)$$

$$\vec{X}^{(3)} = \frac{1}{\sqrt{m}}\begin{Bmatrix}0.18895\\-0.824742\\0.461191\end{Bmatrix}$$

It can be verified that

$$\vec{X}^{(1)T}[m]\vec{X}^{(2)} = (0.176119 \quad 0.554151 \quad 0.774510)\begin{Bmatrix}1.549089\\0.112779\\-0.432883\end{Bmatrix} = 0$$

$$\vec{X}^{(1)T}[m]\vec{X}^{(3)} = (0.176119 \quad 0.554151 \quad 0.774510)\begin{Bmatrix}0.56685\\-0.824742\\0.461191\end{Bmatrix} = 0$$

$$\vec{X}^{(2)T}[m]\vec{X}^{(3)} = (0.516363 \quad 0.112779 \quad -0.432883)\begin{Bmatrix}0.56685\\-0.824742\\0.461191\end{Bmatrix} = 0$$

6.53 For $l_1 = 0.2$ m, $l_2 = 0.3$ m, $l_3 = 0.4$ m, $m_1 = 1$ kg, $m_2 = 2$ kg and $m_3 = 3$ kg, Eq. (E_3) of problem 6.42 gives the equations of motion

$$\begin{bmatrix}6(0.04) & 5(0.06) & 3(0.08)\\5(0.06) & 5(0.09) & 3(0.12)\\3(0.08) & 3(0.12) & 3(0.16)\end{bmatrix}\begin{Bmatrix}\ddot{\theta}_1\\\ddot{\theta}_2\\\ddot{\theta}_3\end{Bmatrix} + \begin{bmatrix}6(9.81)(0.2) & 0 & 0\\0 & 5(9.81)(0.3) & 0\\0 & 0 & 3(9.81)(0.4)\end{bmatrix}\begin{Bmatrix}\theta_1\\\theta_2\\\theta_3\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad \text{---}\ (E_1)$$

For harmonic motion, (E_1) becomes

$$-\omega^2\begin{bmatrix}0.24 & 0.30 & 0.24\\0.30 & 0.45 & 0.36\\0.24 & 0.36 & 0.48\end{bmatrix}\begin{Bmatrix}\Theta_1\\\Theta_2\\\Theta_3\end{Bmatrix} + \begin{bmatrix}11.772 & 0 & 0\\0 & 14.715 & 0\\0 & 0 & 11.772\end{bmatrix}\begin{Bmatrix}\Theta_1\\\Theta_2\\\Theta_3\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad \text{---}\ (E_2)$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

This gives the frequency equation

$$\begin{vmatrix} \omega^2(0.24)-11.772 & \omega^2(0.30) & \omega^2(0.24) \\ \omega^2(0.30) & \omega^2(0.45)-14.715 & \omega^2(0.36) \\ \omega^2(0.24) & \omega^2(0.36) & \omega^2(0.48)-11.772 \end{vmatrix} = 0$$

i.e. $(0.24\omega^2 - 11.772)\left[(0.45\,\omega^2 - 14.715)(0.48\omega^2 - 11.772) - (0.36)^2\omega^4\right]$
$- 0.3\,\omega^2\left[0.3\,\omega^2(0.48\omega^2 - 11.772) - (0.24)(0.36)\,\omega^4\right]$
$+ 0.24\,\omega^2\left[(0.30)(0.36)\;\omega^4 - 0.24\,\omega^2(0.45\omega^2 - 14.715)\right] = 0$

i.e. $\omega^6 - 600.8625\,\omega^4 + 54132.806\,\omega^2 - 590047.6 = 0$

Roots of this equation are

$\omega_1^2 = 12.6335$, $\omega_1 = 3.554364$ rad/s
$\omega_2^2 = 94.6116$, $\omega_2 = 9.726849$ rad/s
$\omega_3^2 = 493.619$, $\omega_3 = 22.217538$ rad/s.

6.54 (a) By replacing l by $\frac{l}{4}$ in problem 6.26, we obtain

$$[a] = \frac{l^3}{64EI}\begin{bmatrix} \frac{9}{64} & \frac{1}{6} & \frac{13}{192} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{6} \\ \frac{13}{192} & \frac{1}{6} & \frac{9}{64} \end{bmatrix}$$

$$[m] = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \; [D] = [a][m] = \frac{ml^3}{EI}\begin{bmatrix} 0.0021975 & 0.0026042 & 0.0010579 \\ 0.0026042 & 0.0052083 & 0.0026042 \\ 0.0010579 & 0.0026042 & 0.0021973 \end{bmatrix}$$

Frequency equation is $\left|[D] - \lambda[I]\right| = 0$ where $\lambda = \frac{1}{\omega^2}$

i.e.
$$\begin{vmatrix} 0.0021973-\alpha & 0.0026042 & 0.0010579 \\ 0.0026042 & 0.0052083-\alpha & 0.0026042 \\ 0.0010579 & 0.0026042 & 0.0021973-\alpha \end{vmatrix} = 0 \quad \text{---}(E_1)$$

where $\alpha = \frac{EI}{ml^3\lambda} = \frac{EI}{ml^3\omega^2}$. Eq. (E_1) gives

$(0.0021973-\alpha)\left[(0.0052083-\alpha)(0.0021973-\alpha) - (0.0026042)^2\right]$
$-(0.0026042)\left[0.0026042(0.0021973-\alpha) - (0.0010579)(0.0026042)\right]$
$+(0.0010579)\left[(0.0026042)^2 - (0.0010579)(0.0052083-\alpha)\right] = 0$

i.e. $\alpha^3 - 0.96029\times10^{-2}\,\alpha^2 + 0.1303355\times10^{-4}\,\alpha - 0.0038623\times10^{-6} = 0$

Roots are:

$\alpha_1 = 0.000421453$, $\omega_1 = 48.71082\sqrt{EI/(ml^3)}$
$\alpha_2 = 0.00113955$, $\omega_2 = 29.62329\sqrt{EI/(ml^3)}$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$\alpha_3 = 0.00804192$, $\omega_3 = 11.15116 \sqrt{EI/(m l^3)}$

(b) $m = 10$ kg, $l = 0.5$ m, $E = 2.07 \times 10^{11}$ N/m^2,

$$I = \frac{\pi}{64} d^4 = \frac{\pi}{64}\left(\frac{2.5}{100}\right)^4 = 1.9175 \times 10^{-8} \text{ m}^4$$

$$\sqrt{\frac{EI}{m l^3}} = \sqrt{\frac{(2.07 \times 10^{11})(1.9175 \times 10^{-8})}{10\,(0.5)^3}} = 56.3505$$

$\omega_3 = 48.71082\,(56.3505) = 2744.8791$ rad/sec

$\omega_2 = 29.62329\,(56.3505) = 1669.2872$ rad/sec

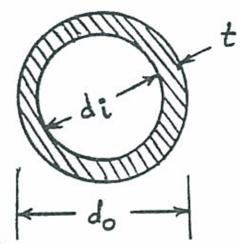
$\omega_1 = 11.15116\,(56.3505) = 628.3734$ rad/sec

(c) In order to have the same natural frequencies, we need to have the same value of I.

(i) For solid circular cross-section of diameter 2.5 cm,

$$I = 1.9175 \times 10^{-8} \text{ m}^4$$

(ii) For hollow circular section:



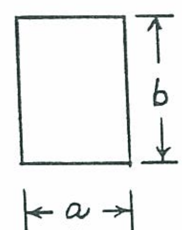
let $d_o = 5t$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(625 - 81)\, t^4$$

$$= 26.7036\, t^4 = 1.9175 \times 10^{-8}$$

$t = 0.5177$ cm, $d_o = 2.5885$ cm, $d_i = 1.5531$ cm.

(iii) For solid rectangular section:

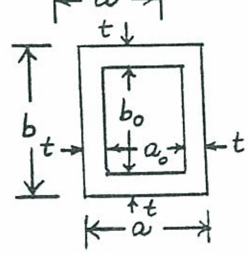


Let $b = 2a$.

$$I = \frac{1}{12}(a)\, b^3 = \frac{2}{3} a^4 = 1.9175 \times 10^{-8}$$

$a = 1.3023$ cm, $b = 2.6046$ cm.

(iv) For hollow rectangular section:



Let $b = 5t$, $b_o = 3t$

$a = 2.5t$, $a_o = 0.5t$

$$I = \frac{1}{12}\left[a b^3 - a_o b_o^3\right]$$

$$= \frac{1}{12}\left[2.5(125)\, t^4 - (0.5)(27)\, t^4\right]$$

$$= 24.9167\, t^4 = 1.9175 \times 10^{-8} \text{ m}^4$$

$t = 0.5267$ cm, $a = 1.3167$ cm, $b = 2.6335$ cm,

$a_o = 0.2634$ cm, $b_o = 1.5801$ cm.

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

Weights:

Weights are proportional to cross-sectional areas.

(i) For solid circular section:

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(2.5)^2 = 4.90875 \text{ cm}^2$$

(ii) For hollow circular section:

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}\left[2.5885^2 - 1.5531^2\right] = 3.3680 \text{ cm}^2$$

(iii) For solid rectangular section:

$$A = ab = (1.3023)(2.6046) = 3.3920 \text{ cm}^2$$

(iv) For hollow rectangular section:

$$A = ab - a_o b_o = (1.3167)(2.6335) - (0.2634)(1.5801) = 3.0513 \text{ cm}^2$$

∴ Least weight beam will have a hollow rectangular section.

6.55

$$\begin{vmatrix} \lambda-5 & -3 & -2 \\ -3 & \lambda-6 & -4 \\ -1 & -2 & \lambda-6 \end{vmatrix} = 0$$

i.e. $(\lambda-5)\left[(\lambda-6)^2 - (-2)(-4)\right] - (-3)\left[-3(\lambda-6) - (-1)(-4)\right]$

$+ (-2)\left[-3(-2) - (-1)(\lambda-6)\right] = 0$

i.e. $\lambda^3 - 17\lambda^2 + 77\lambda - 98 = 0$

Roots give: $\lambda_1 = 2.21398$, $\lambda_2 = 4.16929$, $\lambda_3 = 10.6168$

6.56 From problem 6.24, for $k_i = k$; $i = 1, 2, 3, 4$,

$$[k] = k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad [m] = m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equations of motion:

$$m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + k\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

These become, for harmonic motion,

$$\begin{bmatrix} -m\omega^2 + 2k & -k & 0 \\ -k & -m\omega^2 + 2k & -k \\ 0 & -k & -m\omega^2 + 2k \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--- } (E_1)$$

Frequency equation:

$$(-m\omega^2 + 2k)\left[(-m\omega^2 + 2k)^2 - k^2\right] + k\left[-k(-m\omega^2 + 2k)\right] = 0$$

i.e. $(-\alpha + 2)(\alpha^2 - 4\alpha + 2) = 0$ where $\alpha = \dfrac{m\omega^2}{k}$.

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

This gives $\alpha_1 = 2-\sqrt{2} = 0.585786$, $\alpha_2 = 2$, $\alpha_3 = 3.414214$

$$\omega_1 = 0.765367\sqrt{\frac{k}{m}}, \quad \omega_2 = 1.414214\sqrt{\frac{k}{m}}, \quad \omega_3 = 1.847759\sqrt{\frac{k}{m}}$$

(E_1) gives $\quad X_2^{(j)} = \left(\frac{-m\omega_j^2 + 2k}{k}\right) X_1^{(j)}$

$$-k\,X_1^{(j)} + (-m\omega_j^2 + 2k)\,X_2^{(j)} - k\,X_3^{(j)} = 0$$

or $\quad \left[-k + (-m\omega_j^2 + 2k)^2 \cdot \frac{1}{k}\right] X_1^{(j)} - k\,X_3^{(j)} = 0$

or $\quad X_3^{(j)} = \left\{\frac{(-m\,\omega_j^2 + 2k)^2 - k^2}{k^2}\right\} X_1^{(j)}$

$$j^{th}\text{ mode} = \vec{X}^{(j)} = \begin{Bmatrix} X_1^{(j)} \\ X_2^{(j)} \\ X_3^{(j)} \end{Bmatrix} = X_1^{(j)} \begin{Bmatrix} 1 \\ (-m\,\omega_j^2 + 2k)/k \\ [(-m\omega_j^2 + 2k)^2 - k^2]/k^2 \end{Bmatrix}$$

Mode shapes are:

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.414214 \\ 1 \end{Bmatrix} X_1^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} X_1^{(2)}, \quad \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -1.414214 \\ 1 \end{Bmatrix} X_1^{(3)}$$

6.57 From problem 6.24,

$$[k] = \begin{bmatrix} 2k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & 2k \end{bmatrix}, \quad [m] = \begin{bmatrix} 2m & 0 & 0 \\ 0 & 3m & 0 \\ 0 & 0 & 2m \end{bmatrix}$$

Equations of motion for harmonic motion:

$$\begin{bmatrix} -2m\,\omega^2 + 2k & -k & 0 \\ -k & -3m\,\omega^2 + 2k & -k \\ 0 & -k & -2m\,\omega^2 + 2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad --(E_1)$$

Frequency equation:

$$(-2m\omega^2 + 2k)\left[(-3m\,\omega^2 + 2k)(-2m\,\omega^2 + 2k) - k^2\right] + k\left[-k\,(-2m\,\omega^2 + 2k)\right] = 0$$

i.e. $\quad (-m\omega^2 + k)\left[3m^2\,\omega^4 - 5\,k m\,\omega^2 + k^2\right] = 0$

i.e. $\quad (-\alpha + 1)\,(3\alpha^2 - 5\alpha + 1) = 0 \quad$ where $\alpha = \frac{\omega^2 m}{k}$

$\therefore\ \alpha_1 = 0.232408$, $\quad \omega_1 = 0.482087\sqrt{\frac{k}{m}}$

$\alpha_2 = 1.0$, $\quad \omega_2 = \sqrt{\frac{k}{m}}$

$\alpha_3 = 1.434258$, $\quad \omega_3 = 1.197605\sqrt{\frac{k}{m}}$

(E_1) gives $\quad \frac{X_2^{(j)}}{X_1^{(j)}} = \frac{-2m\,\omega_j^2 + 2k}{k}$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

$$\left(-3m\,\omega_j^2 + 2k\right) X_2^{(j)} - k\, X_3^{(j)} = k\, X_1^{(j)}$$

or $$\left(-3m\,\omega_j^2 + 2k\right)\left(\frac{-2m\,\omega_j^2 + 2k}{k}\right) X_1^{(j)} - k\, X_1^{(j)} = k\, X_3^{(j)}$$

or $$\frac{X_3^{(j)}}{X_1^{(j)}} = \frac{\left(-3m\,\omega_j^2 + 2k\right)\left(-2m\,\omega_j^2 + 2k\right) - k^2}{k^2}$$

$$j^{th}\text{ mode} = \vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ \left(-2m\omega_j^2 + 2k\right)/k \\ \left\{\left(-3m\,\omega_j^2 + 2k\right)\left(-2m\,\omega_j^2 + 2k\right) - k^2\right\}/k^2 \end{Bmatrix}$$

This leads to:

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1 \\ 1.535184 \\ 1 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} -1 \\ -0.868516 \\ 1 \end{Bmatrix}$$

6.58 For $l_i = l$ and $m_i = m$ $(i = 1, 2, 3)$, problem 6.42 gives

$$\begin{bmatrix} 3ml^2 & 2ml^2 & ml^2 \\ 2ml^2 & 2ml^2 & ml^2 \\ ml^2 & ml^2 & ml^2 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{Bmatrix} + \begin{bmatrix} 3mgl & 0 & 0 \\ 0 & 2mgl & 0 \\ 0 & 0 & mgl \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

For harmonic motion,

$$\begin{bmatrix} -3l\omega^2 + 3g & -2l\omega^2 & -l\omega^2 \\ -2l\omega^2 & -2l\omega^2 + 2g & -l\omega^2 \\ -l\omega^2 & -l\omega^2 & -l\omega^2 + g \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

Dividing throughout by $-g$ and defining $\alpha = \frac{\omega^2 l}{g}$, this gives

$$\begin{bmatrix} 3\alpha - 3 & 2\alpha & \alpha \\ 2\alpha & 2\alpha - 2 & \alpha \\ \alpha & \alpha & \alpha - 1 \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{----}\ (E_1)$$

Frequency equation:

$$(3\alpha - 3)\left[(2\alpha - 2)(\alpha - 1) - \alpha^2\right] - 2\alpha\left[2\alpha(\alpha - 1) - \alpha^2\right] + \alpha\left[2\alpha^2 - \alpha(2\alpha - 2)\right] = 0$$

i.e. $$\alpha^3 - 9\alpha^2 + 18\alpha - 6 = 0$$

Roots are:

$$\alpha_1 = 0.415764, \quad \omega_1 = 0.644798\sqrt{\frac{g}{l}}$$
$$\alpha_2 = 2.29431, \quad \omega_2 = 1.514698\sqrt{g/l}$$
$$\alpha_3 = 6.28995, \quad \omega_3 = 2.507977\sqrt{g/l}$$

Mode shapes: (E_1) gives $$\Theta_2^{(j)} = \left\{\frac{-2\alpha_j^2 + 6\alpha_j - 3}{\alpha_j(\alpha_j - 2)}\right\}\Theta_1^{(j)}, \quad \Theta_3^{(j)} = \left(\frac{\alpha_j - 3}{\alpha_j - 2}\right)\Theta_1^{(j)}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$j^{th} \text{ mode} = \vec{\Theta}^{(j)} = \Theta_1^{(j)} \begin{Bmatrix} 1 \\ (-2\alpha_j^2 + 6\alpha_j - 3)/(\alpha_j^2 - 2\alpha_j) \\ (\alpha_j - 3)/(\alpha_j - 2) \end{Bmatrix}$$

Hence

$$\vec{\Theta}^{(1)} = \Theta_1^{(1)} \begin{Bmatrix} 1.0 \\ 1.2922 \\ 1.6312 \end{Bmatrix}, \quad \vec{\Theta}^{(2)} = \Theta_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.3527 \\ -2.3978 \end{Bmatrix}, \quad \vec{\Theta}^{(3)} = \Theta_1^{(3)} \begin{Bmatrix} 1.0 \\ -1.6450 \\ 0.7669 \end{Bmatrix}$$

6.59 From problem 6.27 we find, for $m_1 = m_3 = m$, $m_2 = 2m$, $k_1 = k_2 = k$ and $k_3 = 2k$,

$$[k] = \begin{bmatrix} 2k & -k & 0 \\ -k & 3k & -2k \\ 0 & -2k & 2k \end{bmatrix}, \quad [m] = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & m \end{bmatrix}$$

Equations of motion for harmonic motion

$$\begin{bmatrix} -\omega^2 m + 2k & -k & 0 \\ -k & -2m\omega^2 + 3k & -2k \\ 0 & -2k & -\omega^2 m + 2k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \text{---} (E_1)$$

Frequency equation is

$$\begin{vmatrix} -\omega^2 m + 2k & -k & 0 \\ -k & -2m\omega^2 + 3k & -2k \\ 0 & -2k & -\omega^2 m + 2k \end{vmatrix} = 0$$

or $\quad (-\alpha + 2)(2\alpha^2 - 7\alpha + 1) = 0 \quad$ with $\quad \alpha = \dfrac{m\omega^2}{k}$

$\therefore \quad \alpha_1 = 0.149219, \quad \omega_1 = 0.386289 \sqrt{k/m}$

$\alpha_2 = 2.0, \quad \omega_2 = 1.414214 \sqrt{k/m}$

$\alpha_3 = 3.350781, \quad \omega_3 = 1.830514 \sqrt{k/m}$

Eq. (E_1) gives, for ω_j,

$$\frac{X_2^{(j)}}{X_1^{(j)}} = \frac{-\omega_j^2 m + 2k}{k}$$

$$\frac{X_3^{(j)}}{X_1^{(j)}} = \frac{(-2m\omega_j^2 + 3k)(-m\omega_j^2 + 2k) - k^2}{2k^2}$$

$$j^{th} \text{ mode} = \vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ (-\omega_j^2 m + 2k)/k \\ \{(-2m\omega_j^2 + 3k)(-m\omega_j^2 + 2k) - k^2\}/(2k^2) \end{Bmatrix}$$

Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 1.850781 \\ 2.0 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.0 \\ -0.5 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -1.350781 \\ 2.0 \end{Bmatrix}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.60 For $k_1 = 3k$, $k_2 = k_3 = k$, $m_1 = 4m$, $m_2 = 2m$ and $m_3 = m$, Eq. (E_2) of problem 6.51 gives

$$\begin{bmatrix} -4m\omega^2 + 4k & -k & 0 \\ -k & -2m\omega^2 + 2k & -k \\ 0 & -k & -m\omega^2 + k \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad --- (E_1)$$

Frequency equation:

$$(-4m\omega^2 + 4k)\left\{(-2m\omega^2 + 2k)(-m\omega^2 + k) - k^2\right\} + k\left\{-k(-m\omega^2 + k)\right\} = 0$$

i.e. $\quad (-\alpha + 1)(8\alpha^2 - 16\alpha + 3) = 0 \quad$ with $\alpha = \dfrac{m\omega^2}{k}$

$$\therefore \quad \alpha_1 = 0.209431, \quad \omega_1 = 0.457636\sqrt{k/m}$$

$$\alpha_2 = 1.0, \quad \omega_2 = \sqrt{k/m}$$

$$\alpha_3 = 1.790569, \quad \omega_3 = 1.338121\sqrt{k/m}$$

Eq. (E_1) gives

$$X_2^{(j)} = \left\{\frac{-4m\omega_j^2 + 4k}{k}\right\} X_1^{(j)}$$

$$X_3^{(j)} = \left[-1 + (-2m\omega_j^2 + 2k)\left(\frac{-4m\omega_j^2 + 4k}{k^2}\right)\right] X_1^{(j)}$$

$$j^{th} \text{ mode} = \vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ (-4m\omega_j^2 + 4k)/k \\ \left\{(-2m\omega_j^2 + 2k)(-4m\omega_j^2 + 4k) - k^2\right\}/k^2 \end{Bmatrix}$$

Thus

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1.0 \\ 3.162276 \\ 4.0 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1.0 \\ 0.0 \\ -1.0 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1.0 \\ -3.162276 \\ 4.0 \end{Bmatrix}$$

6.61 For $m_1 = 2m$, $m_2 = m$, $m_3 = 3m$ and $l_i = l$ for all i, problem 6.28 gives

$$a_{11} = \frac{1}{P\left(\frac{1}{l} + \frac{1}{3l}\right)} = \frac{3l}{4P}, \quad a_{21} = \frac{2}{3}a_{11} = \frac{1}{2}\frac{l}{P}, \quad a_{31} = \frac{1}{3}a_{11} = \frac{1}{4}\frac{l}{P}$$

$$a_{22} = \frac{1}{P\left(\frac{1}{2l} + \frac{1}{2l}\right)} = \frac{l}{P}, \quad a_{32} = \frac{1}{2}a_{22} = \frac{1}{2}\frac{l}{P}, \quad a_{33} = \frac{1}{P\left(\frac{1}{3l} + \frac{1}{l}\right)} = \frac{3l}{4P}$$

$$[a] = \frac{l}{4P}\begin{bmatrix} 3 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad [m] = m\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad [a][m] = \frac{lm}{4P}\begin{bmatrix} 6 & 2 & 3 \\ 4 & 4 & 6 \\ 2 & 2 & 9 \end{bmatrix}$$

Equations of motion:

$$[a][m]\ddot{\vec{x}} + [I]\vec{x} = \vec{0}$$

Frequency equation: $\quad \left|-[a][m]\omega^2 + [I]\right| = 0$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

i.e.

$$\left| -\frac{\omega^2 l m}{4P}\begin{bmatrix} 6 & 2 & 3 \\ 4 & 4 & 6 \\ 2 & 2 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0$$

with $\alpha = \omega^2 l m/(4P)$, this equation becomes

$$\begin{vmatrix} 6\alpha-1 & 2\alpha & 3\alpha \\ 4\alpha & 4\alpha-1 & 6\alpha \\ 2\alpha & 2\alpha & 9\alpha-1 \end{vmatrix} = 96\alpha^3 - 88\alpha^2 + 19\alpha - 1 = 0$$

Roots are:

$\alpha_1 = 0.079126$, $\omega_1 = 0.562587\sqrt{\frac{P}{lm}}$

$\alpha_2 = 0.209671$, $\omega_2 = 0.915797\sqrt{\frac{P}{lm}}$

$\alpha_3 = 0.627872$, $\omega_3 = 1.584767\sqrt{\frac{P}{lm}}$

6.62 For $(GJ)_i = GJ$, $J_{di} = J_0$ and $l_i = l$ for all i, problem 6.23 gives

$$[k] = \frac{GJ}{l}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \quad [J_d] = J_0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Equations of motion for harmonic oscillation:

$$\begin{bmatrix} -\omega^2 J_0 + 2\frac{GJ}{l} & -\frac{GJ}{l} & 0 \\ -\frac{GJ}{l} & -\omega^2 J_0 + 2\frac{GJ}{l} & -\frac{GJ}{l} \\ 0 & -\frac{GJ}{l} & -\omega^2 J_0 + 2\frac{GJ}{l} \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--}(E_1)$$

Dividing throughout by GJ/l and defining $\alpha = \frac{\omega^2 J_0 l}{GJ}$, (E_1) gives

$$\begin{bmatrix} -\alpha+2 & -1 & 0 \\ -1 & -\alpha+2 & -1 \\ 0 & -1 & -\alpha+2 \end{bmatrix}\begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{---}(E_2)$$

Frequency equation:

$$\begin{vmatrix} -\alpha+2 & -1 & 0 \\ -1 & -\alpha+2 & -1 \\ 0 & -1 & -\alpha+2 \end{vmatrix} = (-\alpha+2)(\alpha^2 - 4\alpha + 2) = 0$$

Roots are:

$\alpha_1 = 2-\sqrt{2} = 0.585786$, $\alpha_2 = 2$, $\alpha_3 = 2+\sqrt{2} = 3.414214$

$\omega_1 = 0.765367\sqrt{\frac{GJ}{lJ_0}}$, $\omega_2 = 1.414214\sqrt{\frac{GJ}{lJ_0}}$, $\omega_3 = 1.847759\sqrt{\frac{GJ}{lJ_0}}$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

Noting that Eq. (E_1) is similar to Eq. (E_1) of problem 6.56, we can use the same modeshapes:

$$\vec{\Theta}^{(1)} = \Theta_1^{(1)} \begin{Bmatrix} 1.0 \\ 1.414214 \\ 1.0 \end{Bmatrix}, \quad \vec{\Theta}^{(2)} = \Theta_1^{(2)} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}, \quad \vec{\Theta}^{(3)} = \Theta_1^{(3)} \begin{Bmatrix} 1.0 \\ -1.414214 \\ 1.0 \end{Bmatrix}$$

6.63 Equations of motion

$$\frac{\rho A l}{4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \frac{2AE}{l} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--- } (E_1)$$

This gives, for harmonic motion,

$$\begin{bmatrix} \left(-\frac{\omega^2 \rho A l}{4} + \frac{2AE}{l}\right) & -\frac{2AE}{l} & 0 \\ -\frac{2AE}{l} & \left(-\frac{2\omega^2 \rho A l}{4} + \frac{4AE}{l}\right) & -\frac{2AE}{l} \\ 0 & -\frac{2AE}{l} & \left(-\frac{\omega^2 \rho A l}{4} + \frac{2AE}{l}\right) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--- } (E_2)$$

Dividing throughout by $\frac{2AE}{l}$ and defining $\alpha = \frac{\omega^2 \rho A l}{8AE}$,

$$\begin{bmatrix} -\alpha+1 & -1 & 0 \\ -1 & -2\alpha+2 & -1 \\ 0 & -1 & -\alpha+1 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{--- } (E_3)$$

Frequency equation:

$$\begin{vmatrix} -\alpha+1 & -1 & 0 \\ -1 & -2\alpha+2 & -1 \\ 0 & -1 & -\alpha+1 \end{vmatrix} = 2\alpha(-\alpha+1)(\alpha-2) = 0$$

$\therefore$

$$\alpha_1 = 0, \quad \omega_1 = 0$$

$$\alpha_2 = 1, \quad \omega_2 = \sqrt{\frac{8AE}{\rho A l^2}}$$

$$\alpha_3 = 2, \quad \omega_3 = \sqrt{\frac{16AE}{\rho A l^2}}$$

Principal modes:

Eq. (E_3) gives

$$X_2^{(j)} = (-\alpha_j + 1)\, X_1^{(j)}$$

$$X_3^{(j)} = \left[-1 + (-2\alpha_j + 2)(-\alpha_j + 1)\right] X_1^{(j)}$$

jth mode:

$$\vec{X}^{(j)} = X_1^{(j)} \begin{Bmatrix} 1.0 \\ (-\alpha_j + 1) \\ 2(-\alpha_j+1)^2 - 1 \end{Bmatrix}$$

Thus

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\vec{X}^{(1)} = X_1^{(1)} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \quad \vec{X}^{(2)} = X_1^{(2)} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix}, \quad \vec{X}^{(3)} = X_1^{(3)} \begin{Bmatrix} 1 \\ -1 \\ 1 \end{Bmatrix}$$

6.64

For orthonormalization, $\vec{X}^{(i)T}[m]\vec{X}^{(i)} = 1 \; ; \; i = 1, 2, 3$

Let new $\vec{X}^{(1)} = a_1 \begin{Bmatrix} 1 \\ -1 \\ 1 \end{Bmatrix}$, $\vec{X}^{(2)} = a_2 \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$ and $\vec{X}^{(3)} = a_3 \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix}$

$$\vec{X}^{(1)T}[m]\vec{X}^{(1)} = a_1^2 \,(1 \;\; -1 \;\; 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \\ 1 \end{Bmatrix} = 4a_1^2 = 1 \Rightarrow a_1 = \frac{1}{2}$$

$$\vec{X}^{(2)T}[m]\vec{X}^{(2)} = a_2^2 \,(1 \;\; 1 \;\; 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 4a_2^2 = 1 \Rightarrow a_2 = \frac{1}{2}$$

$$\vec{X}^{(3)T}[m]\vec{X}^{(3)} = a_3^2 \,(0 \;\; 1 \;\; 2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 2 \end{Bmatrix} = 6a_3^2 = 1 \Rightarrow a_3 = \frac{1}{\sqrt{6}}$$

$$[m]\text{-orthonormal modal matrix} = [X] = \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & \sqrt{2/3} \\ 1 & 1 & \sqrt{8/3} \end{bmatrix}$$

6.65

Stiffness matrix:

Let $x_1 = 1$, $x_2 = x_3 = 0$. $F_1 = 2+1+1 = 4 = k_{11}$, $F_2 = -1 = k_{21}$, $F_3 = -1 = k_{31}$

Let $x_2 = 1$, $x_1 = x_3 = 0$. $F_2 = 1 + \; = 2 = k_{22}$, $F_1 = -1 = k_{12}$, $F_3 = 0 = k_{32}$

Let $x_3 = 1$, $x_1 = x_2 = 0$. $F_3 = 1+1 = 2 = k_{33}$, $F_1 = -1 = k_{13}$, $F_2 = 0 = k_{23}$

$$\therefore [k] = \begin{bmatrix} 4 & -1 & -1 \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

Mass matrix:

$$[m] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

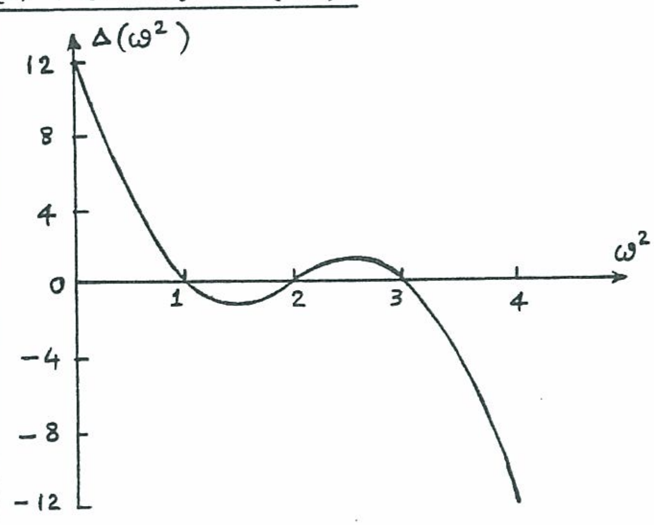
(a) Characteristic polynomial:

$$\begin{vmatrix} (-2\omega^2+4) & -1 & -1 \\ -1 & (-\omega^2+2) & 0 \\ -1 & 0 & (-\omega^2+2) \end{vmatrix} = 0$$

i.e. $2(-\omega^2+1)(-\omega^2+2)(-\omega^2+3) = 0$

$$\therefore \Delta(\omega^2) = 2(-\omega^2+1)(-\omega^2+2)(-\omega^2+3)$$

(b) Plot of $\Delta(\omega^2)$:



