

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

2nd Civil Engineering

By:....

Eng : Ayman abdo

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Introduction to Deflection

Eng : Aymman abdo

Chapter One : Deflections

في الجزء ده سيتم بمشيئة الله دراسة التغير النهائي في شكل المنشأ بعد التحميل النهائي للمنشأ وذلك لمطابقة ما اذا كان الهبوط النهائي للمنشأ في **safe side**.

الطرق المستخدمة لحساب الهبوط في المنشآت :

- ❖ Double Integration Method (2 sections).
- ❖ Conjugate Beam Method (2 sections).
- ❖ Virtual Work Method (2 sections).

Chapter Two : Detergency

يتم دراسة الفرق بين المنشآت المحدده استاتيكا (determinate structures) والمنشآت الغير محدده استاتيكا (indeterminate structures) وكيفية تحديد عدد المعادلات والمجاهيل (1 sections) .

Chapter Three : Consistent Deformation

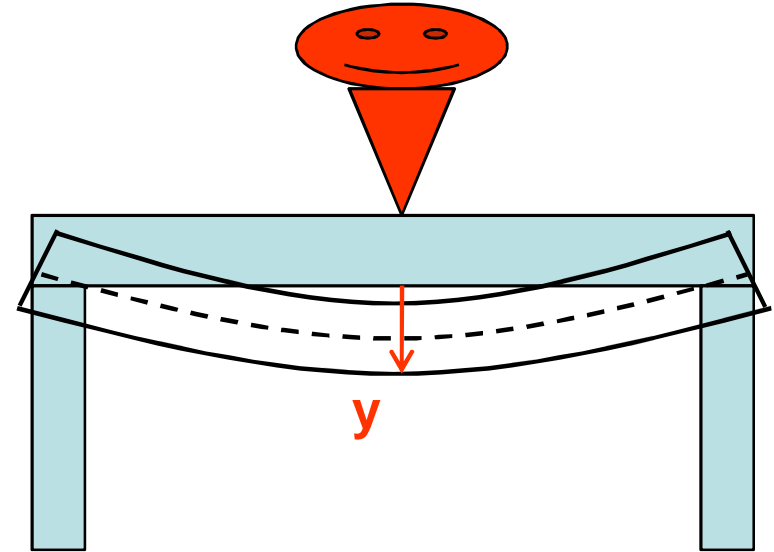
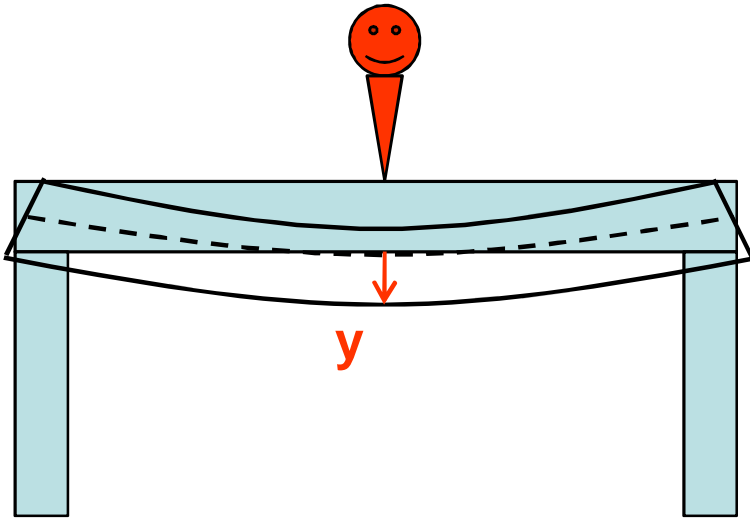
رسم الـ **internal force** للمنشآت الغير محدده استاتيكا (5 sections) .

Drawing without calculation (1 sections)

Deflection

يرتبط الهبوط في المنشآت بالترمين السابقين ارتباطا وثيقا حيث يعتمد اولا على (الاحمال ومدى تأثيرها على العزوم) ثانيا (خواص القطاع والمواد المستخدمة في الكمرة).

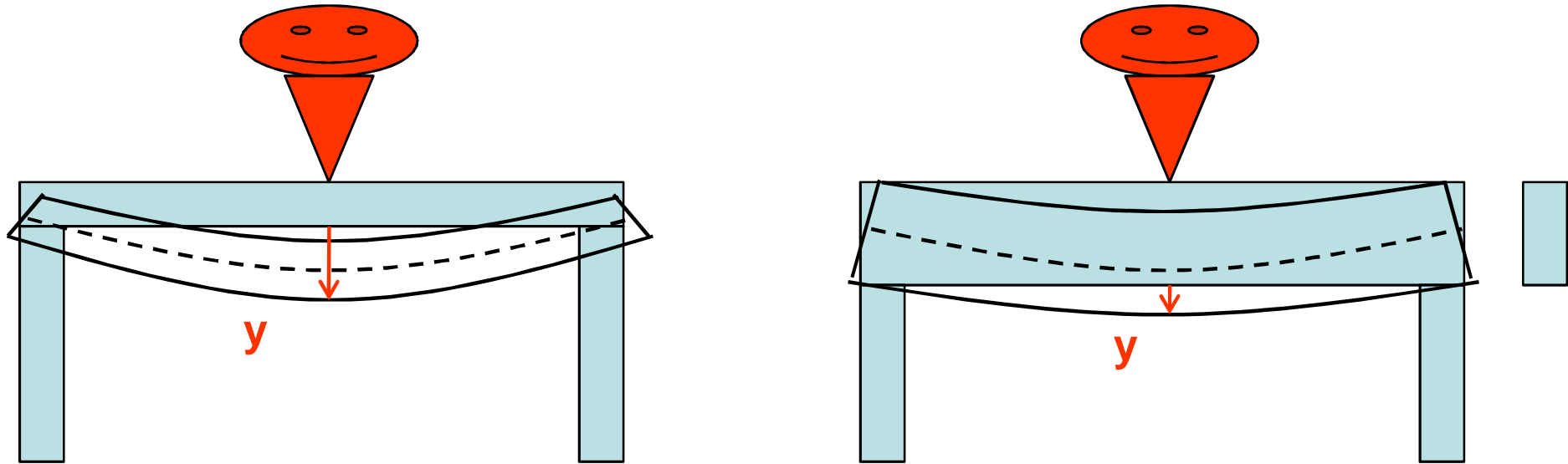
❖ تأثير الاحمال على الهبوط في المنشآت



الهبوط يزيد بزيادة القوى المؤثرة

❖ تأثير خواص القطاع على الهبوط في المنشآت

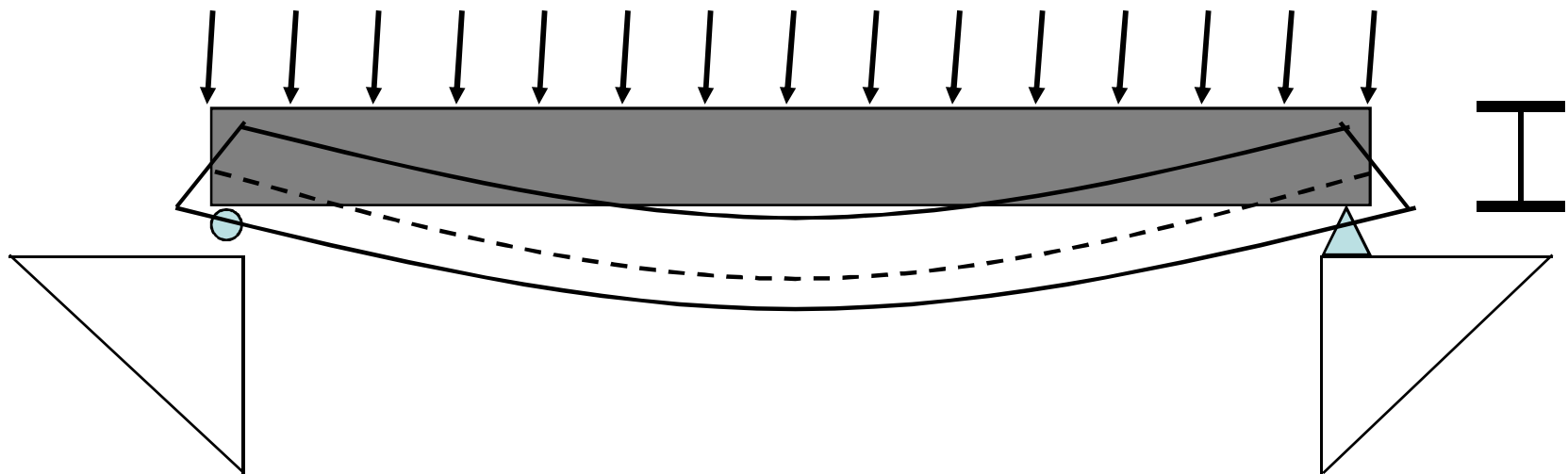
بنفس الاحمال ولكن مع تغيير وضع الكمره وبالتالي تغيير قيمة الـ inertia

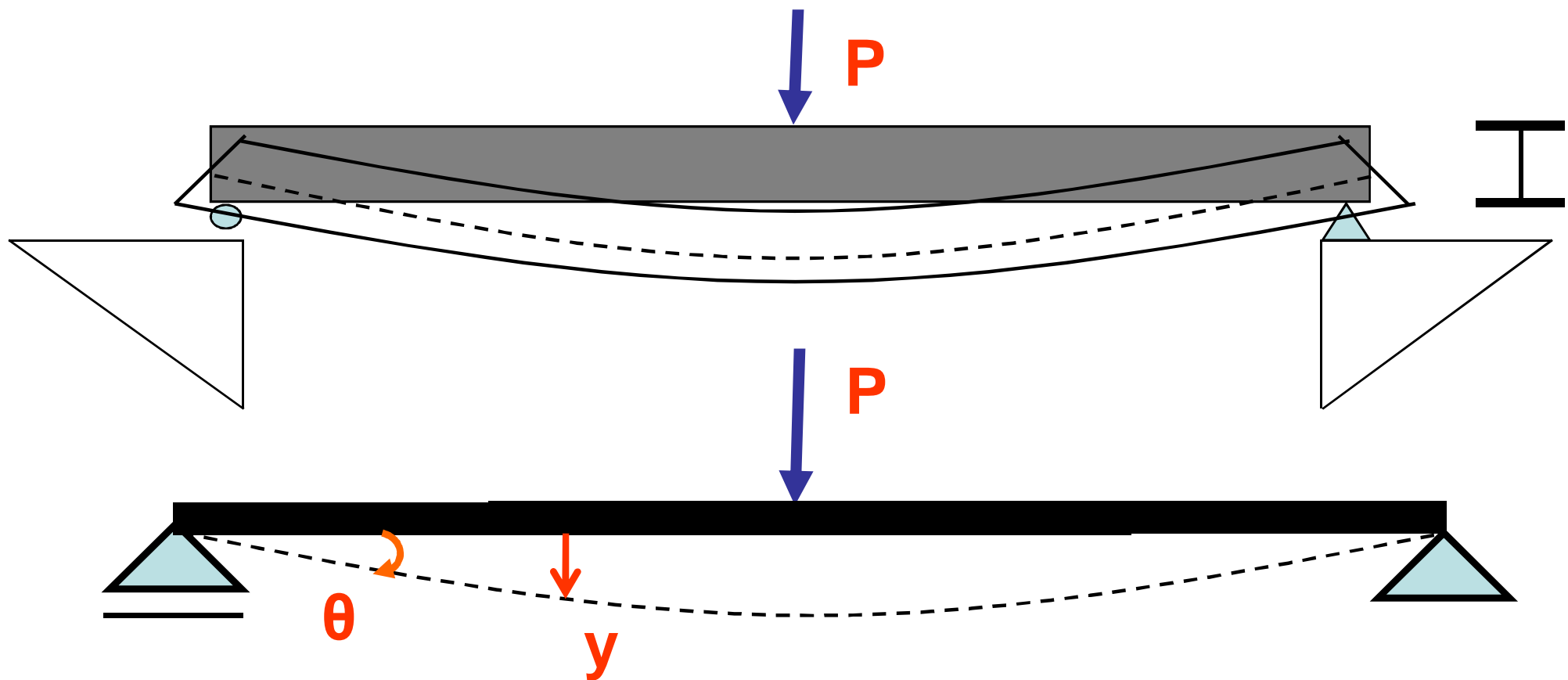


الهبوط يقل بزيادة الـ inertia

ايضا بالتاكيد تتغير قيمة الهبوط بتغيير نوع الماده
الـ modulus of elasticity (E)

Beam Girder





$y \gg \gg$ deflection at any point الهبوط

الهبوط يساوى صفر عند الـ Hinge, Roller and Fixed Support

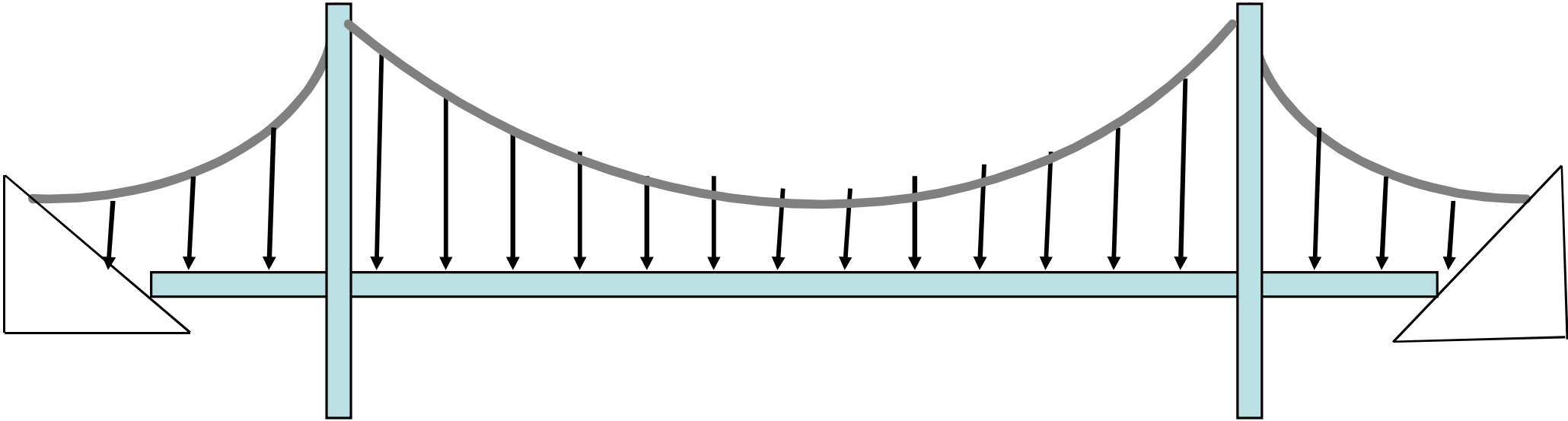
$\theta \gg \gg$ rotation or slope at any point الدوران

الدوران يساوى صفر عند الـ Fixed Support

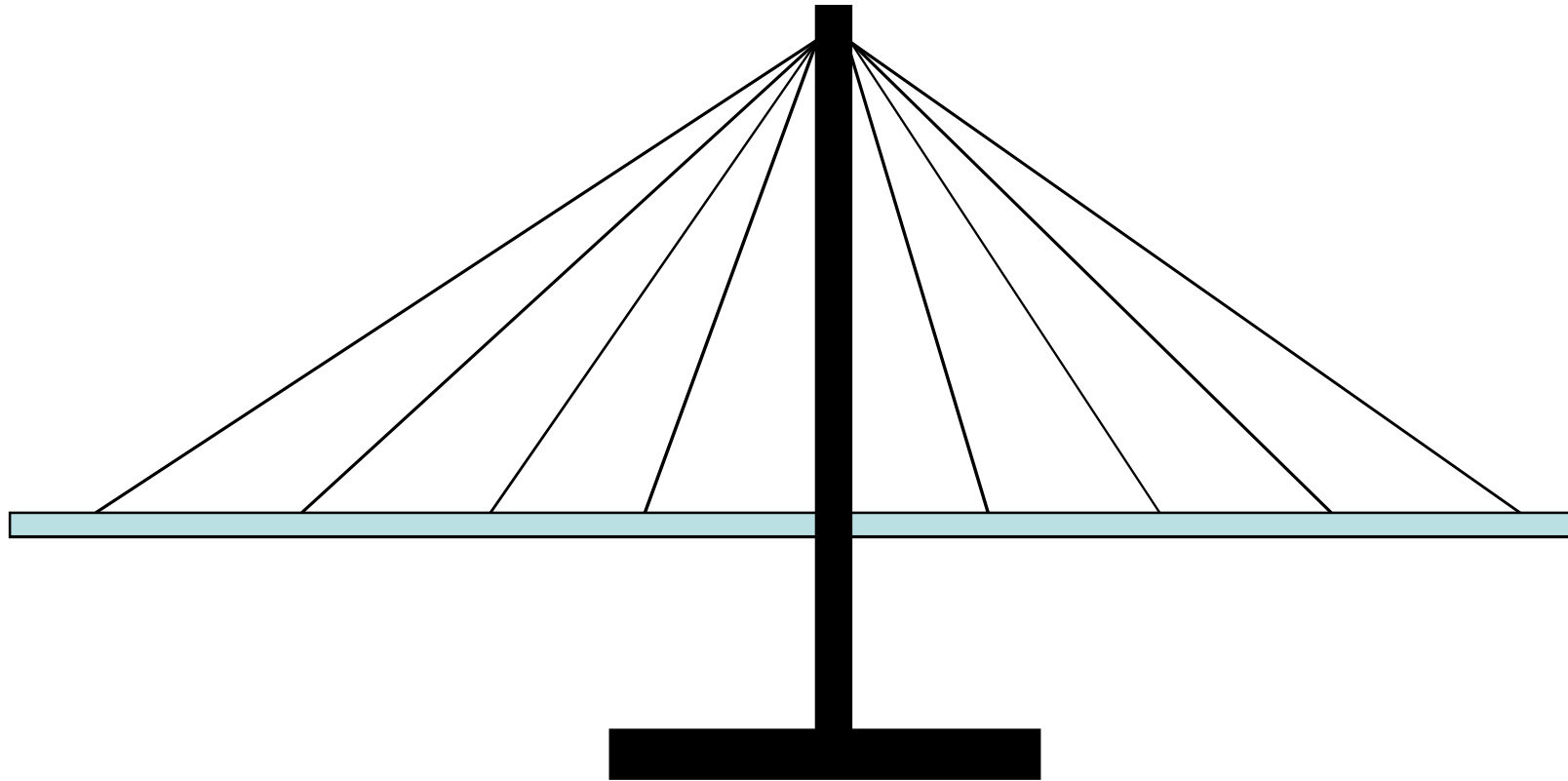
ملاحظة مهمة أن الـ θ = Rotation = تفاضل الـ Deflection $y' =$

بعض الطرق المستخدمة لتلاشى الهبوط الكبير الناتج عن المسافات
الواسعة بين الاعمده

Cable Suspended Structure



Cable Stayed Bridge



مراجعة سريعة لبعض المفاهيم السابق دراستها

Types of Load

1. Dead loads
2. Live loads
3. Dynamic loads (trains, equipment)
4. Wind loads
5. Earthquake loads
6. Thermal loads
7. Settlement loads

Dead Loads

1. weight of the structure itself

floors, beams, roofs, decks, beams/stringers, superstructure

2. loads that are “always there”

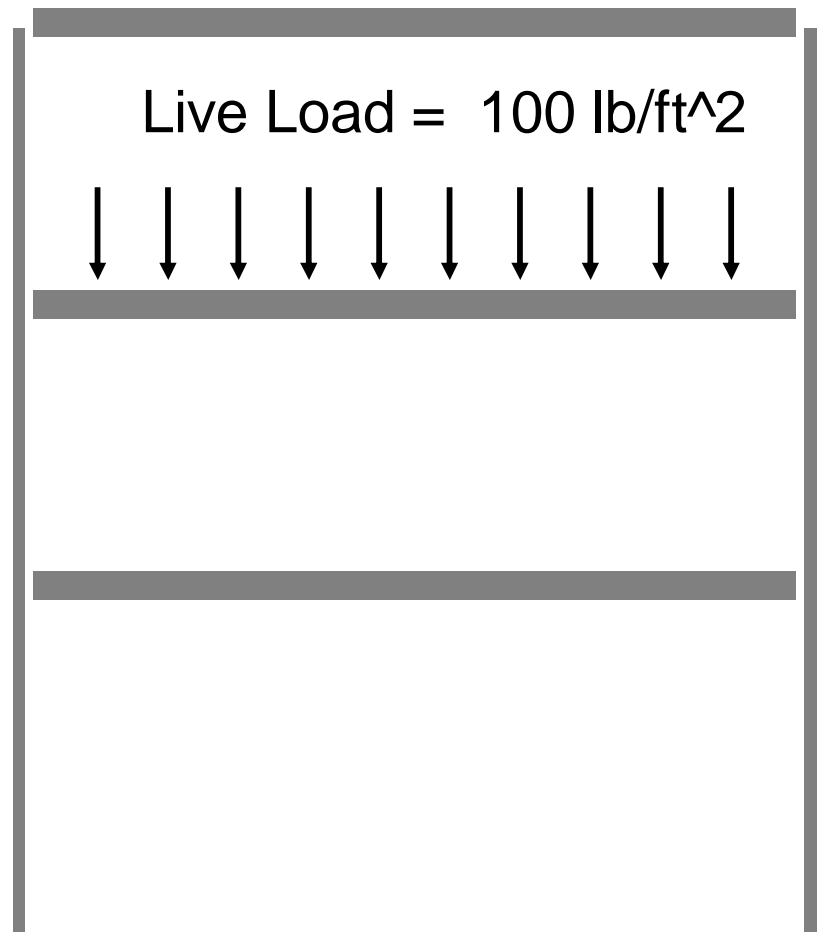
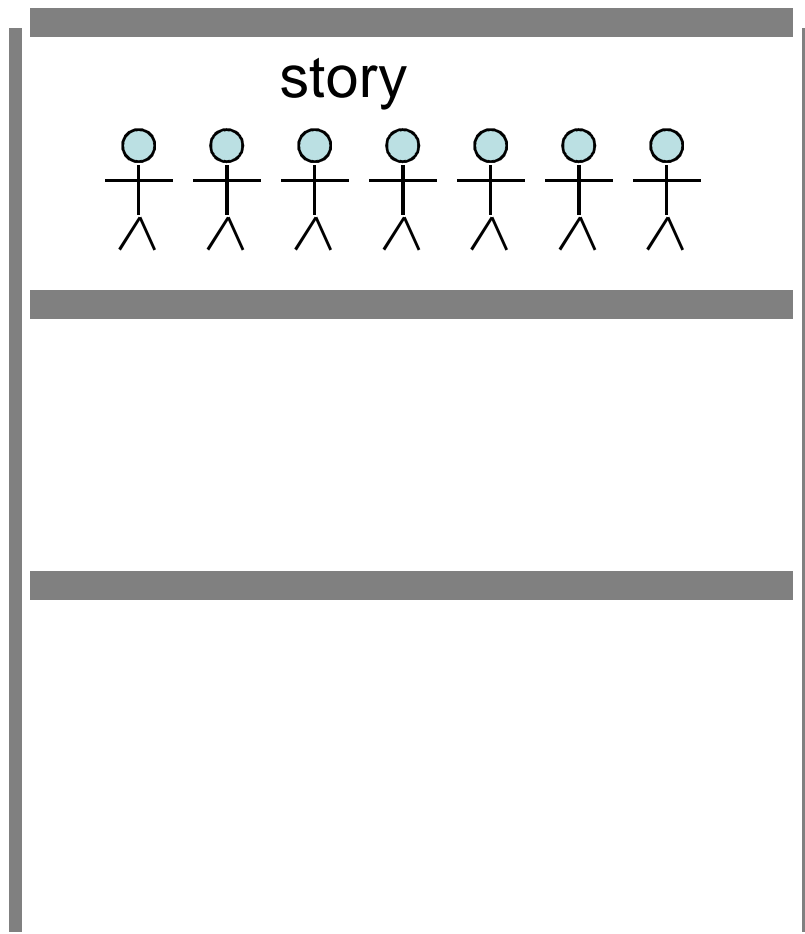


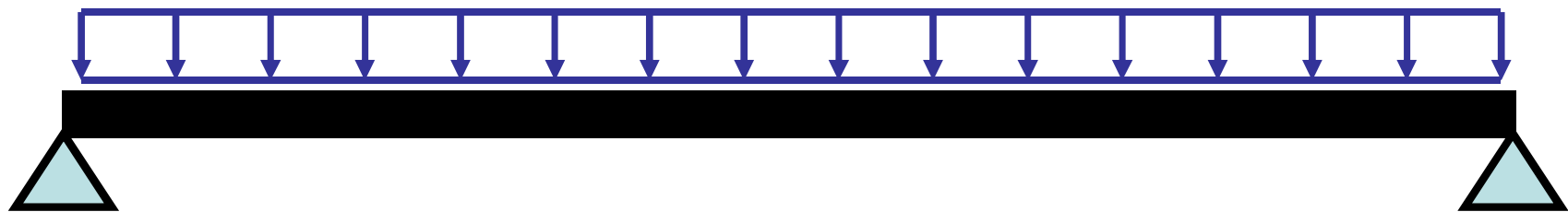
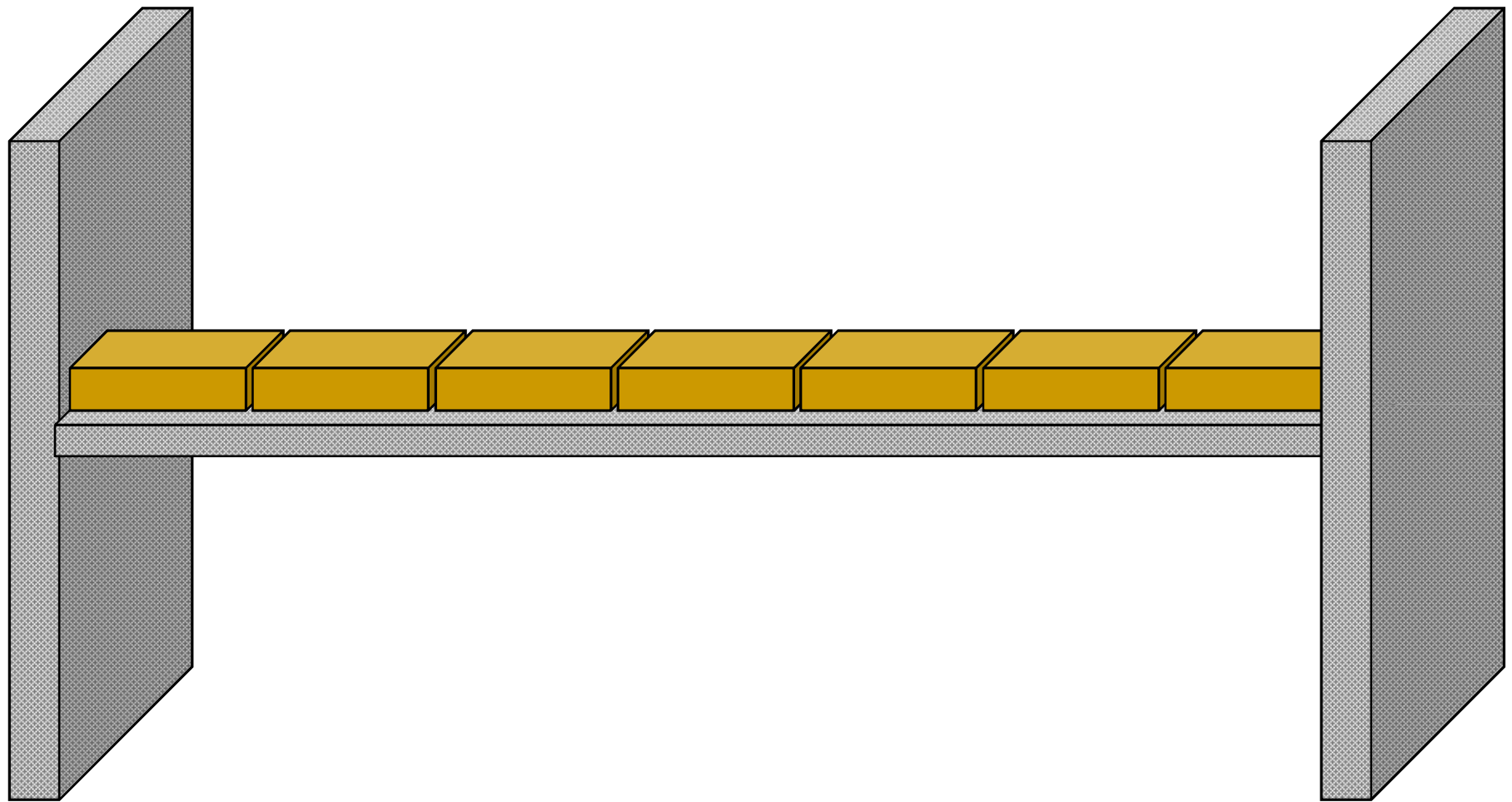
Live Loads

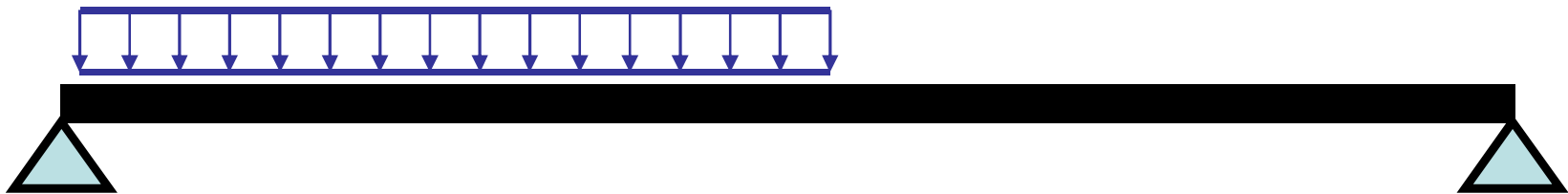
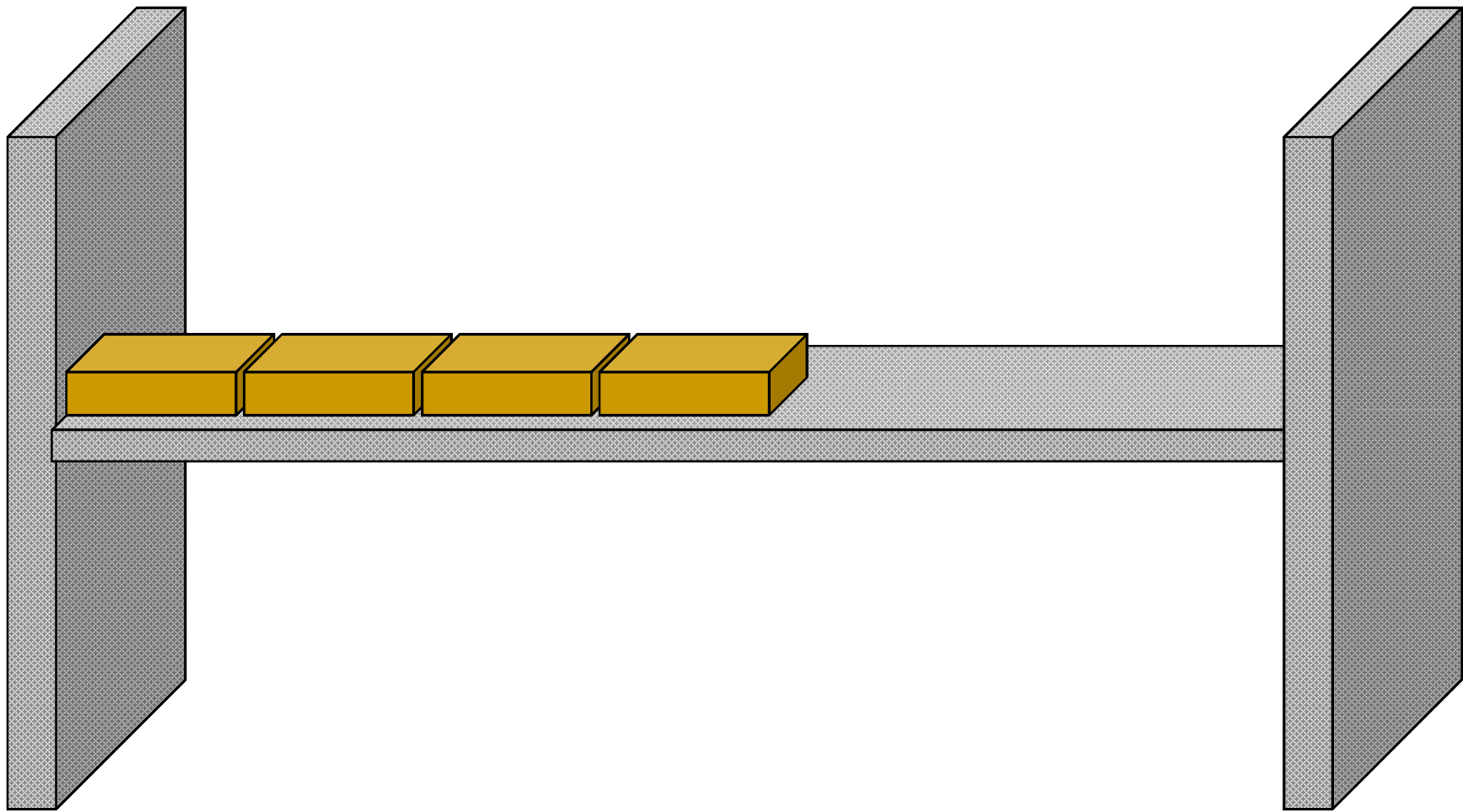
1. People, furniture, equipment
2. Loads that may move or change mass or weight
3. Minimum design loadings are usually specified in the building code