(c) Roots of equation:

$$\left.\begin{aligned} \omega_1^2 &= 1 \\ \omega_2^2 &= 2 \\ \omega_3^2 &= 3 \end{aligned}\right\} \text{from the graph.}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الاتحاد الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.66 (a) $\vec{X}^{(1)} = \begin{Bmatrix} 0.2754946 \\ 0.3994672 \\ 0.4490562 \end{Bmatrix}$, $\vec{X}^{(2)} = \begin{Bmatrix} 0.6916979 \\ 0.2974301 \\ -0.3389320 \end{Bmatrix}$, $[m] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$$\vec{X}^{(1)T} [m] \vec{X}^{(2)} = (0.2754946 \quad 0.3994672 \quad 0.4490562) \begin{Bmatrix} 0.6916979 \\ 0.5948602 \\ -1.0167960 \end{Bmatrix} \simeq 0$$

Let $\vec{X}^{(3)} = \begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}$

Then $\vec{X}^{(3)T} [m] \vec{X}^{(3)} = 1$, $\vec{X}^{(1)T} [m] \vec{X}^{(3)} = 0$, $\vec{X}^{(2)T} [m] \vec{X}^{(3)} = 0$

These relations give

$$a_1^2 + 2 a_2^2 + 3 a_3^2 = 1 \quad \text{--- } (E_1)$$

$$0.2754946 \; a_1 + 0.7989344 \; a_2 + 1.3471686 \; a_3 = 0 \quad \text{--- } (E_2)$$

$$0.6916979 \; a_1 + 0.5948602 \; a_2 - 1.0167960 \; a_3 = 0 \quad \text{--- } (E_3)$$

From (E_2) and (E_3),

$$a_1 = -2.9000002 \; a_2 - 4.89 \; a_3 = -0.86 \; a_2 + 1.4700001 \; a_3$$

or $\quad a_2 = -3.1176468 \; a_3 \quad$ --- (E_4)

and $\quad a_1 = -2.9000002(-3.1176468 \; a_3) - 4.89 \; a_3 = 4.1511763 \; a_3 \quad$ --- (E_5)

(E_1), (E_4) and (E_5) give

$$a_3^2 \, (17.232265 + 9.7197216 + 1) = 1 \Rightarrow a_3 = \pm 0.1891445$$

Hence $\quad a_2 = \mp 0.5896857$, $\quad a_1 = \pm 0.7851722$

$$\therefore \vec{X}^{(3)} = \begin{Bmatrix} 0.7851722 \\ -0.5896857 \\ 0.1891445 \end{Bmatrix}$$

(b) $\omega_i^2 = \vec{X}^{(i)T} [k] \vec{X}^{(i)}$; $[k] = \begin{bmatrix} 6 & -4 & 0 \\ -4 & 10 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$$\omega_1^2 = (0.2754946 \quad 0.3994672 \quad 0.4490562) \begin{Bmatrix} 0.05509889 \\ 2.8926938 \\ 2.6943374 \end{Bmatrix} = 2.3806248$$

$$\omega_2^2 = (0.6916979 \quad 0.2974301 \quad -0.3389320) \begin{Bmatrix} 2.9604671 \\ 0.2075095 \\ -2.0335920 \end{Bmatrix} = 2.7987180$$

$$\omega_3^2 = (0.7851722 \quad -0.5896857 \quad 0.1891445) \begin{Bmatrix} 7.0697761 \\ -9.0375462 \\ 1.1348671 \end{Bmatrix} = 11.094957$$

$$\therefore \omega_1 = 1.5429274, \quad \omega_2 = 1.6729369, \quad \omega_3 = 3.3309095.$$

6.67 From solution of Problem 6.1, we find

$$[m] = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \; ; \; [k] = k \begin{bmatrix} 7 & -1 & -5 \\ -1 & 2 & -1 \\ -5 & -1 & 7 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Frequency equation:

$$\left| -\omega^2 \, [m] + [k] \right| = 0$$

$$\text{or} \quad \begin{vmatrix} (-\alpha+7) & -1 & -5 \\ -1 & (-\alpha+2) & -1 \\ -5 & -1 & (-\alpha+7) \end{vmatrix} = 0$$

where $\alpha = \dfrac{\omega^2 \, m}{k}$. Expansion of the frequency equation gives:

$$(-\alpha+7)\left\{(-\alpha+2)(-\alpha+7)-1\right\} + 1\left\{-(-\alpha+7)-5\right\} - 5\left\{1+5(-\alpha+2)\right\} = 0$$

$$\text{or} \quad \alpha^3 - 16\,\alpha^2 + 50\,\alpha - 24 = 0$$

Roots of this equation give:

$$\alpha_1 = 0.58576 \;;\; \omega_1 = 0.7653\sqrt{\frac{k}{m}}$$

$$\alpha_2 = 4.41428 \;;\; \omega_2 = 1.8478\sqrt{\frac{k}{m}}$$

$$\alpha_3 = 12.0 \;;\; \omega_3 = 3.4641\sqrt{\frac{k}{m}}$$

6.68 From the solution of Problem 6.2, we obtain:

$$[m] = \begin{bmatrix} \frac{2\,m\,\ell^2}{3} & 0 & 0 \\ 0 & 2\,m & 0 \\ 0 & 0 & m \end{bmatrix} = \begin{bmatrix} 0.6667 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[k] = \begin{bmatrix} \frac{25\,k\,\ell^2}{8} & -\frac{k\,\ell}{2} & 0 \\ -\frac{k\,\ell}{2} & 3\,k & -k \\ 0 & -k & k \end{bmatrix} = \begin{bmatrix} 3125 & -500 & 0 \\ -500 & 3000 & -1000 \\ 0 & -1000 & 1000 \end{bmatrix}$$

Frequency equation:

$$\left| -\omega^2 \, [m] + [k] \right| = 0$$

$$\text{or} \quad \begin{vmatrix} (3125 - 0.6667\,\omega^2) & -500 & 0 \\ -500 & (-2\,\omega^2 + 3000) & -1000 \\ 0 & -1000 & (1000 - \omega^2) \end{vmatrix} = 0$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

or $$(3125 - 0.6667\ \omega^2)\left\{(3000 - 2\ \omega^2)\ (1000 - \omega^2) - 1000^2\right\} + 500\left\{-\ 500\ (1000 - \omega^2) - 0\right\} = 0$$

or $$-\ 1.3334\ \omega^6 + 9583.4\ \omega^4 - 16.7083\ (10^6)\ \omega^2 + 6.0\ (10^9) = 0$$

Defining $\alpha = \left(\dfrac{\omega^2}{1000}\right)$, the above equation can be rewritten as

$$\alpha^3 - 7.1872\ \alpha^2 + 12.5306\ \alpha - 4.4998 = 0$$

Roots of this equation are (using Program):

$$\alpha_1 = 0.484831\ \ ;\ \ \omega_1 = 22.0189 \text{ rad/sec}$$
$$\alpha_2 = 1.95501\ \ ;\ \ \omega_2 = 44.2155 \text{ rad/sec}$$
$$\alpha_3 = 4.74738\ \ ;\ \ \omega_3 = 68.9012 \text{ rad/sec}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.75 $[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} m & 0 & 0 \\ 0 & 2m & 0 \\ 0 & 0 & 3m \end{bmatrix}$, $[k] = \begin{bmatrix} k_1 & -k_1 & 0 \\ -k_1 & (k_1+k_2) & -k_2 \\ 0 & -k_2 & k_2 \end{bmatrix} = \begin{bmatrix} k & -k & 0 \\ -k & 2k & -k \\ 0 & -k & k \end{bmatrix}$

Equations of motion for harmonic motion:

$$\begin{bmatrix} (-m\omega^2+k) & -k & 0 \\ -k & (-2m\omega^2+2k) & -k \\ 0 & -k & (-3m\omega^2+k) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E.1)$$

Defining $\alpha = \frac{m\omega^2}{k}$, (E.1) can be rewritten as

$$\begin{bmatrix} (-\alpha+1) & -1 & 0 \\ -1 & (-2\alpha+2) & -1 \\ 0 & -1 & (-3\alpha+1) \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E.2)$$

Frequency equation is

$$2\alpha\,(3\alpha^2 - 7\alpha + 3) = 0$$

Roots are

$\alpha_1 = 0$; $\omega_1 = 0$

$\alpha_2 = 0.565741$; $\omega_2 = 0.752158\sqrt{k/m}$

$\alpha_3 = 1.767592$; $\omega_3 = 1.329508\sqrt{k/m}$

Eqs. (E.2) give $\quad X_2^{(j)} = (-\alpha_j + 1)\,X_1^{(j)}, \quad X_3^{(j)} = \left(\frac{1}{-3\alpha_j+1}\right) X_2^{(j)}$

$$\therefore\ \vec{X}^{(j)} = \begin{Bmatrix} 1.0 \\ (-\alpha_j+1) \\ \left(\frac{-\alpha_j+1}{-3\alpha_j+1}\right) \end{Bmatrix} X_1^{(j)}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Hence

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} X_1^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0.434259 \\ -0.622841 \end{Bmatrix} X_1^{(2)}, \quad \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -0.767592 \\ 0.178395 \end{Bmatrix} X_1^{(3)}$$

6.76 $[k_t] = k_t \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & -2 \\ 0 & -2 & 2 \end{bmatrix}, \quad [J] = J_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

With $\alpha = \dfrac{\omega^2 J_0}{k_t}$, the equations of motion for harmonic motion become

$$\begin{bmatrix} (-\alpha+1) & -1 & 0 \\ -1 & (-\alpha+3) & -2 \\ 0 & -2 & (-\alpha+2) \end{bmatrix} \begin{Bmatrix} \Theta_1 \\ \Theta_2 \\ \Theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E.1)$$

Frequency equation is $\quad \alpha(\alpha^2 - 6\alpha + 6) = 0$

Roots are: $\quad \alpha_1 = 0, \quad \alpha_2 = 1.267949, \quad \alpha_3 = 4.732051$

$$\omega_1 = 0, \quad \omega_2 = 1.126032\sqrt{k_t/J_0}, \quad \omega_3 = 2.175328\sqrt{k_t/J_0}$$

Eq. (E.1) gives

$$\Theta_2^{(j)} = (-\alpha_j + 1)\,\Theta_1^{(j)}, \quad \Theta_3^{(j)} = \frac{2}{(-\alpha_j+2)}\,\Theta_2^{(j)}$$

$$\vec{\Theta}^{(j)} = \begin{Bmatrix} 1 \\ (-\alpha_j+1) \\ \left(\dfrac{-2\alpha_j+2}{-\alpha_j+2}\right) \end{Bmatrix} \Theta_1^{(j)}$$

$$\therefore \quad \vec{\Theta}^{(1)} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \Theta_1^{(1)}, \quad \vec{\Theta}^{(2)} = \begin{Bmatrix} 1 \\ -0.267949 \\ -0.732050 \end{Bmatrix} \Theta_1^{(2)}, \quad \vec{\Theta}^{(3)} = \begin{Bmatrix} 1 \\ -3.732051 \\ 2.732051 \end{Bmatrix} \Theta_1^{(3)}$$

Normalize the eigenvectors with respect to the inertia matrix as

$$\vec{\Theta}^{(1)T}[J]\,\vec{\Theta}^{(1)} = \Theta_1^{(1)^2} J_0 \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 3\,J_0\,\Theta_1^{(1)^2} = 1$$

$$\Theta_1^{(1)} = 0.57735/\sqrt{J_0}$$

$$\vec{\Theta}^{(2)T}[J]\,\vec{\Theta}^{(2)} = \Theta_1^{(2)^2} J_0 \begin{pmatrix} 1 & -0.267949 & -0.732050 \end{pmatrix} \begin{Bmatrix} 1 \\ -0.267949 \\ -0.732050 \end{Bmatrix}$$

$$= \Theta_1^{(2)^2} J_0 (1.607694) = 1$$

$$\Theta_1^{(2)} = 0.788675/\sqrt{J_0}$$

$$\vec{\Theta}^{(3)T}[J]\,\vec{\Theta}^{(3)} = \Theta_1^{(3)^2} J_0 \begin{pmatrix} 1 & -3.732051 & 2.732051 \end{pmatrix} \begin{Bmatrix} 1 \\ -3.732051 \\ 2.732051 \end{Bmatrix}$$

$$= \Theta_1^{(3)^2} J_0 (22.392307)$$

$$\Theta_1^{(3)} = 0.211325/\sqrt{J_0}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

Modal matrix is

$$[X] = \frac{1}{\sqrt{J_0}} \begin{bmatrix} 0.57735 & 0.788675 & 0.211325 \\ 0.57735 & -0.211325 & -0.788676 \\ 0.57735 & -0.577349 & 0.577351 \end{bmatrix}$$

6.77 From solution of Problem 6.57, the natural frequencies and mode shapes are given by:

$$\omega_1 = 0.482087 \sqrt{\frac{k}{m}} \;;\; \omega_2 = \sqrt{\frac{k}{m}} \;;\; \omega_3 = 1.197605 \sqrt{\frac{k}{m}}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.535184 \\ 1.0 \end{Bmatrix} \;;\; \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \;;\; \vec{X}^{(3)} = \begin{Bmatrix} 1.0 \\ 0.868516 \\ -1 \end{Bmatrix}$$

Initial conditions:

$$x_1(0) = x_{10},\; x_2(0) = 0,\; x_3(0) = 0,\; \dot{x}_i(0) = 0 \;;\; i = 1, 2, 3$$

Equations (6.98) and (6.99) yield:

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = x_{10} \quad (1)$$

$$1.5352\, A_1 \cos\phi_1 + 0.8685\, A_3 \cos\phi_3 = 0 \quad (2)$$

$$A_1 \cos\phi_1 - A_2 \cos\phi_2 - A_3 \cos\phi_3 = 0 \quad (3)$$

$$-0.4821 \sqrt{\frac{k}{m}}\, A_1 \sin\phi_1 - \sqrt{\frac{k}{m}}\, A_2 \sin\phi_2 - 1.1976 \sqrt{\frac{k}{m}}\, A_3 \sin\phi_3 = 0 \quad (4)$$

$$-0.4821 \sqrt{\frac{k}{m}}\, (1.5352)\, A_1 \sin\phi_1 - 1.1976 \sqrt{\frac{k}{m}}\, (0.8685)\, A_3 \sin\phi_3 = 0 \quad (5)$$

$$-0.4821 \sqrt{\frac{k}{m}}\, (1.0)\, A_1 \sin\phi_1 - \sqrt{\frac{k}{m}}\, (-1)\, A_2 \sin\phi_2 - 1.1976 \sqrt{\frac{k}{m}}\, (-1)\, A_3 \sin\phi_3 = 0 \quad (6)$$

Solution of Eqs. (4) to (6):

$$\phi_i = 0 \;;\; i = 1, 2, 3$$

Solution of Eqs. (1) to (3) gives:

$$A_1 = 0.5\, x_{10} \;;\; A_2 = 0.3838\, x_{10} \;;\; A_3 = -\,0.8838\, x_{10}$$

Thus the free vibration solution of the system is given by:

$$x_1(t) = x_{10} \left(0.5 \cos 0.4821 \sqrt{\frac{k}{m}}\, t + 0.3838 \cos \sqrt{\frac{k}{m}}\, t - 0.8838 \cos 1.1976 \sqrt{\frac{k}{m}}\, t\right)$$

$$x_2(t) = x_{10} \left\{0.7676 \cos 0.4821 \sqrt{\frac{k}{m}}\, t - 0.7676 \cos 1.1976 \sqrt{\frac{k}{m}}\, t\right\}$$

$$x_3(t) = x_{10} \left(0.5 \cos 0.4821 \sqrt{\frac{k}{m}}\, t - 0.3838 \cos \sqrt{\frac{k}{m}}\, t + 0.8838 \cos 1.1976 \sqrt{\frac{k}{m}}\, t\right)$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.78 From solution of Problem 6.58, we find that:

$$\omega_1 = 0.6448\sqrt{\frac{g}{\ell}} \;;\; \omega_2 = 1.5147\sqrt{\frac{g}{\ell}} \;;\; \omega_3 = 2.5080\sqrt{\frac{g}{\ell}}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.2922 \\ 1.6312 \end{Bmatrix} \;;\; \vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ 0.3527 \\ -2.3978 \end{Bmatrix} \;;\; \vec{X}^{(3)} = \begin{Bmatrix} 1.0 \\ -1.6450 \\ 0.7669 \end{Bmatrix}$$

Equations (6.98) and (6.99) can be written, for the stated initial conditions, as:

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = 0 \tag{1}$$

$$1.2922\, A_1 \cos\phi_1 + 0.3527\, A_2 \cos\phi_2 - 1.6450\, A_3 \cos\phi_3 = 0 \tag{2}$$

$$1.6312\, A_1 \cos\phi_1 - 2.3978\, A_2 \cos\phi_2 + 0.7669\, A_3 \cos\phi_3 = \theta_{30} \tag{3}$$

$$-0.6448\sqrt{\frac{g}{\ell}}\, A_1 \sin\phi_1 - 1.5147\sqrt{\frac{g}{\ell}}\, A_2 \sin\phi_2 - 2.5080\sqrt{\frac{g}{\ell}}\, A_3 \sin\phi_3 = 0 \tag{4}$$

$$-0.6448\sqrt{\frac{g}{\ell}}\,(1.2922)\, A_1 \sin\phi_1 - 1.5147\sqrt{\frac{g}{\ell}}\,(0.3527)\, A_2 \sin\phi_2 - 2.5080\sqrt{\frac{g}{\ell}}\,(-1.6450)\, A_3 \sin\phi_3 = 0 \tag{5}$$

$$-0.6448\sqrt{\frac{g}{\ell}}\,(1.6312)\, A_1 \sin\phi_1 - 1.5147\sqrt{\frac{g}{\ell}}\,(-2.3978)\, A_2 \sin\phi_2 - 2.5080\sqrt{\frac{g}{\ell}}\,(0.7669)\, A_3 \sin\phi_3 = 0 \tag{6}$$

Equations (4) to (6) yield:

$$\phi_i = 0 \;;\; i = 1, 2, 3$$

Equations (1) to (3) give

$$A_1 = 0.1812\, x_{30} \;;\; A_2 = -0.2665\, x_{30} \;;\; A_3 = 0.08524\, x_{30}$$

Thus the free vibration solution can be expressed as (see Eq. (6.96)):

$$x_1(t) = x_{30}\left(0.1812 \cos 0.6448\sqrt{\frac{g}{\ell}}\, t - 0.2665 \cos 1.5147\sqrt{\frac{g}{\ell}}\, t + 0.08524 \cos 2.5080\sqrt{\frac{g}{\ell}}\, t\right) \tag{7}$$

$$x_2(t) = x_{30}\left(0.2341 \cos 0.6448\sqrt{\frac{g}{\ell}}\, t - 0.09399 \cos 1.5147\sqrt{\frac{g}{\ell}}\, t - 0.1402 \cos 2.5080\sqrt{\frac{g}{\ell}}\, t\right) \tag{8}$$

$$x_3(t) = x_{30}\left(0.2956 \cos 0.6448\sqrt{\frac{g}{\ell}}\, t + 0.6390 \cos 1.5147\sqrt{\frac{g}{\ell}}\, t + 0.06537 \cos 2.5080\sqrt{\frac{g}{\ell}}\, t\right) \tag{9}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.79 From solution of Problem 6.61, we obtain

$$\omega_1 = 0.5626\sqrt{\frac{P}{\ell\, m}} \;;\; \omega_2 = 0.9158\sqrt{\frac{P}{\ell\, m}} \;;\; \omega_3 = 1.5848\sqrt{\frac{P}{\ell\, m}}$$

$$\alpha_1 = \frac{\omega_1^2\, \ell\, m}{4\,P} = 0.079126 \;;\; \alpha_2 = \frac{\omega_2^2\, \ell\, m}{4\,P} = 0.209671 \;;\; \alpha_3 = \frac{\omega_3^2\, \ell\, m}{4\,P} = 0.627872$$

The mode shapes can be determined from the equations:

$$(6\,\alpha - 1)\,X_1 + 2\,\alpha\,X_2 + 3\,\alpha\,X_3 = 0 \qquad (1)$$

$$4\,\alpha\,X_1 + (4\,\alpha - 1)\,X_2 + 6\,\alpha\,X_3 = 0 \qquad (2)$$

$$2\,\alpha\,X_1 + 2\,\alpha\,X_2 + (9\,\alpha - 1)\,X_3 = 0 \qquad (3)$$

$$\text{Let}\quad X_1 = 1 \qquad (4)$$

Then Eqs. (1) and (2) yield

$$X_2 = 2 - 8\,\alpha \qquad (5)$$

$$X_3 = \frac{1 - 10\,\alpha + 16\,\alpha^2}{3\,\alpha} \qquad (6)$$

Using the values of α_1, α_2 and α_3, we obtain, from Eqs. (4) to (6):

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.3670 \\ 1.3014 \end{Bmatrix} \;;\; \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0.3226 \\ -0.6253 \end{Bmatrix} \;;\; \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -3.0230 \\ 0.5462 \end{Bmatrix}$$

The stated initial conditions give, using Eqs. (6.98) and (6.99),

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = 0 \qquad (7)$$

$$1.3670\,A_1 \cos\phi_1 + 0.3226\,A_2 \cos\phi_2 - 3.0230\,A_3 \cos\phi_3 = x_{20} \qquad (8)$$

$$1.3014\,A_1 \cos\phi_1 - 0.6253\,A_2 \cos\phi_2 + 0.5462\,A_3 \cos\phi_3 = 0 \qquad (9)$$

$$0.5626\sqrt{\frac{P}{\ell\, m}}\,A_1 \sin\phi_1 + 0.9158\sqrt{\frac{P}{\ell\, m}}\,A_2 \sin\phi_2 + 1.5848\sqrt{\frac{P}{\ell\, m}}\,A_3 \sin\phi_3 = 0 \qquad (10)$$

$$0.5626\sqrt{\frac{P}{\ell\, m}}\,(1.3670)\,A_1 \sin\phi_1 + 0.9158\sqrt{\frac{P}{\ell\, m}}\,(0.3226)\,A_2 \sin\phi_2 - 1.5848\sqrt{\frac{P}{\ell\, m}}\,(3.0230)\,A_3 \sin\phi_3 = 0 \qquad (11)$$

$$0.5626\sqrt{\frac{P}{\ell\, m}}\,(1.3014)\,A_1 \sin\phi_1 - 0.9158\sqrt{\frac{P}{\ell\, m}}\,(0.6253)\,A_2 \sin\phi_2 + 1.5848\sqrt{\frac{P}{\ell\, m}}\,(0.5462)\,A_3 \sin\phi_3 = 0 \qquad (12)$$

Equations (10) to (12) yield:

$$\phi_i = 0 \;;\; i = 1,\, 2,\, 3$$

Equations (7) to (9) give

$$A_1 = 0.1527\,x_{20} \;;\; A_2 = 0.09847\,x_{20} \;;\; A_3 = -\,0.2512\,x_{20}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$x_1(t) = x_{20}\ (0.1527 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\ t + 0.09847 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\ t - 0.2512 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\ t) \tag{13}$$

$$x_2(t) = x_{20}\ (0.2087 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\ t + 0.03177 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\ t + 0.7594 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\ t) \tag{14}$$

$$x_3(t) = x_{20}\ (0.1987 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\ t - 0.06157 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\ t - 0.1372 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\ t) \tag{15}$$

6.80 From solution of Problem 6.51, we obtain

$$\omega_1 = 0.3376 \sqrt{\frac{k}{m}}\ ;\ \omega_2 = 1.4142 \sqrt{\frac{k}{m}}\ ;\ \omega_3 = 2.0943 \sqrt{\frac{k}{m}}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.4430 \\ 1.6286 \end{Bmatrix}\ ;\ \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0.5 \\ -0.5 \end{Bmatrix}\ ;\ \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -0.693 \\ 0.2047 \end{Bmatrix}$$

Initial conditions:

$$x_i(0) = 0\ ;\ i = 1,\ 2,\ 3\ ;\ \dot{x}_1(0) = \dot{x}_{10}\ ,\ \dot{x}_2(0) = \dot{x}_3(0) = 0$$

Equations (6.98) and (6.99) can be expressed as:

$$A_1 \cos \phi_1 + A_2 \cos \phi_2 + A_3 \cos \phi_3 = 0 \tag{1}$$

$$1.4430\ A_1 \cos \phi_1 + 0.5\ A_2 \cos \phi_2 - 0.6930\ A_3 \cos \phi_3 = 0 \tag{2}$$

$$1.6286\ A_1 \cos \phi_1 - 0.5\ A_2 \cos \phi_2 + 0.2047\ A_3 \cos \phi_3 = 0 \tag{3}$$

$$-0.3376 \sqrt{\frac{k}{m}}\ A_1 \sin \phi_1 - 1.4142 \sqrt{\frac{k}{m}}\ A_2 \sin \phi_2 - 2.0943 \sqrt{\frac{k}{m}}\ A_3 \sin \phi_3 = \dot{x}_{10} \tag{4}$$

$$-0.4872 \sqrt{\frac{k}{m}}\ A_1 \sin \phi_1 - 0.7071 \sqrt{\frac{k}{m}}\ A_2 \sin \phi_2 + 1.4513 \sqrt{\frac{k}{m}}\ A_3 \sin \phi_3 = 0 \tag{5}$$

$$-0.5498 \sqrt{\frac{k}{m}}\ A_1 \sin \phi_1 + 0.7071 \sqrt{\frac{k}{m}}\ A_2 \sin \phi_2 - 0.4287 \sqrt{\frac{k}{m}}\ A_3 \sin \phi_3 = 0 \tag{6}$$

Equations (1) to (3) give:

$$\phi_i = \frac{\pi}{2}\ ;\ i = 1,\ 2,\ 3$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

Treating $\sqrt{\frac{k}{m}}\ A_i \sin \phi_i$ (i = 1, 2, 3) as unknowns and noting that all $\phi_i = \frac{\pi}{2}$, Eqs. (4) to (6) can be solved to obtain

$$\sqrt{\frac{k}{m}}\ A_1 = -0.2257\ \dot{x}_{10}\ ;\ A_1 = -0.2257\sqrt{\frac{m}{k}}\ \dot{x}_{10}$$
$$\sqrt{\frac{k}{m}}\ A_2 = -0.3143\ \dot{x}_{10}\ ;\ A_2 = -0.3143\sqrt{\frac{m}{k}}\ \dot{x}_{10}$$
$$\sqrt{\frac{k}{m}}\ A_3 = -0.2289\ \dot{x}_{10}\ ;\ A_3 = -0.2289\sqrt{\frac{m}{k}}\ \dot{x}_{10}$$

The free vibration solution of the system can be expressed, using Eq. (6.96), as

$$x_1(t) = \dot{x}_{10}\sqrt{\frac{m}{k}}\ \Big(-0.2257 \cos\Big(0.3376\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big) - 0.3143 \cos\Big(1.4142\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big)$$
$$- 0.2289 \cos\Big(2.0943\sqrt{\frac{k}{m}} + \frac{\pi}{2}\Big)\Big) \qquad (7)$$

$$x_2(t) = \dot{x}_{10}\sqrt{\frac{m}{k}}\ \Big(-0.3257 \cos\Big(0.3376\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big) - 0.1571 \cos\Big(1.4142\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big)$$
$$+ 0.1586 \cos\Big(2.0943\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big)\Big) \qquad (8)$$

$$x_3(t) = \dot{x}_{10}\sqrt{\frac{m}{k}}\ \Big(-0.3676 \cos\Big(0.3376\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big) + 0.1571 \cos\Big(1.4142\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big)$$
$$- 0.0469 \cos\Big(2.0943\sqrt{\frac{k}{m}}\ t + \frac{\pi}{2}\Big)\Big) \qquad (9)$$

6.81 From solution of Problem 6.59, we find:

$$\omega_1 = 0.3863\sqrt{\frac{k}{m}}\ ;\ \omega_2 = 1.4142\sqrt{\frac{k}{m}}\ ;\ \omega_3 = 1.8305\sqrt{\frac{k}{m}}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1.8508 \\ 2 \end{Bmatrix}\ ;\ \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -0.5 \end{Bmatrix}\ ;\ \vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -1.3508 \\ 2 \end{Bmatrix}$$

Initial conditions:

$$x_i(0) = 0\ ,\ i = 1,\ 2,\ 3\ ;\ \dot{x}_1(0) = \dot{x}_2(0) = 0,\ \ \dot{x}_3(0) = \dot{x}_{30}$$

Equations (6.98) and (6.99) can be expressed as:

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = 0 \qquad (1)$$
$$1.8508\ A_1 \cos\phi_1 - 1.3508\ A_3 \cos\phi_3 = 0 \qquad (2)$$
$$2\ A_1 \cos\phi_1 - 0.5\ A_2 \cos\phi_2 + 2\ A_3 \cos\phi_3 = 0 \qquad (3)$$
$$-0.3863\sqrt{\frac{k}{m}}\ A_1 \sin\phi_1 - 1.4142\sqrt{\frac{k}{m}}\ A_2 \sin\phi_2$$
$$- 1.8305\sqrt{\frac{k}{m}}\ A_3 \sin\phi_3 = 0 \qquad (4)$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$-0.7150\sqrt{\frac{k}{m}}\,A_1 \sin\phi_1 + 2.4726\sqrt{\frac{k}{m}}\,A_3 \sin\phi_3 = 0 \qquad (5)$$

$$-0.7726\sqrt{\frac{k}{m}}\,A_1 \sin\phi_1 + 0.7071\sqrt{\frac{k}{m}}\,A_2 \sin\phi_2 - 3.6610\sqrt{\frac{k}{m}}\,A_3 \sin\phi_3 = \dot{x}_{30} \qquad (6)$$

Equations (1) to (3) yield:

$$\phi_i = \frac{\pi}{2} \ ; \ i = 1, 2, 3$$

and $\sqrt{\frac{k}{m}}\,A_i \sin\phi_i$ $(i = 1, 2, 3)$ are the unknowns in Eqs. (4) to (6). The solution of Eqs. (4) to (6) gives, with $\phi_i = \frac{\pi}{2}$:

$$\sqrt{\frac{k}{m}}\,A_1 = -0.4369\,\dot{x}_{30} \ ; \ A_1 = -0.4369\,\dot{x}_{30}\sqrt{\frac{m}{k}}$$

$$\sqrt{\frac{k}{m}}\,A_2 = 0.2828\,\dot{x}_{30} \ ; \ A_2 = 0.2828\,\dot{x}_{30}\sqrt{\frac{m}{k}}$$

$$\sqrt{\frac{k}{m}}\,A_3 = -0.1263\,\dot{x}_{30} \ ; \ A_3 = -0.1263\,\dot{x}_{30}\sqrt{\frac{m}{k}}$$

Thus the free vibration response of the system can be expressed as (see Eq. (6.96)):

$$x_1(t) = \dot{x}_{30}\sqrt{\frac{m}{k}}\left(-0.4369\cos\left(0.3863\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right) + 0.2828\cos\left(1.4142\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right) - 0.1263\cos\left(1.8305\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right)\right)$$

$$x_2(t) = \dot{x}_{30}\sqrt{\frac{m}{k}}\left\{-0.8086\cos\left(0.3863\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right) - 0.1706\cos\left(1.8305\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right)\right\}$$

$$x_3(t) = \dot{x}_{30}\sqrt{\frac{m}{k}}\left(-0.8738\cos\left(0.3863\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right) - 0.1414\cos\left(1.4142\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right) - 0.2526\cos\left(1.8305\sqrt{\frac{k}{m}}\,t + \frac{\pi}{2}\right)\right)$$

6.82 From Example 6.14, we obtain:

$$[m] = m\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} \ ; \ [k] = k\begin{bmatrix}1 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 1\end{bmatrix}$$

The natural frequencies and mode shapes are given by:

$$\omega_1 = 0 \ ; \ \omega_2 = \sqrt{\frac{k}{m}} \ ; \ \omega_3 = \sqrt{\frac{3\,k}{m}}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

$$\vec{X}^{(1)} = a \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} \; ; \; \vec{X}^{(2)} = b \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \; ; \; \vec{X}^{(3)} = c \begin{Bmatrix} 1 \\ -2 \\ 1 \end{Bmatrix}$$