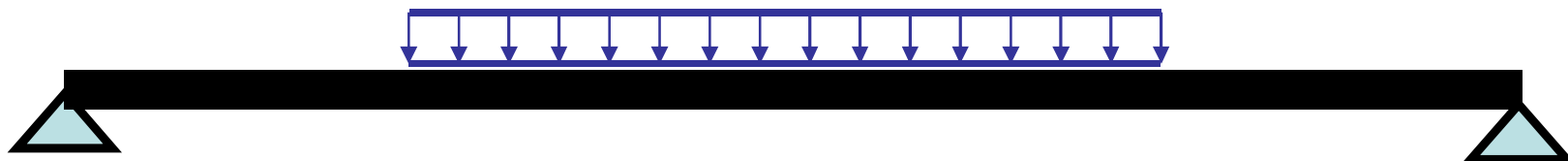
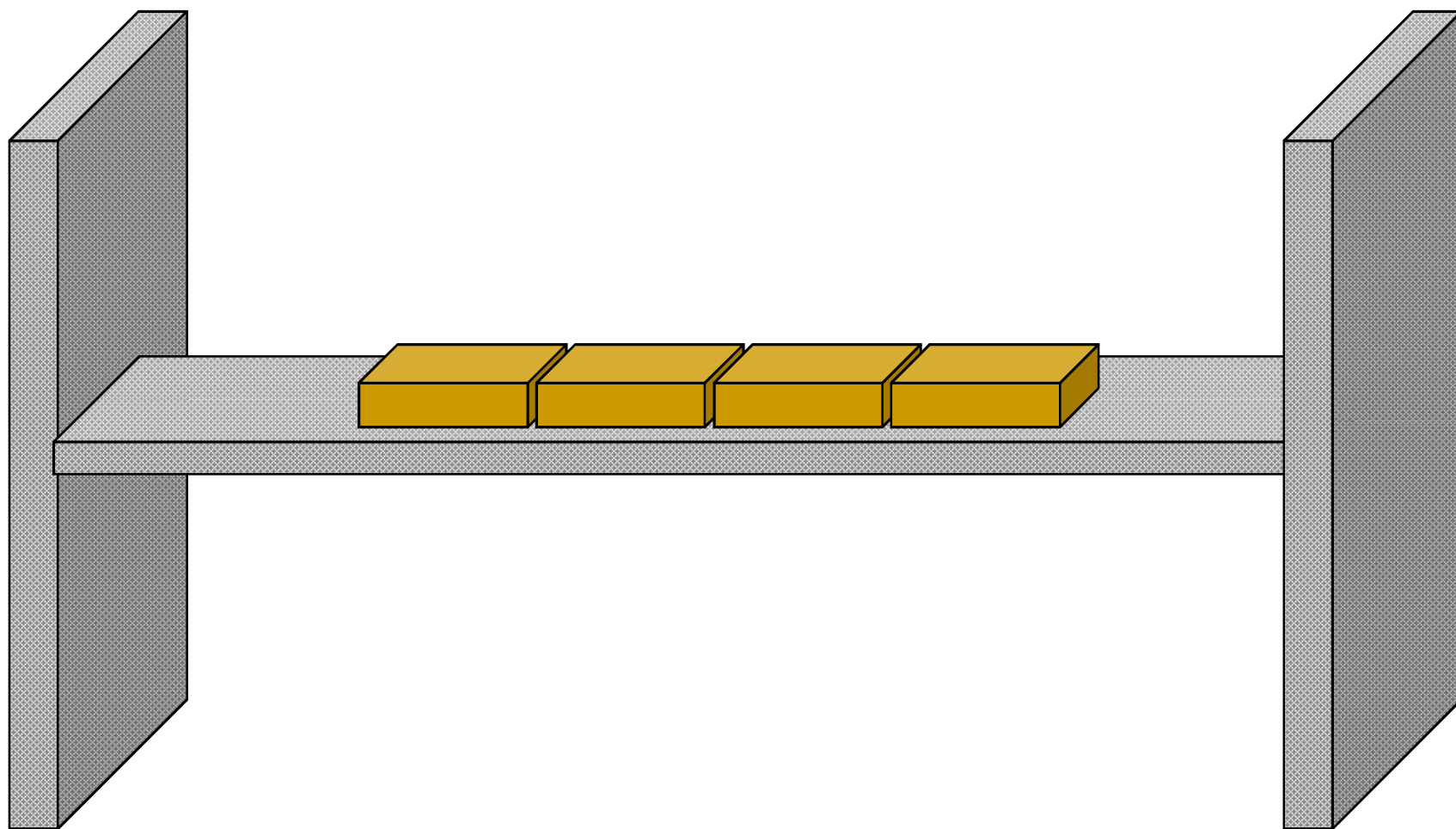


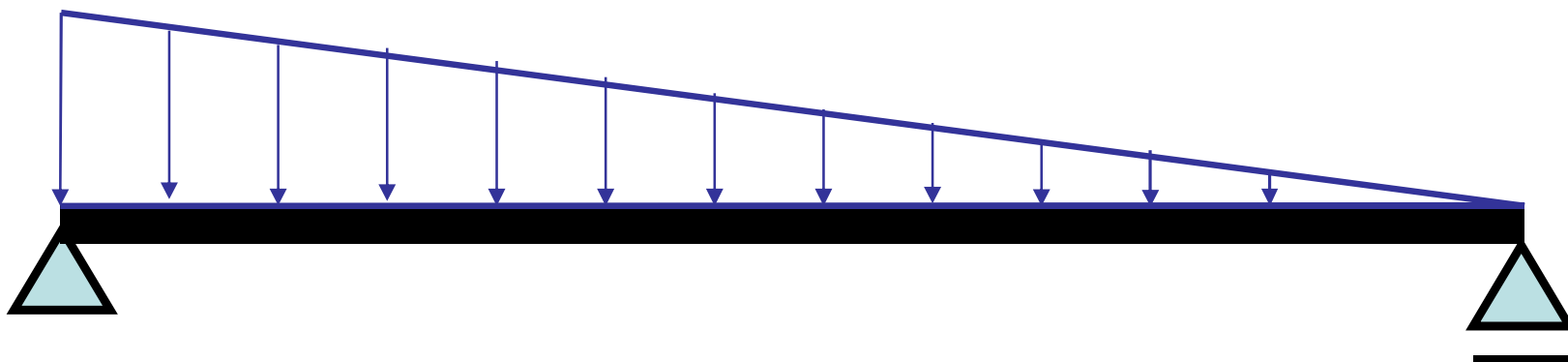
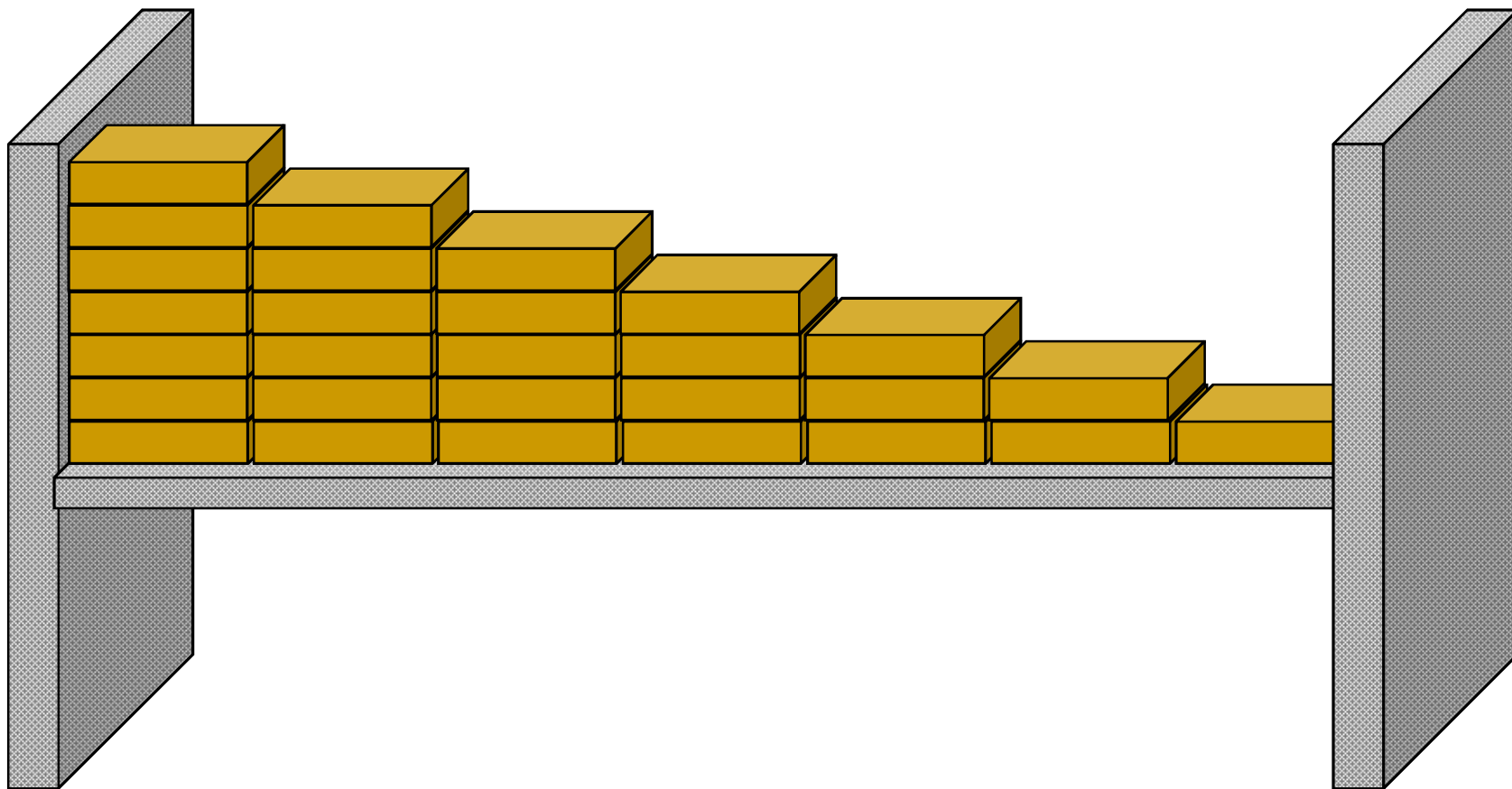
Load Example: Live Load

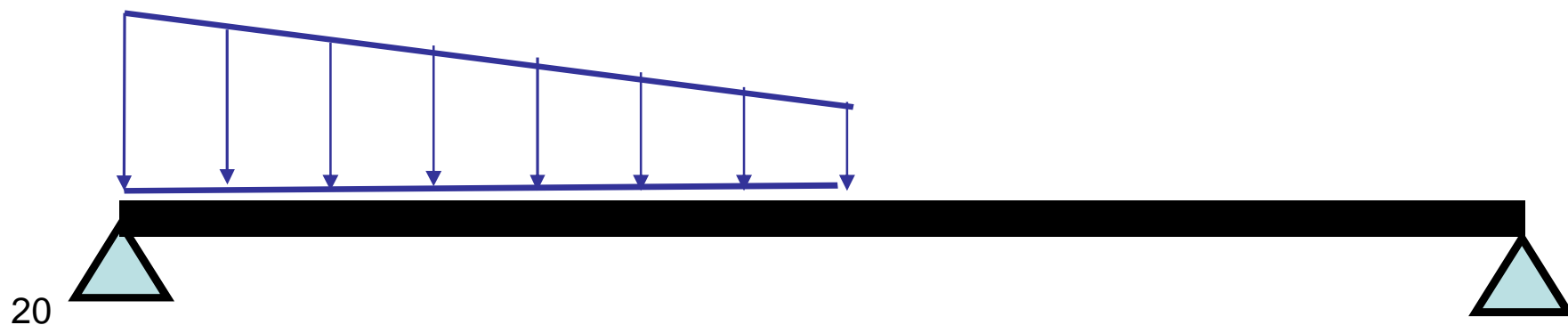
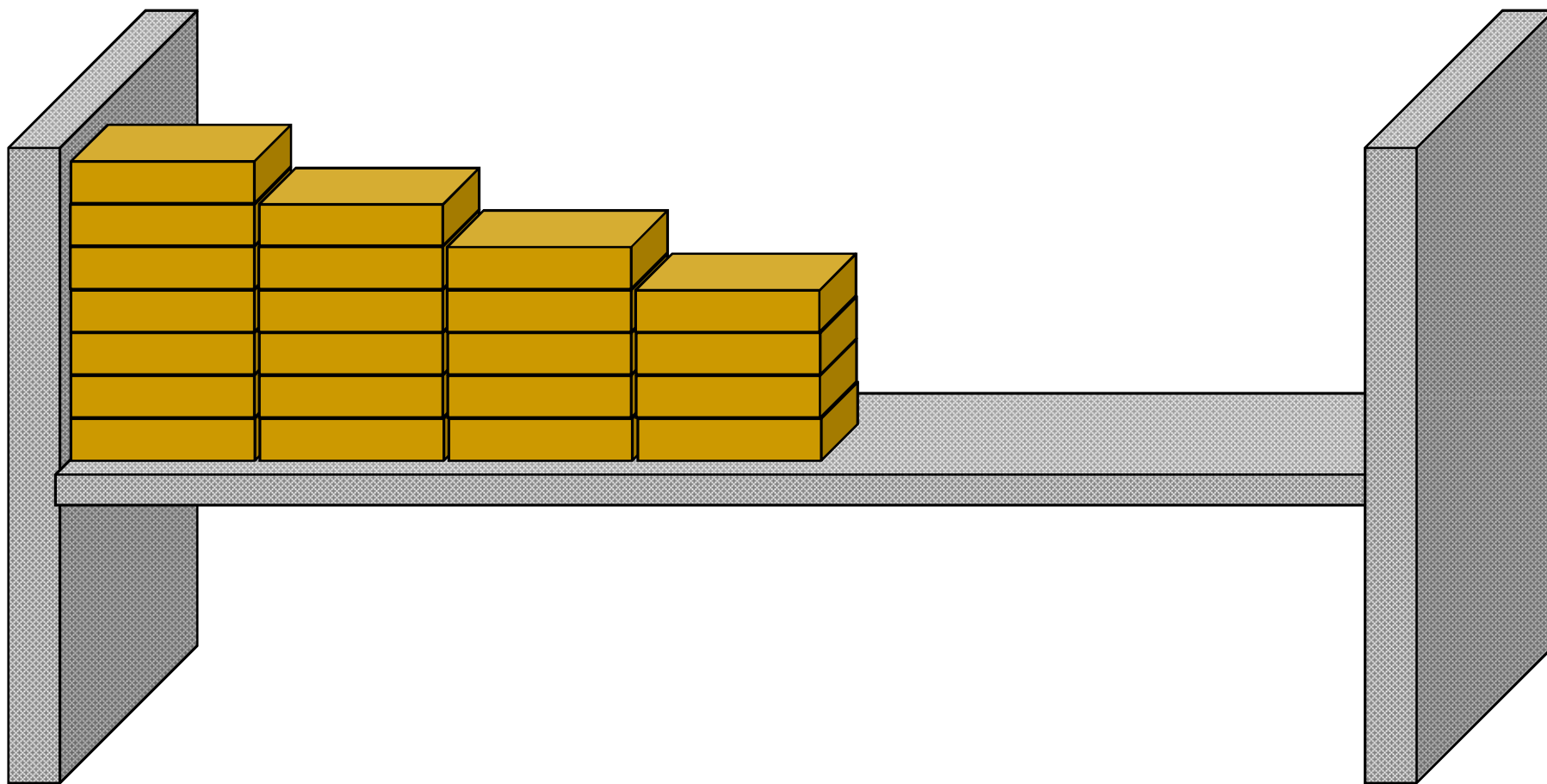