Orthonormalization of mode shapes:

$$\vec{X}^{(1)T} \, [m] \, \vec{X}^{(1)} = a^2 \, (1 \quad 1 \quad 1) \, m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = 3 \, a^2 \, m = 1 \quad \text{or} \quad a = \sqrt{\frac{1}{3 \, m}}$$

$$\vec{X}^{(2)T} \, [m] \, \vec{X}^{(2)} = b^2 \, m \, (1 \quad 0 \quad -1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} = 2 \, b^2 \, m = 1 \quad \text{or} \quad b = \sqrt{\frac{1}{2 \, m}}$$

$$\vec{X}^{(3)T} \, [m] \, \vec{X}^{(3)} = c^2 \, m \, (1 \quad -2 \quad 1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ -2 \\ 1 \end{Bmatrix} = 6 \, c^2 \, m = 1 \quad \text{or} \quad c = \sqrt{\frac{1}{6 \, m}}$$

Modal matrix: $$[X] = [\vec{X}^{(1)} \quad \vec{X}^{(2)} \quad \vec{X}^{(3)}] = \frac{1}{\sqrt{m}} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \qquad (1)$$

Solution is given by Eq. (6.104):

$$\vec{x}(t) = [X] \, \vec{q}(t) \qquad (2)$$

where $\vec{q}(t)$ is given by Eqs. (6.113):

$$\ddot{q}_i(t) + \omega_i^2 \, q_i(t) = Q_i(t) \; ; \; i = 1, 2, 3 \qquad (3)$$

where

$$\vec{Q}(t) = [X]^T \, \vec{F}(t) = \vec{0} \quad \text{(no external forces)}$$

For the rigid body mode, $\omega_1^2 = 0$ and Eq. (3) reduces to:

$$\ddot{q}_1(t) = 0 \qquad (4)$$

whose solution can be written as

$$q_1(t) = c_1 + c_2 \, t \qquad (5)$$

where c_1 and c_2 are constants given by

$$c_1 = q_1(t = 0) \quad ; \quad c_2 = \dot{q}_1(t = 0) \qquad (6)$$

Initial conditions of the problem:

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\vec{x}(0) = \begin{Bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{7}$$

$$\text{and} \quad \dot{\vec{x}}(0) = \begin{Bmatrix} \dot{x}_0 \\ 0 \\ 0 \end{Bmatrix} \tag{8}$$

Using Eqs. (6.115) and (6.116), we find

$$\vec{q}(0) = [X]^T \; [m] \; \vec{x}(0) = \vec{0} \tag{9}$$

$$\dot{\vec{q}}(0) = [X]^T \; [m] \; \dot{\vec{x}}(0) = \sqrt{m} \; \dot{x}_0 \begin{Bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} \end{Bmatrix} \tag{10}$$

From Eqs. (6), (9) and (10), we obtain

$$c_1 = q_1(0) = 0 \quad ; \quad c_2 = \dot{q}_1(0) = \sqrt{\frac{m}{3}} \; \dot{x}_0 \tag{11}$$

and hence, from Eq. (5), we find

$$q_1(t) = \sqrt{\frac{m}{3}} \; \dot{x}_0 \; t \tag{12}$$

Solution of Eqs. (3) for $q_2(t)$ and $q_3(t)$ can be expressed as

$$q_2(t) = q_2(0) \cos \omega_2 \; t + \frac{\dot{q}_2(0)}{\omega_2} \sin \omega_2 \; t = \frac{m \, \dot{x}_0}{\sqrt{2 \, k}} \sin \sqrt{\frac{k}{m}} \; t \tag{13}$$

$$q_3(t) = q_3(0) \cos \omega_3 \; t + \frac{\dot{q}_3(0)}{\omega_3} \sin \omega_3 \; t = \frac{m \, \dot{x}_0}{3 \sqrt{2 \, k}} \sin \sqrt{\frac{3 \, k}{m}} \; t \tag{14}$$

The free vibration of the system is given by Eq. (2):

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{Bmatrix} = \frac{1}{\sqrt{m}} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{Bmatrix} \sqrt{\frac{m}{3}} \; \dot{x}_0 \; t \\ \frac{m \, \dot{x}_0}{\sqrt{2 \, k}} \sin \sqrt{\frac{k}{m}} \; t \\ \frac{m \, \dot{x}_0}{3 \sqrt{2 \, k}} \sin \sqrt{\frac{3 \, k}{m}} \; t \end{Bmatrix} \tag{15}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.83 For given system, $m = 10$, $k = 100$ and hence $\sqrt{k/m} = \sqrt{10} = 3.1623$. For free vibration response, the initial conditions lead to Eqs. (6.98) and (6.99), which can be expressed as:

$$A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 = x_1(0) = 0.1 \qquad (1)$$

$$1.8019\, A_1 \cos\phi_1 + 0.4450\, A_2 \cos\phi_2 - 1.2468\, A_3 \cos\phi_3 = x_2(0) = 0.1 \qquad (2)$$

$$2.2470\, A_1 \cos\phi_1 - 0.8020\, A_2 \cos\phi_2 + 0.5544\, A_3 \cos\phi_3 = x_3(0) = 0.1 \qquad (3)$$

$$-0.44504\sqrt{\frac{k}{m}}\, A_1 \sin\phi_1 - 1.2471\sqrt{\frac{k}{m}}\, A_2 \sin\phi_2 - 1.8025\sqrt{\frac{k}{m}}\, A_3 \sin\phi_3 = \dot{x}_1(0) = 0$$

or

$$-1.4073\, A_1 \sin\phi_1 - 3.9437\, A_2 \sin\phi_2 - 5.7000\, A_3 \sin\phi_3 = 0 \qquad (4)$$

$$-0.10192\sqrt{\frac{k}{m}}\, A_1 \sin\phi_1 - 0.55496\sqrt{\frac{k}{m}}\, A_2 \sin\phi_2 + 2.2474\sqrt{\frac{k}{m}}\, A_3 \sin\phi_3 = \dot{x}_2(0) = 0$$

or

$$-0.3223\, A_1 \sin\phi_1 - 1.7549\, A_2 \sin\phi_2 + 7.1069\, A_3 \sin\phi_3 = 0 \qquad (5)$$

$$-\sqrt{\frac{k}{m}}\, A_1 \sin\phi_1 + \sqrt{\frac{k}{m}}\, A_2 \sin\phi_2 - \sqrt{\frac{k}{m}}\, A_3 \sin\phi_3 = \dot{x}_3(0) = 0$$

or

$$-A_1 \sin\phi_1 + A_2 \sin\phi_2 - A_3 \sin\phi_3 = 0 \qquad (6)$$

Solution of Eqs. (1) to (3) is given by

$$A_1 \cos\phi_1 = 0.0543, \quad A_2 \cos\phi_2 = 0.0349, \quad A_3 \cos\phi_3 = 0.0108 \qquad (7)$$

Solution of Eqs. (4) to (6) is given by

$$A_1 \sin\phi_1 = 0, \quad A_2 \sin\phi_2 = 0, \quad A_3 \sin\phi_3 = 0 \qquad (8)$$

Equations (7) and (8) yield

$$\left.\begin{array}{l} A_1 = 0.0543, \quad A_2 = 0.0349, \quad A_3 = 0.0108 \\ \phi_1 = 0, \quad \phi_2 = 0, \quad \phi_3 = 0 \end{array}\right\} \qquad (9)$$

Thus the free vibration response of the system is given by Eq. (6.96):

$$x_1(t) = 0.0543 \cos\omega_1 t + 0.0349 \cos\omega_2 t + 0.0108 \cos\omega_3 t$$

$$x_2(t) = 1.8019(0.0543) \cos\omega_1 t + 0.4450(0.0349) \cos\omega_2 t$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$- 1.2468\,(0.0108)\cos\omega_3 t$$

$$x_3(t) = 2.2470\,(0.0543)\cos\omega_1 t - 0.8020\,(0.0349)\cos\omega_2 t + 0.5544\,(0.0108)\cos\omega_3 t$$

where $\omega_1 = 0.44504\sqrt{10} = 1.4073$ rad/sec, $\omega_2 = 1.2471\sqrt{10} = 3.9437$ rad/sec, and $\omega_3 = 1.8025\sqrt{10} = 5.7000$ rad/sec.

6.84 From the solution of Problem 5.28, the natural frequencies and normal modes are given by

$$\omega_1 = 2, \quad \vec{X}^{(1)} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} X_1^{(1)} \tag{1}$$

$$\omega_2 = \sqrt{12}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} X_1^{(2)} \tag{2}$$

where $X_1^{(1)}$ and $X_1^{(2)}$ are arbitrary constants. By orthogonalizing the normal modes with respect to the mass matrix, we can find the values of $X_1^{(1)}$ and $X_1^{(2)}$:

$$\vec{X}^{(1)T}[m]\vec{X}^{(1)} = 1 \Rightarrow \left(X_1^{(1)}\right)^2 \{1 \quad 1\} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} = 1$$

or $\quad X_1^{(1)} = \frac{1}{2}$

$$\vec{X}^{(2)T}[m]\vec{X}^{(2)} = 1 \Rightarrow \left(X_1^{(2)}\right)^2 \{1 \quad -1\} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} = 1$$

or $\quad X_1^{(2)} = \frac{1}{2}$

Thus the modal matrix becomes

$$[X] = \left[\vec{X}^{(1)} \quad \vec{X}^{(2)}\right] = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{3}$$

Using $\vec{x} = [X]\vec{q}$, the equations of motion can be written as

$$\ddot{\vec{q}} + [\omega^2_{\diagdown}]\,\vec{q} = \vec{Q} = \vec{0} \tag{4}$$

or

$$\ddot{q}_i + \omega_i^2\, q_i = 0\,; \quad i = 1,2 \tag{5}$$

Solution of Eqs. (5):

$$q_i(t) = q_{i0}\cos\omega_i t + \frac{\dot{q}_{i0}}{\omega_i}\sin\omega_i t \tag{6}$$

where q_{i0} and $\dot{q}_{i0}$ denote the initial values of q_i

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

and $\dot{q}_i$, respectively. From given initial conditions, we find:

$$\vec{q}(0) = \begin{Bmatrix} q_{10}(0) \\ q_{20}(0) \end{Bmatrix} = [X]^T [m] \vec{x}(0) = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$$\dot{\vec{q}}(0) = \begin{Bmatrix} \dot{q}_{10}(0) \\ \dot{q}_{20}(0) \end{Bmatrix} = [X]^T [m] \dot{\vec{x}}(0) = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\begin{Bmatrix} 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

Thus Eqs. (6) become

$$q_1(t) = \cos 2t + \frac{1}{2}\sin 2t$$

$$q_2(t) = \cos\sqrt{12}\, t - \frac{1}{\sqrt{12}}\sin\sqrt{12}\, t$$

The physical displacements can be found as

$$\vec{x}(t) = [X]\vec{q}(t) = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix}$$

or

$$x_1(t) = \frac{1}{2}(q_1 + q_2), \qquad x_2(t) = \frac{1}{2}(q_1 - q_2)$$

$$\therefore x_1(t) = \frac{1}{2}\left\{\cos 2t + \frac{1}{2}\sin 2t + \cos\sqrt{12}\, t - \frac{1}{\sqrt{12}}\sin\sqrt{12}\, t\right\}$$

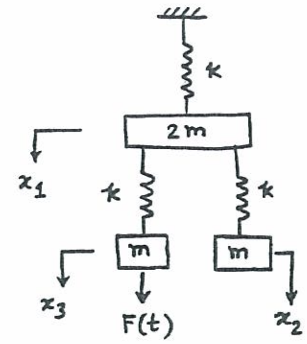
$$x_2(t) = \frac{1}{2}\left\{\cos 2t + \frac{1}{2}\sin 2t - \cos\sqrt{12}\, t + \frac{1}{\sqrt{12}}\sin\sqrt{12}\, t\right\}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

6.89 Equations of motion are

$$\begin{bmatrix} 2m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} 3k & -k & -k \\ -k & k & 0 \\ -k & 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F(t) \end{Bmatrix}$$



where $m = 1$ kg, $k = 1000$ N/m, $F(t) = 5 \sin 10t$ N.

Eigenvalue analysis:

Frequency equation is

$$\left| -\omega^2 m \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + k \begin{bmatrix} 3 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \right| = 0$$

or
$$\begin{vmatrix} -2\lambda+3 & -1 & -1 \\ -1 & -\lambda+1 & 0 \\ -1 & 0 & -\lambda+1 \end{vmatrix} = 0 \qquad \text{where} \quad \lambda = \frac{\omega^2 m}{k}$$

or $2\lambda^3 - 7\lambda^2 + 6\lambda - 1 = 0$

Roots are

$\lambda_1 = 0.219220$, $\quad \lambda_2 = 1.0$, $\quad \lambda_3 = 2.28078$

$\omega_1 = 0.4682094 \sqrt{\frac{k}{m}}$, $\quad \omega_2 = 1.0 \sqrt{\frac{k}{m}}$, $\quad \omega_3 = 1.5102251 \sqrt{\frac{k}{m}}$

Mode shapes are given by

$$\begin{bmatrix} -2\lambda_i+3 & -1 & -1 \\ -1 & -\lambda_i+1 & 0 \\ -1 & 0 & -\lambda_i+1 \end{bmatrix} \begin{Bmatrix} X_1^{(i)} \\ X_2^{(i)} \\ X_3^{(i)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

or $\quad X_1^{(i)} = (-\lambda_i+1)\, X_2^{(i)}$, $\quad X_3^{(i)} = (-2\lambda_i+3)\, X_1^{(i)} - X_2^{(i)}$

$$\vec{X}^{(i)} = \begin{Bmatrix} -\lambda_i+1 \\ 1 \\ (-2\lambda_i+3)(-\lambda_i+1) - 1 \end{Bmatrix} X_2^{(i)}$$

$$\vec{X}^{(1)} = \begin{Bmatrix} 0.78078 \\ 1 \\ 1 \end{Bmatrix} X_2^{(1)}, \quad \vec{X}^{(2)} = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} X_2^{(2)}, \quad \vec{X}^{(3)} = \begin{Bmatrix} -1.28078 \\ 1 \\ 1 \end{Bmatrix} X_2^{(3)}$$

Normalization of mode shapes with respect to $[m]$:

$$\vec{X}^{(1)T} [m]\, \vec{X}^{(1)} = (0.78078 \quad 1 \quad 1) \begin{Bmatrix} 1.56156 \\ 1 \\ 1 \end{Bmatrix} \left(X_2^{(1)}\right)^2 = 3.21923 \left(X_2^{(1)}\right)^2 = 1$$

$$X_2^{(1)} = 0.55734 \; ; \qquad \vec{X}^{(1)} = \begin{Bmatrix} 0.43516 \\ 0.55734 \\ 0.55734 \end{Bmatrix}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

$$\vec{X}^{(2)T}[m]\,\vec{X}^{(2)} = (0 \quad 1 \quad -1)\begin{Bmatrix}0\\1\\-1\end{Bmatrix}\left(X_2^{(2)}\right)^2 = 2\left(X_2^{(2)}\right)^2 = 1$$

$$X_2^{(2)} = 0.70711\ ; \qquad \vec{X}^{(2)} = \begin{Bmatrix}0\\0.70711\\-0.70711\end{Bmatrix}$$

$$\vec{X}^{(3)T}[m]\,\vec{X}^{(3)} = (-1.28078 \quad 1 \quad 1)\begin{Bmatrix}-2.56156\\1\\1\end{Bmatrix}\left(X_2^{(3)}\right)^2 = 5.28079\left(X_2^{(3)}\right)^2 = 1$$

$$X_2^{(3)} = 0.43516\ ; \qquad \vec{X}^{(3)} = \begin{Bmatrix}-0.55734\\0.43516\\0.43516\end{Bmatrix}$$

$\vec{Q} = [X]^T\,\vec{F}(t) =$ vector of generalized forces

$$= \begin{bmatrix}0.43516 & 0.55734 & 0.55734\\0 & 0.70711 & -0.70711\\-0.55734 & 0.43516 & 0.43516\end{bmatrix}\begin{Bmatrix}0\\0\\F_0 \sin\omega t\end{Bmatrix} = \begin{Bmatrix}2.7867\\-3.5356\\2.1758\end{Bmatrix}\sin 10t$$

Uncoupled equations of motion are

$$\left.\begin{aligned}\ddot{q}_1 + 219.22\, q_1 &= 2.7867 \sin 10t\\ \ddot{q}_2 + 1000.00\, q_2 &= -3.5356 \sin 10t\\ \ddot{q}_3 + 2280.78\, q_3 &= 2.1758 \sin 10t\end{aligned}\right\} \qquad (E.1)$$

Particular solutions of (E.1) are

$$q_1(t) = \left(\frac{2.7867}{219.22-100}\right)\sin 10t = 0.0233744 \sin 10t$$

$$q_2(t) = \left(\frac{-3.5356}{1000-100}\right)\sin 10t = -0.0039284 \sin 10t$$

$$q_3(t) = \left(\frac{2.1758}{2280.78-100}\right)\sin 10t = 0.0009977 \sin 10t$$

Since $\vec{x} = [X]\,\vec{q} = \begin{bmatrix}0.43516 & 0 & -0.55734\\0.55734 & 0.70711 & 0.43516\\0.55734 & -0.70711 & 0.43516\end{bmatrix}\vec{q}$, we get

$$x_1(t) = 0.0096155 \sin 10t \quad m$$

$$x_2(t) = 0.0095333 \sin 10t \quad m$$

$$x_3(t) = 0.0162395 \sin 10t \quad m.$$

6.90 (a) From Example 6.8, for $k_{t1} = k_{t2} = k_{t3} = k_t$ and $J_1 = J_2 = J_3 = J_0$,

$$[m] = J_0\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}, \qquad [k] = k_t\begin{bmatrix}2 & -1 & 0\\-1 & 2 & -1\\0 & -1 & 1\end{bmatrix}$$

Eigenvalue problem becomes

$$\begin{bmatrix}-\lambda+2 & -1 & 0\\-1 & -\lambda+2 & -1\\0 & -1 & -\lambda+1\end{bmatrix}\begin{Bmatrix}\Theta_1\\\Theta_2\\\Theta_3\end{Bmatrix} = \begin{Bmatrix}0\\0\\0\end{Bmatrix} \quad \text{where } \lambda = \frac{\omega^2 J_0}{k_t}. \qquad \text{--- (E.1)}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

Frequency equation is

$$\lambda^3 - 5\lambda^2 + 6\lambda - 1 = 0$$

Roots are

$$\lambda_1 = 0.198, \quad \lambda_2 = 1.555, \quad \lambda_3 = 3.247$$

$$\omega_1 = 0.44497\sqrt{\frac{k_t}{J_0}}, \quad \omega_2 = 1.24700\sqrt{\frac{k_t}{J_0}}, \quad \omega_3 = 1.80194\sqrt{\frac{k_t}{J_0}}$$

(E.1) gives $\Theta_2^{(j)} = (-\lambda_j + 2)\,\Theta_1^{(j)}$, $\Theta_3^{(j)} = \left(\frac{-\lambda_j+2}{-\lambda_j+1}\right)\Theta_1^{(j)}$

$$\vec{\Theta}^{(j)} = \begin{Bmatrix} 1 \\ -\lambda_j + 2 \\ \left(\frac{-\lambda_j+2}{-\lambda_j+1}\right) \end{Bmatrix} \Theta_1^{(j)}$$

$$\vec{\Theta}^{(1)} = \begin{Bmatrix} 1 \\ 1.802 \\ 2.247 \end{Bmatrix}\Theta_1^{(1)}, \quad \vec{\Theta}^{(2)} = \begin{Bmatrix} 1 \\ 0.445 \\ -0.802 \end{Bmatrix}\Theta_1^{(2)}, \quad \vec{\Theta}^{(3)} = \begin{Bmatrix} 1 \\ -1.247 \\ 0.555 \end{Bmatrix}\Theta_1^{(3)}$$

Normalization:

$$\vec{\Theta}^{(1)T}[m]\,\vec{\Theta}^{(1)} = (1 \quad 1.802 \quad 2.247)\begin{Bmatrix} 1 \\ 1.802 \\ 2.247 \end{Bmatrix} J_0 \left(\Theta_1^{(1)}\right)^2 = 9.2962\left(\Theta_1^{(1)}\right)^2 J_0 = 1$$

$$\Theta_1^{(1)} = 0.328/\sqrt{J_0}$$

$$\vec{\Theta}^{(2)T}[m]\,\vec{\Theta}^{(2)} = (1 \quad 0.445 \quad -0.802)\begin{Bmatrix} 1 \\ 0.445 \\ -0.802 \end{Bmatrix} J_0 \left(\Theta_1^{(2)}\right)^2 = 1.8412\left(\Theta_1^{(2)}\right)^2 J_0 = 1$$

$$\Theta_1^{(2)} = 0.737/\sqrt{J_0}$$

$$\vec{\Theta}^{(3)T}[m]\,\vec{\Theta}^{(3)} = (1 \quad -1.247 \quad 0.555)\begin{Bmatrix} 1 \\ -1.247 \\ 0.555 \end{Bmatrix} J_0 \left(\Theta_1^{(3)}\right)^2 = 2.863\left(\Theta_1^{(3)}\right)^2 J_0 = 1$$

$$\Theta_1^{(3)} = 0.591/\sqrt{J_0}$$

[m] - orthonormal modal matrix is

$$[X] = \frac{1}{\sqrt{J_0}}\begin{bmatrix} 0.328 & 0.737 & 0.591 \\ 0.591 & 0.328 & -0.737 \\ 0.737 & -0.591 & 0.328 \end{bmatrix}$$

For given data,

$$\omega_1 = 4.4497 \text{ rad/s}, \quad \omega_2 = 12.4700 \text{ rad/s}, \quad \omega_3 = 18.0194 \text{ rad/s}$$

$$[X] = \begin{bmatrix} 0.328 & 0.737 & 0.591 \\ 0.591 & 0.328 & -0.737 \\ 0.737 & -0.591 & 0.328 \end{bmatrix}$$

(b) According to modal analysis, uncoupled equations of motion are

$$\ddot{q}_i(t) + \omega_i^2\, q_i(t) = Q_i(t)\ ; \quad i = 1, 2, 3 \qquad \text{(E.2)}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

where $\vec{Q}(t) = [X]^T \vec{M}_t(t) = [X]^T \begin{Bmatrix} 0 \\ 0 \\ M_{t0} \cos\omega t \end{Bmatrix}$

$$= \begin{Bmatrix} Q_{10} \\ Q_{20} \\ Q_{30} \end{Bmatrix} \cos 100t \equiv \begin{Bmatrix} 368.5 \\ -295.5 \\ 164.0 \end{Bmatrix} \cos 100t$$

Steady state solution of (E.2) is $q_i(t) = \left(\frac{Q_{i0}}{\omega_i^2 - \omega^2}\right) \cos\omega t$

$$\therefore \; q_1(t) = -0.03692 \cos 100t$$
$$q_2(t) = 0.03002 \cos 100t$$
$$q_3(t) = -0.01695 \cos 100t$$

Angular deflections are

$$\vec{\theta}(t) = [X]\vec{q}(t) = \begin{Bmatrix} -0.0000025 \\ 0.0005190 \\ -0.0505115 \end{Bmatrix} \cos 100t \quad \text{radian}$$

6.91 From problem 6.56, $\vec{X}^{(1)} = \begin{Bmatrix} 1 \\ \sqrt{2} \\ 1 \end{Bmatrix} X_1^{(1)}$, $\vec{X}^{(2)} = \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} X_1^{(2)}$, $\vec{X}^{(3)} = \begin{Bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{Bmatrix} X_1^{(3)}$

$$[m] = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Normalization gives

$$\vec{X}^{(1)T}[m]\vec{X}^{(1)} = (1 \;\; \sqrt{2} \;\; 1) \begin{Bmatrix} 1 \\ \sqrt{2} \\ 1 \end{Bmatrix} \left(X_1^{(1)}\right)^2 m = 1 \;\Rightarrow\; X_1^{(1)} = \frac{1}{2\sqrt{m}}$$

$$\vec{X}^{(2)T}[m]\vec{X}^{(2)} = (1 \;\; 0 \;\; -1) \begin{Bmatrix} 1 \\ 0 \\ -1 \end{Bmatrix} \left(X_1^{(2)}\right)^2 m = 1 \;\Rightarrow\; X_1^{(2)} = \frac{1}{\sqrt{2m}}$$

$$\vec{X}^{(3)T}[m]\vec{X}^{(3)} = (1 \;\; -\sqrt{2} \;\; 1) \begin{Bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{Bmatrix} \left(X_1^{(3)}\right)^2 m = 1 \;\Rightarrow\; X_1^{(3)} = \frac{1}{2\sqrt{m}}$$

Modal matrix is

$$[X] = \frac{1}{\sqrt{m}} \begin{bmatrix} 1/2 & 1/\sqrt{2} & 1/2 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 1/2 & -1/\sqrt{2} & 1/2 \end{bmatrix}$$

original coupled equations are $[m]\ddot{\vec{x}} + [k]\vec{x} = \vec{0}$

uncoupled equations with $\vec{x} = [X]\vec{q}$ are

$$\ddot{\vec{q}} + [\omega^2]\vec{q} = \vec{0}$$

or

$$\begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{Bmatrix} + \begin{bmatrix} \omega_1^2 & 0 & 0 \\ 0 & \omega_2^2 & 0 \\ 0 & 0 & \omega_3^2 \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

with $\omega_1^2 = 0.585786 \frac{k}{m}$, $\omega_2^2 = 2 \frac{k}{m}$, $\omega_3^2 = 3.414214 \frac{k}{m}$.

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

6.92 $[k]^{-1}\ \vec{F}(t) = \frac{1}{k}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} F_0 \cos\omega t = \frac{1}{k}\begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix} F_0 \cos\omega t$

From Example 6.19,

$$q_{10} = \frac{Q_{10}}{\omega_1^2}\frac{1}{\left|1-\left(\frac{\omega}{\omega_1}\right)^2\right|} = \frac{1.6561\ \frac{F_0}{\sqrt{m}}}{\left(0.19806\ \frac{k}{m}\right)} * \frac{1}{\left|1-\left(\frac{1.75}{0.44504}\right)^2\right|}$$

$$= 0.57816\ F_0\sqrt{m}/k$$

and $q_1(t) = q_{10}\cos\omega t$; $\ddot{q}_1 = -q_{10}\,\omega^2\cos\omega t$

$$-\frac{1}{\omega_1^2}\vec{X}^{(1)}\ddot{q}_1(t) = q_{10}\left(\frac{\omega^2}{\omega_1^2}\right)\frac{1}{\sqrt{m}}\begin{Bmatrix} 0.3280 \\ 0.5911 \\ 0.7370 \end{Bmatrix}\cos\omega t$$

$$= 8.93979\ \frac{F_0}{k}\begin{Bmatrix} 0.3280 \\ 0.5911 \\ 0.7370 \end{Bmatrix}\cos\omega t$$

Eq. (E.3) in problem statement gives

$$\vec{x}(t) = \begin{Bmatrix} 3 \\ 5 \\ 6 \end{Bmatrix}\frac{F_0}{k}\cos\omega t + \begin{Bmatrix} 2.93225 \\ 5.28431 \\ 6.58863 \end{Bmatrix}\frac{F_0}{k}\cos\omega t$$

$$= \begin{Bmatrix} 5.93225 \\ 10.28431 \\ 12.58863 \end{Bmatrix}\frac{F_0}{k}\cos\omega t$$

6.93 Equations of motion for free vibration

$$m\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}\ddot{\vec{x}} + k\begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix}\vec{x} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

From problem 6.51,

$\omega_1 = 0.337627$, $\omega_2 = 1.414221$, $\omega_3 = 2.094278$

$$\vec{X}^{(1)} = \begin{Bmatrix} 1.0 \\ 1.443004 \\ 1.628659 \end{Bmatrix} X_1^{(1)},\quad \vec{X}^{(2)} = \begin{Bmatrix} 1.0 \\ 0.49999 \\ -0.49998 \end{Bmatrix} X_1^{(2)},\quad \vec{X}^{(3)} = \begin{Bmatrix} 1.0 \\ -0.693 \\ 0.204666 \end{Bmatrix} X_1^{(3)}$$

Free vibratory motion is given by (see section 5.3)

$$\vec{x}(t) = \vec{x}^{(1)}(t) + \vec{x}^{(2)}(t) + \vec{x}^{(3)}(t)$$

$$= \begin{Bmatrix} X_1^{(1)}\cos(\omega_1 t+\phi_1) + X_1^{(2)}\cos(\omega_2 t+\phi_2) + X_1^{(3)}\cos(\omega_3 t+\phi_3) \\ 1.443\ X_1^{(1)}\cos(\omega_1 t+\phi_1) + 0.5\ X_1^{(2)}\cos(\omega_2 t+\phi_2) - 0.693\ X_1^{(3)}\cos(\omega_3 t+\phi_3) \\ 1.6287\ X_1^{(1)}\cos(\omega_1 t+\phi_1) - 0.5\ X_1^{(2)}\cos(\omega_2 t+\phi_2) + 0.2047\ X_1^{(3)}\cos(\omega_3 t+\phi_3) \end{Bmatrix} \quad \text{--- (E.1)}$$

Known initial conditions give

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
MechMuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{Bmatrix} = \begin{Bmatrix} 1 \\ 2 \\ 1 \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.443 & 0.5 & -0.693 \\ 1.6287 & -0.5 & 0.2047 \end{bmatrix} \begin{Bmatrix} X_1^{(1)} \cos\phi_1 \\ X_1^{(2)} \cos\phi_2 \\ X_1^{(3)} \cos\phi_3 \end{Bmatrix} \quad \text{--- (E.2)}$$

$$\begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \\ \dot{x}_3(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ -1 \end{Bmatrix} = \begin{bmatrix} -\omega_1 & -\omega_2 & -\omega_3 \\ -1.443\,\omega_1 & -0.5\,\omega_2 & 0.693\,\omega_3 \\ -1.6287\,\omega_1 & 0.5\,\omega_2 & -0.2047\,\omega_3 \end{bmatrix} \begin{Bmatrix} X_1^{(1)} \sin\phi_1 \\ X_1^{(2)} \sin\phi_2 \\ X_1^{(3)} \sin\phi_3 \end{Bmatrix} \quad \text{--- (E.3)}$$

Solution of (E.2) and (E.3) gives

$X_1^{(1)} \cos\phi_1 = 0.8884$, $X_1^{(1)} \sin\phi_1 = 0.4514$

$X_1^{(2)} \cos\phi_2 = 0.6667$, $X_1^{(2)} \sin\phi_2 = -0.7857$

$X_1^{(3)} \cos\phi_3 = -0.5551$, $X_1^{(3)} \sin\phi_3 = 0.4578$

Hence $X_1^{(1)} = 0.9965$, $X_1^{(2)} = 1.0304$, $X_1^{(3)} = 0.7195$

$\phi_1 = 26.9353^\circ$, $\phi_2 = -49.6840^\circ$, $\phi_3 = 140.4870^\circ$

and the final solution is given by (E.1).

6.94

$$\vec{x}(t) = [X]\,\vec{q}(t) \quad ; \quad \dot{\vec{x}}(t) = [X]\,\dot{\vec{q}}(t) \tag{1}$$

If $\vec{x}(0)$ and $\dot{\vec{x}}(0)$ are the known initial conditions in terms of physical coordinates, to find the corresponding values of $\vec{q}(0)$ and $\dot{\vec{q}}(0)$, we premultiply both sides of Eq. (1) by $[X]^T\,[m]$ to obtain at $t = 0$:

$$[X]^T\,[m]\,\vec{x}(0) = [X]^T\,[m]\,[X]\,\vec{q}(0) \tag{2}$$

$$[X]^T\,[m]\,\dot{\vec{x}}(0) = [X]^T\,[m]\,[X]\,\dot{\vec{q}}(0) \tag{3}$$

Since the normal modes are normalized with respect to the mass matrix as

$$[X]^T\,[m]\,[X] = [I] \tag{4}$$

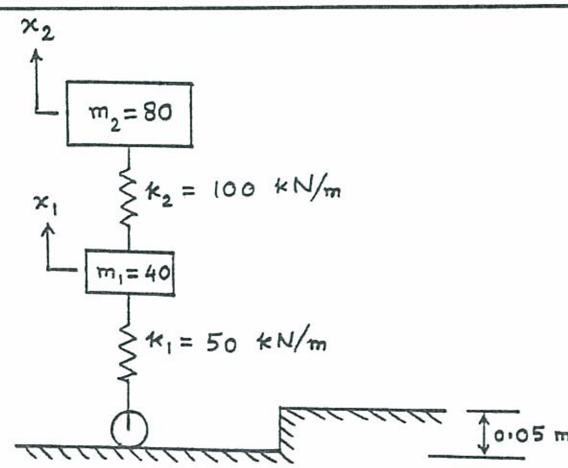
Eqs. (2) and (3) reduce to:

$$\vec{q}(0) = [X]^T\,[m]\,\vec{x}(0) \quad ; \quad \dot{\vec{q}}(0) = [X]^T\,[m]\,\dot{\vec{x}}(0) \tag{5}$$

6.95

$$[k] = \begin{bmatrix} 150 & -100 \\ -100 & 100 \end{bmatrix} (10^3)\ \text{N/m}$$

$$[m] = \begin{bmatrix} 40 & 0 \\ 0 & 80 \end{bmatrix}\ \text{kg}$$



PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

Natural frequencies are given by (see Eq. (3) in the solution of Problem 5.5):

$$\omega_{1,2}^2 = \frac{k_1 + k_2}{2\, m_1} + \frac{k_2}{2\, m_2} \pm \left\{ \frac{1}{4} \left(\frac{k_1 + k_2}{m_1} + \frac{k_2}{m_2} \right)^2 - \frac{k_1\, k_2}{m_1\, m_2} \right\}^{\frac{1}{2}}$$

$$= \left[\frac{150}{80} + \frac{100}{160} \pm \left\{ \frac{1}{4} \left(\frac{150}{40} + \frac{100}{80} \right)^2 - \frac{(100)\,(50)}{(40)\,(80)} \right\}^{\frac{1}{2}} \right] (10^3)$$

$$= 334.9365,\ 4665.1$$

$$\omega_1 = 18.3013 \text{ rad/sec} \;\; ; \;\; \omega_2 = 68.3015 \text{ rad/sec}$$

Mode shapes are defined by Eqs. (4) and (5) in the solution of Problem 5.1:

$$\frac{X_2^{(1)}}{X_1^{(1)}} = \frac{k_2}{-\, m_2\, \omega_1^2 + k_2} = \frac{100\, (10^3)}{-\,(80)\,(334.9365) + (100)\,(10^3)} = 1.3660$$

$$\vec{X}^{(1)} = a \begin{Bmatrix} 1.0 \\ 1.366 \end{Bmatrix}$$

where a is a constant.

$$\frac{X_2^{(2)}}{X_1^{(2)}} = \frac{k_2}{-\, m_2\, \omega_2^2 + k_2} = \frac{(100)\,(10^3)}{-\,(80)\,(4665.1) + (100)\,(10^3)} = -\,0.3660$$

$$\vec{X}^{(2)} = b \begin{Bmatrix} 1.0 \\ -0.366 \end{Bmatrix}$$

where b is a constant.

Orthogonalization of modes:

$$\vec{X}^{(1)T}\, [m]\, \vec{X}^{(1)} = a^2\, (1.0 \quad 1.366) \begin{bmatrix} 40 & 0 \\ 0 & 80 \end{bmatrix} \begin{Bmatrix} 1.0 \\ 1.366 \end{Bmatrix} = 189.2765\, a^2 = 1$$

$$a = 0.07269$$

$$\vec{X}^{(2)T}\, [m]\, \vec{X}^{(2)} = b^2\, (1.0 \quad -0.366) \begin{bmatrix} 40 & 0 \\ 0 & 80 \end{bmatrix} \begin{Bmatrix} 1.0 \\ -0.366 \end{Bmatrix} = 50.7165\, b^2 = 1$$

$$b = 0.14042$$

Modal matrix:

$$[X] = \begin{bmatrix} 0.07269 & 0.14042 \\ 0.09929 & -0.05139 \end{bmatrix}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Due to the elevation of 0.05 m, spring k_1 and hence m_1 will be subjected to additional compression of k_1 (0.05) = 2500 N.

$$\vec{F}(t) = \begin{Bmatrix} 2500 \\ 0 \end{Bmatrix} \text{ N}$$

Equation (6.111) gives:

$$\vec{Q}(t) = [X]^T \vec{F}(t) = \begin{Bmatrix} 181.725 \\ 351.05 \end{Bmatrix}$$

Solution is given by (without initial conditions) Eq. (6.114):

$$q_i(t) = \frac{1}{\omega_i} \int_0^t Q_i(\tau) \sin \omega_i (t - \tau)\, d\tau \quad ; \quad i = 1, 2$$

$$\text{Since} \quad \int_{\tau=0}^{t} \sin \Omega (t - \tau)\, d\tau = - \int_{\tau' = t - \tau = t}^{\tau' = t - \tau = 0} \sin \Omega \tau'\, d\tau' = \frac{1}{\Omega} (1 - \cos \Omega t)$$

we find

$$q_1(t) = \frac{181.725}{(18.3013^2)} (1 - \cos 18.3013\, t) = 0.5426\, (1 - \cos 18.3013\, t)$$

$$q_2(t) = \frac{351.05}{(68.3015^2)} (1 - \cos 68.3015\, t) = 0.07525\, (1 - \cos 68.3015\, t)$$

Response of the masses can be found from Eq. (6.104):

$$\vec{x}(t) = [X]\, \vec{q}(t) = \begin{bmatrix} 0.07269 & 0.14042 \\ 0.09929 & -0.05139 \end{bmatrix} \begin{Bmatrix} 0.5426\, (1 - \cos 18.3013\, t) \\ 0.07525\, (1 - \cos 68.3015\, t) \end{Bmatrix}$$

$$= \begin{Bmatrix} 0.03944\, (1 - \cos 18.3013\, t) + 0.01057\, (1 - \cos 68.3015\, t) \\ 0.05387\, (1 - \cos 18.3013\, t - 0.00387\, (1 - \cos 68.3015\, t) \end{Bmatrix}$$

Note:

This problem can also be solved by specifying the initial conditions as

$$\vec{x}(0) = \begin{Bmatrix} 0.05 \\ 0.05 \end{Bmatrix} \text{ m} \quad ; \quad \dot{\vec{x}}(0) = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

and solving the free vibration problem.

6.96 $\ell_i = 0.5$ m ; $m_i = 1$ kg (i = 1, 2, 3). Assume $m_1 = m_2 = m_3 = m = 1$; $\ell_1 = \ell_2 = \ell_3 = \ell = 0.5$.

From solution of Problem 6.42, we obtain:

$$[m] = m\, \ell^2 \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 0.25 \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

$$[k] = m\,g\,\ell \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4.905 \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

From solution of Problem 6.58, the natural frequencies and mode shapes can be found as

$$\omega_1 = 0.644798 \sqrt{\frac{g}{\ell}} \;;\; \omega_2 = 1.514698 \sqrt{\frac{g}{\ell}} \;;\; \omega_3 = 2.507977 \sqrt{\frac{g}{\ell}}$$

Since $\sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.81}{0.5}} = 4.4294$, we find

$$\omega_1 = 2.8561 \text{ rad/sec} \;;\; \omega_2 = 6.7092 \text{ rad/sec} \;;\; \omega_3 = 11.1088 \text{ rad/sec}$$

$$\vec{X}^{(1)} = a \begin{Bmatrix} 1.0 \\ 1.2922 \\ 1.6312 \end{Bmatrix} \;;\; \vec{X}^{(2)} = b \begin{Bmatrix} 1.0 \\ 0.3527 \\ -2.3978 \end{Bmatrix} \;;\; \vec{X}^{(3)} = c \begin{Bmatrix} 1.0 \\ -1.6450 \\ 0.7669 \end{Bmatrix}$$

Orthonormalization of mode shapes:

$$\begin{aligned} \vec{X}^{(1)^T} [m]\, \vec{X}^{(1)} &= 5.4118\, a^2 = 1 \quad \text{or} \quad a = 0.4299 \\ \vec{X}^{(2)^T} [m]\, \vec{X}^{(2)} &= 0.9805\, b^2 = 1 \quad \text{or} \quad b = 1.0099 \\ \vec{X}^{(3)^T} [m]\, \vec{X}^{(3)} &= 0.3577\, c^2 = 1 \quad \text{or} \quad c = 1.6720 \end{aligned}$$

Modal matrix:

$$[X] = \left[\vec{X}^{(1)} \quad \vec{X}^{(2)} \quad \vec{X}^{(3)}\right] = \begin{bmatrix} 0.4299 & 1.0099 & 1.6720 \\ 0.5555 & 0.3562 & -2.7504 \\ 0.7012 & -2.4215 & 1.2822 \end{bmatrix} \tag{1}$$

$$\vec{F}(t) = \begin{Bmatrix} 0 \\ 0 \\ M_{t3}(t) \end{Bmatrix} \tag{2}$$

$$\vec{Q}(t) = [X]^T\, \vec{F}(t) = \begin{Bmatrix} 0.7012\, M_{t3}(t) \\ -\, 2.4215\, M_{t3}(t) \\ 1.2822\, M_{t3}(t) \end{Bmatrix} \tag{3}$$

Solution of $q_i(t)$ without initial conditions:

$$q_i(t) = \frac{1}{\omega_i} \int_0^t Q_i(\tau) \sin \omega_i\, (t - \tau)\, d\tau \;;\; i = 1,\, 2,\, 3 \tag{4}$$

We denote $M_{t3}(t)$ as

$$M_{t3}(t) = M_0 \left(u(t) - u(t - t_0)\right) \tag{5}$$

where $M_0 = 0.1$ N-m, $t_0 = 0.1$ sec and $u(t)$ and $u(t - t_0)$ are the unit step functions:

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

$$u(t) = \begin{cases} 0 & ;\ t < 0 \\ 1 & ;\ t > 0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 0 & ;\ t < t_0 \\ 1 & ;\ t > t_0 \end{cases}$$

Thus we obtain

$$q_1(t) = \frac{1}{\omega_1} \int_0^t Q_1(\tau) \sin \omega_1 \ (t - \tau) \ d\tau$$

$$= \frac{1}{2.8561} \int_0^t 0.7012 \ (0.1) \left\{ u(\tau) - u(\tau - 0.1) \right\} \sin 2.8561 \ (t - \tau) \ d\tau$$

$$= 0.02455 \left\{ \int_0^t u(\tau) \sin 2.8561 \ (t - \tau) \ d\tau - \int_0^t u(\tau - 0.1) \sin 2.8561 \ (t - \tau) \ d\tau \right\}$$

By noting that

$$\int_0^t u(\tau) \sin \Omega \ (t - \tau) \ d\tau = \frac{1}{\Omega} \ (1 - \cos \Omega \ t) \tag{6}$$

$$\text{and} \quad \int_0^t u(\tau - t_0) \sin \Omega \ (t - \tau) \ d\tau = \frac{1}{\Omega} \left[1 - \cos \Omega \ (t - t_0) \right] \tag{7}$$

we can derive

$$\begin{aligned} q_1(t) &= 0.008596 \left[1 - \cos 2.8561 \ t \right] \quad ; \ 0 < t < 0.1 \text{ sec} \\ &= 0.008596 \left[\cos 2.8561 \ (t - 0.1) - \cos 2.8561 \ t \right] \quad ; \ t \geq 0.1 \text{ sec} \end{aligned} \tag{8}$$

Similarly, we can derive

$$\begin{aligned} q_2(t) &= -\ 0.005379 \left[1 - \cos 6.7092 \ t \right] \quad ; \ 0 < t < 0.1 \text{ sec} \\ &= -\ 0.005379 \left[\cos 6.7092 \ (t - 0.1) - \cos 6.7092 \ t \right] \quad ; \ t \geq 0.1 \text{ sec} \end{aligned} \tag{9}$$

$$\begin{aligned} q_3(t) &= 0.001039 \left[1 - \cos 11.1088 \ t \right] \quad ; \ 0 < t < 0.1 \text{ sec} \\ &= 0.001039 \left[\cos 11.1088 \ (t - 0.1) - \cos 11.1088 \ t \right] \quad : \ t \geq 0.1 \text{ sec} \end{aligned} \tag{10}$$

Thus the angular (physical) displacements of the pendulum can be expressed as:

$$\vec{x}(t) = [X] \ \vec{q}(t) \tag{11}$$

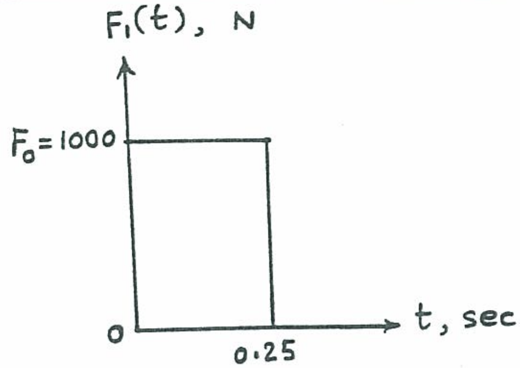
where $[X]$ is given by Eq. (1) and $\vec{q}(t)$ is given by Eqs. (8) to (10). Hence

$$\begin{aligned} x_1(t) &= 0.003675 \ (1 - \cos 2.8561 \ t) - 0.005432 \ (1 - \cos 6.7092 \ t) \\ &\quad + 0.001737 \ (1 - \cos 11.1088 \ t) \quad 0 < t < 0.1 \text{ sec} \\ &= 0.003695 \left[\cos 2.8561 \ (t - 0.1) - \cos 2.8561 \ t \right] \\ &\quad - 0.005432 \left[\cos 6.7092 \ (t - 0.1) - \cos 6.7092 \ t \right] \\ &\quad + 0.001737 \left[\cos 11.1088 \ (t - 0.1) - \cos 11.1088 \ t \right] \quad t \geq 0.1 \text{ sec} \end{aligned}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
لجنة الميكانيك - الإتجاه الإسلامي

$$x_2(t) = 0.004775\ (1 - \cos 2.8561\ t) - 0.001916\ (1 - \cos 6.7092\ t)$$
$$- 0.002858\ (1 - \cos 11.1088\ t) \quad 0 < t < 0.1 \text{ sec}$$
$$= 0.004775 \left[\cos 2.8561\ (t - 0.1) - \cos 2.8561\ t\right]$$
$$- 0.001916 \left[\cos 6.7092\ (t - 0.1) - \cos 6.7092\ t\right]$$
$$- 0.002858 \left[\cos 11.1088\ (t - 0.1) - \cos 11.1088\ t\right] \quad t \geq 0.1 \text{ sec}$$
$$x_3(t) = 0.006027\ (1 - \cos 2.8561\ t) + 0.013025\ (1 - \cos 6.7092\ t)$$
$$+ 0.001332\ (1 - \cos 11.1088\ t) \quad 0 < t < 0.1 \text{ sec}$$
$$= 0.006027 \left[\cos 2.8561\ (t - 0.1) - \cos 2.8561\ t\right]$$
$$+ 0.013025 \left[\cos 6.7092\ (t - 0.1) - \cos 6.7092\ t\right]$$
$$+ 0.001332 \left[\cos 11.1088\ (t - 0.1) - \cos 11.1088\ t\right] \quad t \geq 0.1 \text{ sec}$$

6.97 $m = 2$ kg, $k = 10{,}000$ N/m,



$$\sqrt{\frac{k}{m}} = \sqrt{\frac{10000}{2}} = 70.7107.$$

$$[m] = 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$[k] = 10^4 \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix}$$

From the solution of Problem 6.51, we find the natural frequencies and mode shapes as:

$$\omega_1 = 0.337627 \sqrt{\frac{k}{m}} \ ; \ \omega_2 = 1.414221 \sqrt{\frac{k}{m}} \ ; \ \omega_3 = 2.094278 \sqrt{\frac{k}{m}}$$

or $\omega_1 = 23.8738$ rad/sec ; $\omega_2 = 100.0006$ rad/sec ; $\omega_3 = 148.0879$ rad/sec

$$\vec{X}^{(1)} = a \begin{Bmatrix} 1.0 \\ 1.4430 \\ 1.6286 \end{Bmatrix} \ ; \ \vec{X}^{(2)} = b \begin{Bmatrix} 1.0 \\ 0.5 \\ -0.5 \end{Bmatrix} \ ; \ \vec{X}^{(3)} = c \begin{Bmatrix} 1.0 \\ -0.6930 \\ 0.2047 \end{Bmatrix}$$

Orthonormalization of modes:

$$\vec{X}^{(1)T}\ [m]\ \vec{X}^{(1)} = 26.2430\ a^2 = 1 \quad \text{or} \quad a = 0.1952$$

$$\vec{X}^{(2)T}\ [m]\ \vec{X}^{(2)} = 4.5\ b^2 = 1 \quad \text{or} \quad b = 0.4714$$

$$\vec{X}^{(3)T}\ [m]\ \vec{X}^{(3)} = 4.1724\ c^2 = 1 \quad \text{or} \quad c = 0.4896$$

Modal matrix:

$$[X] = \left[\vec{X}^{(1)} \quad \vec{X}^{(2)} \quad \vec{X}^{(3)}\right] = \begin{bmatrix} 0.1952 & 0.4714 & 0.4896 \\ 0.2817 & 0.2357 & -0.3393 \\ 0.3179 & -0.2357 & 0.1002 \end{bmatrix} \qquad (1)$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$\vec{F}(t) = \begin{Bmatrix} F_1(t) \\ 0 \\ 0 \end{Bmatrix} \tag{2}$$

$$\vec{Q}(t) = [X]^T \vec{F}(t) = \begin{Bmatrix} 0.1952\ F_1(t) \\ 0.4714\ F_1(t) \\ 0.4896\ F_1(t) \end{Bmatrix} \tag{3}$$

Solution of $q_i(t)$ without initial conditions:

$$q_i(t) = \frac{1}{\omega_i} \int_0^t Q_i(\tau) \sin \omega_i (t - \tau)\, d\tau \ ; \ i = 1,\ 2,\ 3 \tag{4}$$

$$F_1(t) = F_0 \left\{ u(t) - u(t - t_0) \right\}$$

where $F_0 = 1000$ N, $t_0 = 0.25$ sec, and $u(t)$ and $u(t - t_0)$ are unit step functions:

$$u(t) = \begin{cases} 0 & ;\ t < 0 \\ 1 & ;\ t > t_0 \end{cases}$$

$$u(t - t_0) = \begin{cases} 0 & ;\ t < t_0 \\ 1 & ;\ t > t_0 \end{cases}$$

Equations (3) give:

$$\vec{Q}(t) = \begin{Bmatrix} Q_1(t) \\ Q_2(t) \\ Q_3(t) \end{Bmatrix} = \begin{Bmatrix} 195.2 \left\{ u(t) - u(t - 0.25) \right\} \\ 471.4 \left\{ u(t) - u(t - 0.25) \right\} \\ 489.6 \left\{ u(t) - u(t - 0.25) \right\} \end{Bmatrix} \tag{5}$$

and Eqs. (4) yield:

$$q_1(t) = 0.3425\ (1 - \cos 23.8738\ t) \ ; \ 0 < t < 0.25 \text{ sec}$$
$$= 0.3425 \left[\cos 23.8738\ (t - 0.25) - \cos 23.8738\ t \right] \ ; \ t \geq 0.25 \text{ sec}$$

$$q_2(t) = 0.04714\ (1 - \cos 100.0006\ t) \ ; \ 0 < t < 0.25 \text{ sec}$$
$$= 0.04714 \left[\cos 100.0006\ (t - 0.25) - \cos 100.0006\ t \right] \ \ t \geq 0.25 \text{ sec}$$

$$q_3(t) = 0.02232\ (1 - \cos 148.0879\ t) \ ; \ 0 < t < 0.25 \text{ sec}$$
$$= 0.02232 \left[\cos 148.0879\ (t - 0.25) - \cos 148.0879\ t \right] \ ; \ t \geq 0.25 \text{ sec}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

The physical displacements of the masses are given by:

$$\vec{x}(t) = [X]\,\vec{q}(t)$$

which can be explicitly expressed as:

$$\begin{aligned}
x_1(t) &= 0.06686\,(1 - \cos 23.8738\,t) + 0.02222\,(1 - \cos 100.0006\,t) \\
&\quad + 0.01093\,(1 - \cos 148.0879\,t) \;;\; 0 < t < 0.25 \text{ sec} \\
&= 0.06686\left[\cos 23.8738\,(t - 0.25) - \cos 23.8738\,t\right] \\
&\quad + 0.02222\left[\cos 100.0006\,(t - 0.25) - \cos 100.0006\,t\right] \\
&\quad + 0.01093\left[\cos 148.0879\,(t - 0.25) - \cos 148.0879\,t\right] \;;\; t \geq 0.25 \text{ sec} \\
x_2(t) &= 0.09648\,(1 - \cos 23.8738\,t) + 0.01111\,(1 - \cos 100.0006\,t) \\
&\quad - 0.007573\,(1 - \cos 148.0879\,t) \;;\; 0 < t < 0.25 \text{ sec} \\
&= 0.09648\left[\cos 23.8738\,(t - 0.25) - \cos 23.8738\,t\right] \\
&\quad + 0.01111\left[\cos 100.0006\,(t - 0.25) - \cos 100.0006\,t\right] \\
&\quad - 0.007573\left[\cos 148.0879\,(t - 0.25) - \cos 148.0879\,t\right] \;;\; t \geq 0.25 \text{ sec} \\
x_3(t) &= 0.1089\,(1 - \cos 23.8738\,t) - 0.01111\,(1 - \cos 100.0006\,t) \\
&\quad + 0.002236\,(1 - \cos 148.0879\,t) \;;\; 0 < t < 0.25 \text{ sec} \\
&= 0.1089\left[\cos 23.8738\,(t - 0.25) - \cos 23.8738\,t\right] \\
&\quad - 0.01111\left[\cos 100.0006\,(t - 0.25) - \cos 100.0006\,t\right] \\
&\quad + 0.002236\left[\cos 148.0879\,(t - 0.25) - \cos 148.0879\,t\right] \;;\; t \geq 0.25 \text{ sec}
\end{aligned}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.99 Equations of motion $[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{F}$ --- (E.1)

where

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad [c] = \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3+c_4 \end{bmatrix},$$

$$[k] = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3+k_4 \end{bmatrix}, \quad \vec{x} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix}, \quad \vec{F} = \begin{Bmatrix} F_1 = F_0 \cos \omega t \\ F_2 = 0 \\ F_3 = 0 \end{Bmatrix}$$

Since $F_j(t) = \text{Re}\left[F_{j0}\, e^{i\omega t}\right]$ with $F_{10} = F_0$, and $F_{20} = F_{30} = 0$,

we assume $x_j(t) = X_j\, e^{i\omega t}$; $j = 1, 2, 3$. Then (E.1) becomes

$$\left[Z_{rs}(i\omega)\right] \vec{X} = \vec{F_0} \qquad \text{--- (E.2)}$$

where

$$\begin{aligned}
Z_{11}(i\omega) &= -m_1\omega^2 + (c_1+c_2)\, i\omega + (k_1+k_2) = -\omega^2 + 2i\omega + 200 \\
Z_{12}(i\omega) &= Z_{21}(i\omega) = -c_2\, i\omega - k_2 = -i\omega - 100 \\
Z_{13}(i\omega) &= Z_{31}(i\omega) = 0 \\
Z_{22}(i\omega) &= -m_2\omega^2 + (c_2+c_3)\, i\omega + (k_2+k_3) = -\omega^2 + 2i\omega + 200 \quad \text{--- (E.3)} \\
Z_{23}(i\omega) &= Z_{32}(i\omega) = -c_3\, i\omega - k_3 = -i\omega - 100 \\
Z_{33}(i\omega) &= -m_3\omega^2 + (c_3+c_4)\, i\omega + (k_3+k_4) = -\omega^2 + 2i\omega + 200
\end{aligned}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

(E.2) becomes

$$(2i+199)\,X_1 - (i+100)\,X_2 + (0)\,X_3 = 10$$
$$-(i+100)\,X_1 + (2i+199)\,X_2 - (i+100)\,X_3 = 0 \qquad \text{--- (E.4)}$$
$$(0)\,X_1 - (i+100)\,X_2 + (2i+199)\,X_3 = 0$$

Solution of (E.4) can be expressed as

$$X_j = \frac{\Delta_j}{\Delta} \; ; \quad j = 1, 2, 3 \qquad \text{--- (E.5)}$$

where

$$\Delta_1 = \begin{vmatrix} 10 & -(i+100) & 0 \\ 0 & (2i+199) & -(i+100) \\ 0 & -(i+100) & (2i+199) \end{vmatrix} = 295\,980 + 5960\,i$$

$$\Delta_2 = \begin{vmatrix} (2i+199) & 10 & 0 \\ -(i+100) & 0 & -(i+100) \\ 0 & 0 & (2i+199) \end{vmatrix} = 198\,980 + 3990\,i$$

$$\Delta_3 = \begin{vmatrix} (2i+199) & -(i+100) & 10 \\ -(i+100) & (2i+199) & 0 \\ 0 & -(i+100) & 0 \end{vmatrix} = 99\,990 + 2000\,i$$

$$\Delta = \begin{vmatrix} (2i+199) & -(i+100) & 0 \\ -(i+100) & (2i+199) & -(i+100) \\ 0 & -(i+100) & (2i+199) \end{vmatrix} = 118\,002\,i + 3\,899\,409$$

Using (E.5), we get

$$X_1 = \frac{87639.682}{1154850.392 + 11\,685.754\,i} \; ; \quad \text{Amplitude} = 0.0758845 \text{ m}, \quad \text{Phase angle} = 0.5798^\circ$$

$$X_2 = \frac{39\,608.961}{776375.228 + 7921.396\,i} \; ; \quad \text{Amplitude} = 0.0510152 \text{ m}, \quad \text{Phase angle} = 0.5845^\circ$$

$$X_3 = \frac{10\,002.000}{390\,137.914 + 4000.202\,i} \; ; \quad \text{Amplitude} = 0.0256357 \text{ m}, \quad \text{Phase angle} = 0.5874^\circ$$

Thus the steady state responses are:

$$x_1(t) = 0.0758845 \cos(\omega t + 0.5798^\circ) \text{ m}$$
$$x_2(t) = 0.0510152 \cos(\omega t + 0.5845^\circ) \text{ m}$$
$$x_3(t) = 0.0256357 \cos(\omega t + 0.5874^\circ) \text{ m}$$

6.100 Modal matrix can be expressed as $\underset{12\times3}{[X]} = [\vec{X}^{(1)} \;\; \vec{X}^{(2)} \;\; \vec{X}^{(3)}]$

Using Eq. (6.122), $\underset{12\times1}{\vec{x}(t)} = \underset{12\times3}{[X]}\,\underset{3\times1}{\vec{q}(t)}$ where $\vec{q}(t) = \begin{Bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \end{Bmatrix}$

equations of motion can be written as

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

$$[m][X]\ddot{\vec{q}} + [c][X]\dot{\vec{q}} + [k][X]\vec{q} = -[m]\vec{u}_1\,\ddot{x}_0(t)$$

Premultiplication by $[X]^T$ gives

$$[X]^T[m][X]\ddot{\vec{q}} + [X]^T[c][X]\dot{\vec{q}} + [X]^T[k][X]\vec{q} = -[X]^T[m]\vec{u}_1\,\ddot{x}_0(t) \quad \text{--- (E.1)}$$

Assuming that the mass matrix is diagonal and the damping matrix is proportional, (E.1) can be expressed in scalar form as

$$\underline{m}_{ii}\,\ddot{q}_i + \underline{c}_{ii}\,\dot{q}_i + \underline{k}_{ii}\,q_i = -\ddot{x}_0(t)\sum_{j=1}^{12} m_j X_j^{(i)}\ ;\quad i = 1,2,3 \quad \text{--- (E.2)}$$

where $\underline{m}_{ii}$, $\underline{c}_{ii}$ and $\underline{k}_{ii}$ are generalized mass, generalized damping, and generalized stiffness, m_j is the mass at the j^{th} d.o.f. and $X_j^{(i)}$ is the j^{th} component of the vector $\vec{X}^{(i)}$.

Here $\underline{m}_{ii} = \sum_{j=1}^{12} m_j \left(X_j^{(i)}\right)^2 = m \sum_{j=1}^{12}\left(X_j^{(i)}\right)^2$

$$\underline{c}_{ii} = 2\zeta_i\,\omega_i \quad \text{and} \quad \frac{\underline{k}_{ii}}{\underline{m}_{ii}} = \omega_i^2\ ;\quad i = 1,2,3$$

where m = mass at each d.o.f.