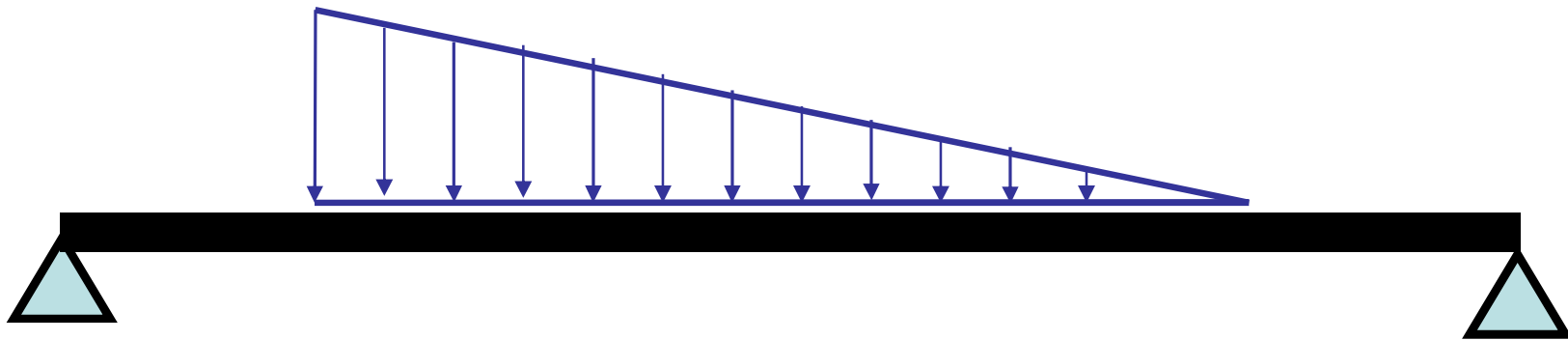
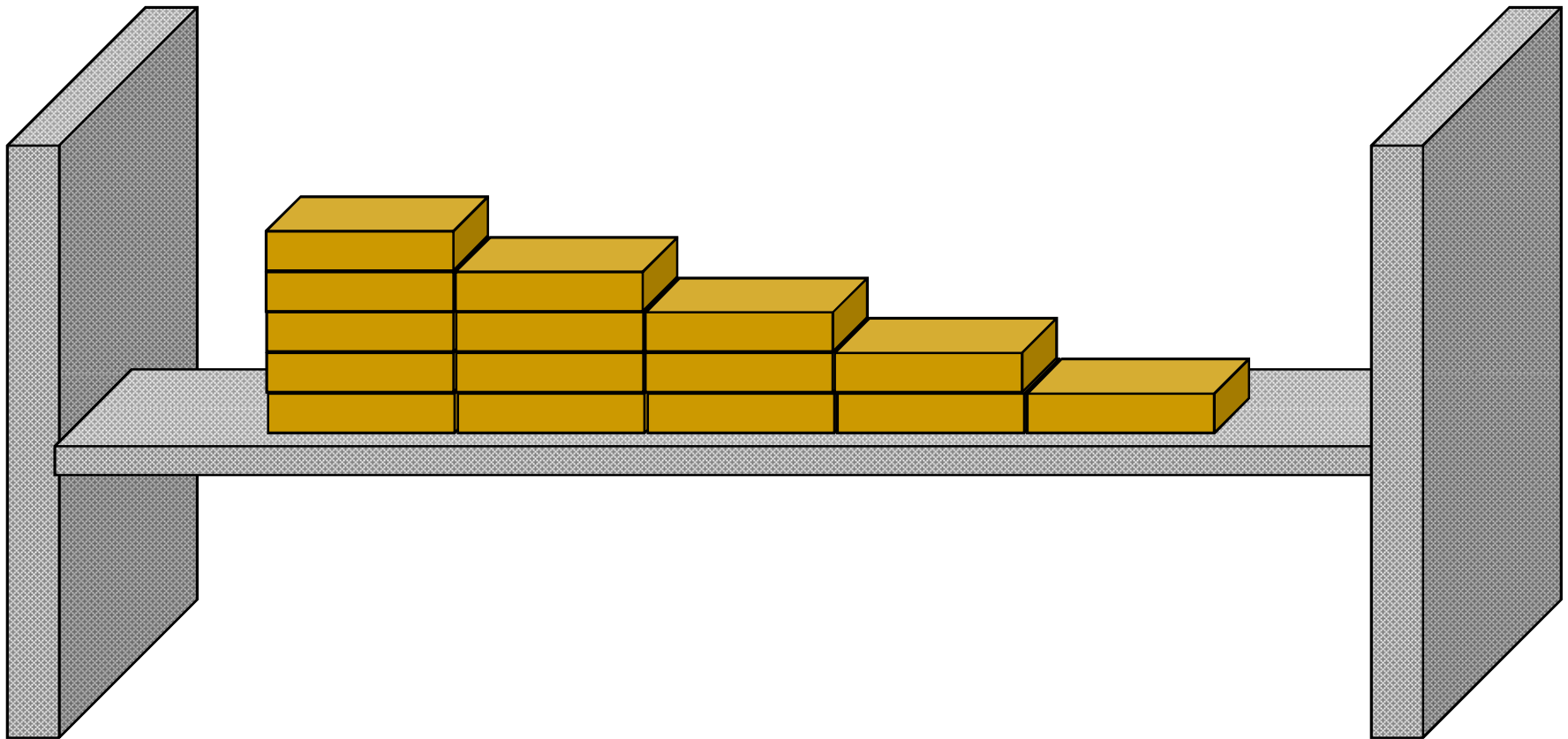


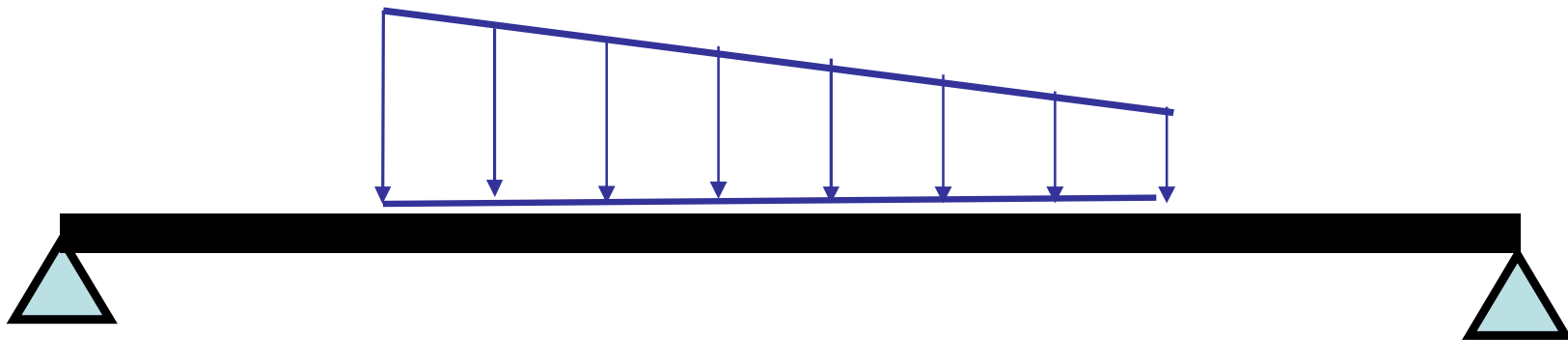
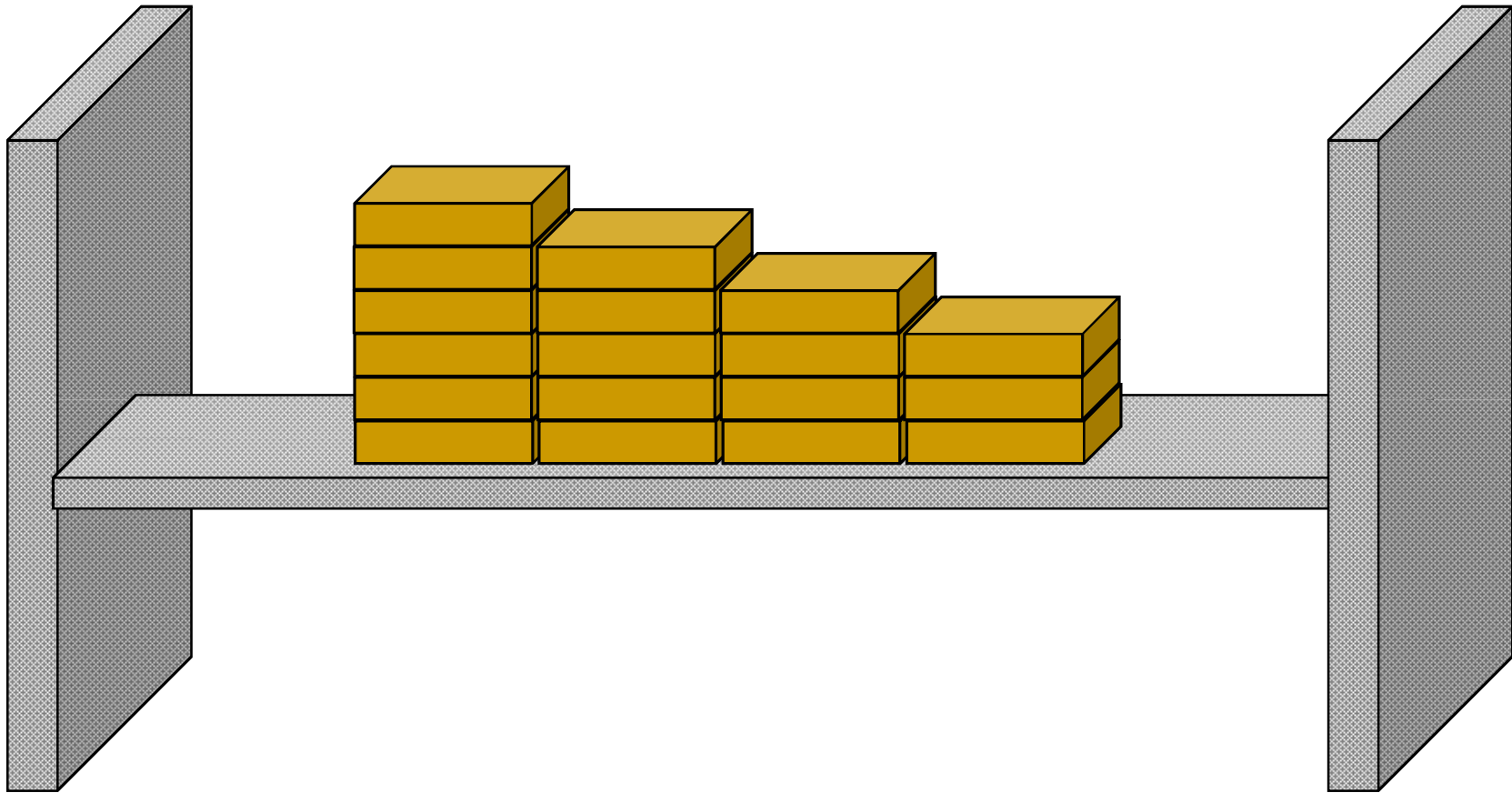




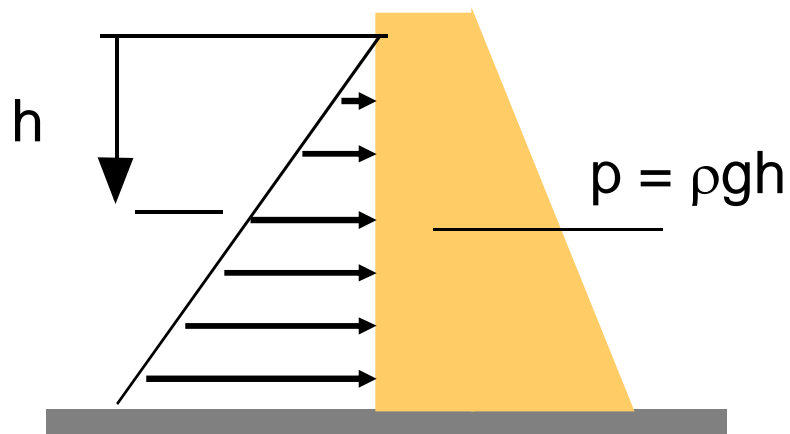
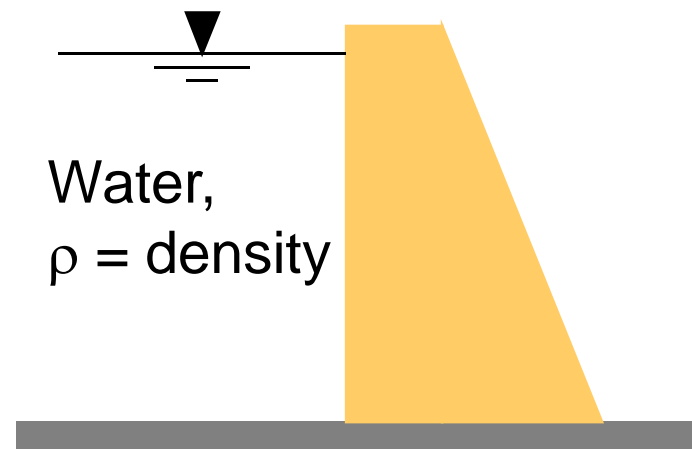