Eq. (E.2) can be written, noting that there is no damping in the system, as

$$\ddot{q}_i + \omega_i^2\,q_i = -\frac{\ddot{x}_0(t)\left\{\sum_{j=1}^{12} m_j X_j^{(i)}\right\}}{\left\{\sum_{j=1}^{12} m_j \left(X_j^{(i)}\right)^2\right\}} = -\ddot{x}_0(t)\,\frac{\sum_{j=1}^{12} X_j^{(i)}}{\sum_{j=1}^{12}\left(X_j^{(i)}\right)^2}\ ;\quad i = 1,2,3 \quad \text{--- (E.3)}$$

By noting that

$\sum_{j=1}^{12} X_j^{(i)} = 7.964$ for $i = 1$, -2.67 for $i = 2$, 1.618 for $i = 3$,

$\sum_{j=1}^{12}\left(X_j^{(i)}\right)^2 = 6.275658$ for $i = 1$, 6.48612 for $i = 2$, 6.90962 for $i = 3$,

Eqs. (E.3) can be reduced to

$$\begin{aligned}
\ddot{q}_1(t) + 50625.0\,q_1(t) &= -1.26903\,\ddot{x}_0(t)\\
\ddot{q}_2(t) + 435600.0\,q_2(t) &= 0.411648\,\ddot{x}_0(t) \quad \text{--- (E.4)}\\
\ddot{q}_3(t) + 1210000.0\,q_3(t) &= -0.234166\,\ddot{x}_0(t)
\end{aligned}$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.101 Eigenvalue problem:

$$\lambda [m] \vec{X} = [k] \vec{X}$$

with $\lambda = \omega^2$, $[m] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $[k] = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$

Solution using MATLAB:

```
% Ex6_101.m
>> k = [1 -2 1;-2 4 -2; 1 -2 1]

k =

     1    -2     1
    -2     4    -2
     1    -2     1

>> m = [1 0 0; 0 2 0;0 0 1]

m =

     1     0     0
     0     2     0
     0     0     1

>> [V, D] = eig(k, m)

V =

   -0.6439   -0.5792    0.5000
   -0.4876    0.1105   -0.5000
   -0.3314    0.8001    0.5000


D =

   -0.0000         0         0
         0    0.0000         0
         0         0    4.0000
```

6.102

$$x_1(t) = x_{20}\,\Big(0.1527 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\, t + 0.09847 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\, t - 0.2512 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\, t\Big) \qquad (1)$$

$$x_2(t) = x_{20}\,\Big(0.2087 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\, t + 0.03177 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\, t + 0.7594 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\, t\Big) \qquad (2)$$

$$x_3(t) = x_{20}\,\Big(0.1987 \cos 0.5626 \sqrt{\frac{P}{\ell\, m}}\, t - 0.06157 \cos 0.9158 \sqrt{\frac{P}{\ell\, m}}\, t - 0.1372 \cos 1.5848 \sqrt{\frac{P}{\ell\, m}}\, t\Big) \qquad (3)$$

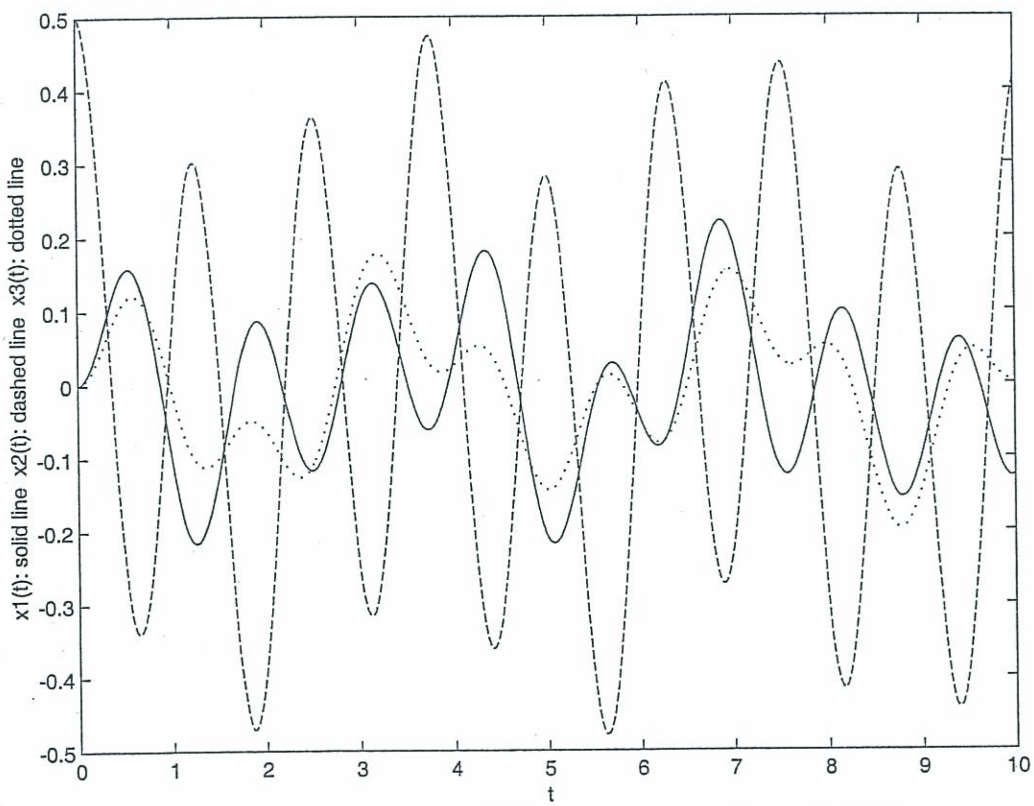
Data: $x_{20} = 0.5$, $P = 100$, $\ell = 5$, $m = 2$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

MATLAB Solution of Eqs. (1) – (3):

```
% Ex6_102.m
x20 = 0.5;
p = 100;
l = 5;
m = 2;
c = sqrt(p/(l*m));
for i = 1: 501
    t(i) = 10*(i-1)/500;
    x1(i) = x20 * ( 0.1527*cos(0.5626*c*t(i)) + ...
        0.09847*cos(0.9158*c*t(i)) - 0.2512*cos(1.5848*c*t(i)) );
    x2(i) = x20 * ( 0.2087*cos(0.5626*c*t(i)) + ...
        0.03177*cos(0.9158*c*t(i)) + 0.7594*cos(1.5848*c*t(i)) );
    x3(i) = x20 * ( 0.1987*cos(0.5626*c*t(i)) - ...
        0.06157*cos(0.9158*c*t(i)) - 0.1372*cos(1.5848*c*t(i)) );
end
plot(t,x1);
hold on;
plot(t,x2,'--');
plot(t,x3,':');
xlabel('t');
ylabel('x1(t): solid line  x2(t): dashed line  x3(t): dotted line');
```



6.103 Equations of motion:

$$2m\ddot{x}_1 + 3kx_1 - kx_2 - kx_3 = 0 \quad (1)$$

$$m\ddot{x}_2 - kx_1 + kx_2 = 0 \quad (2)$$

$$m\ddot{x}_3 - kx_1 + kx_3 = F(t) = F_0 \sin\omega t \quad (3)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Using the data: $m = 1$, $k = 1000$, $F_0 = 5$, $\omega = 10$,
Eqs. (1) – (3) can be expressed as

$$\ddot{x}_1 = -1500\, x_1 + 500\, x_2 + 500\, x_3 \tag{4}$$

$$\ddot{x}_2 = 1000\, x_1 - 1000\, x_2 \tag{5}$$

$$\ddot{x}_3 = 1000\, x_1 - 1000\, x_3 + 5 \sin 10t \tag{6}$$

Let

$$\vec{Y} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_3 \end{Bmatrix} \quad \text{and} \quad \vec{Y}(0) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

Eqs. (4) – (6) can be rewritten as

$$\frac{d\vec{Y}}{dt} = \begin{Bmatrix} y_2 \\ -1500\, y_1 + 500\, y_3 + 500\, y_5 \\ y_4 \\ 1000\, y_1 - 1000\, y_3 \\ y_6 \\ 1000\, y_1 - 1000\, y_5 + 5 \sin 10t \end{Bmatrix} \tag{7}$$

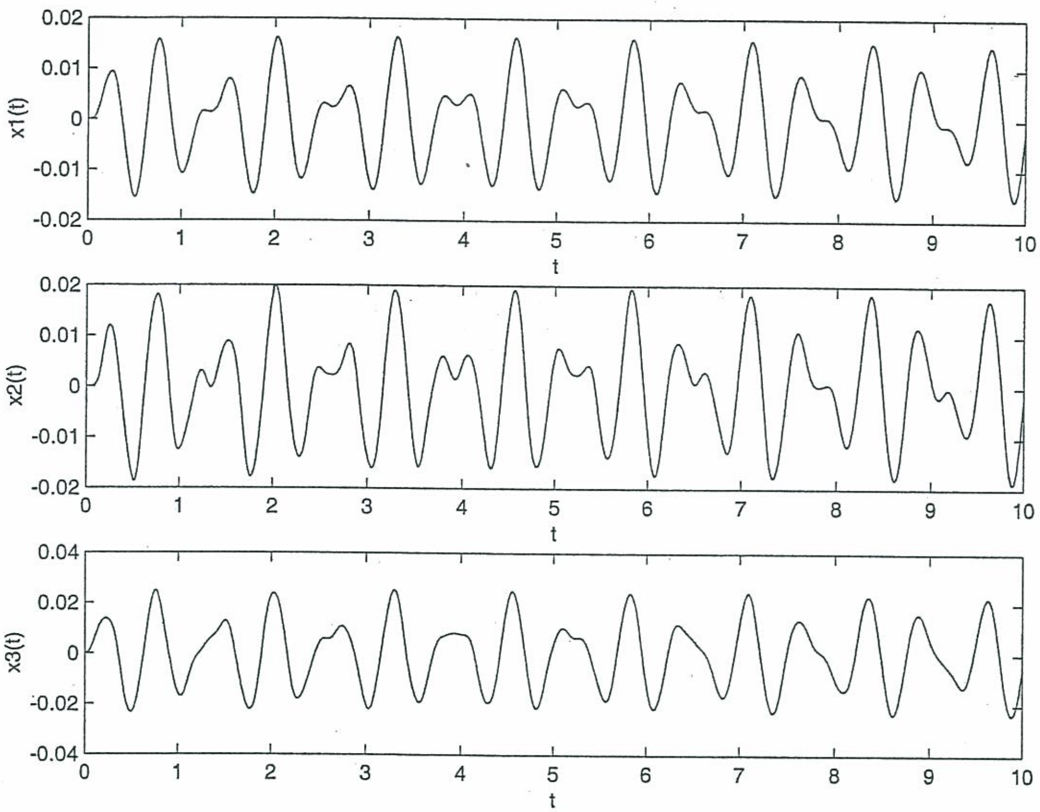
Solution of Eq. (7) using MATLAB:

```
% Ex6_103.m
% This program will use the function dfunc6_103.m ,they should
% be in the same folder
tspan = [0: 0.01: 10];
x0 = [0.0; 0.0; 0.0; 0.0; 0.0; 0.0];
[t,x] = ode23('dfunc6_103', tspan, x0);
subplot(311);
plot(t,x(:,1));
xlabel('t');
ylabel('x1(t)');
subplot(312);
plot(t,x(:,3));
xlabel('t');
ylabel('x2(t)');
subplot(313);
plot(t,x(:,5));
xlabel('t');
ylabel('x3(t)');

% dfunc6_103.m
function f = dfunc6_103(t,x)
F0 = 5;
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
w = 10;
m = 1;
k = 1000;
f = zeros(6,1);
f(1) = x(2);
f(2) = -1500*x(1) + 500*x(3) + 500*x(5);
f(3) = x(4);
f(4) = 1000*x(1) - 1000*x(3);
f(5) = x(6);
f(6) = 1000*x(1) - 1000*x(5) + 5*sin(10*t);
```



6.104 Roots of $f(x) = x^{12} - 2 = 0$ using MATLAB:

```
% Ex6_104.m
>> x = roots([1 zeros(1,11) -2])

x =

  -1.0595
  -0.9175 + 0.5297i
  -0.9175 - 0.5297i
  -0.5297 + 0.9175i
  -0.5297 - 0.9175i
   0.0000 + 1.0595i
   0.0000 - 1.0595i
   0.5297 + 0.9175i
   0.5297 - 0.9175i
   1.0595
   0.9175 + 0.5297i
   0.9175 - 0.5297i
```


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.105 Equations of motion can be rewritten as

$$\ddot{x}_1 = -1.5\,\dot{x}_1 + 0.5\,\dot{x}_2 - 14\,x_1 + 6\,x_2 + 0.5\cos 2t \qquad (1)$$

$$\ddot{x}_2 = 0.25\,\dot{x}_1 - \dot{x}_2 + 0.75\,\dot{x}_3 + 3x_1 - 5x_2 + 2x_3 \qquad (2)$$

$$\ddot{x}_3 = 0.5\,\dot{x}_2 - 0.5\,\dot{x}_3 + 1.3333\,x_2 - 1.3333\,x_3 \qquad (3)$$

Let $\vec{Y} = \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{Bmatrix} = \begin{Bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \\ x_3 \\ \dot{x}_3 \end{Bmatrix}$ and $\vec{Y}(0) = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$

Eqs. (1) – (3) can be expressed as

$$\frac{d\vec{Y}}{dt} = \begin{Bmatrix} y_2 \\ -1.5\,y_2 + 0.5\,y_4 - 14\,y_1 + 6\,y_3 + 0.5\cos 2t \\ y_4 \\ 0.25\,y_2 - y_4 + 0.75\,y_6 + 3y_1 - 5y_3 + 2y_5 \\ y_6 \\ 0.5\,y_4 - 0.5\,y_6 + 1.3333\,y_3 - 1.3333\,y_5 \end{Bmatrix} \qquad (4)$$

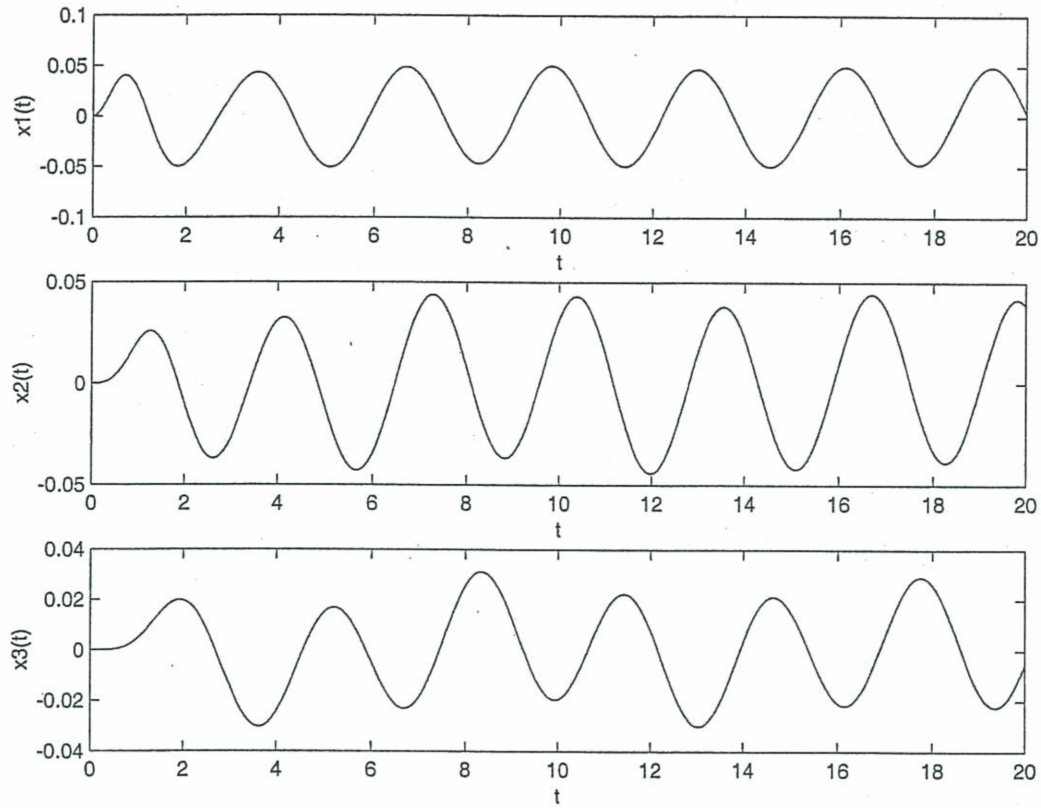
Solution of Eq. (4) using MATLAB:

```
% Ex6_105.m
% This program will use the function dfunc6_105.m, they should
% be in the same folder
tspan = [0: 0.01: 20];
x0 = [0.0; 0.0; 0.0; 0.0; 0.0; 0.0];
[t,x] = ode23('dfunc6_105', tspan, x0);
subplot(311);
plot(t,x(:,1));
xlabel('t');
ylabel('x1(t)');
subplot(312);
plot(t,x(:,3));
xlabel('t');
ylabel('x2(t)');
subplot(313);
plot(t,x(:,5));
xlabel('t');
ylabel('x3(t)');
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
% dfunc6_105.m
function f = dfunc6_105(t,x)
f = zeros(6,1);
f(1) = x(2);
f(2) = -1.5*x(2) + 0.5*x(4) - 14*x(1) + 6*x(3) + 0.5*cos(2*t);
f(3) = x(4);
f(4) = 0.25*x(2) - x(4) + 0.75*x(6) + 3*x(1) - 5*x(3) + 2*x(5);
f(5) = x(6);
f(6) = 0.5*x(4) - 0.5*x(6) + 1.3333*x(3) - 1.3333*x(5);
```



6.106 Equations of motion:

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\ddot{\vec{x}} + \begin{bmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{bmatrix}\dot{\vec{x}} + \begin{bmatrix}200&-100&0\\-100&200&-100\\0&-100&200\end{bmatrix}\vec{x} = \begin{Bmatrix}10\cos t\\0\\0\end{Bmatrix}$$

i.e.,

$$\ddot{x}_1 = -2\dot{x}_1 + \dot{x}_2 - 200x_1 + 100x_2 + 10\cos t \qquad (1)$$

$$\ddot{x}_2 = \dot{x}_1 - 2\dot{x}_2 + \dot{x}_3 + 100x_1 - 200x_2 + 100x_3 \qquad (2)$$

$$\ddot{x}_3 = \dot{x}_2 - 2\dot{x}_3 + 100x_2 - 200x_3 \qquad (3)$$

Defining $\vec{Y} = \begin{Bmatrix}y_1\\y_2\\\vdots\\y_6\end{Bmatrix} = \begin{Bmatrix}x_1\\\dot{x}_1\\x_2\\\vdots\\\dot{x}_3\end{Bmatrix}$ and using $\vec{Y}(0) = \vec{0}$

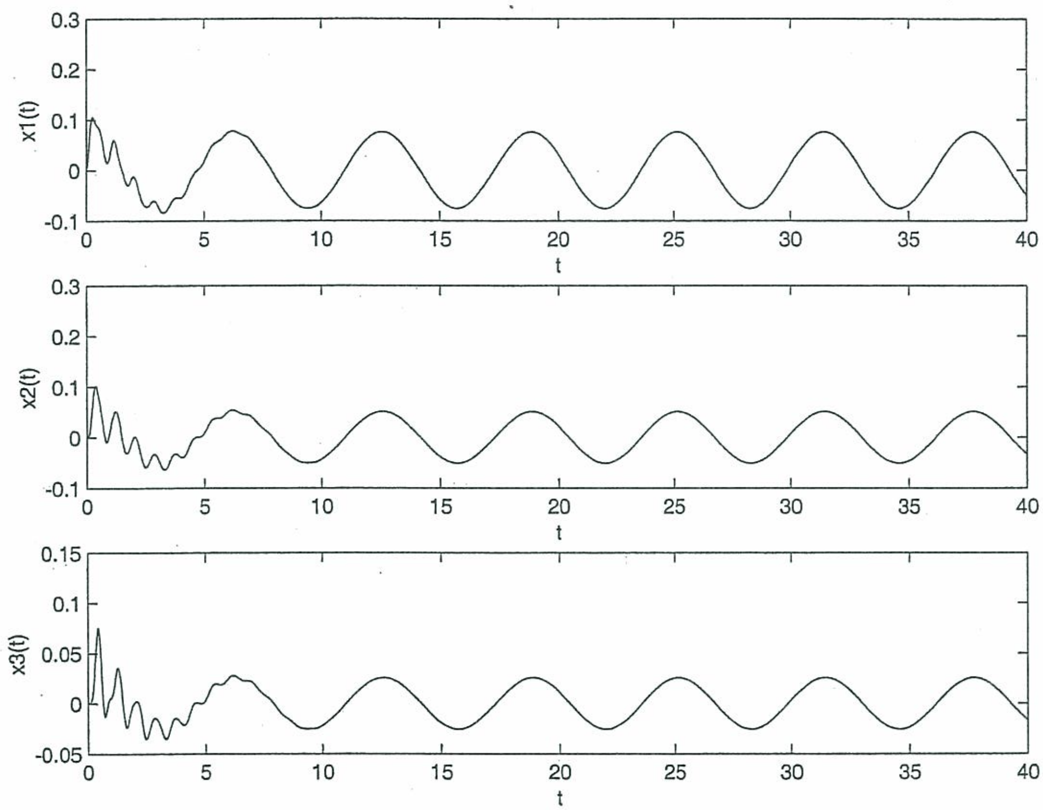
PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Eqs. (1) – (4) can be rewritten as

$$\frac{d\vec{Y}}{dt} = \begin{Bmatrix} y_2 \\ -2y_2 + y_4 - 200y_1 + 100y_3 + 10\cos t \\ y_4 \\ y_2 - 2y_4 + y_6 + 100y_1 - 200y_3 + 100y_5 \\ y_6 \\ y_4 - 2y_6 + 100y_3 - 200y_5 \end{Bmatrix} \qquad (4)$$

Solution of Eq. (4) using MATLAB:




```
% dfunc6_106.m
function f = dfunc6_106(t,x)
f = zeros(6,1);
f(1) = x(2);
f(2) = -2*x(2) + x(4) - 200*x(1) + 100*x(3) + 10*cos(t);
f(3) = x(4);
f(4) = x(2) - 2*x(4) + 100*x(1) - 200*x(3) + 100*x(5);
f(5) = x(6);
f(6) = x(4) - 2*x(6) + 100*x(3) - 200*x(5);
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
% Ex6_106.m
% This program will use the function dfunc6_106.m ,they should
% be in the same folder
tspan = [0: 0.01: 40];
x0 = [0.0; 0.0; 0.0; 0.0; 0.0; 0.0];
[t,x] = ode23('dfunc6_106', tspan, x0);
subplot(311);
plot(t,x(:,1));
xlabel('t');
ylabel('x1(t)');
subplot(312);
plot(t,x(:,3));
xlabel('t');
ylabel('x2(t)');
subplot(313);
plot(t,x(:,5));
xlabel('t');
ylabel('x3(t)');
```

6.107

```
Results of Ex6_107
*****************
>> program7

polynomial expansion of a determinantal equation


data: determinant A:

   5.000000e+000      3.000000e+000      2.000000e+000
   3.000000e+000      6.000000e+000      4.000000e+000
   1.000000e+000      2.000000e+000      6.000000e+000

result: polynomial coefficients in
pcf(np)*(x^n)+pcf(n)*(x^(n-1))+...+pcf(2)+pcf(1)=0

-9.800000e+001   7.700000e+001   -1.700000e+001  1.000000e+000
```

6.108

```
%====================================================================
%
% Program8.m
% Main program which calls the function MODAL
%
%====================================================================
% Run "Program8" in MATLAB command window, Progrm8.m and modal.m
% should be in the same folder,and set the path to this folder
% following line contain problem-dependent data
n=3;
nvec=3;
nstep=300;
delt=0.01;
xm=[41.4 0.0 0.0; 0.0 38.8 0.0; 0.0 0.0 25.88];
omf=[5 10 20];
om=[25.076 53.578 110.907];
z=[0.001 0.001 0.001];
x0=[0.0 0.0 0.0];
xd0=[0.0 0.0 0.0];
ev=[1.0 1.0 1.0;1.303 0.860 -1.0;1.947 -1.685 0.183];
%end of problem-dependent data
```


لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
for i=1:nstep
   time=i*delt;
   f(1,i)=5000*cos(5*time);
   f(2,i)=10000*cos(10*time);
   f(3,i)=20000*cos(20*time);
end
for i=1:nvec
   for j=1:n
      evt(i,j)=ev(i,j);
   end
end
[x,t]=modal(xm,om,omf,z,x0,xd0,f,delt,ev,evt,nstep,n,nvec);
fprintf('\n Response of system using modal analysis \n\n');
for i=1:n
   fprintf('\n Coordinate %2.0f \n',i);
   fprintf(' %8.5e  %8.5e  %8.5e  %8.5e  %8.5e\n',x(i,1:nstep));
end
for i = 1: n
   plot(t,x(i,1:nstep));
   hold on;
end
xlabel('t');
ylabel('x');
gtext('Coordinate 3');
gtext('Coordinate 2');
gtext('Coordinate 1' );
%===================================================================
%
% function modal.m
%
%===================================================================
function [x,t]=modal(xm,om,omf,z,x0,xd0,f,delt,ev,evt,nstep,n,nvec);
t(1)=delt;
for i=2:nstep
   t(i)=t(i-1)+delt;
end
% normalization of modal matrix with respect to the mass matrix
% xmx=matmul(xm,ev,n,n,nvec);
% xtmx=matmul(evt,xmx,nvec,n,nvec);
xmx=xm*ev;
xtmx=evt*xmx;
for i=1:nvec
   for j=1:n
      ev(j,i)=ev(j,i)/sqrt(xtmx(i,i));
   end
end
% conversion of information to normal coordinates
for i=1:nvec
   y0(i)=0.0;
   yd0(i)=0.0;
end
for i=1:nvec
   for j=1:n
      yo(i)=y0(i)+ev(j,i)*x0(j);
      yd0(i)=yd0(i)+ev(j,i)*xd0(j);
   end
end
```


Mech.MuslimEngineer.Net

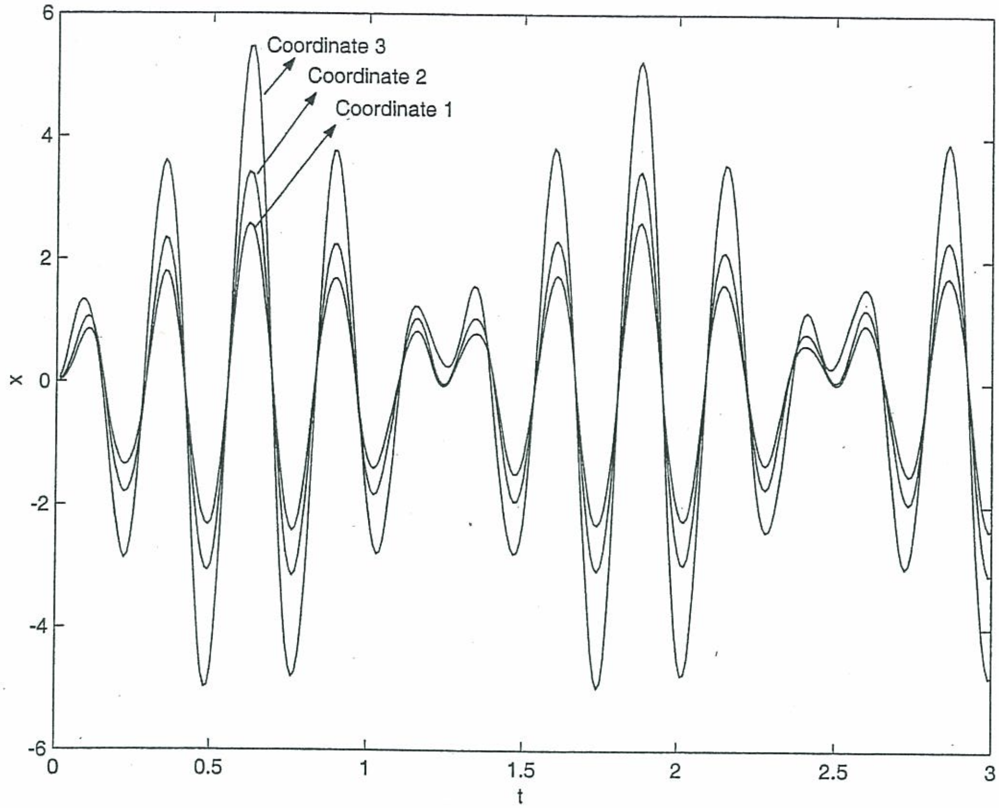
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
for i=1:nvec
   for j=1:n
      evt(i,j)=ev(j,i);
   end
end
% q=matmul(evt,f,nvec,n,nstep);
q=evt*f;
for i=1:nvec
   r=omf(i)/om(i);
   pp=y0(i);
   qq=yd0(i);
   zi=z(i);
   omeg=om(i);
   omd=omeg*sqrt(1-zi^2);
   for j=1:nstep
      if j~=1
         pp=u(i,j-1);
         qq=v(i,j-1);
      end
      c1=exp(-zi*omeg*delt);
      c2=cos(omd*delt);
      c3=sin(omd*delt);
      c4=(qq+omeg*zi*pp)/omd;
      c5=omeg*zi/omd;
      c6=q(i,j)/(omeg^2);
      u(i,j)=c1*(pp*c2+c3*c4)+c6*(1-c1*(c2+c3*c5));
      v(i,j)=omd*c1*(-pp*c3+c2*c4-c5*(pp*c2+c3*c4))+c6*omd*c1*c3*(1+c5^2);
   end
end
% finding the solution in the original coordinates
% x=matmul(ev,u,n,nvec,nstep);
x=ev*u;

%=====================================================================
%
% function matmul.m
%
%=====================================================================
function [a]=matmul(b,c,l,m,n)
% Matrix multipication subroutine: A=B*C
% b(l,m) and c(m,n) are input matrices, A(l,n) is output matrix
for i=1:l
   for j=1:n
      a(i,j)=0;
      for k=1:m
         a(i,j)=a(i,j)+b(i,k)*c(k,j);
      end
   end
end
```


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.




```
Results of Ex6_108
******************
>> program8

 Response of system using modal analysis


 Coordinate  1
 1.16587e-002 4.77899e-002 1.10778e-001 2.01670e-001 3.17703e-001
 4.51127e-001 5.89314e-001 7.15610e-001 8.10748e-001 8.55251e-001
 8.33049e-001 7.35446e-001 5.63789e-001 3.29513e-001 5.16989e-002
 -2.46502e-001 -5.41192e-001 -8.10289e-001 -1.03560e+000 -1.20447e+000
 :
 .


 Coordinate  2
 1.94937e-002 7.72115e-002 1.71039e-001 2.97350e-001 4.49827e-001
 6.17690e-001 7.84560e-001 9.29302e-001 1.02905e+000 1.06321e+000
 1.01695e+000 8.83330e-001 6.64302e-001 3.71083e-001 2.35527e-002
 -3.51910e-001 -7.26226e-001 -1.07196e+000 -1.36651e+000 -1.59315e+000
 :
 .


 Coordinate  3
 5.07839e-002 1.94324e-001 4.06865e-001 6.55062e-001 9.01966e-001
 1.11328e+000 1.26237e+000 1.33292e+000 1.31883e+000 1.22192e+000
 1.04849e+000 8.06186e-001 5.01856e-001 1.41193e-001 -2.69837e-001
 -7.22160e-001 -1.20031e+000 -1.68000e+000 -2.12777e+000 -2.50340e+000
 :
 .
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.110

```
C ===================================================================
C
C SOLUTION OF PROBLEM 6.110
C
C ===================================================================
C
C N        = NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
C NM       = N-1
C XK(N,N)  = STIFFNESS MATRIX
C XM(N,N)  = MASS MATRIX
C OM(N)    = VECTOR OF NATURAL FREQUENCIES
C X(N,N)   = MATRIX OF EIGENVECTORS, J TH EIGENVECTOR IS STORED IN THE
C            J TH COLUMN OF THE MATRIX X
C DIMENSIONS OF OTHER MATRICES: A(NM,NM),B(NM),LA(NM),LB(NM,2),S(NM)
C
      DIMENSION XK(3,3),XM(3,3),OM(3),X(3,3),A(2,2),B(2),LA(2),LB(2,2),
     2 S(2)
      DATA XK/2.,-1.,0.,-1.,2.,-1.,0.,-1.,2./
      DATA XM/2.,0.,0.,0.,3.,0.,0.,0.,2./
      DATA OM/.482087,1.,1.197605/
      N=3
      NM=N-1
      IND=1
      DO 10 I=1,N
      DO 20 J=1,NM
      DO 20 K=1,NM
      A(J,K)=XK(J,K+1)-(OM(I)**2)*XM(J,K+1)
20    B(J)=-XK(J,1)+(OM(I)**2)*XM(J,1)
      CALL SIMUL (A,B,NM,IND,LA,LB,S)
      X(1,I)=1.0
      DO 30 J=1,NM
30    X(J+1,I)=B(J)
      WRITE (44,110) OM(I),(X(J,I),J=1,N)
110   FORMAT (//,2X,19H NATURAL FREQUENCY=,E15.8,/,2X,13H EIGENVECTOR=,
     2 /,(2X,4E15.8))
10    CONTINUE
      STOP
      END
```

```
NATURAL FREQUENCY= 0.48208699E+00
EIGENVECTOR=
0.10000000E+01 0.15351844E+01 0.10000019E+01

NATURAL FREQUENCY= 0.10000000E+01
EIGENVECTOR=
0.10000000E+01 0.00000000E+00-0.10000000E+01

NATURAL FREQUENCY= 0.11976050E+01
EIGENVECTOR=
0.10000000E+01-0.86851549E+00 0.99999440E+00
```


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.111

```
C ======================================================================
C
C PROBLEM  6.111
C
C ======================================================================
C N       = NUMBER OF DEGREES OF FREEDOM OF THE SYSTEM
C XM(N,N)= MASS MATRIX
C X(N,N) = MATRIX CONTAINING THE ORIGINAL I TH NORMAL MODE IN I TH
C           COLUMN
C XN(N,N)= MATRIX CONTAINING THE [M]-ORTHONORMAL I TH NORMAL MODE
C           IN I TH COLUMN
C DIMENSIONS OF OTHER VECTORS: A(N),B(N)
      DIMENSION XM(3,3),X(3,3),XN(3,3),A(3),B(3)
      N=3
      DATA XM /1.0,0.0,0.0,0.0,2.0,0.0,0.0,0.0,1.0/
      DATA X /1.0,-1.0,1.0,1.0,1.0,1.0,0.0,1.0,2.0/
      DO 10 I=1,N
      DO 20 J=1,N
 20   A(J)=X(J,I)
      CALL MULT (XM,A,B,N)
      SUM=0.0
      DO 30 K=1,N
 30   SUM=SUM+X(K,I)*B(K)
      SUM=SQRT(1.0/SUM)
      DO 40 K=1,N
 40   XN(K,I)=X(K,I)*SUM
      PRINT 50, I,(XN(J,I),J=1,N)
 50   FORMAT (/,2X,13H EIGENVECTOR:,I5,/,2X,(4E15.8))
 10   CONTINUE
      STOP
      END
C ======================================================================
C
C SUBROUTINE MULT
C
C ======================================================================
      SUBROUTINE MULT (XM,A,B,N)
      DIMENSION XM(N,N),A(N),B(N)
      DO 10 I=1,N
      B(I)=0.0
      DO 20 J=1,N
 20   B(I)=B(I)+XM(I,J)*A(J)
 10   CONTINUE
      RETURN
      END
```

```
 EIGENVECTOR:     1
 0.50000000E+00-0.50000000E+00 0.50000000E+00

 EIGENVECTOR:     2
 0.50000000E+00 0.50000000E+00 0.50000000E+00

 EIGENVECTOR:     3
 0.00000000E+00 0.40824831E+00 0.81649661E+00
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