Load Example: Water in a dam

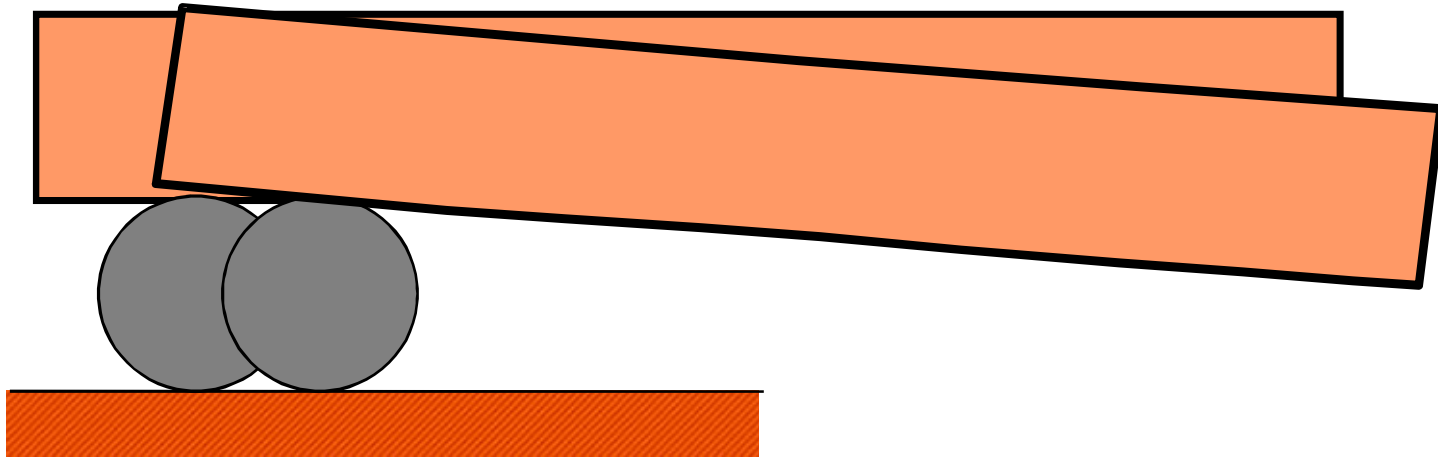


Types of Supports

ROLLER SUPPORT



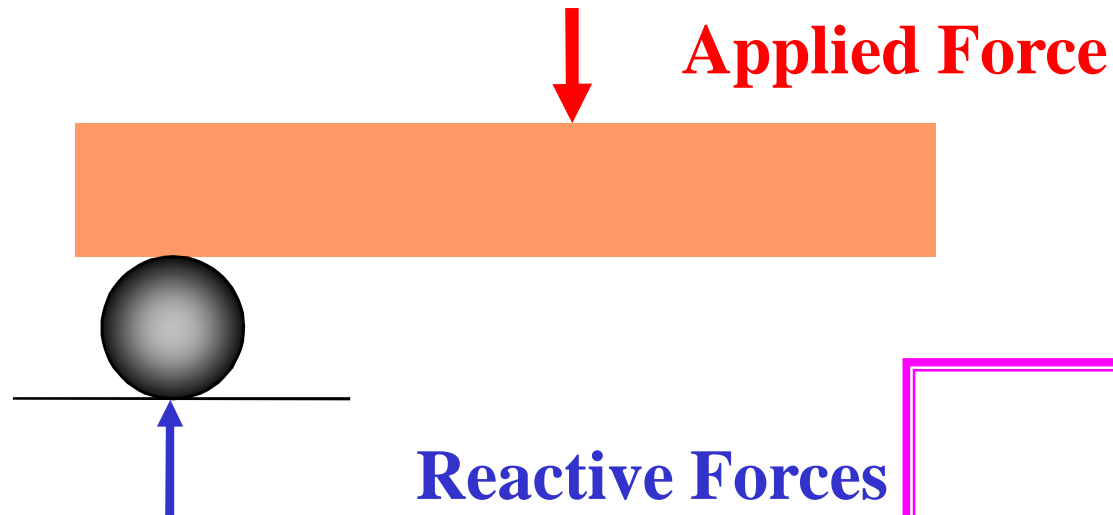
Roller Support



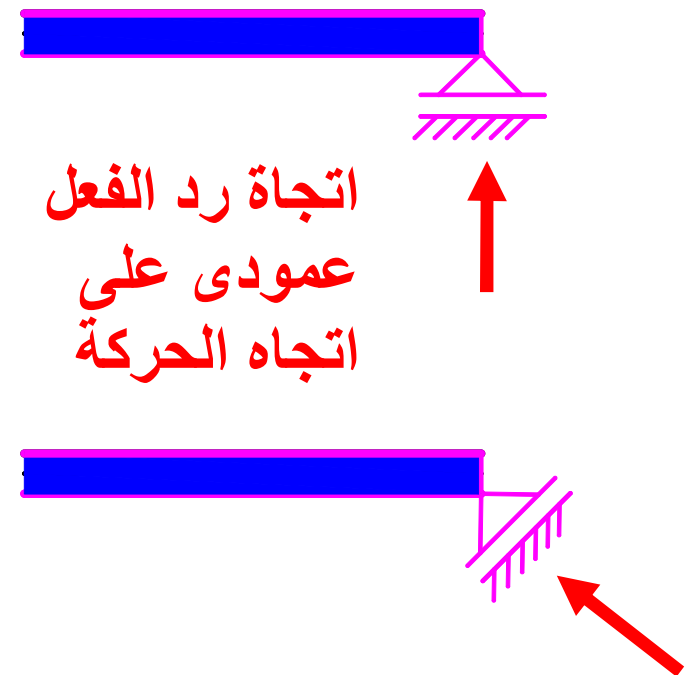
Roller Support

يسمح بالحركة في احد الاتجاهين والدوران

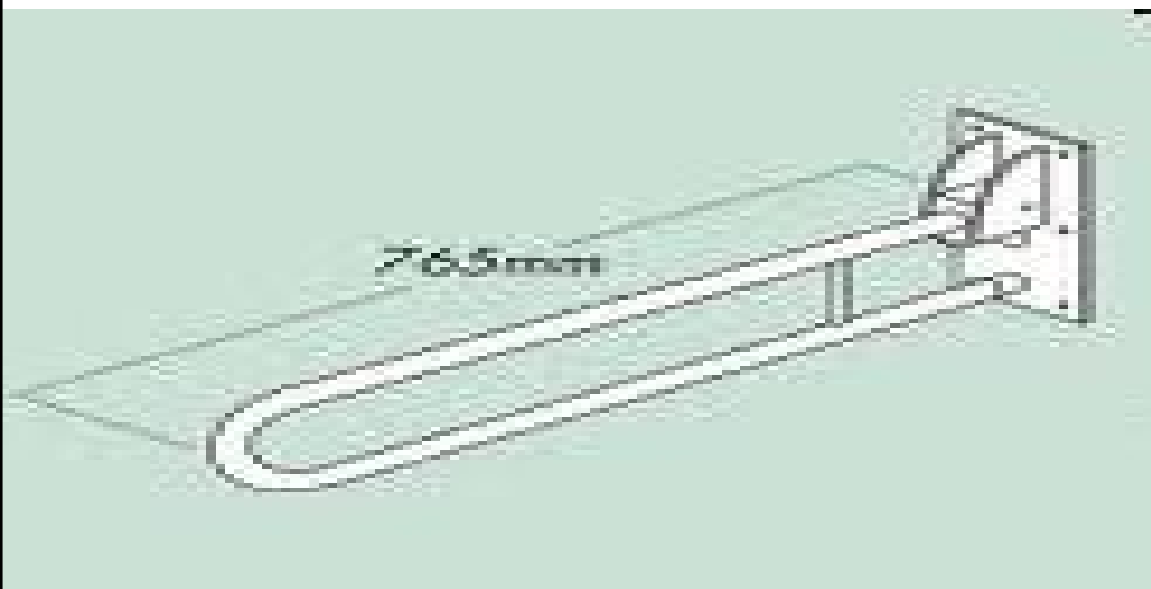
ROLLER SUPPORT



The Reactive Force must always be perpendicular to the surface of **Rolling**

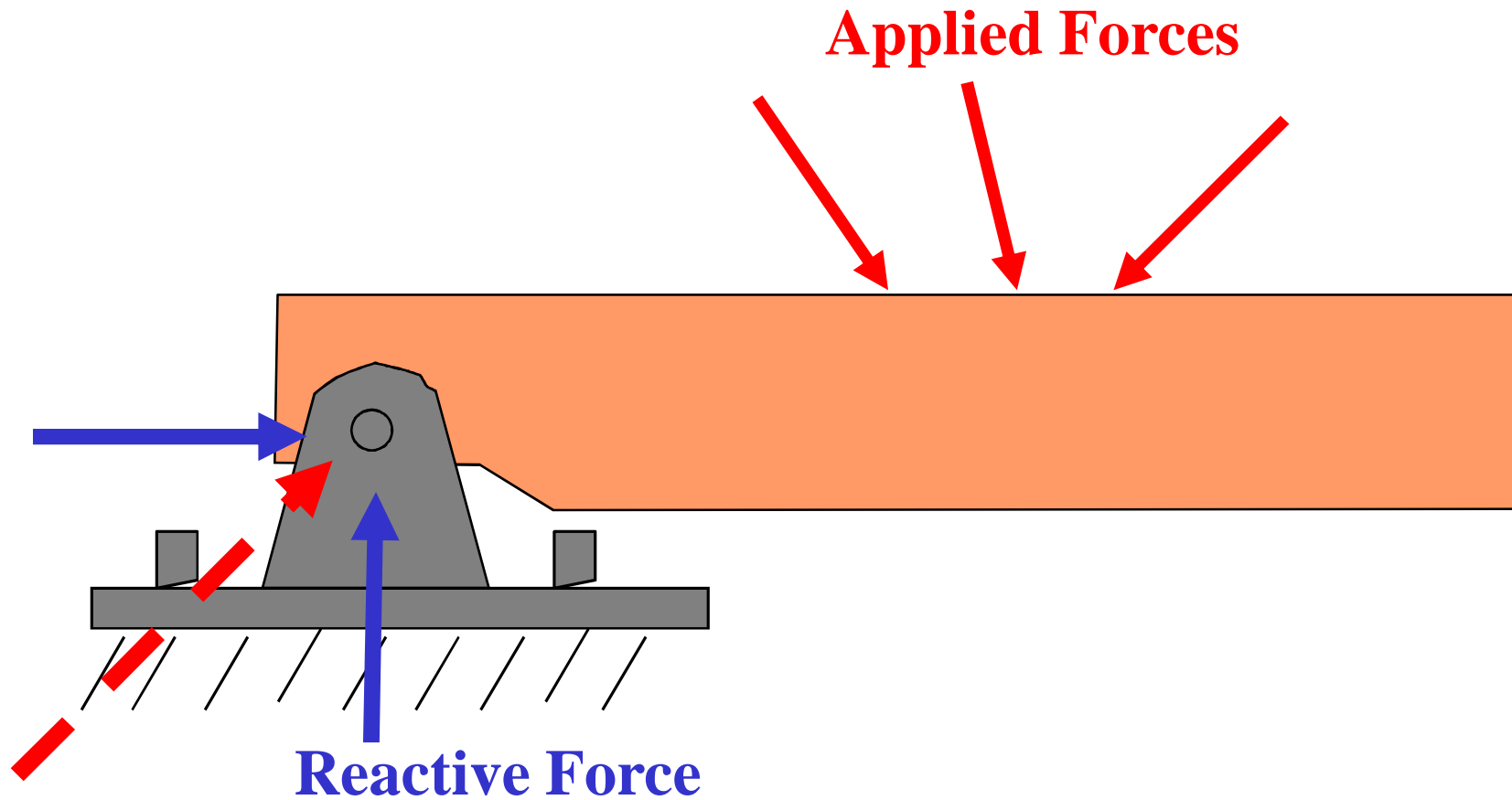


HINGED SUPPORT

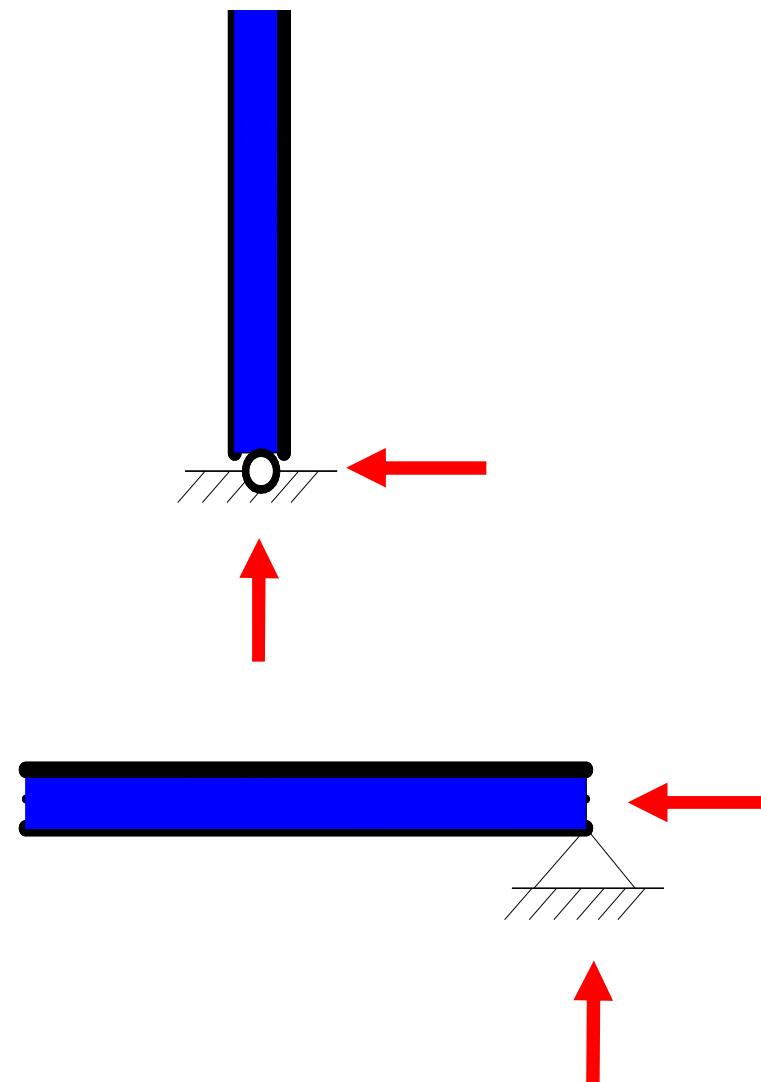
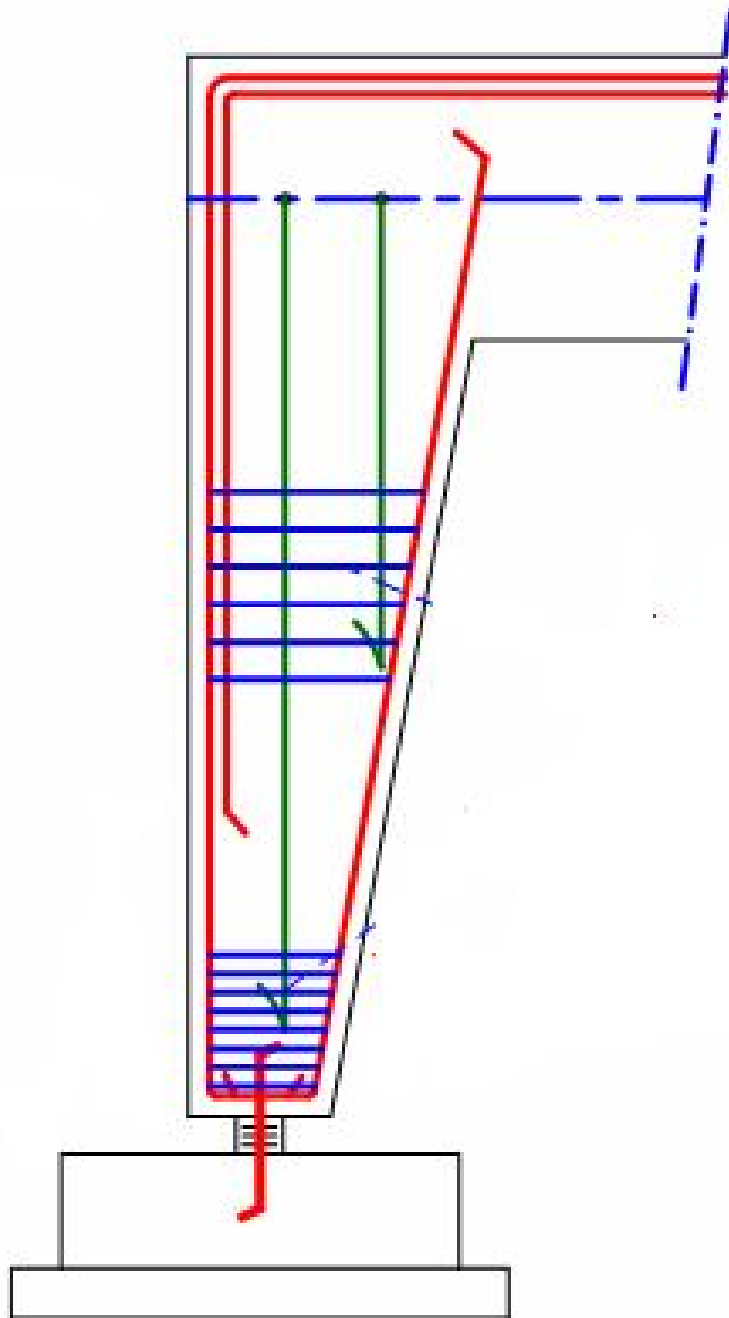