6.112 The main program and results are shown below.

```
C =====================================================================
C
C MAIN PROGRAM FOR CALLING THE SUBROUTINE MODAL
C
C =====================================================================
C
      DIMENSION XM(3,3),OM(3),Z(3),XO(3),XDO(3),YO(3),YDO(3),EV(3,3),
     2 EVT(3,3),XMX(3,3),XIMX(3,3),T(40),F(3,40),X(3,40),U(3,40),
     3 V(3,40),Q(3,40)
      DATA N,NVEC,NSTEP,DELT/3,3,40,0.1/
      DATA XM/2.0,0.0,0.0,0.0,2.0,0.0,0.0,0.0,2.0/
      OMF=5.0
      DATA OM/1.530734,2.828428,3.695518/
      DATA Z/0.0,0.0,0.0/
      DATA XO/0.0,0.0,0.0/
      DATA XDO/0.0,0.0,0.0/
      DATA (EV(I,1),I=1,3)/1.0,1.414214,1.0/
      DATA (EV(I,2),I=1,3)/1.0,0.0,-1.0/
      DATA (EV(I,3),I=1,3)/1.0,-1.414214,1.0/
      DO 5 I=1,NSTEP
      TIME=REAL(I)*DELT
5     F(1,I)=10.0*SIN(5.0*TIME)
      DO 10 I=1,NSTEP
      F(2,I)=0.0
10    F(3,I)=0.0
      DO 20 I=1,NVEC
      DO 20 J=1,N
20    EVT(I,J)=EV(J,I)
      CALL MODAL (XM,OM,OMF,T,Z,XO,XDO,YO,YDO,Q,F,DELT,EV,EVT,XMX,
     2 XTMX,X,U,V,NSTEP,N,NVEC)
      WRITE (29,30)
30    FORMAT (//,40H RESPONSE OF SYSTEM USING MODAL ANALYSIS,/)
      DO 40 I=1,N
40    WRITE (29,50) I,(X(I,J),J=1,NSTEP)
50    FORMAT (/,11H COORDINATE,I5,/,(1X,5E14.6))
      STOP
      END
```

```
RESPONSE OF SYSTEM USING MODAL ANALYSIS


COORDINATE     1
  0.119060E-01  0.556720E-01  0.140716E+00  0.262090E+00  0.400585E+00
  0.526842E+00  0.608511E+00  0.618689E+00  0.543493E+00  0.386804E+00
  0.170858E+00 -0.676210E-01 -0.284971E+00 -0.440109E+00 -0.503791E+00
 -0.465406E+00 -0.335646E+00 -0.144346E+00  0.660445E-01  0.249893E+00
  0.368518E+00  0.398672E+00  0.337414E+00  0.202231E+00  0.264541E-01
 -0.148937E+00 -0.285117E+00 -0.354495E+00 -0.346914E+00 -0.271541E+00
 -0.154034E+00 -0.297635E-01  0.652391E-01  0.103368E+00  0.723514E-01
 -0.216811E-01 -0.155688E+00 -0.295363E+00 -0.403527E+00 -0.449197E+00

COORDINATE     2
  0.397433E-04  0.655754E-03  0.356972E-02  0.119145E-01  0.297943E-01
  0.612223E-01  0.108686E+00  0.171738E+00  0.246047E+00  0.323279E+00
  0.391991E+00  0.439459E+00  0.454134E+00  0.428213E+00  0.359732E+00
  0.253658E+00  0.121640E+00 -0.196366E-01 -0.151233E+00 -0.255357E+00
 -0.318689E+00 -0.334940E+00 -0.306091E+00 -0.242021E+00 -0.158585E+00
 -0.745270E-01 -0.784475E-02  0.276338E-01  0.247827E-01 -0.156785E-01
 -0.856190E-01 -0.171361E+00 -0.256531E+00 -0.325272E+00 -0.365137E+00
 -0.369097E+00 -0.336327E+00 -0.271711E+00 -0.184302E+00 -0.851769E-01

COORDINATE     3
  0.526197E-07  0.338722E-05  0.398979E-04  0.232946E-03  0.912614E-03
  0.274891E-02  0.685317E-02  0.147839E-01  0.284018E-01  0.495493E-01
  0.795763E-01  0.118784E+00  0.165904E+00  0.217749E+00  0.269161E+00
```

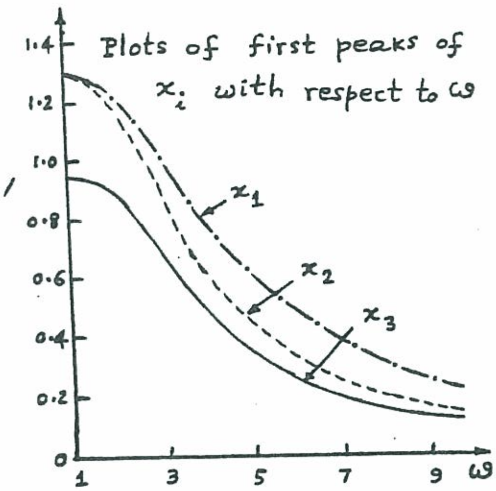

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

0.313352E+00	0.342632E+00	0.349486E+00	0.327819E+00	0.274185E+00
0.188751E+00	0.758134E-01	-0.563065E-01	-0.195996E+00	-0.329661E+00
-0.443429E+00	-0.524977E+00	-0.565224E+00	-0.559583E+00	-0.508616E+00
-0.417951E+00	-0.297528E+00	-0.160267E+00	-0.204086E-01	0.108245E+00
0.213908E+00	0.288030E+00	0.326005E+00	0.327354E+00	0.295404E+00

6.113 The main program used to generate the numerical results and the graph are given.

```
C ==================================================================
C
C MAIN PROGRAM FOR CALLING THE SUBROUTINE MODAL
C
C ==================================================================
C
      DIMENSION XM(3,3),OM(3),Z(3),X0(3),XD0(3),Y0(3),YD0(3),EV(3,3),
    2 EVT(3,3),XMX(3,3),XTMX(3,3),T(40),F(3,40),X(3,40),U(3,40),
    3 V(3,40),Q(3,40)
      DATA N,NVEC,NSTEP,DELT/3,3,40,0.1/
      DATA XM/2.0,0.0,0.0,0.0,2.0,0.0,0.0,0.0,2.0/
      DATA OM/1.530734,2.828428,3.695518/
      DATA Z/0.0,0.0,0.0/
      DATA X0/0.0,0.0,0.0/
      DATA XD0/0.0,0.0,0.0/
      DATA (EV(I,1),I=1,3)/1.0,1.414214,1.0/
      DATA (EV(I,2),I=1,3)/1.0,0.0,-1.0/
      DATA (EV(I,3),I=1,3)/1.0,-1.414214,1.0/
      OMF=0.0
      DO 55 IJK=1,10
      OMF=OMF+1.0
      DO 5 I=1,NSTEP
      TIME=REAL(I)*DELT
 5    F(1,I)=10.0*SIN(OMF*TIME)
      DO 10 I=1,NSTEP
      F(2,I)=0.0
 10   F(3,I)=0.0
      DO 20 I=1,NVEC
      DO 20 J=1,N
 20   EVT(I,J)=EV(J,I)
      CALL MODAL (XM,OM,OMF,T,Z,X0,XD0,Y0,YD0,Q,F,DELT,EV,EVT,XMX,
    2 XTMX,X,U,V,NSTEP,N,NVEC)
      WRITE (29,30)
 30   FORMAT (//,40H RESPONSE OF SYSTEM USING MODAL ANALYSIS,/)
      DO 40 I=1,N
 40   WRITE (29,50) I,(X(I,J),J=1,NSTEP)
 50   FORMAT (/,11H COORDINATE,I5,/,(1X,5E14.6))
 55   CONTINUE
      STOP
      END
```



6.114 (a) Equation of motion:

$$\begin{bmatrix} m_f & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & m_h \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F_3(t) \end{Bmatrix}$$

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Forcing function can be written as

$$F_3(t) = \mathrm{Re}\left(F_0\, e^{i\omega t}\right) = F_{30}\cos\omega t \equiv 1000\cos 60t \ \ lb. \qquad (E_1)$$

steady-state solution can be assumed as

$$x_j(t) = X_j\, e^{i\omega t}, \quad j = 1,2,3 \qquad (E_2)$$

Equations of motion become

$$\begin{bmatrix} -m_f\omega^2 + (c_1+c_2)i\omega + k_1 + k_2 & -c_2 i\omega - k_2 & 0 \\ -c_2 i\omega - k_2 & -m_b\omega^2 + (c_2+c_3)i\omega + k_2 + k_3 & -c_3 i\omega - k_3 \\ 0 & -c_3 i\omega - k_3 & -m_h\omega^2 + c_3 i\omega + k_3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ F_{30} \end{Bmatrix} \qquad (E_3)$$

For known data, Eq. (E_3) becomes

$$[Z_{ij}]\vec{X} = \vec{F}_0 \qquad (E_4)$$

where

$$Z_{11} = -50\,\omega^2 + i\,20\omega + 5500 = -174\,500 + i\,1200$$
$$Z_{12} = Z_{21} = -i\,10\omega - 500 = -500 - i\,600$$
$$Z_{13} = Z_{31} = 0 \qquad (E_5)$$
$$Z_{22} = -10\,\omega^2 + i\,20\,\omega + 2500 = -33\,500 + i\,1200$$
$$Z_{23} = Z_{32} = -i\,10\omega - 2000 = -2000 - i\,600$$
$$Z_{33} = -2\,\omega^2 + i\,10\omega + 2000 = -5200 + i\,600$$

Solution of Eq. (E_4) can be expressed as

$$\vec{X} = [Z_{ij}]^{-1}\vec{F}_0 \qquad (E_6)$$

Using Cramer's rule, we get

$$X_1 = \begin{vmatrix} 0 & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ F_{30} & Z_{32} & Z_{33} \end{vmatrix} \cdot \frac{1}{\det[Z_{ij}]} = \left(-0.1230\times10^{-4} - i\,0.5284\times10^{-4}\right)$$

$$X_2 = \begin{vmatrix} Z_{11} & 0 & Z_{13} \\ Z_{21} & 0 & Z_{23} \\ Z_{31} & F_{30} & Z_{33} \end{vmatrix} \cdot \frac{1}{\det[Z_{ij}]} = \left(0.1087\times10^{-1} + i\,0.5373\times10^{-2}\right)$$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك - الإتجاه الإسلامي

$$X_3 = \begin{vmatrix} Z_{11} & Z_{12} & 0 \\ Z_{21} & Z_{22} & 0 \\ Z_{31} & Z_{32} & F_{30} \end{vmatrix} \cdot \frac{1}{\det[Z_{ij}]} = (-0.1929 - i\,0.02558)$$

$$\therefore x_1(t) = X_1 (\cos 60t + i \sin 60t)$$

$$= (-0.1230 \times 10^{-4} \cos 60t + 0.5284 \times 10^{-4} \sin 60t)$$

$$- i\,(0.5284 \times 10^{-4} \cos 60t + 0.1230 \times 10^{-4} \sin 60t)$$

Actual response of m_f = Real $[x_1(t)]$

$$= (-0.1230 \times 10^{-4} \cos 60t + 0.5284 \times 10^{-4} \sin 60t) \text{ in.}$$

Similarly, we find:

Actual response of m_b = Real $[x_2(t)]$

$$= (0.01087 \cos 60t - 0.005373 \sin 60t) \text{ in.}$$

Actual response of m_h = Real $[x_3(t)]$

$$= (-0.1929 \cos 60t + 0.02558 \sin 60t) \text{ in.}$$

(b) Change the stiffness k_2 from 100 lb/in in increments of 100 lb/in and find the response of the tool head (x_3) using the procedure outlined in part (a). Find the value of k_2 for which the maximum response of x_3 is 25% lower than the value found in part (a).

(c) Change the values of c_1, c_2, c_3, k_1 and k_3 individually and find whether any of these quantities can be used to achieve the goal of part (b).

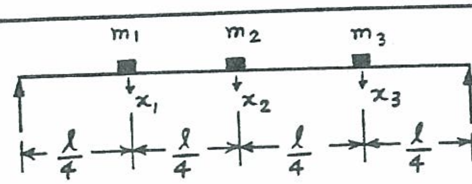
© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Chapter 7

Determination of Natural Frequencies and Mode Shapes



7.1 From Example 6.6,

$$a_{11} = a_{33} = \frac{3}{256}\frac{l^3}{EI}$$

$$a_{22} = \frac{1}{48}\frac{l^3}{EI}$$

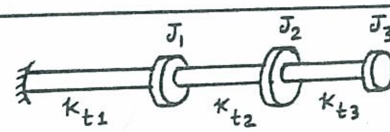
(a) Eq. (7.6) gives

$$\frac{1}{\omega_1^2} \simeq \frac{m l^3}{EI}\left(\frac{3\times 5}{256} + \frac{1}{48}\times 1 + \frac{3\times 5}{256}\right) = \frac{106}{768}\frac{m l^3}{EI} = 0.13802\ \frac{m l^3}{EI}$$

$$\omega_1 \simeq 2.6917\sqrt{\frac{EI}{m l^3}}$$

(b) $$\frac{1}{\omega_1^2} \simeq \frac{m l^3}{EI}\left(\frac{3}{256} + \frac{1\times 5}{48} + \frac{3}{256}\right) = \frac{98}{758}\frac{m l^3}{EI} = 0.12760\ \frac{m l^3}{EI}$$

$$\omega_1 \simeq 2.7994\sqrt{\frac{EI}{m l^3}}$$



7.2 Flexibility influence coefficients:

a_{11} = rotation of J_1 when a unit torque is applied to $J_1 = 1/k_{t1}$

a_{22} = rotation of J_2 when a unit torque is applied to J_2

$$= \frac{1}{k_{teq}} = \frac{1}{k_{t1}} + \frac{1}{k_{t2}}$$

a_{33} = rotation of J_3 when a unit torque is applied to J_3

$$= \frac{1}{k_{teq}} = \frac{1}{k_{t1}} + \frac{1}{k_{t2}} + \frac{1}{k_{t3}}$$

(a) Eq. (7.6) gives $\frac{1}{\omega_1^2} \simeq a_{11}J_1 + a_{22}J_2 + a_{33}J_3 = \frac{J_0}{k_t}(1+2+3)$

$$\omega_1 \simeq 0.4082\sqrt{k_t/J_0}$$

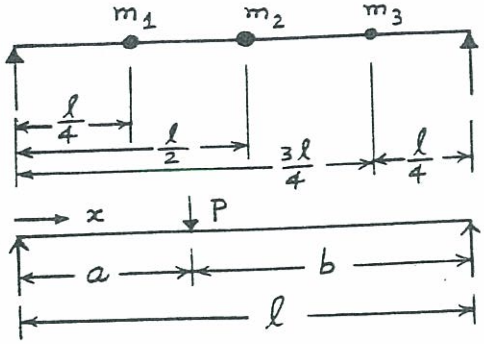
(b) $$\frac{1}{\omega_1^2} \simeq \frac{J_1}{k_{t1}} + J_2\left(\frac{1}{k_{t1}} + \frac{1}{k_{t2}}\right) + J_3\left(\frac{1}{k_{t1}} + \frac{1}{k_{t2}} + \frac{1}{k_{t3}}\right)$$

$$\simeq \frac{J_0}{k_t} + 2J_0\left(\frac{1}{k_t} + \frac{1}{2k_t}\right) + 3J_0\left(\frac{1}{k_t} + \frac{1}{2k_t} + \frac{1}{3k_t}\right) = \frac{9.5\ J_0}{k_t}$$

$$\omega_1 \simeq 0.3244\sqrt{k_t/J_0}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.



7.3 $m_1 = m, \quad m_2 = 2m, \quad m_3 = 3m;$

$l_1 = l_2 = l_3 = l_4 = l/4$

$$w(x) = \frac{Pbx}{6EIl}\left(l^2 - b^2 - x^2\right); \quad 0 \le x \le a$$

$$= -\frac{Pa(l-x)}{6EIl}\left(a^2 + x^2 - 2lx\right); \quad a \le x \le l$$

Deflection due to weight of m_1: $(P = mg)$

At location of m_1 $\left(x = \frac{l}{4},\ b = \frac{3l}{4},\ l = l\right)$

$$w_1' = \frac{(mg)\left(\frac{3l}{4}\right)\left(\frac{l}{4}\right)}{6EIl}\left\{l^2 - \frac{9}{16}l^2 - \frac{1}{16}l^2\right\} = \frac{3mgl^3}{256\,EI}$$

At location of m_2 $\left(a = \frac{l}{4},\ x = \frac{l}{2},\ l = l\right)$

$$w_2' = -\frac{(mg)\left(\frac{l}{4}\right)\left(l - \frac{l}{2}\right)}{6EIl}\left(\frac{l^2}{16} + \frac{l^2}{4} - l^2\right) = \frac{11\,mgl^3}{768\,EI}$$

At location of m_3 $\left(a = \frac{l}{4},\ x = \frac{3l}{4},\ l = l\right)$

$$w_3' = -\frac{(mg)\left(\frac{l}{4}\right)\left(l - \frac{3l}{4}\right)}{6EIl}\left(\frac{l^2}{16} + \frac{9}{16}l^2 - \frac{6}{4}l^2\right) = \frac{7\,mgl^3}{768\,EI}$$

Deflection due to weight of m_2: $(P = 2mg)$

At location of m_1 $\left(x = \frac{l}{4},\ b = \frac{l}{2},\ l = l\right)$

$$w_1'' = \frac{(2mg)\left(\frac{l}{2}\right)\left(\frac{l}{4}\right)}{6EIl}\left(l^2 - \frac{l^2}{4} - \frac{1}{16}l^2\right) = \frac{11\,mgl^3}{384\,EI}$$

At location of m_2 $\left(x = \frac{l}{2},\ b = \frac{l}{2},\ l = l\right)$

$$w_2'' = \frac{(2mg)\left(\frac{l}{2}\right)\left(\frac{l}{2}\right)}{6EIl}\left(l^2 - \frac{l^2}{4} - \frac{l^2}{4}\right) = \frac{mgl^3}{24\,EI}$$

At location of m_3 $\left(x = \frac{3l}{4},\ a = \frac{l}{2},\ b = \frac{l}{2},\ l = l\right)$

$$w_3'' = -\frac{(2mg)\left(\frac{l}{2}\right)\left(l - \frac{3l}{4}\right)}{6EIl}\left(\frac{l^2}{4} + \frac{9}{16}l^2 - \frac{6}{4}l^2\right) = \frac{11\,mgl^3}{1024\,EI}$$

Deflection due to weight of m_3: $(P = 3mg)$

At location of m_1 $\left(x = \frac{l}{4},\ b = \frac{l}{4},\ l = l\right)$

$$w_1''' = \frac{(3mg)\left(\frac{l}{4}\right)\left(\frac{l}{4}\right)}{6EIl}\left(l^2 - \frac{l^2}{16} - \frac{l^2}{16}\right) = \frac{7mgl^3}{256\,EI}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

At location of m_2 $\left(x = \frac{l}{2},\ b = \frac{l}{4},\ l = l\right)$

$$w_2''' = \frac{(3mg)\left(\frac{l}{4}\right)\left(\frac{l}{2}\right)}{6EIl}\left(l^2 - \frac{l^2}{16} - \frac{l^2}{4}\right) = \frac{11\, m g\, l^3}{256\ EI}$$

At location of m_3 $\left(x = \frac{3l}{4},\ b = \frac{l}{4},\ l = l\right)$

$$w_3''' = \frac{(3mg)\left(\frac{l}{4}\right)\left(\frac{3l}{4}\right)}{6EIl}\left(l^2 - \frac{1}{16}l^2 - \frac{9}{16}l^2\right) = \frac{9\, m g\, l^3}{256\ EI}$$

Total deflection of masses:

$$w_1 = w_1' + w_1'' + w_1''' = \frac{mgl^3}{EI}\left(\frac{3}{256} + \frac{11}{384} + \frac{7}{256}\right) = \frac{13}{192}\,\frac{mgl^3}{EI}$$

$$w_2 = w_2' + w_2'' + w_2''' = \frac{mgl^3}{EI}\left(\frac{11}{768} + \frac{1}{24} + \frac{11}{256}\right) = \frac{19}{192}\,\frac{mgl^3}{EI}$$

$$w_3 = w_3' + w_3'' + w_3''' = \frac{mgl^3}{EI}\left(\frac{7}{768} + \frac{11}{1024} + \frac{9}{256}\right) = \frac{169}{3072}\,\frac{mgl^3}{EI}$$

$$\omega = \left\{\frac{g\,\frac{m^2 g l^3}{EI}\left(\frac{13}{192} + 2\times\frac{19}{192} + 3\times\frac{169}{3072}\right)}{\frac{m^2 g^2 l^6 m}{E^2 I^2}\left\{\left(\frac{13}{192}\right)^2 + 2\left(\frac{19}{192}\right)^2 + 3\left(\frac{169}{3072}\right)^2\right\}}\right\}^{1/2}$$

$$= 3.5987\sqrt{\frac{EI}{ml^3}}$$

7.4 ω_{11} = natural frequency of wing itself $= 20\ Hz = 125.664\ \frac{rad}{sec}$

ω_{22} = natural frequency of weapon attached at the tip of the wing (neglecting the effect of mass of wing)

$$= \sqrt{\frac{k}{m}} = \sqrt{\frac{50000}{12}\left(\frac{386.4}{2000}\right)} = 28.3725\ \frac{rad}{sec}$$

New frequency of vibration of the wing with weapon is given by

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{1}{125.664^2} + \frac{1}{28.3725^2}$$

$$= 130.5563 \times 10^{-5}$$

$$\therefore\ \omega_1 = 27.6759\ \frac{rad}{sec} = 4.4047\ Hz$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.5 For a simply supported beam, natural frequency is given by (assuming its mass to be concentrated at the middle)

$$\omega_{11} = \sqrt{\frac{k}{m}} \quad \text{where} \quad k = \frac{48EI}{\ell^3}$$

Neglecting the mass of beam, if trolley is placed at the middle of girder, its natural frequency is given by

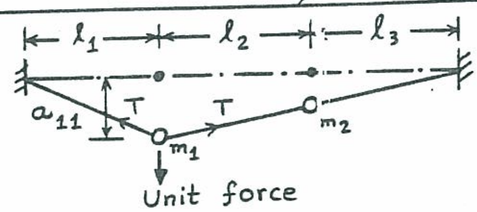
$$\omega_{22} = \sqrt{\frac{k}{10m}}$$

Fundamental natural frequency of the combined system is given by

$$\frac{1}{\omega_1^2} = \frac{1}{\omega_{11}^2} + \frac{1}{\omega_{22}^2} = \frac{m}{k} + \frac{10m}{k} = \frac{11m}{k}$$

$$\therefore \omega_1 = 0.3015\sqrt{\frac{k}{m}}$$ = 30.15% of the natural frequency of the girder (without the trolley)

7.6 Flexibility coefficients:



Let T = tension in string

a_{11} = deflection of m_1 due to unit force applied to m_1

Unit force = sum of components of tension in vertical direction

i.e. $$1 = T\left(\frac{a_{11}}{\ell_1}\right) + T\left(\frac{a_{11}}{\ell_2+\ell_3}\right) = T a_{11}\left(\frac{1}{\ell} + \frac{1}{2\ell}\right) = \frac{3Ta_{11}}{2\ell}$$

$$a_{11} = \frac{2\ell}{3T} = a_{22} \text{ (by symmetry)}$$

$$\frac{1}{\omega_1^2} \simeq a_{11}m_1 + a_{22}m_2 = \frac{2\ell}{3T}(m+m) = \frac{4m\ell}{3T}$$

$\omega_1 \simeq 0.866\sqrt{\dfrac{T}{m\ell}}$; Exact solution is $\omega_1 = \sqrt{\dfrac{T}{m\ell}}$ (Problem 5.22).

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.7 From Example 7.3, $\omega = 0.028222\sqrt{EI}$

Since $E = 2.07 \times 10^{11}$ N/m², $\omega = 12840.2346\sqrt{I}$ rad/sec

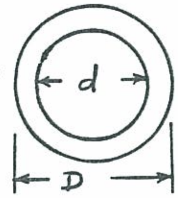
In order to have $\omega = 0.5$ Hz $= 3.1416$ rad/sec,

$12840.2346\sqrt{I} = 3.1416 \Rightarrow I = 59862.43022 \times 10^{-12}$ m^4

For a tubular section,

$$I = \frac{\pi}{64}(D^4 - d^4) = 59862.43022 \times 10^{-12}$$

i.e. $D^4 - d^4 = 1.219504 \times 10^{-6}$ m^4



To minimize weight, we need to minimize $(D^2 - d^2)$.

Problem is: Find D and d to minimize $(D^2 - d^2)$

subject to $D^4 - d^4 = 1.219504 \times 10^{-6}$

or Find d and $r = D/d$

to minimize $f = d^2(r^2 - 1)$ (E1)

subject to $d^4(r^4 - 1) = 121.9504 \times 10^{-8}$ (E2)

Eq. (E2) gives $$d^2 = \frac{11.0431 \times 10^{-4}}{\sqrt{r^4-1}}$$

Eq. (E1) becomes $$f = \frac{11.0431 \times 10^{-4}(r^2-1)}{\sqrt{r^4-1}}$$

For minimum of f, $$\frac{df}{dr} = \frac{d}{dr}\left(\frac{r^2-1}{\sqrt{r^4-1}}\right) = 0$$

i.e., $$\frac{(r^2-1)\frac{1}{2}(r^4-1)^{-\frac{1}{2}}(4r^3) - \sqrt{r^4-1}\,(2r)}{r^4-1} = 0$$

i.e., $r^2(r^2-1) = (r^2+1)(r^2-1)$

i.e., $r^2 = 1$ or $r^2 = r^2 + 1$

i.e., $r = 1$ is the only feasible solution.

i.e., A solid circular section.

Since $I = \frac{\pi D^4}{64} = 0.059862 \times 10^{-6}$, $D = 0.03323$ m

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.10 From Example 6.3,

$$[k] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = k \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix}$$

$$[m] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Assuming the mode shape as $\vec{X} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$, Rayleigh's quotient becomes

$$R(\vec{X}) = \omega^2 = \frac{\vec{X}^T [k] \vec{X}}{\vec{X}^T [m] \vec{X}}$$

$$= \frac{(1 \quad 2 \quad 3)\, k \begin{bmatrix} 3 & -2 & 0 \\ -2 & 5 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}}{(1 \quad 2 \quad 3)\, m \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}} = \frac{1}{6} \frac{k}{m}$$

$$\omega_1 = 0.4082 \sqrt{\frac{k}{m}}$$

Exact value is $\omega_1 = 0.3376 \sqrt{\frac{k}{m}}$ (Problem 6.51).

(7.11) Stiffness matrix:

Give each disc a unit angular displacement holding other discs with zero rotation. Torque required will give stiffness coefficients.

$\theta_1 = 1,\ \theta_2 = \theta_3 = 0$: $\quad M_{t1} = k_{t1} + k_{t2}, \quad M_{t2} = -k_{t2}, \quad M_{t3} = 0$

$\theta_2 = 1,\ \theta_1 = \theta_3 = 0$: $\quad M_{t2} = k_{t2} + k_{t3}, \quad M_{t1} = -k_{t2}, \quad M_{t3} = -k_{t3}$

$\theta_3 = 1,\ \theta_1 = \theta_2 = 0$: $\quad M_{t3} = k_{t3}, \quad M_{t2} = -k_{t3}, \quad M_{t1} = 0$

$$[k] = \begin{bmatrix} k_{t1} + k_{t2} & -k_{t2} & 0 \\ -k_{t2} & k_{t2} + k_{t3} & -k_{t3} \\ 0 & -k_{t3} & k_{t3} \end{bmatrix} = k_t \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$[m] = \begin{bmatrix} J_1 & 0 & 0 \\ 0 & J_2 & 0 \\ 0 & 0 & J_3 \end{bmatrix} = J_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Assume the mode shape as $\vec{\Theta} = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$

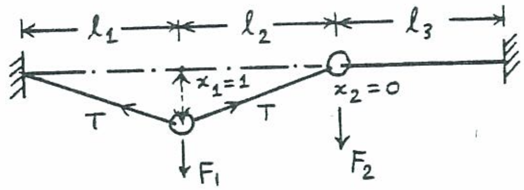
$$R(\vec{\Theta}) = \omega^2 = \frac{(1 \quad 2 \quad 3)\, k_t \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}}{(1 \quad 2 \quad 3)\, J_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}} = \frac{1}{12} \frac{k_t}{J_0}$$

$$\omega_1 \simeq 0.2887 \sqrt{\frac{k_t}{J_0}}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

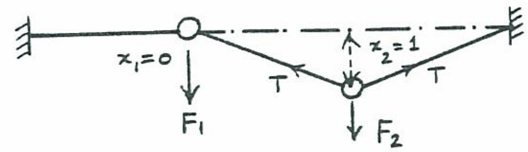
(7.12) Stiffness matrix:



When $x_1 = 1$, $x_2 = 0$:

$$F_1 = k_{11} = \frac{T}{l_1} + \frac{T}{l_2}$$

$$F_2 = k_{21} = -\frac{T}{l_2}$$



When $x_1 = 0$, $x_2 = 1$:

$$F_2 = \frac{T}{l_2} + \frac{T}{l_3}$$

$$F_1 = k_{12} = -\frac{T}{l_2}$$

$$[k] = T\begin{bmatrix} \left(\frac{1}{l_1} + \frac{1}{l_2}\right) & -\frac{1}{l_2} \\ -\frac{1}{l_2} & \left(\frac{1}{l_2} + \frac{1}{l_3}\right) \end{bmatrix} = \frac{T}{l}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

$$[m] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Assume the mode shape as $\vec{X} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

$$R(\vec{X}) = \omega^2 = \frac{(1 \quad 2)\, \frac{T}{l}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{(1 \quad 2)\, m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}} = \frac{6}{5}\,\frac{T}{l m}$$

$$\omega_1 \simeq 1.0954\sqrt{\frac{T}{l m}}$$

Exact value is $\omega_1 = \sqrt{\frac{T}{l m}}$ (Problem 5.22).

(7.13) From problem 7.12, for $l_1 = l_2 = l_3 = l$,

$$[k] = \frac{T}{l}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

and for $m_1 = m$, $m_2 = 5m$, $[m] = m\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$

Let $\vec{X} = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$

$$R(\vec{X}) = \omega^2 = \frac{(1 \quad 2)\, \frac{T}{l}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}}{(1 \quad 2)\, m\begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}\begin{Bmatrix} 1 \\ 2 \end{Bmatrix}} = \frac{2}{7}\,\frac{T}{m l}$$

$$\omega_1 \simeq 0.5345\sqrt{\frac{T}{m l}}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.14 Apply a unit load to masses m_1 and m_2 along x_1 and x_2, respectively:

$$a_{11} = \frac{1}{2k_1} \quad ; \quad a_{22} = \frac{1}{2k_1} + \frac{1}{2k_2} = \frac{k_2 + k_1}{2k_1 k_2}$$

Dunkerley's equation gives

$$\frac{1}{\omega_1^2} = a_{11} m_1 + a_{22} m_2 = \frac{m_1}{2k_1} + m_2 \left(\frac{k_2 + k_1}{2 k_1 k_2} \right) \qquad (E_1)$$

Since $k_1 = k_2 = 3EI/h^3 \equiv k$, $m_1 = 2m$, $m_2 = m$ and hence (E_1) gives

$$\omega_1 = \sqrt{\frac{3EI}{2mh^3}}$$

7.15 Eq. (7.21) gives, for $r = n$,

$$R(\vec{x}) = \frac{c_n^2 \omega_n^2 + c_n^2 \sum_{i=1}^{n-1} \left(\frac{c_i}{c_n} \right)^2 \omega_i^2}{c_n^2 + c_n^2 \sum_{i=1}^{n-1} \left(\frac{c_i}{c_n} \right)^2} \qquad (E_1)$$

Let $\left| \frac{c_i}{c_n} \right| = \epsilon_i << 1$. Then Eq. (E_1) becomes

$$R(\vec{x}) = \frac{\omega_n^2 + \sum_{i=1}^{n-1} \epsilon_i^2 \omega_i^2}{1 + \sum_{i=1}^{n-1} \epsilon_i^2} \simeq \left(\omega_n^2 + \sum_{i=1} \epsilon_i^2 \omega_i^2 \right) \left(1 - \sum_{i=1}^{n-1} \epsilon_i^2 \right)$$

$$\simeq \omega_n^2 + \sum_{i=1}^{n-1} \epsilon_i^2 \omega_i^2 - \omega_n^2 \sum_{i=1}^{n-1} \epsilon_i^2$$

$$\simeq \omega_n^2 + \sum_{i=1}^{n-1} \left(\omega_i^2 - \omega_n^2 \right) \epsilon_i^2 \qquad (E_2)$$

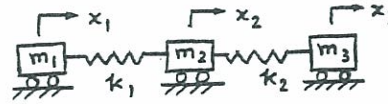
Since $\omega_i^2 < \omega_n^2$, in general, (E_2) shows that

$$R(\vec{x}) \le \omega_n^2$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.19 Equations of motion



$$m_1 \ddot{x}_1 + k_1 (x_1 - x_2) = 0 \quad \text{--- (E.1)}$$

$$m_2 \ddot{x}_2 + k_1 (x_2 - x_1) + k_2 (x_2 - x_3) = 0 \quad \text{--- (E.2)}$$

$$m_3 \ddot{x}_3 + k_2 (x_3 - x_2) = 0 \quad \text{--- (E.3)}$$

With $x_i(t) = X_i \cos \omega t$, Eqs. (E.1) and (E.2) give

$$m_1 \omega^2 X_1 = k_1 (X_1 - X_2) \ ; \qquad m_2 \omega^2 X_2 = -\omega^2 m_1 X_1 + k_2 (X_2 - X_3)$$

or $$X_2 = X_1 - \frac{\omega^2 m_1 X_1}{k_1} \ ; \quad X_3 = X_2 - \frac{\omega^2}{k_2} (m_1 X_1 + m_2 X_2)$$

Since the system is free - free, the resultant force applied to mass 3,

$$F = \sum_{i=1}^{3} \omega^2 m_i X_i \ ,$$ must be zero.