**KNEE
HINGE**

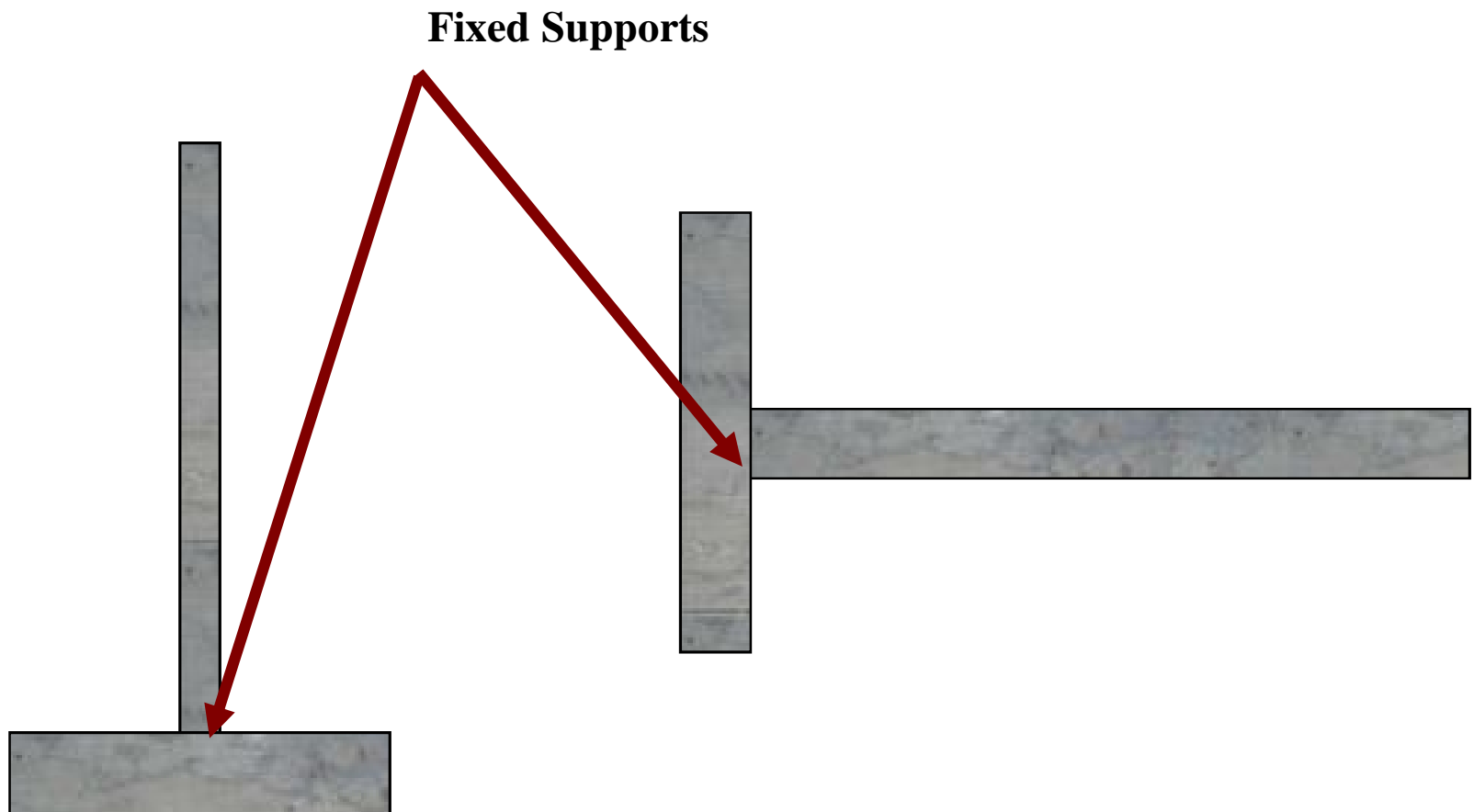
PIN or HINGE SUPPORT



**The Reactive Force can be in
any direction**

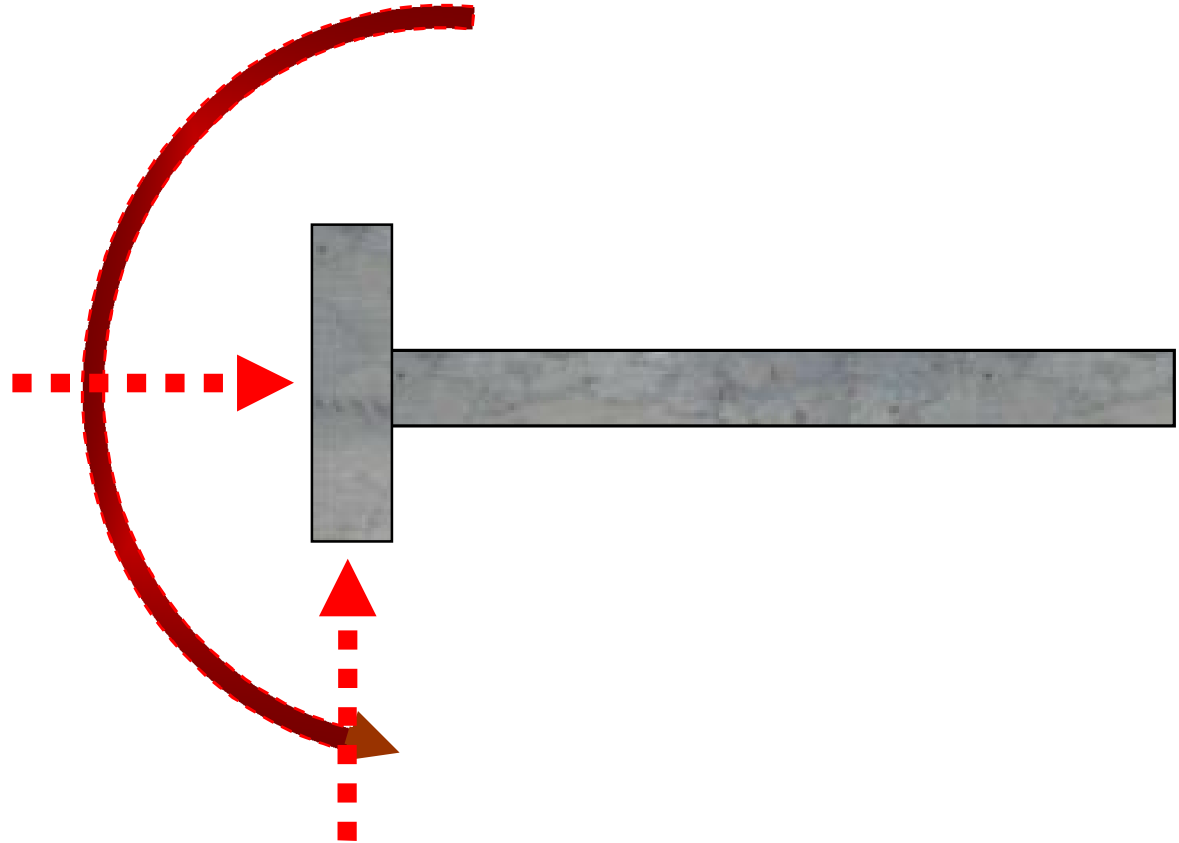


FIXED SUPPORT



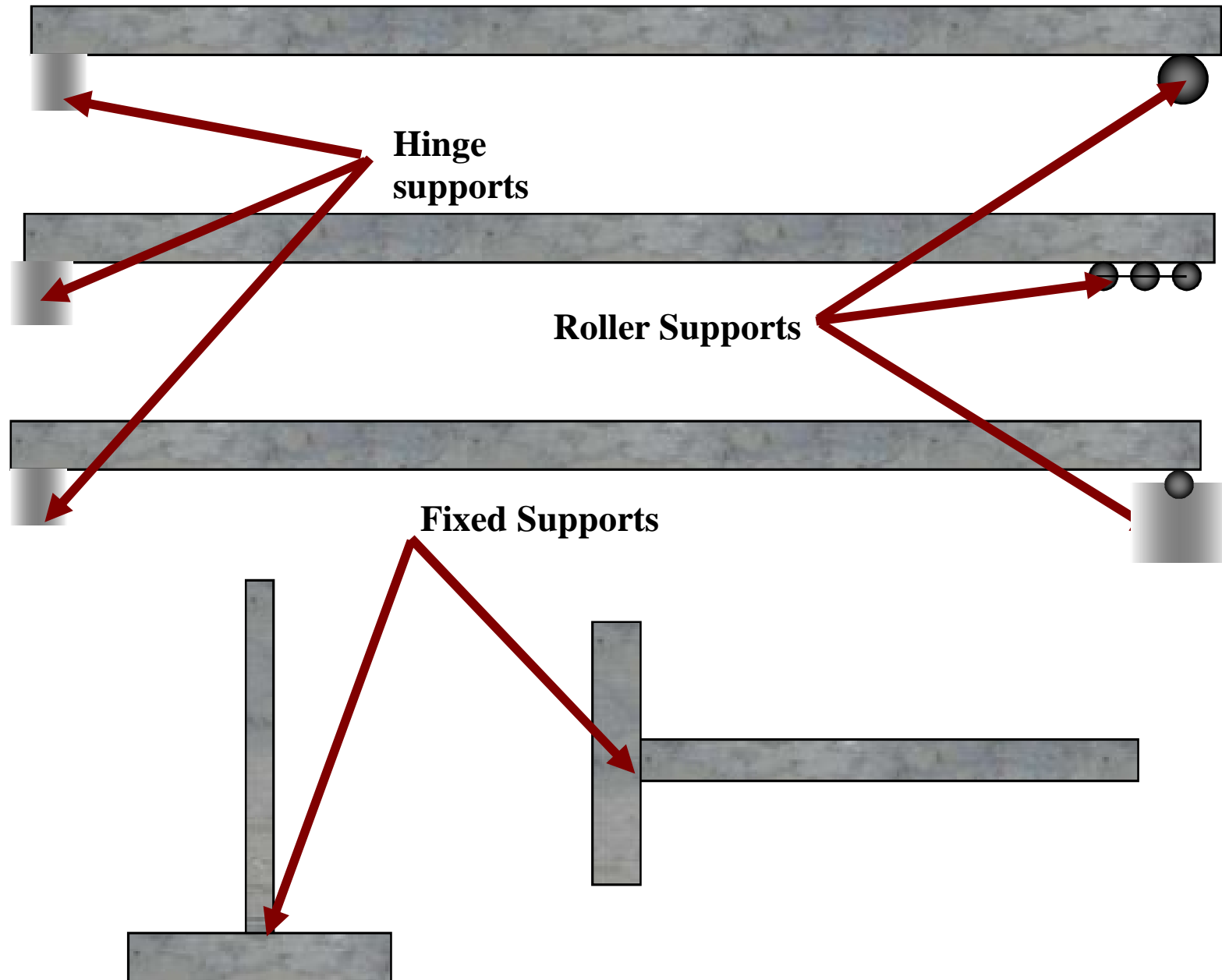
Fixed Support

لا يسمح بالحركة فى الاتجاهين والدوران

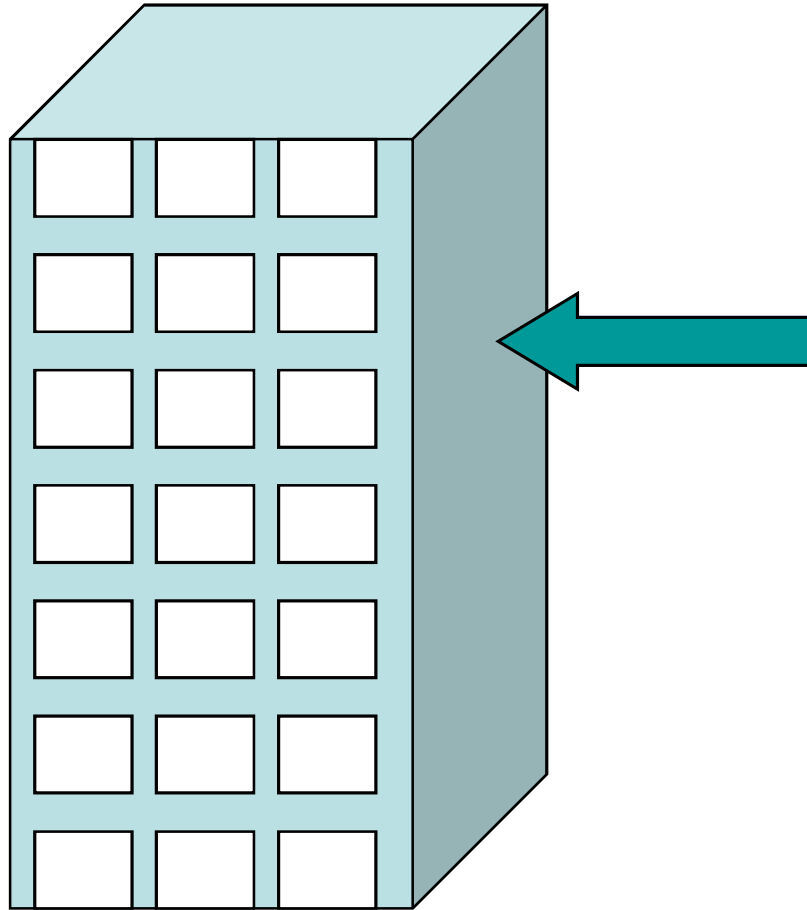


The Reactive Force can be in any direction Also there is reaction in the rotation direction

Roller, Hinge and Fixed Supports



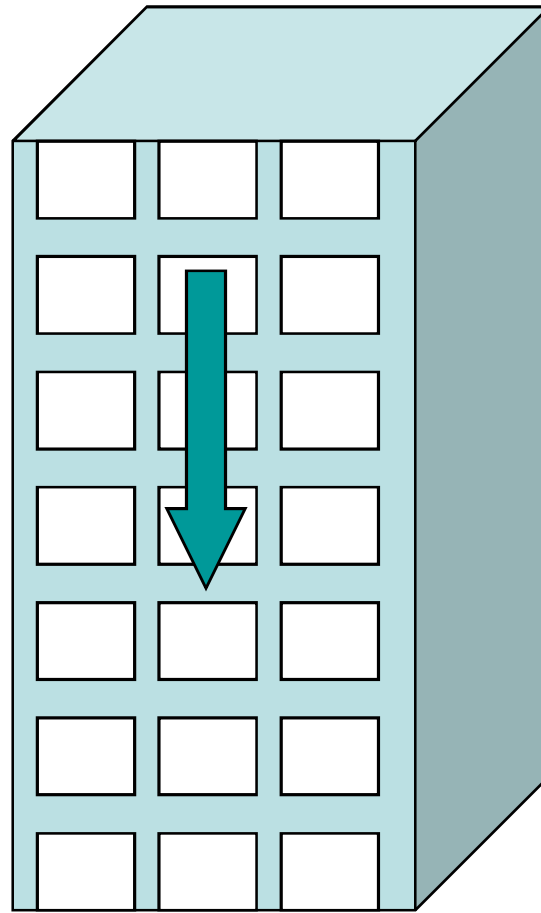
Reactions



LATERAL: WIND, EARTHQUAKE

$$\sum \mathbf{F}_x = 0.0$$

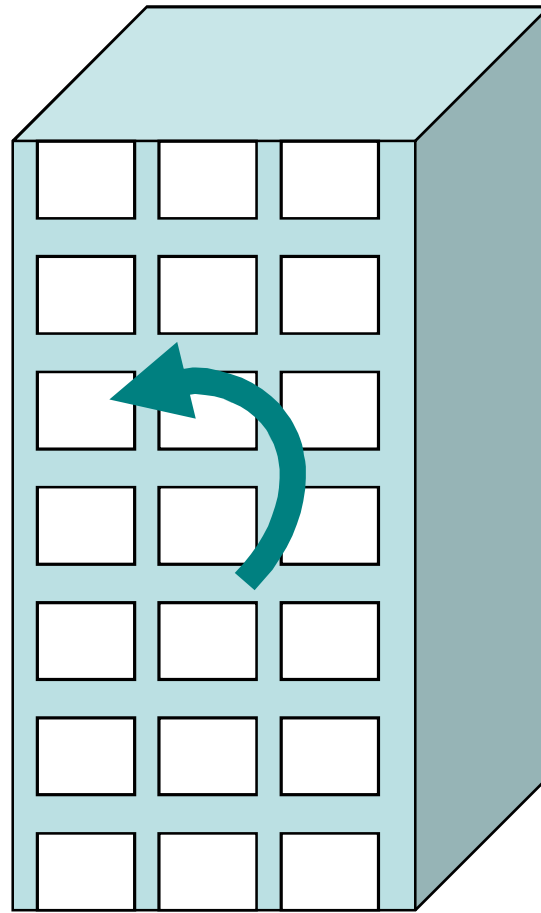
$$>>>>> = <<<<<$$



VERTICAL: GRAVITY

$$\sum \mathbf{F}_Y = 0.0$$

$$\wedge \wedge \wedge = \vee \vee \vee$$

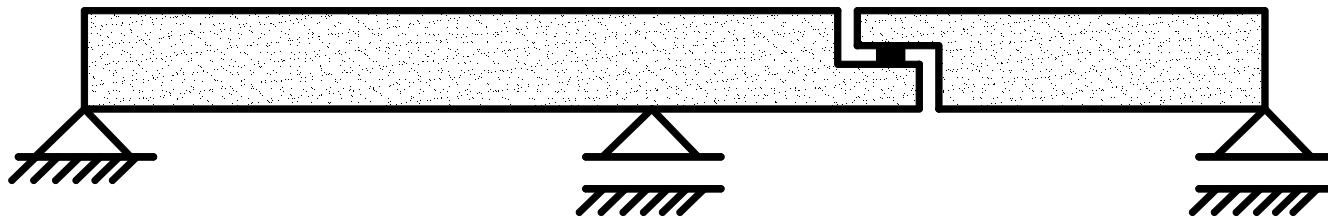
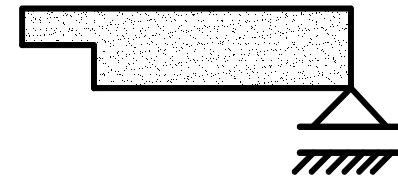
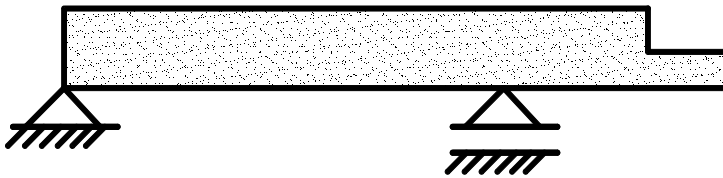
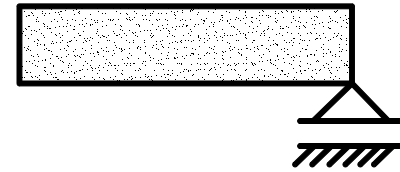
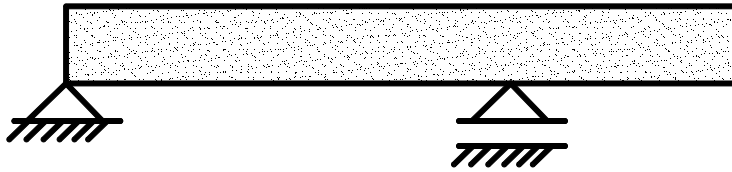


ROTATION: OVERTURNING LOADS

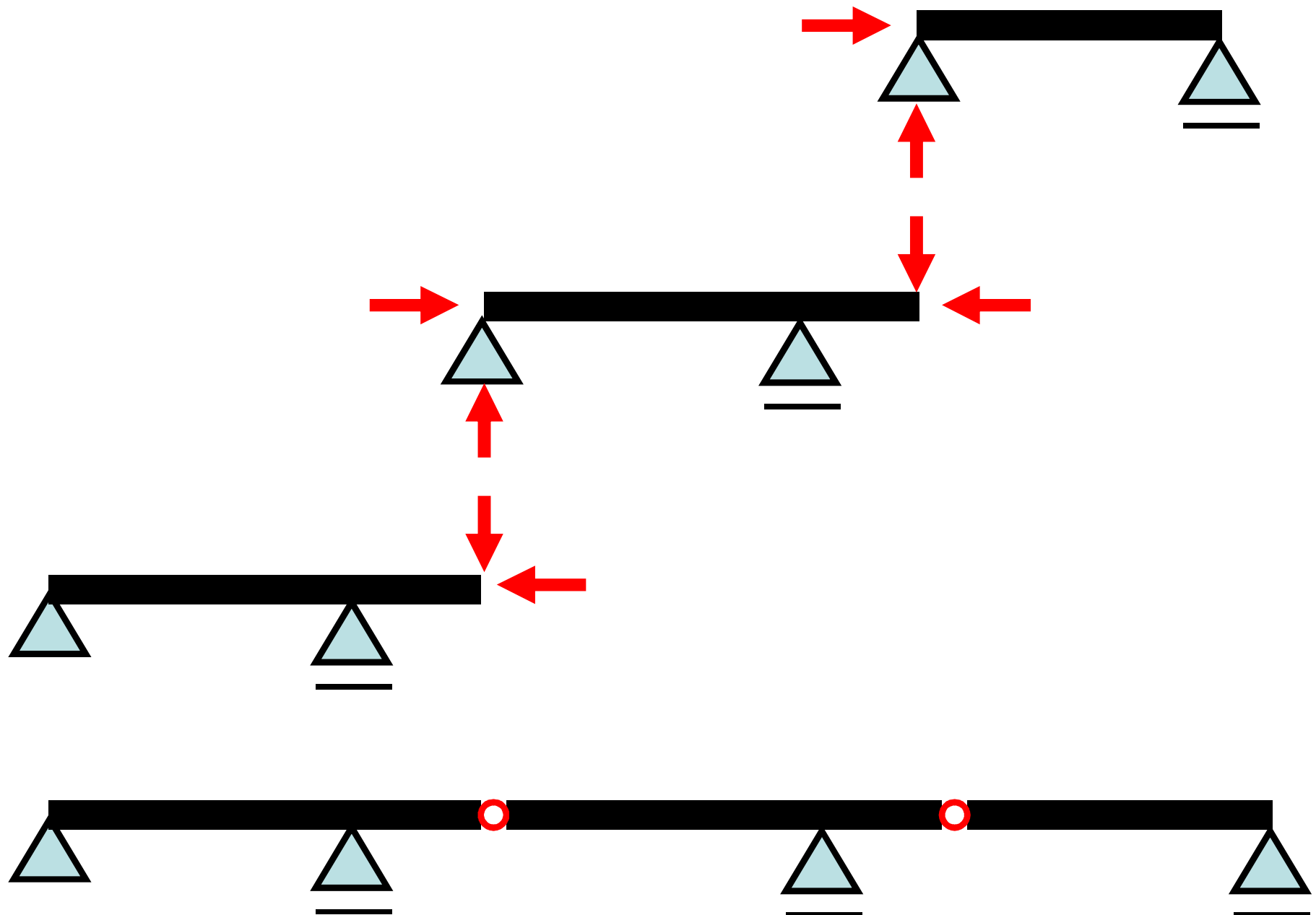
$$\Sigma \mathbf{M} = 0.0$$

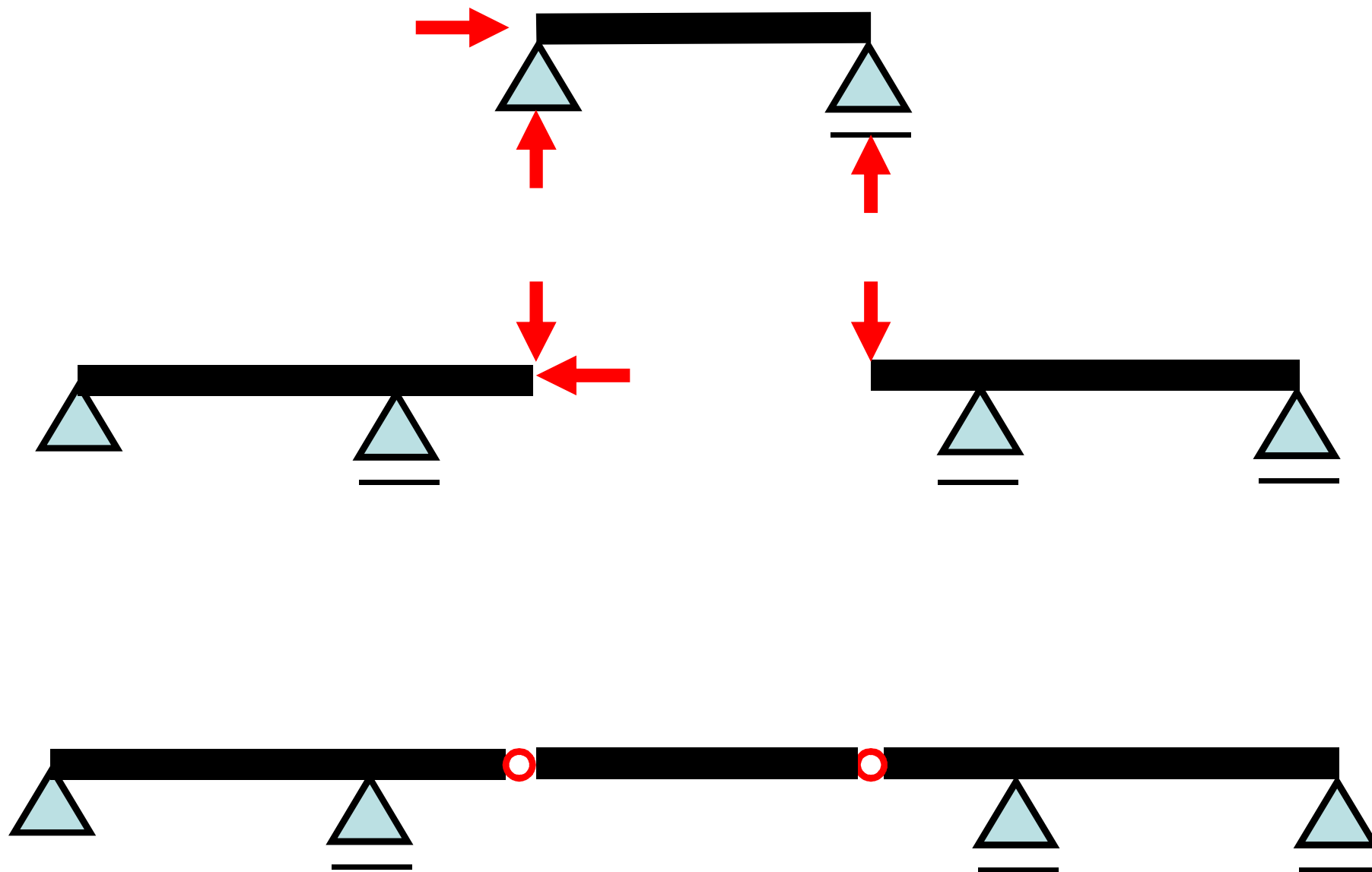
A diagram showing two thick, dark red curved arrows pointing in opposite directions (one clockwise, one counter-clockwise) separated by a blue equals sign, representing a balanced or zero net moment state.

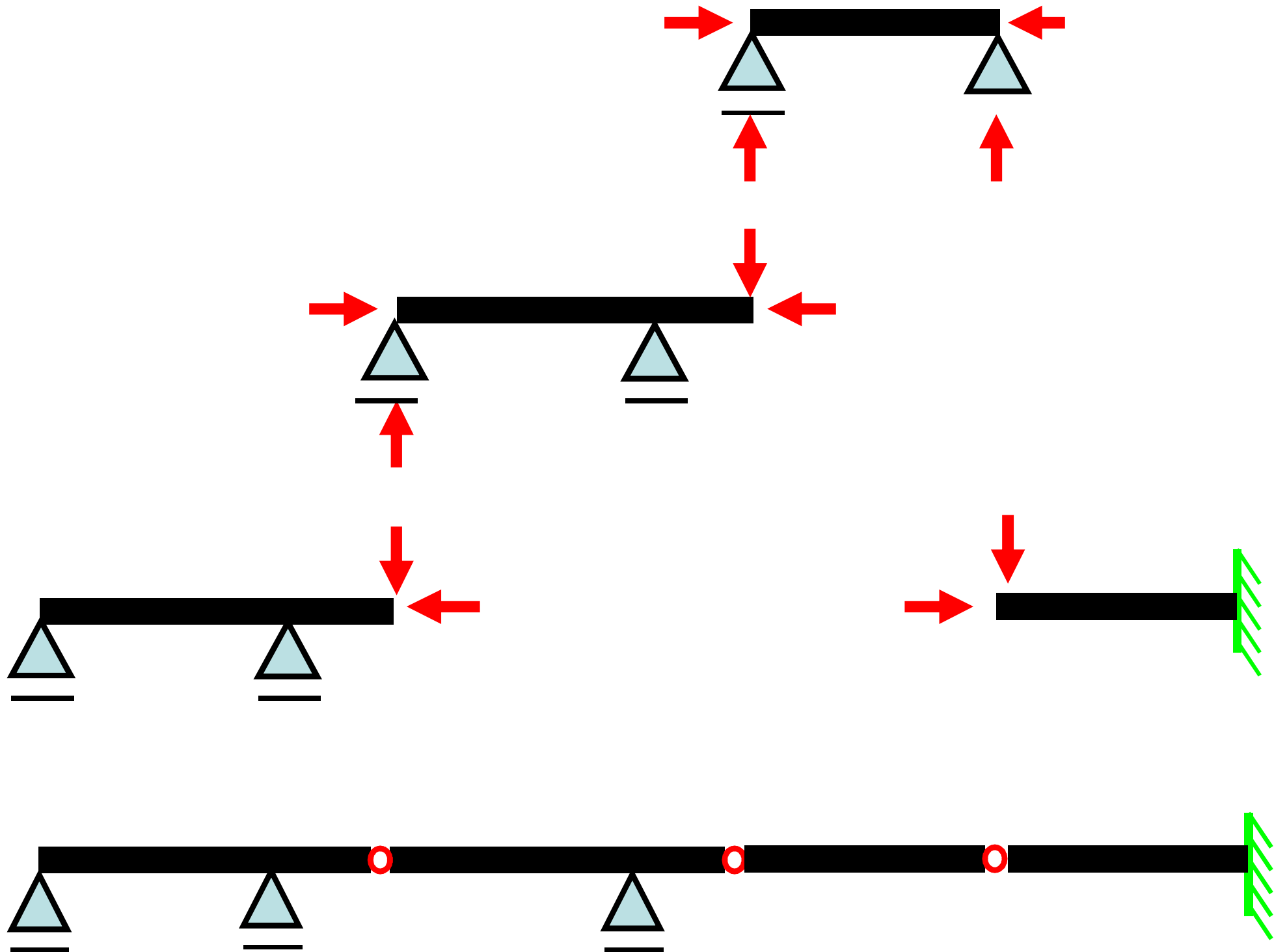
**compined beams with
intermediate hinge**



انواع الفصل

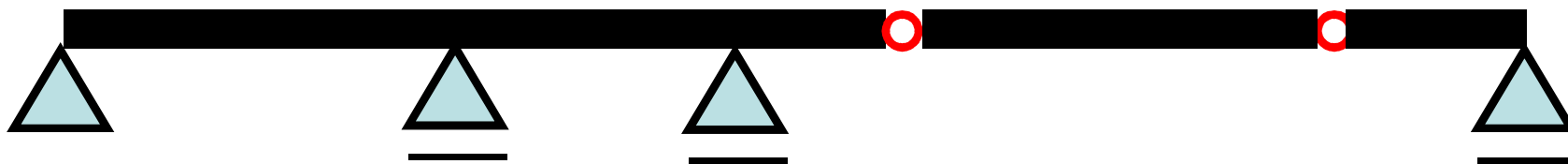
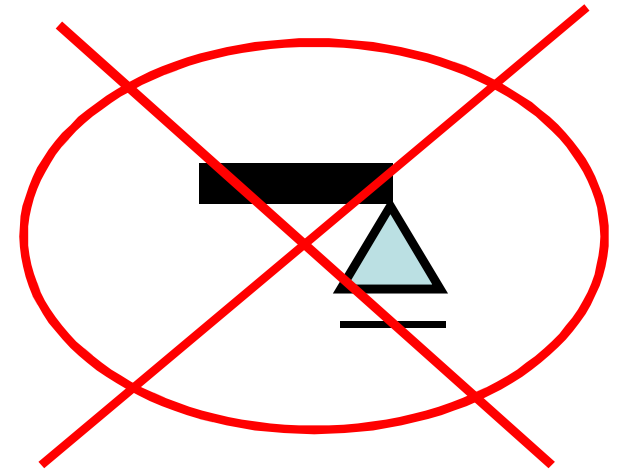
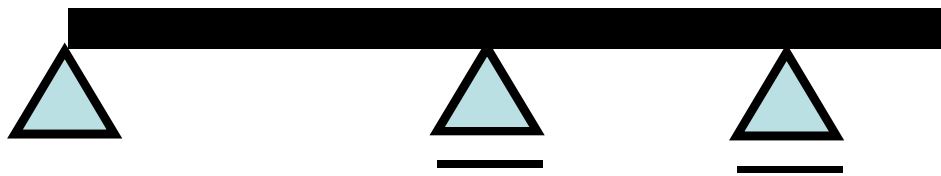


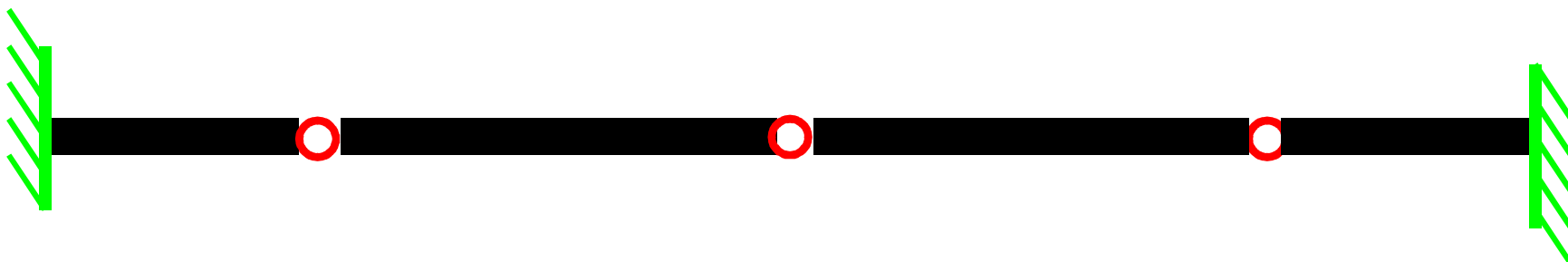
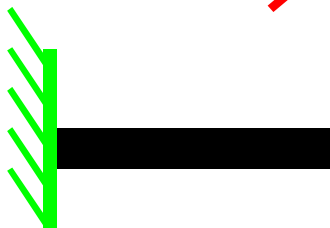
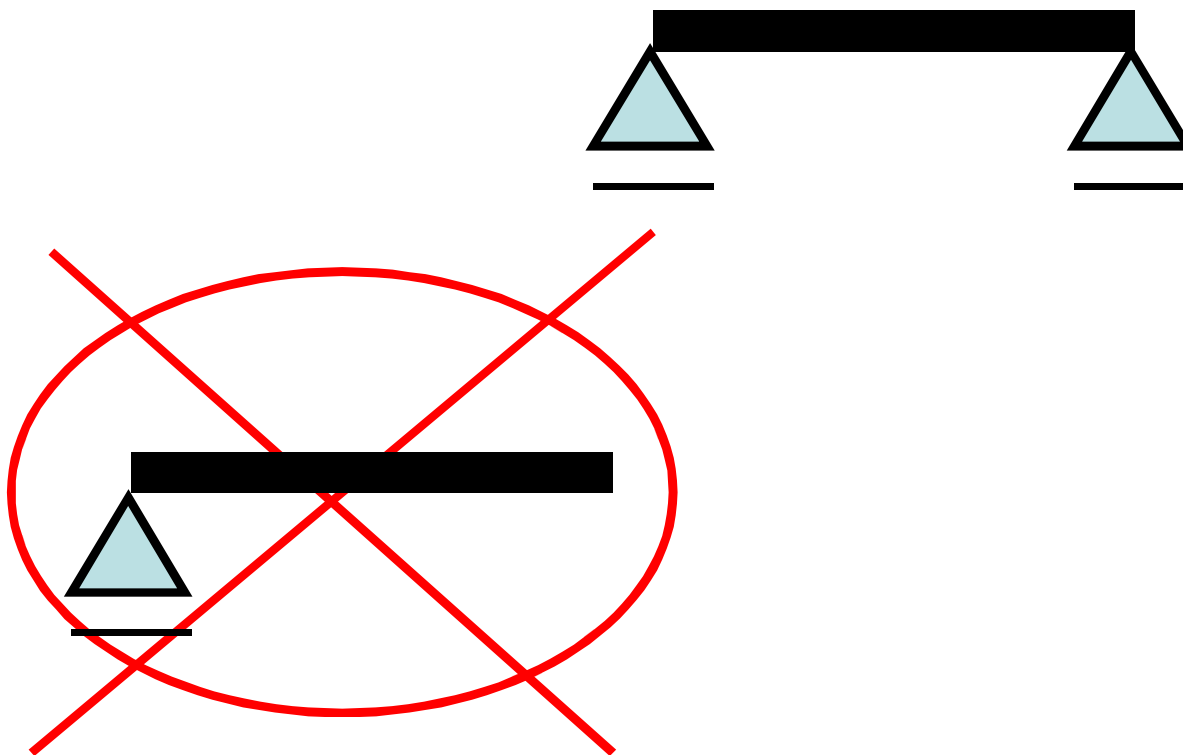