The computer program, with the subroutine FUN and the output, are given below.

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
C ======================================================================
C
C HOLZER METHOD
C THIS PROGRAM REQUIRES USER SUPPLIED SUBROUTINE FUN
C
C ======================================================================
C NT = NUMBER OF TIMES INCREMENT OF OM CHANGED
C FOLLOWING LINE CONTAINS PROBLEM-DEPENDENT DATA
C NR = NUMBER OF ROOTS REQUIRED
      NR=2   <-----
C END OF PROBLEM-DEPENDENT DATA
      NFUN=1
      OM=0.1 <----
      CALL FUN (OM,F)
      PRINT 60,NFUN,OM,F
      FB=F
      INR=0
      OM=0.0
  100 DEL=10.0 <----
      NT=0
  200 CONTINUE
      F1=FB
   10 OM=OM+DEL
      CALL FUN (OM,F)
      NFUN=NFUN+1
      PRINT 60,NFUN,OM,F
      F2=F1*F
      IF (F2 .LT. 0.0) GO TO 20
      F1=F
      GO TO 10
   20 NT=NT+1
      PRINT 30, OM,F,DEL,NFUN
   30 FORMAT (//,31H CHANGE OF SIGN DETECTED AT OM=,E15.8,/,3H F=,
     2 E15.8,/,5H DEL=,E15.8,/,6H NFUN=,I5,/)
      IF (NT .EQ. 6) GO TO 40
      IF (NT .EQ. 1) OMB=OM
      OM=OM-DEL
      DEL=DEL/10.0
      GO TO 200
   40 INR=INR+1
      IF (INR .EQ. NR) GO TO 50
      OM=OMB
      CALL FUN (OM,F)
      NFUN=NFUN+1
      PRINT 60,NFUN,OM,F
      FB=F
      GO TO 100
   50 CONTINUE
   60 FORMAT (2X,6H NFUN=,I4,2X,4H OM=,E15.8,2X,3H F=,E15.8)
      STOP
      END
C ======================================================================
C
C SUBROUTINE FUN
C
C ======================================================================
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك   الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
      SUBROUTINE FUN (OM,F)
      XM1=100.0
      XM2=20.0
      XM3=200.0
      XK1=8000.0
      XK2=4000.0
      OMS=OM**2
      X1=1.0
      X2=(1.0-(OMS*XM1/XK1))*X1
      X3=X2-(OMS/XK2)*(XM1*X1+XM2*X2)
      F=OMS*(XM1*X1+XM2*X2+XM3*X3)
      RETURN
      END
```

$\omega_1 = 0$

```
NFUN=    1    OM= 0.10000000E+00    F= 0.31991253E+01
NFUN=    2    OM= 0.10000000E+02    F=-0.43000000E+05

CHANGE OF SIGN DETECTED AT OM= 0.10000000E+02
F=-0.43000000E+05
DEL= 0.10000000E+02
NFUN=    2

NFUN=    3    OM= 0.10000000E+01    F= 0.31126251E+03
NFUN=    4    OM= 0.20000000E+01    F= 0.11408000E+04
NFUN=    5    OM= 0.30000000E+01    F= 0.21803625E+04
NFUN=    6    OM= 0.40000000E+01    F= 0.29312000E+04
NFUN=    7    OM= 0.50000000E+01    F= 0.27265625E+04
NFUN=    8    OM= 0.60000000E+01    F= 0.76320044E+03
NFUN=    9    OM= 0.70000000E+01    F=-0.38581377E+04

CHANGE OF SIGN DETECTED AT OM= 0.70000000E+01
F=-0.38581377E+04
DEL= 0.10000000E+01
NFUN=    9
   :
CHANGE OF SIGN DETECTED AT OM= 0.62220016E+01
F=-0.28649828E+00
DEL= 0.99999990E-01
NFUN=   27
```

← $\omega_2 = 6.2220016$

```
NFUN=   28    OM= 0.10000000E+02    F=-0.43000000E+05
NFUN=   29    OM= 0.20000000E+02    F=-0.47200000E+06
NFUN=   30    OM= 0.30000000E+02    F= 0.23130000E+07

CHANGE OF SIGN DETECTED AT OM= 0.30000000E+02
F= 0.23130000E+07
DEL= 0.10000000E+02
NFUN=   30

NFUN=   31    OM= 0.21000000E+02    F=-0.48851225E+06
NFUN=   32    OM= 0.22000000E+02    F=-0.47761113E+06
NFUN=   33    OM= 0.23000000E+02    F=-0.42888006E+06
```


Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
:
CHANGE OF SIGN DETECTED AT OM= 0.25715595E+02
F= 0.21467506E+02
DEL= 0.99999990E-04
NFUN=    58
```

$\leftarrow \omega_3 = 25.715595$

7.20 Eigenvalue problem is

$$\begin{bmatrix} (\lambda-2) & 1 & 0 \\ 1 & (\lambda-2) & 1 \\ 0 & 1 & (2\lambda-3) \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad (E_1)$$

where $\lambda = \frac{m\omega^2}{k}$.

If $x_1 = 1$, (E_1) gives $x_2 = -(\lambda-2)\,x_1$, $x_3 = -x_1 - (\lambda-2)\,x_2$,

$E = x_2 + (2\lambda - 3)\,x_3$ (should be zero)

The computer program and results are given.

```
C ====================================================================
C
C HOLZER METHOD
C THIS PROGRAM REQUIRES USER SUPPLIED SUBROUTINE FUN
C
C ====================================================================
C NT = NUMBER OF TIMES INCREMENT OF OM CHANGED
C FOLLOWING LINE CONTAINS PROBLEM-DEPENDENT DATA
C NR = NUMBER OF ROOTS REQUIRED
      NR=3 <-----
C END OF PROBLEM-DEPENDENT DATA
      NFUN=1
      OM=0.01 <-----
      CALL FUN (OM,F)
      WRITE(13,60) NFUN,OM,F
      FB=F
      INR=0
      OM=0.0
100   DEL=0.25 <-----
      NT=0
200   CONTINUE
      F1=FB
10    OM=OM+DEL
      CALL FUN (OM,F)
      NFUN=NFUN+1
      WRITE(13,60) NFUN,OM,F
      F2=F1*F
      IF (F2 .LT. 0.0) GO TO 20
      F1=F
      GO TO 10
20    NT=NT+1
      WRITE(13,30) OM,F,DEL,NFUN
30    FORMAT (//,31H CHANGE OF SIGN DETECTED AT OM=,E15.8,/,3H F=,
     2 E15.8,/,5H DEL=,E15.8,/,6H NFUN=,I5,/)
      IF (NT .EQ. 6) GO TO 40
      IF (NT .EQ. 1) OMB=OM
      OM=OM-DEL
      DEL=DEL/10.0
      GO TO 200
40    INR=INR+1
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
      IF (INR .EQ. NR) GO TO 50
      OM=OMB
      CALL FUN (OM,F)
      NFUN=NFUN+1
      WRITE(13,60) NFUN,OM,F
      FB=F
      GO TO 100
50    CONTINUE
60    FORMAT (2X,6H NFUN=,I4,2X,4H OM=,E15.8,2X,3H F=,E15.8)
      STOP
      END
C =====================================================================
C
C SUBROUTINE FUN
C
C =====================================================================
      SUBROUTINE FUN (OM,F)
      X1=1.0
      X2=-(OM-2.0)*X1
      X3=-X1-(OM-2.0)*X2
      F=X2+(2.0*OM-3.0)*X3
      RETURN
      END
```

```
  NFUN=    1    OM= 0.99999998E-02    F=-0.68310986E+01
  NFUN=    2    OM= 0.25000000E+00    F=-0.34062500E+01
  NFUN=    3    OM= 0.50000000E+00    F=-0.10000000E+01
  NFUN=    4    OM= 0.75000000E+00    F= 0.40625000E+00

CHANGE OF SIGN DETECTED AT OM= 0.75000000E+00
F= 0.40625000E+00
DEL= 0.25000000E+00
NFUN=    4

  NFUN=    5    OM= 0.52499998E+00    F=-0.81746900E+00
  NFUN=    6    OM= 0.54999995E+00    F=-0.64475036E+00
  NFUN=    7    OM= 0.57499993E+00    F=-0.48165655E+00
  NFUN=    8    OM= 0.59999990E+00    F=-0.32800055E+00
  NFUN=    9    OM= 0.62499988E+00    F=-0.18359447E+00
  NFUN=   10    OM= 0.64999986E+00    F=-0.48250675E-01
  NFUN=   11    OM= 0.67499983E+00    F= 0.78217864E-01
  :
  NFUN=   28    OM= 0.65932715E+00    F=-0.38385391E-04
  NFUN=   29    OM= 0.65932965E+00    F=-0.25749207E-04
  NFUN=   30    OM= 0.65933216E+00    F=-0.12755394E-04
  NFUN=   31    OM= 0.65933466E+00    F=-0.11920929E-06
  NFUN=   32    OM= 0.65933716E+00    F= 0.12636185E-04

CHANGE OF SIGN DETECTED AT OM= 0.65933716E+00
F= 0.12636185E-04
DEL= 0.24999997E-05
NFUN=   32

  NFUN=   33    OM= 0.75000000E+00    F= 0.40625000E+00
```

← $\lambda_1 = 0.65933716$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
NFUN=  34   OM= 0.10000000E+01   F= 0.10000000E+01
NFUN=  35   OM= 0.12500000E+01   F= 0.96875000E+00
NFUN=  36   OM= 0.15000000E+01   F= 0.50000000E+00
NFUN=  37   OM= 0.17500000E+01   F=-0.21875000E+00
 :
NFUN=  63   OM= 0.16789526E+01   F= 0.32067299E-04
NFUN=  64   OM= 0.16789551E+01   F= 0.24497509E-04
NFUN=  65   OM= 0.16789576E+01   F= 0.16927719E-04
NFUN=  66   OM= 0.16789601E+01   F= 0.93579292E-05
NFUN=  67   OM= 0.16789626E+01   F= 0.17881393E-05
NFUN=  68   OM= 0.16789651E+01   F=-0.57816505E-05

CHANGE OF SIGN DETECTED AT OM= 0.16789651E+01
F=-0.57816505E-05
DEL= 0.24999997E-05
NFUN=   68
```

← $\lambda_2 = 1.6789651$

```
NFUN=  69   OM= 0.17500000E+01   F=-0.21875000E+00
NFUN=  70   OM= 0.20000000E+01   F=-0.10000000E+01
NFUN=  71   OM= 0.22500000E+01   F=-0.16562500E+01
NFUN=  72   OM= 0.25000000E+01   F=-0.20000000E+01
NFUN=  73   OM= 0.27500000E+01   F=-0.18437500E+01
NFUN=  74   OM= 0.30000000E+01   F=-0.10000000E+01
NFUN=  75   OM= 0.32500000E+01   F= 0.71875000E+00
 :
NFUN=  95   OM= 0.31615264E+01   F=-0.13036728E-02
NFUN=  96   OM= 0.31615515E+01   F=-0.11179447E-02
NFUN=  97   OM= 0.31615765E+01   F=-0.93221664E-03
NFUN=  98   OM= 0.31616015E+01   F=-0.74636936E-03
NFUN=  99   OM= 0.31616266E+01   F=-0.56064129E-03
NFUN= 100   OM= 0.31616516E+01   F=-0.37479401E-03
NFUN= 101   OM= 0.31616766E+01   F=-0.18906593E-03
NFUN= 102   OM= 0.31617017E+01   F=-0.32186508E-05
NFUN= 103   OM= 0.31617267E+01   F= 0.18215179E-03

CHANGE OF SIGN DETECTED AT OM= 0.31617267E+01
F= 0.18215179E-03
DEL= 0.24999998E-04
NFUN=  103
```

← $\lambda_3 = 3.1617267$

7.21 The program listed in Problems 7.19 and 7.20 is used with NR = 1, initial value of OM = 0.01, DEL = 0.25 and

```
C ==================================================================
C
C SUBROUTINE FUN
C
C ==================================================================
      SUBROUTINE FUN (OM,F)
      X1=1.0
      X2=(2.0-OM)*X1
      X3=-X1+(2.0-OM)*X2
      F=-X2+(1.0-OM)*X3
```

$\Theta_1 = 1$

$\Theta_2 = (-\lambda + 2)\Theta_1$; $\lambda = \left(\frac{J_0\omega^2}{k_t}\right)$

$\Theta_3 = -\Theta_1 + (-\lambda + 2)\Theta_2$

$E = -\Theta_2 + (-\lambda + 1)\Theta_3$

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
RETURN
END
```

The output of the program is given below.

```
NFUN=   1   OM= 0.99999998E-02   F= 0.94049907E+00
NFUN=   2   OM= 0.25000000E+00   F=-0.20312500E+00

CHANGE OF SIGN DETECTED AT OM= 0.25000000E+00
F=-0.20312500E+00
DEL= 0.25000000E+00
NFUN=    2

NFUN=   3   OM= 0.25000000E-01   F= 0.85310948E+00
NFUN=   4   OM= 0.50000001E-01   F= 0.71237516E+00
NFUN=   5   OM= 0.75000003E-01   F= 0.57770300E+00
NFUN=   6   OM= 0.10000000E+00   F= 0.44899976E+00
NFUN=   7   OM= 0.12500000E+00   F= 0.32617188E+00
NFUN=   8   OM= 0.15000001E+00   F= 0.20912516E+00
NFUN=   9   OM= 0.17500001E+00   F= 0.97765565E-01
NFUN=  10   OM= 0.20000002E+00   F=-0.80001354E-02

CHANGE OF SIGN DETECTED AT OM= 0.20000002E+00
F=-0.80001354E-02
DEL= 0.25000000E-01
NFUN=   10

NFUN=  11   OM= 0.17750001E+00   F= 0.86938858E-01
NFUN=  12   OM= 0.18000001E+00   F= 0.76168060E-01
NFUN=  13   OM= 0.18250000E+00   F= 0.65452933E-01
NFUN=  14   OM= 0.18500000E+00   F= 0.54793477E-01
NFUN=  15   OM= 0.18750000E+00   F= 0.44189453E-01
NFUN=  16   OM= 0.19000000E+00   F= 0.33641100E-01
NFUN=  17   OM= 0.19250000E+00   F= 0.23147941E-01
NFUN=  18   OM= 0.19499999E+00   F= 0.12710214E-01
NFUN=  19   OM= 0.19749999E+00   F= 0.23275614E-02
NFUN=  20   OM= 0.19999999E+00   F=-0.79998970E-02

CHANGE OF SIGN DETECTED AT OM= 0.19999999E+00
F=-0.79998970E-02
DEL= 0.24999999E-02
NFUN=   20

NFUN=  21   OM= 0.19774999E+00   F= 0.12923479E-02
NFUN=  22   OM= 0.19799998E+00   F= 0.25773048E-03
NFUN=  23   OM= 0.19824998E+00   F=-0.77641010E-03

CHANGE OF SIGN DETECTED AT OM= 0.19824998E+00
F=-0.77641010E-03
DEL= 0.24999998E-03
NFUN=   23
```

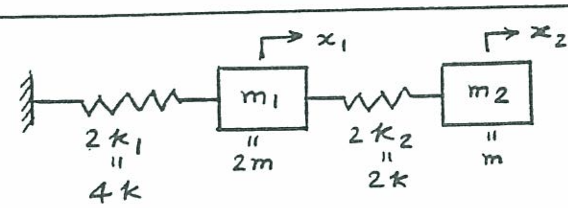

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

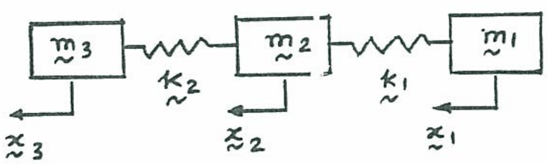
```
NFUN=  24    OM= 0.19802499E+00    F= 0.15425682E-03
NFUN=  25    OM= 0.19804999E+00    F= 0.50902367E-04
NFUN=  26    OM= 0.19807500E+00    F=-0.52571297E-04

CHANGE OF SIGN DETECTED AT OM= 0.19807500E+00
F=-0.52571297E-04
DEL= 0.24999998E-04
NFUN=   26

NFUN=  27    OM= 0.19805250E+00    F= 0.40531158E-04
NFUN=  28    OM= 0.19805500E+00    F= 0.30040741E-04
NFUN=  29    OM= 0.19805750E+00    F= 0.19669533E-04
NFUN=  30    OM= 0.19806001E+00    F= 0.92983246E-05
NFUN=  31    OM= 0.19806251E+00    F=-0.10728836E-05

CHANGE OF SIGN DETECTED AT OM= 0.19806251E+00
F=-0.10728836E-05
DEL= 0.24999997E-05
NFUN=   31
```

$\leftarrow \lambda_1 = 0.19806251$

7.22 The system can be modeled as shown.



The system can be redrawn as follows:



Here $\underset{\sim}{m}_1 = m$, $\underset{\sim}{k}_1 = 2k$, $\underset{\sim}{m}_2 = 2m$, $\underset{\sim}{k}_2 = 4k$, $\underset{\sim}{x}_1 = x_2$, $\underset{\sim}{x}_2 = x_1$, $\underset{\sim}{x}_3 = 0$, $\underset{\sim}{m}_3$ = any value.

Let $\underset{\sim}{X}_1 = 1$

$$\underset{\sim}{X}_2 = \underset{\sim}{X}_1 - \frac{\omega^2 \underset{\sim}{m}_1 \underset{\sim}{X}_1}{\underset{\sim}{k}_1}$$

$$= \underset{\sim}{X}_1 \left(1 - \frac{m\omega^2}{2k}\right)$$

$$\underset{\sim}{X}_3 = \underset{\sim}{X}_2 - \frac{\omega^2}{\underset{\sim}{k}_2}\left(\underset{\sim}{m}_1 \underset{\sim}{X}_1 + \underset{\sim}{m}_2 \underset{\sim}{X}_2\right)$$

$$= \underset{\sim}{X}_2 \left(1 - \frac{m\omega^2}{2k}\right) - \underset{\sim}{X}_1 \left(\frac{m\omega^2}{4k}\right)$$

$$= \underset{\sim}{X}_1 \left(1 + \frac{m^2\omega^4}{4k^2} - \frac{5m\omega^2}{4k}\right)$$

Holzer's procedure involves assuming different values for ω^2 and finding, out of those values, the correct frequency ω as the one which gives $\underset{\sim}{X}_3 = 0$ (boundary condition to be satisfied).

From the expression for $\underset{\sim}{X}_3$, we can find the correct

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

frequency (without trial and error) by setting

$$\omega^4\left(\frac{m^2}{4k^2}\right) - \omega^2\left(\frac{5m}{4k}\right) + 1 = 0$$

or $$\omega^2 = \frac{\left(\frac{5m}{4k}\right) \pm \sqrt{\frac{25m^2}{16k^2} - \frac{m^2}{k^2}}}{(m^2/2k^2)} = \frac{\frac{5m}{4k} \pm \frac{3m}{4k}}{(m^2/2k^2)}$$

Thus the first natural frequency is given by

$$\omega_1^2 = \left(\frac{5m}{4k} - \frac{3m}{4k}\right)\Big/\left(\frac{m^2}{2k^2}\right) \quad \text{or} \quad \omega_1 = \sqrt{\frac{k}{m}}.$$

7.23 Eqs. (7.32) to (7.35) give

$$\Theta_2 = \Theta_1 - \frac{\omega^2 J_1}{k_{t1}}\Theta_1$$

$$\Theta_3 = \Theta_2 - \frac{\omega^2}{k_{t2}}(J_1\Theta_1 + J_2\Theta_2)$$

$$E = \sum_{i=1}^{3}\omega^2 J_i \Theta_i = \text{sum of inertia torques (should be zero)}$$

The computer program of Problems 7.19 and 7.20 is used with NR = 3, OM = 0.01 and DEL = 10.0.

The subroutine FUN and results are given.

```
C =====================================================================
C
C SUBROUTINE FUN
C
C =====================================================================
      SUBROUTINE FUN (OM,F)
      XJ1=10.0
      XJ2=5.0
      XJ3=1.0
      XKT1=1.0E+06
      XKT2=1.0E+06
      X1=1.0
      OMS=OM**2
      X2=X1-(OMS*XJ1/XKT1)*X1
      X3=X2-(OMS/XKT2)*(XJ1*X1+XJ2*X2)
      F=OMS*(XJ1*X1+XJ2*X2+XJ3*X3)
      RETURN
      END
```

$\omega_1 = 0$

```
NFUN=   1   OM= 0.99999998E-02   F= 0.16000000E-02
NFUN=   2   OM= 0.10000000E+02   F= 0.15992500E+04
NFUN=   3   OM= 0.20000000E+02   F= 0.63880034E+04
NFUN=   4   OM= 0.30000000E+02   F= 0.14339287E+05
NFUN=   5   OM= 0.40000000E+02   F= 0.25408205E+05
NFUN=   6   OM= 0.50000000E+02   F= 0.39532031E+05
NFUN=   7   OM= 0.60000000E+02   F= 0.56630336E+05
NFUN=   8   OM= 0.70000000E+02   F= 0.76605133E+05
NFUN=   9   OM= 0.80000000E+02   F= 0.99341109E+05
NFUN=  10   OM= 0.90000000E+02   F= 0.12470582E+06
   :
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

```
NFUN=  50    OM= 0.49000000E+03    F= 0.21006358E+06
NFUN=  51    OM= 0.50000000E+03    F= 0.93750000E+05
NFUN=  52    OM= 0.51000000E+03    F=-0.32486393E+05

CHANGE OF SIGN DETECTED AT OM= 0.51000000E+03
F=-0.32486393E+05
DEL= 0.10000000E+02
NFUN=   52
  :
NFUN=  69    OM= 0.50750012E+03    F= 0.93337460E+01
NFUN=  70    OM= 0.50750021E+03    F= 0.79214058E+01
NFUN=  71    OM= 0.50750031E+03    F= 0.66932836E+01
NFUN=  72    OM= 0.50750040E+03    F= 0.57721915E+01
NFUN=  73    OM= 0.50750049E+03    F= 0.45440674E+01
NFUN=  74    OM= 0.50750058E+03    F= 0.33159423E+01
NFUN=  75    OM= 0.50750067E+03    F= 0.20878162E+01
NFUN=  76    OM= 0.50750076E+03    F= 0.92109573E+00
NFUN=  77    OM= 0.50750085E+03    F=-0.30703202E+00

CHANGE OF SIGN DETECTED AT OM= 0.50750085E+03        ← ω2 = 507.50085
F=-0.30703202E+00
DEL= 0.99999990E-04
NFUN=   77

NFUN=  78    OM= 0.51000000E+03    F=-0.32486393E+05
NFUN=  79    OM= 0.52000000E+03    F=-0.16878158E+06
NFUN=  80    OM= 0.53000000E+03    F=-0.31524272E+06
  :
NFUN= 137    OM= 0.11000000E+04    F=-0.18694431E+07
NFUN= 138    OM= 0.11100000E+04    F=-0.62095219E+06
NFUN= 139    OM= 0.11200000E+04    F= 0.74758494E+06

CHANGE OF SIGN DETECTED AT OM= 0.11200000E+04
F= 0.74758494E+06
DEL= 0.10000000E+02
NFUN=  139
  :
NFUN= 168    OM= 0.11146489E+04    F=-0.37916328E+02
NFUN= 169    OM= 0.11146490E+04    F=-0.23697710E+02
NFUN= 170    OM= 0.11146492E+04    F=-0.94790859E+01
NFUN= 171    OM= 0.11146493E+04    F= 0.47395439E+01

CHANGE OF SIGN DETECTED AT OM= 0.11146493E+04        ← ω3 = 1114.6493
F= 0.47395439E+01
DEL= 0.99999990E-04
NFUN=  171
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك   الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.27 Eigenvector $\vec{X}^{(1)}$ corresponding to $\lambda_1 = 10.38068 \left(= \frac{1}{\omega_1^2}\right)$ is given by

$$\begin{bmatrix} (2.5-\lambda_1) & -1 & 0 \\ -1 & (5-\lambda_1) & -\sqrt{2} \\ 0 & -\sqrt{2} & (10-\lambda_1) \end{bmatrix} \begin{Bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ x_3^{(1)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

i.e. $x_2^{(1)} = -7.88068\, x_1^{(1)}$ and $x_3^{(1)} = -3.71497\, x_2^{(1)} = 29.27649\, x_1^{(1)}$

$$\vec{X}^{(1)} = \alpha \begin{Bmatrix} 1 \\ -7.88068 \\ 29.27649 \end{Bmatrix} \quad \text{where } \alpha = \text{value of } x_1^{(1)}$$

when $\vec{X}^{(1)}$ is normalized as $\vec{X}^{(1)T} [m] \vec{X}^{(1)} = 1$, $\alpha = 0.03296$

$$\therefore \vec{X}^{(1)} = \begin{Bmatrix} 0.03296 \\ -0.25975 \\ 0.96495 \end{Bmatrix}$$

$$[D_2] = [D_1] - \lambda_1 \vec{X}^{(1)} \vec{X}^{(1)T} [m]$$

$$= \begin{bmatrix} 2.5 & -1.0 & 0.0 \\ -1.0 & 5.0 & -1.4142 \\ 0.0 & -1.4142 & 10.0 \end{bmatrix} - (10.38068) \begin{Bmatrix} 0.03296 \\ -0.25975 \\ 0.96495 \end{Bmatrix} \{0.03296 \;\; -0.25975 \;\; 0.96495\}$$

$$= \begin{bmatrix} 2.4887228 & -0.9111273 & -0.3301549 \\ -0.9111273 & 4.2996149 & 1.1876738 \\ -0.3301549 & 1.1876738 & 0.3342530 \end{bmatrix}$$

If $\vec{X}_0 = \begin{Bmatrix} 1.0 \\ -7.88068 \\ 29.27649 \end{Bmatrix}$ is used, the iterative procedure $\vec{X}_{i+1} = [D_2] \vec{X}_i$ gives the following results (with $x_{1,i+1} = 1$):

Iter. No. (i)		$\vec{X}_{i+1}$ as a row vector (with $x_{1,i+1} = 1$)
ITER=	0	0.10000000E+01-0.78806801E+01 0.29276489E+02
ITER=	1	0.10000000E+01-0.10666667E+02 0.29333334E+02
ITER=	2	0.10000000E+01-0.47272644E+01-0.13066854E+01
ITER=	3	0.10000000E+01-0.31531227E+01-0.88291872E+00
ITER=	4	0.10000000E+01-0.27448514E+01-0.77301961E+00
ITER=	5	0.10000000E+01-0.25989382E+01-0.73374254E+00
ITER=	6	0.10000000E+01-0.25411220E+01-0.71817952E+00
ITER=	7	0.10000000E+01-0.25172873E+01-0.71176374E+00
ITER=	8	0.10000000E+01-0.25073018E+01-0.70907575E+00
ITER=	9	0.10000000E+01-0.25030899E+01-0.70794201E+00
ITER=	10	0.10000000E+01-0.25013082E+01-0.70746231E+00
ITER=	11	0.10000000E+01-0.25005538E+01-0.70725930E+00
ITER=	12	0.10000000E+01-0.25002341E+01-0.70717323E+00
ITER=	13	0.10000000E+01-0.25000987E+01-0.70713681E+00
ITER=	14	0.10000000E+01-0.25000415E+01-0.70712131E+00
ITER=	15	0.10000000E+01-0.25000172E+01-0.70711482E+00

converged value of $\lambda_2 = 5.00004216$ (or $\omega_2 = 0.44721171$)

By using a similar procedure, $[D_3]$ is found. With the same $\vec{X}_0$, the results obtained from $\vec{X}_{i+1} = [D_3] \vec{X}_i$ are given below.

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Iter. no. (i)		$\vec{X}_{i+1}$ as a row vector (with $X_{1,i+1}=1$)		
ITER=	0	0.10000000E+01	-0.78806801E+01	0.29276489E+02
ITER=	1	0.10000000E+01	0.19600000E+02	-0.72824997E+02
ITER=	2	0.10000000E+01	0.38067123E+00	0.68348452E-01
ITER=	3	0.10000000E+01	0.38067168E+00	0.68312615E-01
ITER=	4	0.10000000E+01	0.38067159E+00	0.68312593E-01

converged value of $\lambda_3 = 2.11932243$ ($\omega_3 = 0.68691260$).

7.28 $[k]^{-1} = \frac{1}{k}\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & 2\\ 1 & 2 & 2.5\end{bmatrix}$, $[D] = [k]^{-1}[m] = \frac{m}{k}\begin{bmatrix}1 & 1 & 2\\ 1 & 2 & 4\\ 1 & 2 & 5\end{bmatrix}$

Using $\vec{X}_0 = \begin{Bmatrix}1\\1\\1\end{Bmatrix}$, the following results are obtained:

Iteration number (i)		$\vec{X}_{i+1} = [D]\vec{X}_i$ with $X_{1,i}=1$ (given as row vector)		
ITER=	0	0.10000000E+01	0.10000000E+01	0.10000000E+01
ITER=	1	0.10000000E+01	0.17500000E+01	0.20000000E+01
ITER=	2	0.10000000E+01	0.18518518E+01	0.21481481E+01
ITER=	3	0.10000000E+01	0.18601036E+01	0.21606219E+01
ITER=	4	0.10000000E+01	0.18607503E+01	0.21616161E+01
ITER=	5	0.10000000E+01	0.18608015E+01	0.21616952E+01
ITER=	6	0.10000000E+01	0.18608055E+01	0.21617017E+01 ← $\vec{X}^{(1)}$

converged frequency $= 0.373088 = \omega_1\sqrt{\frac{m}{k}}$

Repetition of the procedure with $[D_2]$ and $[D_3]$ gives the following results:

$$\omega_2\sqrt{\frac{m}{k}} = 1.321324$$

$$\vec{X}^{(2)} = \{1.000000 \quad 0.254098 \quad -0.340706\}$$

$$\omega_3\sqrt{\frac{m}{k}} = 2.028523$$

$$\vec{X}^{(3)} = \{1.000000 \quad -2.114801 \quad 0.678847\}$$

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.29 $[k]^{-1} = \frac{1}{k}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.5 & 1.5 \\ 1 & 1.5 & 1.833 \end{bmatrix}$, $[k]^{-1}[m] = \frac{m}{k}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.5 & 1.5 \\ 1 & 1.5 & 1.833 \end{bmatrix}$

$$[D] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1.5 & 1.5 \\ 1 & 1.5 & 1.833 \end{bmatrix}$$

With $\vec{X}_0 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$, the following results are obtained (Program 10.F):

Iteration number (i)	$\vec{X}_{i+1} = [D]\vec{X}_i$ with $x_{1,i} = 1$ (given as row vector)		
0	0.10000000E+01	0.10000000E+01	0.10000000E+01
1	0.10000000E+01	0.13333334E+01	0.14443334E+01
2	0.10000000E+01	0.13676430E+01	0.14949605E+01
3	0.10000000E+01	0.13705537E+01	0.14994360E+01
4	0.10000000E+01	0.13708007E+01	0.14998223E+01
5.	0.10000000E+01	0.13708220E+01	0.14998555E+01
6	0.10000000E+01	0.13708237E+01	0.14998584E+01 ← $\vec{X}^{(1)}$

converged frequency = 0.50828409 = ω_1

0	0.10000000E+01	0.10000000E+01	0.10000000E+01
1	0.10000000E+01	-0.56849640E-01	-0.61459929E+00
2	0.10000000E+01	-0.22656711E-01	-0.64602280E+00
3	0.10000000E+01	-0.91092410E-02	-0.65840477E+00
4	0.10000000E+01	-0.38191630E-02	-0.66323972E+00
5	0.10000000E+01	-0.17635337E-02	-0.66511846E+00
6	0.10000000E+01	-0.96627331E-03	-0.66584712E+00
7	0.10000000E+01	-0.65728393E-03	-0.66612953E+00
8	0.10000000E+01	-0.53756207E-03	-0.66623896E+00
9	0.10000000E+01	-0.49118389E-03	-0.66628140E+00
10	0.10000000E+01	-0.47320157E-03	-0.66629785E+00
11	0.10000000E+01	-0.46624581E-03	-0.66630417E+00
12	0.10000000E+01	-0.46355074E-03	-0.66630661E+00
13	0.10000000E+01	-0.46252197E-03	-0.66630769E+00
14	0.10000000E+01	-0.46209697E-03	-0.66630799E+00
15	0.10000000E+01	-0.46194042E-03	-0.66630810E+00
16	0.10000000E+01	-0.46188448E-03	-0.66630810E+00
17	0.10000000E+01	-0.46187337E-03	-0.66630816E+00
18	0.10000000E+01	-0.46186216E-03	-0.66630822E+00
19	0.10000000E+01	-0.46185096E-03	-0.66630816E+00
20	0.10000000E+01	-0.46185098E-03	-0.66630822E+00 ← $\vec{X}^{(2)}$

converged frequency = 1.7323176 = ω_2

0	0.10000000E+01	0.10000000E+01	0.10000000E+01
1	0.10000000E+01	-0.23654659E+01	0.15024464E+01
2	0.10000000E+01	-0.23733659E+01	0.15024524E+01
3	0.10000000E+01	-0.23733664E+01	0.15024524E+01 ← $\vec{X}^{(3)}$

converged frequency = 2.783294 = ω_3

7.30 From problem 6.23, $[k] = \frac{GJ}{l}\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$, $[J_d] = J_0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$[D] = \begin{bmatrix} 0.75 & 0.50 & 0.25 \\ 0.50 & 1.00 & 0.50 \\ 0.25 & 0.50 & 0.75 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

With $\vec{X}_0 = \begin{Bmatrix} 1 \\ 2 \\ 3 \end{Bmatrix}$, the following results are obtained (Program 10.F):

Iteration number (i)	$\vec{X}_{i+1} = [D]\vec{X}_i$ with $x_{1,i} = 1$ (given as row vector)
0	0.10000000E+01 0.20000000E+01 0.30000000E+01
1	0.10000000E+01 0.16000000E+01 0.14000000E+01
2	0.10000000E+01 0.14736842E+01 0.11052632E+01
3	0.10000000E+01 0.14328358E+01 0.10298507E+01
4	0.10000000E+01 0.14199134E+01 0.10086581E+01
5	0.10000000E+01 0.14159292E+01 0.10025285E+01
6	0.10000000E+01 0.14147242E+01 0.10007399E+01
7	0.10000000E+01 0.14143645E+01 0.10002166E+01
8	0.10000000E+01 0.14142580E+01 0.10000634E+01
9	0.10000000E+01 0.14142267E+01 0.10000186E+01
10	0.10000000E+01 0.14142175E+01 0.10000055E+01
11	0.10000000E+01 0.14142147E+01 0.10000015E+01 ← $\vec{X}^{(1)}$

converged frequency = 0.76536608 = ω_1

0	0.10000000E+01 0.20000000E+01 0.30000000E+01
1	0.10000000E+01 0.29291141E+00-0.14141805E+01
2	0.10000000E+01 0.15801974E+00-0.12234681E+01
3	0.10000000E+01 0.88471927E-01-0.11251128E+01
4	0.10000000E+01 0.50517395E-01-0.10714371E+01
5	0.10000000E+01 0.29161688E-01-0.10412357E+01
⋮	
27	0.10000000E+01 0.19669531E-05-0.99999768E+00
28	0.10000000E+01 0.18924472E-05-0.99999750E+00
29	0.10000000E+01 0.18179417E-05-0.99999750E+00
30	0.10000000E+01 0.18030403E-05-0.99999738E+00
31	0.10000000E+01 0.17732382E-05-0.99999744E+00
32	0.10000000E+01 0.17732380E-05-0.99999726E+00 ← $\vec{X}^{(2)}$

converged frequency = 1.4142134 = ω_2

0	0.10000000E+01 0.20000000E+01 0.30000000E+01
1	0.10000000E+01-0.14143940E+01 0.10000018E+01
2	0.10000000E+01-0.14142135E+01 0.10000004E+01
3	0.10000000E+01-0.14142135E+01 0.10000004E+01 ← $\vec{X}^{(3)}$

converged frequency = 1.8477590 = ω_3

7.31 From Problem 7.12, $[k] = \frac{T}{\ell}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, $[m] = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $[k]^{-1} = \frac{\ell}{3T}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$

$$[D] = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

With $\vec{X}_0 = \begin{Bmatrix} 1 \\ 2 \end{Bmatrix}$, the following results are obtained (Program 10.F):

Iteration number (i)	$\vec{X}_{i+1} = [D]\vec{X}_i$ with $x_{1,i} = 1$ (given as row vector)
0	0.10000000E+01 0.20000000E+01
1	0.10000000E+01 0.12500000E+01
2	0.10000000E+01 0.10769231E+01
3	0.10000000E+01 0.10250001E+01
4	0.10000000E+01 0.10082645E+01
5	0.10000000E+01 0.10027474E+01
6	0.10000000E+01 0.10009151E+01
7	0.10000000E+01 0.10003049E+01

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

8	0.10000000E+01	0.10001017E+01
9	0.10000000E+01	0.10000340E+01
10	0.10000000E+01	0.10000113E+01
11	0.10000000E+01	0.10000038E+01 ← $\vec{X}^{(1)}$

Converged frequency = 0.99999809 = $\omega_1 \sqrt{ml/T}$

0	0.10000000E+01	0.20000000E+01
1	0.10000000E+01	-0.99991989E+00
2	0.10000000E+01	-0.99998856E+00
3	0.10000000E+01	-0.99998856E+00 ← $\vec{X}^{(2)}$

Converged frequency = 1.7320508 = $\omega_2 \sqrt{ml/T}$

7.32 $[k]^{-1} = \frac{1}{k} \begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1.0 & 1.0 & 1.0 \\ 0.5 & 1.0 & 2.0 & 2.0 \\ 0.5 & 1.0 & 2.0 & 3.0 \end{bmatrix}$, $[D] = \frac{m}{k} \begin{bmatrix} 1.5 & 1.0 & 0.5 & 0.5 \\ 1.5 & 2.0 & 1.0 & 1.0 \\ 1.5 & 2.0 & 2.0 & 2.0 \\ 1.5 & 2.0 & 2.0 & 3.0 \end{bmatrix}$

With $\vec{X}_0 = \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$, the following results are obtained for $\vec{X}^{(1)}$ and ω_1.

Iteration number (i)	$\vec{X}_{i+1} = [D]\vec{X}_i$ with $X_{1,i} = 1$ (given as row vector)			
0	0.10000000E+01	0.10000000E+01	0.10000000E+01	0.10000000E+01
1	0.10000000E+01	0.15714285E+01	0.21428571E+01	0.24285715E+01
2	0.10000000E+01	0.17200000E+01	0.25733335E+01	0.30266669E+01
3	0.10000000E+01	0.17508308E+01	0.26810634E+01	0.31838319E+01
4	0.10000000E+01	0.17574103E+01	0.27059193E+01	0.32208292E+01
5	0.10000000E+01	0.17588729E+01	0.27116063E+01	0.32293591E+01
6	0.10000000E+01	0.17592046E+01	0.27129092E+01	0.32313194E+01
7	0.10000000E+01	0.17592806E+01	0.27132087E+01	0.32317696E+01
8	0.10000000E+01	0.17592978E+01	0.27132771E+01	0.32318730E+01
9	0.10000000E+01	0.17593019E+01	0.27132931E+01	0.32318966E+01

Converged frequency = 0.40058133 = $\omega_1 \sqrt{m/k}$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.37 The problem-dependent data for Program 9.F and output are given.

```
C FOLLOWING 3 LINES CONTAIN PROBLEM-DEPENDENT DATA
      DIMENSION D(3,3),E(3,3)
      DATA N,ITMAX,EPS/3,200,1.0E-05/
      DATA D/3.0,-2.0,0.0,-2.0,5.0,-3.0,0.0,-3.0,3.0/
C END OF PROBLEM-DEPENDENT DATA
---------------
EIGENVALUE SOLUTION BY JACOBI METHOD

GIVEN MATRIX
  0.300000E+01  -0.200000E+01   0.000000E+00
 -0.200000E+01   0.500000E+01  -0.300000E+01
  0.000000E+00  -0.300000E+01   0.300000E+01

 EIGEN VALUES ARE
  0.774166E+01   0.300000E+01   0.258343E+00

 EIGEN VECTORS
     FIRST           SECOND          THIRD
  0.335734E+00   0.832048E+00   0.441564E+00
 -0.796032E+00  -0.343903E-05   0.605254E+00
  0.503602E+00  -0.554704E+00   0.662336E+00
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.38 The problem-dependent data for Program 9.F and output are given.

```
C FOLLOWING 3 LINES CONTAIN PROBLEM-DEPENDENT DATA
        DIMENSION D(3,3),E(3,3)
        DATA N,ITMAX,EPS/3,200,1.0E-05/
        DATA D/3.0,2.0,1.0,2.0,2.0,1.0,1.0,1.0,1.0/
C END OF PROBLEM-DEPENDENT DATA
-----------------
EIGENVALUE SOLUTION BY JACOBI METHOD

GIVEN MATRIX
  0.300000E+01   0.200000E+01   0.100000E+01
  0.200000E+01   0.200000E+01   0.100000E+01
  0.100000E+01   0.100000E+01   0.100000E+01

 EIGEN VALUES ARE
  0.504892E+01   0.643104E+00   0.307979E+00

 EIGEN VECTORS
     FIRST            SECOND          THIRD
   0.736978E+00  -0.591004E+00   0.327991E+00
   0.591007E+00   0.327977E+00  -0.736982E+00
   0.327985E+00   0.736984E+00   0.590999E+00
```


PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.39 The problem-dependent data to be used in Program 9.F and results are given.

```
C FOLLOWING 3 LINES CONTAIN PROBLEM-DEPENDENT DATA
      DIMENSION D(4,4),E(4,4)
      DATA N,ITMAX,EPS/4,200,1.0E-05/
      DATA D/4.,-2.,6.,4.,-2.,2.,-1.,3.,6.,-1.,22.,13.,4.,3.,13.,46./
C END OF PROBLEM-DEPENDENT DATA
```

```
      EIGENVALUE SOLUTION BY JACOBI METHOD

      GIVEN MATRIX
        0.400000E+01   -0.200000E+01    0.600000E+01
        0.400000E+01
       -0.200000E+01    0.200000E+01   -0.100000E+01
        0.300000E+01
        0.600000E+01   -0.100000E+01    0.220000E+02
        0.130000E+02
        0.400000E+01    0.300000E+01    0.130000E+02
        0.460000E+02

        EIGEN VALUES ARE
         0.525424E+02    0.178109E+02    0.346931E+01    0.177413E+00

        EIGEN VECTORS
            FIRST            SECOND           THIRD
          0.123182E+00    -0.274316E+00    0.718788E+00
          0.626834E+00     0.407039E-01    0.167114E+00
         -0.614583E+00     0.769873E+00    0.407591E+00
         -0.851753E+00    -0.316805E+00   -0.895647E-01
          0.903902E+00     0.413933E+00    0.725754E-01
         -0.797051E-01
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.40 The main program which calls DECOMP and results are given.

```
C ===============================================================
C
C PROGRAM
C MAIN PROGRAM WHICH CALLS DECOMP
C
C ===============================================================
      DIMENSION A(4,4),U(4,4)
      DATA A/4.0,-2.0,6.0,4.0,-2.0,2.0,-1.0,3.0,6.0,-1.0,22.0,13.0,
     2 4.0,3.0,13.0,46.0/
      N=4
      CALL DECOMP (A,U,N)
      WRITE (17,10)
10    FORMAT (/,25H UPPER TRIANGULAR MATRIX:,/)
      DO 30 I=1,N
      WRITE (17,20) (U(I,J),J=1,N)
20    FORMAT (3E15.8)
30    CONTINUE
      STOP
      END
-------------------------
UPPER TRIANGULAR MATRIX:

0.20000000E+01-0.10000000E+01 0.30000000E+01
0.20000000E+01
0.00000000E+00 0.10000000E+01 0.20000000E+01
0.00000000E+00
0.00000000E+00 0.00000000E+00 0.30000000E+01
0.23333333E+01
0.00000000E+00 0.00000000E+00 0.00000000E+00
0.60461192E+01
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.41 From Eq. (7.84), $u_{11} = \sqrt{5} = 2.236068$, $u_{12} = \frac{a_{12}}{u_{11}} = \frac{-1}{u_{11}} = -0.44721359$,

$$u_{13} = \frac{a_{13}}{u_{11}} = \frac{1}{u_{11}} = 0.44721359$$

$$u_{22} = \left(a_{22} - u_{12}^2\right)^{1/2} = 2.408319, \quad u_{23} = \frac{1}{u_{22}}\left(a_{23} - u_{12}u_{13}\right) = -1.5778641$$

$$u_{33} = \left(a_{33} - u_{13}^2 - u_{23}^2\right)^{1/2} = 0.55708611$$

$$[U] = \begin{bmatrix} 2.236068 & -0.44721359 & 0.44721359 \\ 0 & 2.408319 & -1.5778641 \\ 0 & 0 & 0.55708611 \end{bmatrix}$$

$$\alpha_{11} = \frac{1}{u_{11}} = 0.44721359 \qquad \alpha_{12} = -\frac{1}{u_{11}}\left(u_{12}\alpha_{22}\right) = 0.083045475$$

$$\alpha_{22} = \frac{1}{u_{22}} = 0.41522738 \qquad \alpha_{23} = -\frac{1}{u_{22}}\left(u_{23}\alpha_{33}\right) = 1.1760702$$

$$\alpha_{33} = \frac{1}{u_{33}} = 1.7950547 \qquad \alpha_{13} = -\frac{1}{u_{11}}\left(u_{12}\alpha_{23} + u_{13}\alpha_{33}\right) = -0.12379687$$

$$[U]^{-1} = [\alpha_{ij}] = \begin{bmatrix} 0.44721359 & 0.083045475 & -0.12379687 \\ 0 & 0.41522738 & 1.1760702 \\ 0 & 0 & 1.7950547 \end{bmatrix}$$

7.42 From Eqs. (7.84) and (7.86),

$$u_{11} = \sqrt{2} = 1.4142135, \quad u_{12} = \frac{a_{12}}{u_{11}} = \frac{5}{u_{11}} = 3.5355339, \quad u_{13} = \frac{a_{13}}{u_{11}} = 5.6568542$$

$$u_{22} = \left(a_{22} - u_{12}^2\right)^{1/2} = 1.8708287, \quad u_{23} = \frac{1}{u_{22}}\left(a_{23} - u_{12}u_{13}\right) = 4.2761798$$

$$u_{33} = \left(a_{33} - u_{13}^2 - u_{23}^2\right)^{1/2} = 1.9272485$$

$$[U] = \begin{bmatrix} 1.4142135 & 3.5355339 & 5.6568542 \\ 0 & 1.8708287 & 4.2761798 \\ 0 & 0 & 1.9272485 \end{bmatrix}$$

$$\alpha_{11} = \frac{1}{u_{11}} = 0.70710677 \qquad \alpha_{12} = -\frac{1}{u_{11}}\left(u_{12}\alpha_{22}\right) = -1.3363062$$

$$\alpha_{22} = \frac{1}{u_{22}} = 0.53452247 \qquad \alpha_{23} = -\frac{1}{u_{22}}\left(u_{23}\alpha_{33}\right) = -1.1859988$$

$$\alpha_{33} = \frac{1}{u_{33}} = 0.51887447 \qquad \alpha_{13} = -\frac{1}{u_{11}}\left(u_{12}\alpha_{23} + u_{13}\alpha_{33}\right) = 0.88949931$$

$$[U]^{-1} = [\alpha_{ij}] = \begin{bmatrix} 0.70710677 & -1.3363062 & 0.88949931 \\ 0 & 0.53452247 & -1.1859988 \\ 0 & 0 & 0.51887447 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.43 $[k] = k \begin{bmatrix} 4 & -2 & 0 & 0 \\ -2 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$, $[m] = m \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Eq. (7.84) gives, for $[A] = [k]$,

$$[U] = \begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 1.41421 & -0.707107 & 0 \\ 0 & 0 & 1.22474 & -0.816497 \\ 0 & 0 & 0 & 0.577350 \end{bmatrix} \sqrt{k}$$

Eq. (7.86) gives

$$[U]^{-1} = \begin{bmatrix} 0.5 & 0.35355338 & 0.20412414 & 0.28867510 \\ 0 & 0.70710677 & 0.40824828 & 0.57735020 \\ 0 & 0 & 0.81649655 & 1.1547004 \\ 0 & 0 & 0 & 1.7320507 \end{bmatrix} \frac{1}{\sqrt{k}}$$

standard eigenproblem is $\quad [D]\vec{Y} = \lambda \vec{Y}$

where

$$[D] = \left([U]^{-1}\right)^T [m] [U]^{-1}$$

$$= \frac{m}{k} \begin{bmatrix} 0.75 & 0.53033006 & 0.30618620 & 0.43301266 \\ 0.53033006 & 1.3749999 & 0.79385662 & 1.1226827 \\ 0.30618620 & 0.79385662 & 1.125 & 1.5909899 \\ 0.43301266 & 1.1226827 & 1.5909899 & 5.2499990 \end{bmatrix}$$

$\lambda = 1/\omega^2$

and $\vec{Y} = [U]\vec{X}$

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.44 From Eq. (7.84),

$$u_{11} = \sqrt{a_{11}} = \sqrt{16} = 4, \quad u_{12} = \frac{a_{12}}{u_{11}} = -\frac{20}{4} = -5, \quad u_{13} = \frac{a_{13}}{u_{11}} = -\frac{24}{4} = -6$$

$$u_{22} = \left(a_{22} - u_{12}^2\right)^{1/2} = (89 - 25)^{1/2} = 8$$

$$u_{23} = \frac{1}{u_{22}}\left(a_{23} - u_{12}u_{13}\right) = \frac{1}{8}\left(-50 - (-5)(-6)\right) = -10$$

$$u_{33} = \left(a_{33} - u_{13}^2 - u_{23}^2\right)^{1/2} = (280 - 36 - 100)^{1/2} = 12$$

$$[A] = [U]^T [U]$$

with

$$[U] = \begin{bmatrix} 4 & -5 & -6 \\ 0 & 8 & -10 \\ 0 & 0 & 12 \end{bmatrix}$$

PRELIMINARY VERSION 12/14/2010

Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.45

```
% Ex7_45.m
>> A = [3 -2 0; -2 5 -3; 0 -1 1]

A =

     3    -2     0
    -2     5    -3
     0    -1     1

>> [V, D] = eig(A)

V =

   -0.4765   -0.8962    0.4154
    0.8656   -0.3444    0.5950
   -0.1537    0.2797    0.6880


D =

    6.6334         0         0
         0    2.2315         0
         0         0    0.1351
```

7.46

```
% Ex7_46.m
>> A = [-5 2 1; 1 -9 -1; 2 -1 7]

A =

    -5     2     1
     1    -9    -1
     2    -1     7

>> [V, D] = eig(A)

V =

   -0.0723   -0.9572    0.4172
    0.0570   -0.2514   -0.9027
   -0.9958    0.1431   -0.1048


D =

    7.2024         0         0
         0   -4.6241         0
         0         0   -9.5783
```

PRELIMINARY VERSION 12/14/2010

لجنة الميكانيك   الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.47

```
% Results of Ex7_47
>> program9
Eigenvalue solution by Jacobi Method

Given matrix
2.50000000e+000     -1.00000000e+000     0.00000000e+000
-1.00000000e+000     5.00000000e+000    -1.41421356e+000
0.00000000e+000     -1.41421356e+000     1.00000000e+001

Eigen values are
1.03806779e+001      5.00000000e+000     2.11932209e+000

Eigen vactors are
   First               Second               Third
3.29649826e-002      3.59210604e-001     9.32674140e-001
-2.59786425e-001    -8.98026512e-001     3.55048443e-001
9.65103271e-001     -2.54000247e-001     6.37146104e-002
```

7.48

```
% Results of Ex7_48
>> program10
Solution of eigenvalue problem by
matrix iteration method

Natural frequencies:

    4.450417e-001     1.246977e+000     1.801938e+000

Mode shapes (Columnwise):

    1.000000e+000     1.000000e+000     1.000000e+000
    8.019379e-001    -5.549503e-001    -2.246941e+000
    4.450421e-001    -1.246987e+000     1.801867e+000
```

7.49

```
% Results of Ex7_49
>> program11
Upper triangular matrix [U]:

 2.000000e+000    -1.000000e+000     0.000000e+000
 0.000000e+000     1.414214e+000    -7.071068e-001
 0.000000e+000     0.000000e+000     1.224745e+000
 0.000000e+000     0.000000e+000     0.000000e+000

Inverse of the upper triangular matrix:

 5.000000e-001     3.535534e-001     2.041241e-001
 0.000000e+000     7.071068e-001     4.082483e-001
 0.000000e+000     0.000000e+000     8.164966e-001
 0.000000e+000     0.000000e+000     0.000000e+000

Matrix [UMU]=[UTI][M][UI]:

 7.500000e-001     5.303301e-001     3.061862e-001
 5.303301e-001     1.375000e+000     7.938566e-001
 3.061862e-001     7.938566e-001     1.125000e+000
 4.330127e-001     1.122683e+000     1.590990e+000

Eigenvalues:

 6.231904e+000     1.431905e+000     5.000000e-001     3.361911e-001

Eigenvectors(Columnwise):

 4.804506e-001    -4.370337e-001     2.672613e-001    -8.209847e-002
 8.452583e-001    -4.162514e-001    -2.672612e-001     2.021007e-001
 1.303605e+000     2.067115e-001    -2.672612e-001    -4.318039e-001
 1.552770e+000     6.853100e-001     2.672613e-001     2.186930e-001
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.50

```
Results of Ex7_50
*********************

Please input N:
3

Please input matrix D row by row:
2 2 2
2 5 5
2 5 12


EIGENVALUE SOLUTION BY JACOBI METHOD

 GIVEN MATRIX
    2.000000    2.000000     2.000000
    2.000000    5.000000     5.000000
    2.000000    5.000000    12.000000

EIGEN VALUES ARE
     15.15868265      2.87890731       0.96241005

EIGEN VECTORS
           FIRST              SECOND              THIRD
      0.20164232         -0.49011982         0.84801117
      0.46419929         -0.71456420        -0.52337082
      0.86247284          0.49917989         0.08342689
```


PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

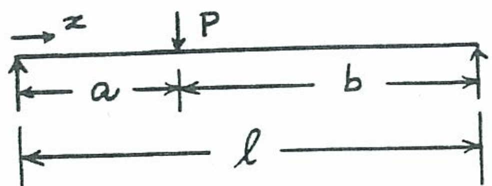
7.51

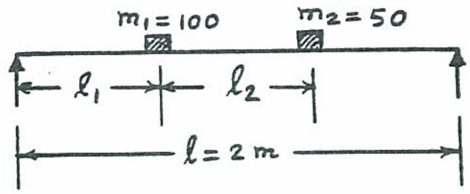
```
Results of Ex7_51
***************************************************************
Please input ND:
3
Please input BK matrix row by row:
10 -4 0
-4 6 -2
0 -2 2
Please input BM matrix row by row:
3 0 0
0 2 0
0 0 1

UPPER TRIANGULAR MATRIX [U]:

      3.16227766     -1.26491106      0.00000000
      0.00000000      2.09761770     -0.95346259
      0.00000000      0.00000000      1.04446594

INVERSE OF THE UPPER TRIANGULAR MATRIX, [UI],

      0.31622777      0.19069252      0.17407766
      0.00000000      0.47673129      0.43519414
      0.00000000      0.00000000      0.95742711

MATRIX [UMU] = [UTI][M][UI]:

      0.30000000      0.18090681      0.16514456
      0.18090681      0.56363636      0.51452725
      0.16514456      0.51452725      1.38636364


EIGENVALUES:

      1.67156962      0.38337151      0.19505887

EIGENVECTORS (COLUMNWISE):

      0.28923075     -0.23770397      0.16281604
      0.59330466     -0.12923308     -0.21898643
      0.84651373      0.42480537      0.14007703
```


PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.52





Basic relationship:

$$w(x) = \begin{cases} \dfrac{P b x}{6EIl}(l^2 - b^2 - x^2); & 0 \le x \le a \qquad (E_1) \\ -\dfrac{Pa(l-x)}{6EIl}(a^2 + x^2 - 2lx); & a \le x \le l \qquad (E_2) \end{cases}$$

Deflection of mass m_1 due to load $m_1 g$:

Using $x = l_1$, $b = 2 - l_1$ and $l = 2$ in (E_1):

$$w_1' = \frac{(100 \times 9.81)(2 - l_1) l_1}{6EI(2)} \{4 - (2-l_1)^2 - l_1^2\} = \frac{981\, l_1^2 (2-l_1)^2}{6EI} \qquad (E_3)$$

Deflection of mass m_2 due to load $m_1 g$:

Using $x = l_1 + l_2$, $a = l_1$, $b = 2 - l_1$, $l = 2$ in (E_2):

$$w_2' = -\frac{(100 \times 9.81)\, l_1 (2 - l_1 - l_2)}{6EI(2)} \{l_1^2 + (l_1 + l_2)^2 - 2(2)(l_1 + l_2)\}$$

$$= -\frac{981\, l_1 (2 - l_1 - l_2)}{12EI}(2l_1^2 + l_2^2 + 2 l_1 l_2 - 4 l_1 - 4 l_2) \qquad (E_4)$$

Deflection of mass m_1 due to load $m_2 g$:

Using $x = l_1$, $l = 2$, $b = (2 - l_1 - l_2)$ in (E_1):

$$w_1'' = \frac{(50 \times 9.81)(2 - l_1 - l_2)\, l_1}{6EI(2)} \{4 - (2 - l_1 - l_2)^2 - l_1^2\}$$

$$= \frac{490.5\, l_1 (2 - l_1 - l_2)(-2l_1^2 - l_2^2 + 4l_1 + 4l_2 - 2l_1 l_2)}{12EI} \qquad (E_5)$$

لجنة الميكانيك الإتجاه الإسلامي

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

Deflection of mass m_2 due to load $m_2 g$:

Using $x = l_1 + l_2$, $l = 2$ and $b = 2 - l_1 - l_2$ in (E_1):

$$w_2'' = \frac{(50 \times 9.81)(2 - l_1 - l_2)(l_1 + l_2)}{6EI(2)} \{4 - (2 - l_1 - l_2)^2 - (l_1 + l_2)^2\}$$

$$= \frac{490.5\ (l_1 + l_2)(2 - l_1 - l_2)(-2l_1^2 - 2l_2^2 + 4l_1 + 4l_2 - 4l_1 l_2)}{12EI} \qquad (E_6)$$

Total deflection of masses m_1 and m_2 are:

$$w_1 = w_1' + w_1'' = \frac{981\ l_1^2 (2 - l_1)^2}{6EI} + \frac{490.5\, l_1\ (2 - l_1 - l_2)(-2l_1^2 - l_2^2 + 4l_1 + 4l_2 - 2l_1 l_2)}{12EI} \qquad (E_7)$$

$$w_2 = w_2' + w_2'' = \frac{-981\ l_1 (2 - l_1 - l_2)(2l_1^2 + l_2^2 + 2l_1 l_2 - 4l_1 - 4l_2)}{12EI} + \frac{490.5\ (l_1 + l_2)(2 - l_1 - l_2)(-2l_1^2 - 2l_2^2 + 4l_1 + 4l_2 - 4l_1 l_2)}{12EI} \qquad (E_8)$$

Fundamental natural frequency is given by

$$\omega = \left\{ \frac{g(m_1 w_1 + m_2 w_2)}{(m_1 w_1^2 + m_2 w_2^2)} \right\}^{\frac{1}{2}} = 3.1321 \left(\frac{2w_1 + w_2}{2w_1^2 + w_2^2} \right) \qquad (E_9)$$

To maximize ω, we can maximize ω^2.

Problem is: Find l_1 and l_2

to maximize $f = \left(\frac{2w_1 + w_2}{2w_1^2 + w_2^2} \right)$

where w_1 and w_2 are given by (E_7) and (E_8).

Problem can be solved as follows:

Treat f as a function of l_1 and l_2.

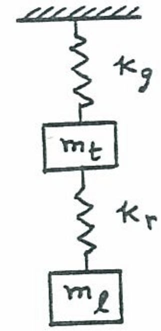
Set $\frac{\partial f}{\partial l_1} = 0$ and $\frac{\partial f}{\partial l_2} = 0$ $\qquad (E_{10})$

Solve Eqs. (E_{10}) for l_1 and l_2.

PRELIMINARY VERSION 12/14/2010
Mech.MuslimEngineer.Net

© 2011 Pearson Education, Inc., Upper Saddle River, NJ. All rights reserved. This publication is protected by Copyright and written permission should be obtained from the publisher prior to any prohibited reproduction, storage in a retrieval system, or transmission in any form or by any means, electronic, mechanical, photocopying, recording, or likewise. For information regarding permission(s), write to: Rights and Permissions Department, Pearson Education, Inc., Upper Saddle River, NJ 07458.

7.53 Stiffness of one girder (simply supported beam)



$$k_{g1} = \frac{48\,EI}{\ell^3} = \frac{48\,(30\times10^6)\left(\frac{1}{12}a^4\right)}{(30\times12)^3} = 2.572\,a^4$$

where a = width and depth of cross-section (inch) of girder.

$$k_g = 2\,k_{g1} = 5.144\,a^4 \quad \text{lb/in}$$

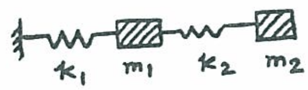
$$m_t = \text{mass of trolley} = \frac{40000}{386.4} = 103.5197 \ \text{lb-sec}^2/\text{in}$$

$$\text{stiffness of rope} = k_r = \frac{AE}{\ell} = \frac{\frac{\pi d^2}{4}(30\times10^6)}{(20\times12)} = 98175\,d^2$$

where d = diameter of the rope (inch).

$$m_\ell = \text{mass of lifted load} = \frac{10000}{386.4} = 25.8799 \ \text{lb-sec}^2/\text{in}$$

From section 5.3, the natural frequencies of a 2 d.o.f. system (shown in adjacent figure) are given by



$$\omega_{1,2}^2 = \frac{1}{2}\left\{\frac{(k_1+k_2)\,m_2 + k_2 m_1}{m_1 m_2}\right\} \mp \left[\left\{\frac{(k_1+k_2)\,m_2 + k_2 m_1}{m_1 m_2}\right\}^2 - 4\left\{\frac{(k_1+k_2)\,k_2 - k_2^2}{m_1 m_2}\right\}\right]^{1/2} \qquad (E_1)$$

Here $k_1 = k_g$, $k_2 = k_r$, $m_1 = m_t$, $m_2 = m_\ell$ and hence ω_1^2 and ω_2^2 can be expressed as functions of a and d:

$$\omega_1^2 = \omega_1^2(a,\,d) \quad ; \quad \omega_2^2 = \omega_2^2(a,\,d) \qquad (E_2)$$

Requirement is

$$\omega_1^2(a,\,d) > (157.08)^2 \quad ; \quad \omega_2^2(a,\,d) > (157.08)^2 \qquad (E_3)$$

since 1500 rpm = 157.08 rad/sec.

choose a and d such that the inequalities (E_3) are satisfied. A trial and error procedure can be used for this purpose.

PRELIMINARY VERSION 12/14/2010
لجنة الميكانيك الإتجاه الإسلامي