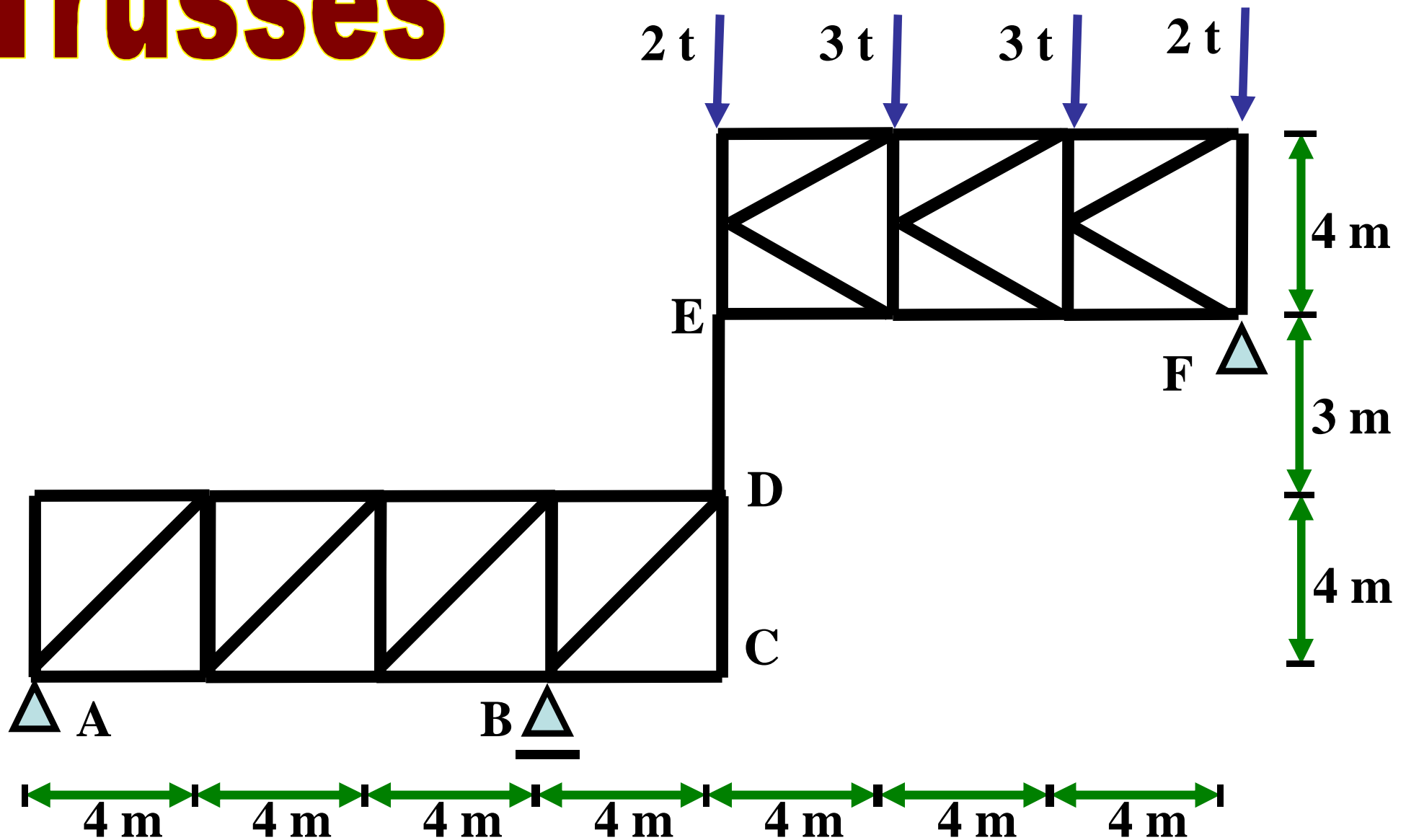
الحالات التي تصعب فيها الكمرة غير متزنه.

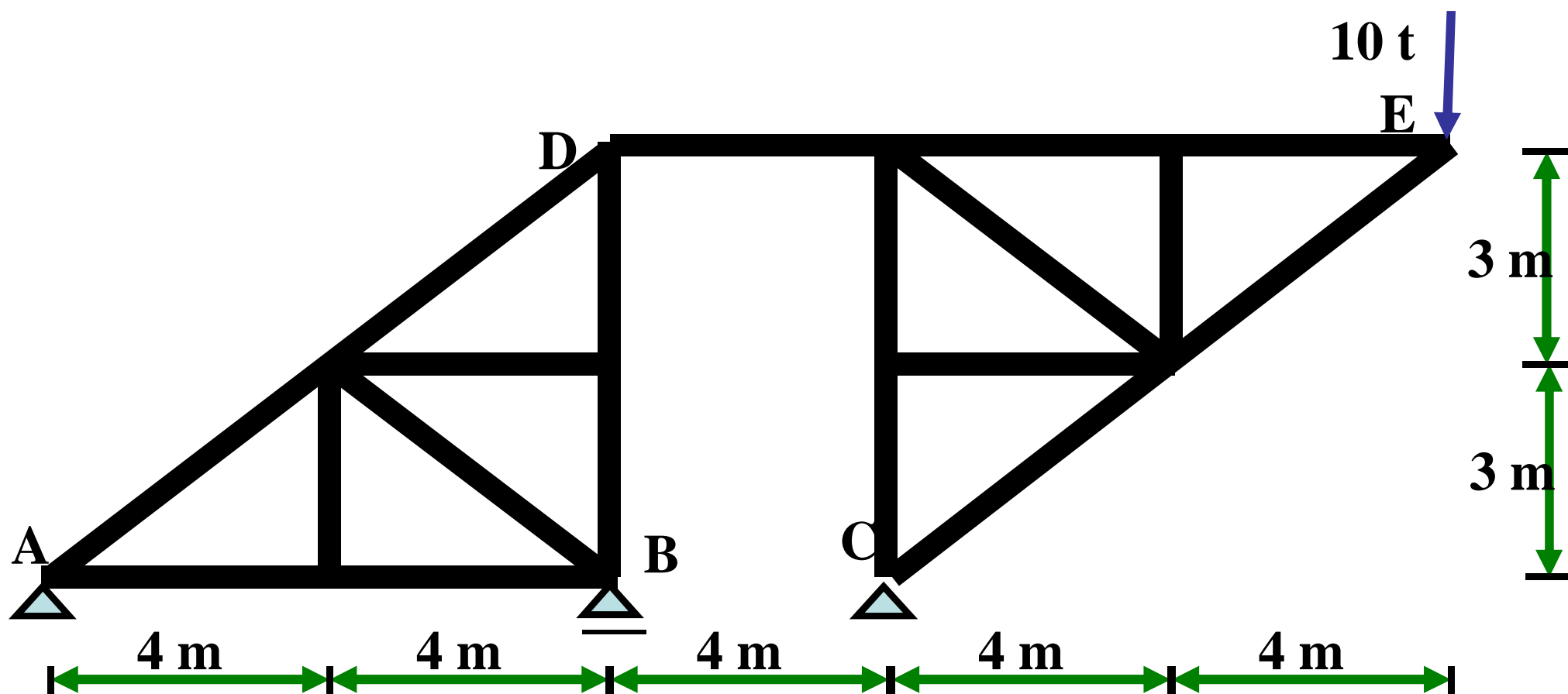
Unstable structure.

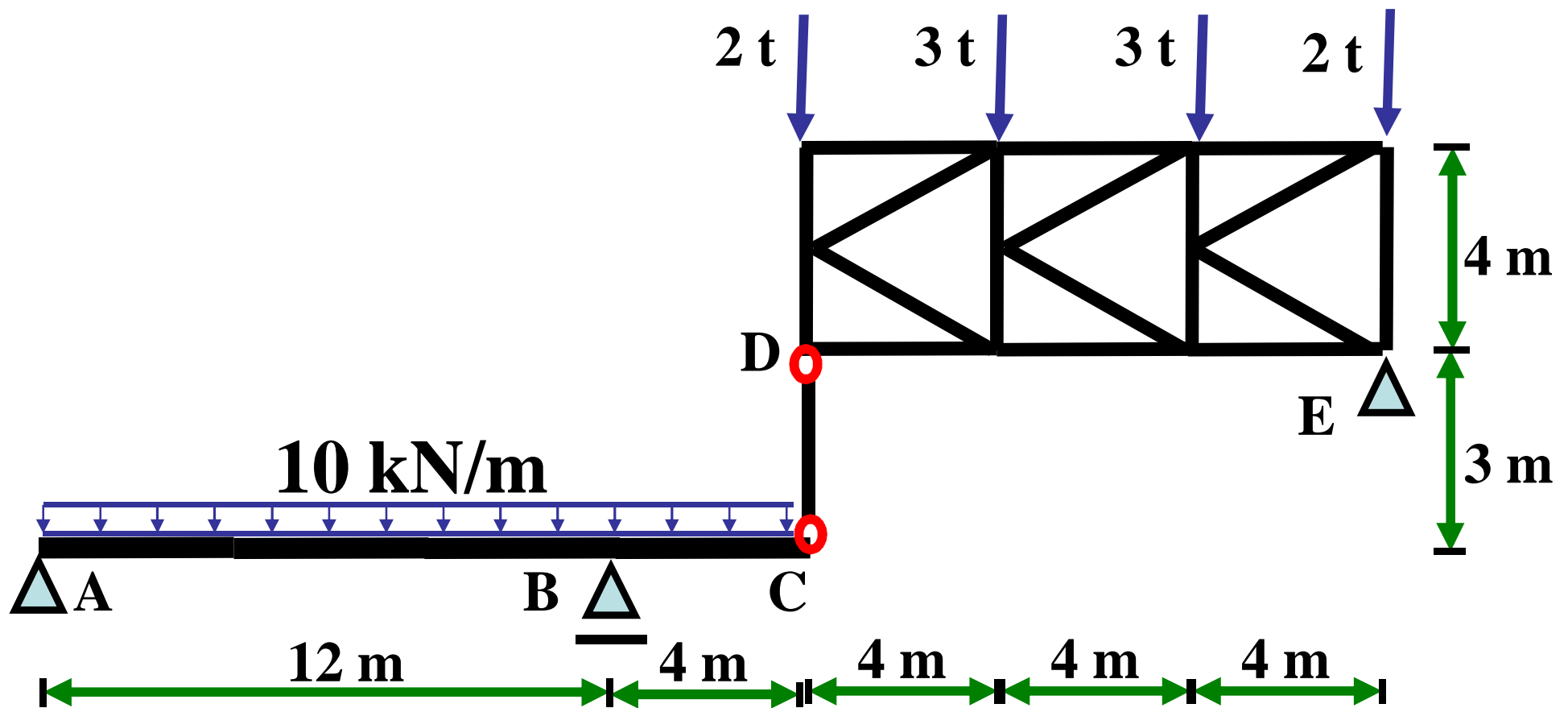




Trusses

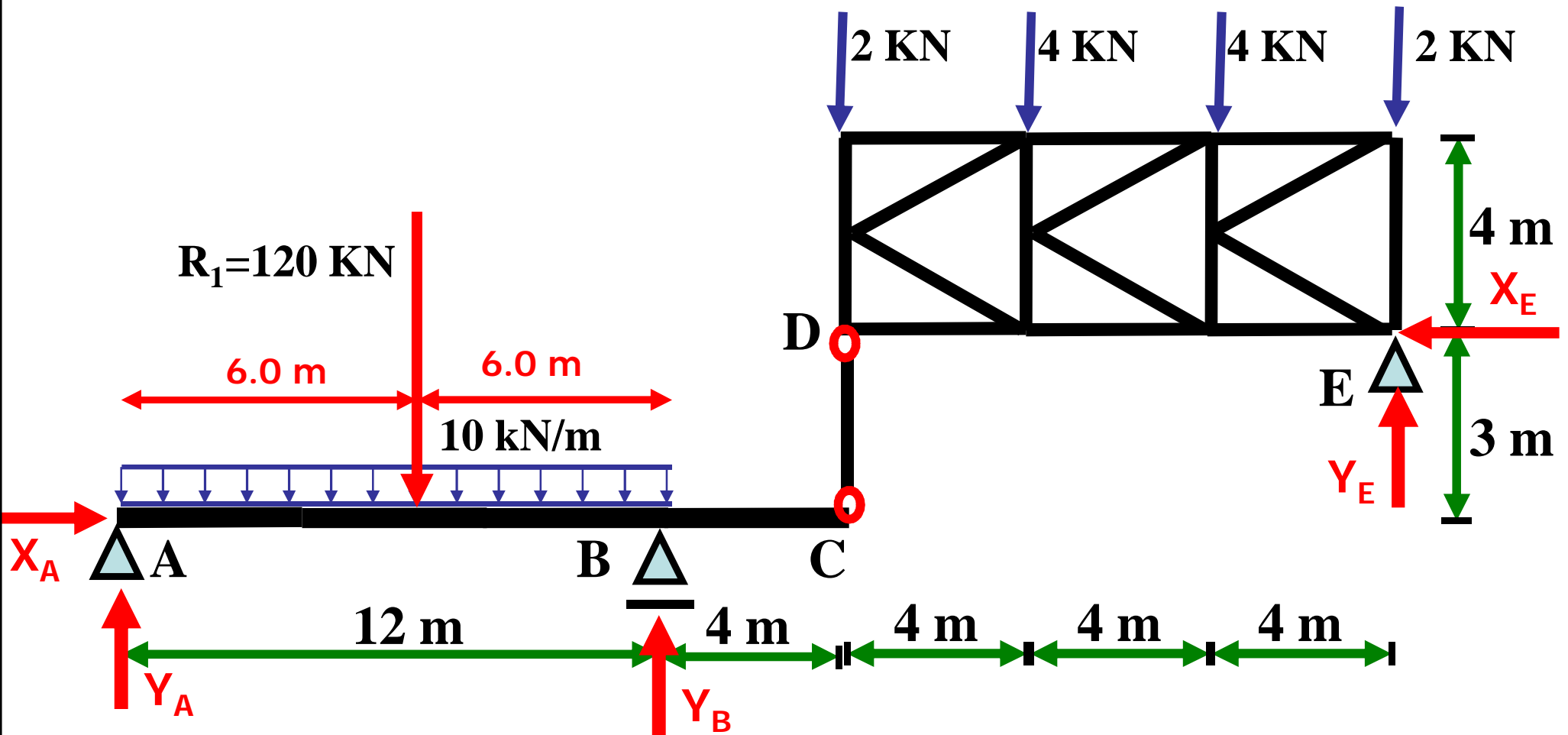


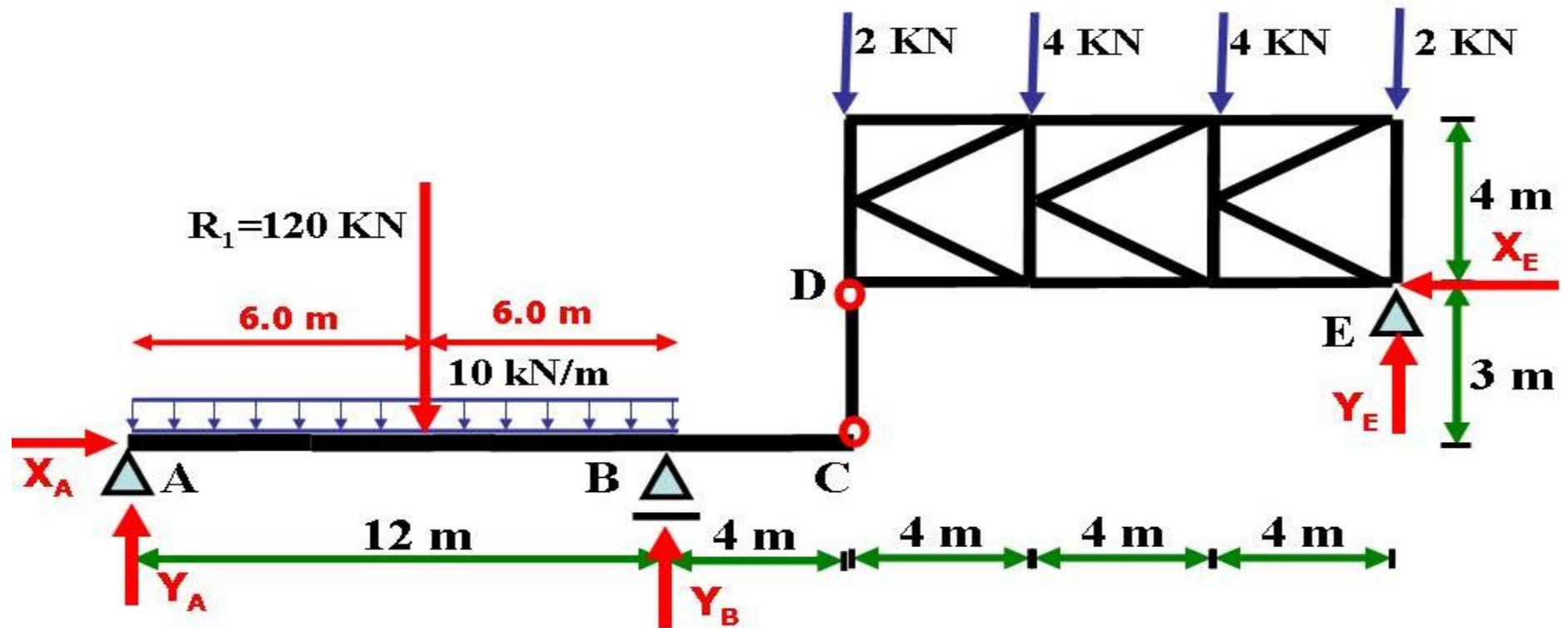




Determine the reaction for the following truss

first method





$$\sum \mathbf{M}_{DR} = 0.0$$

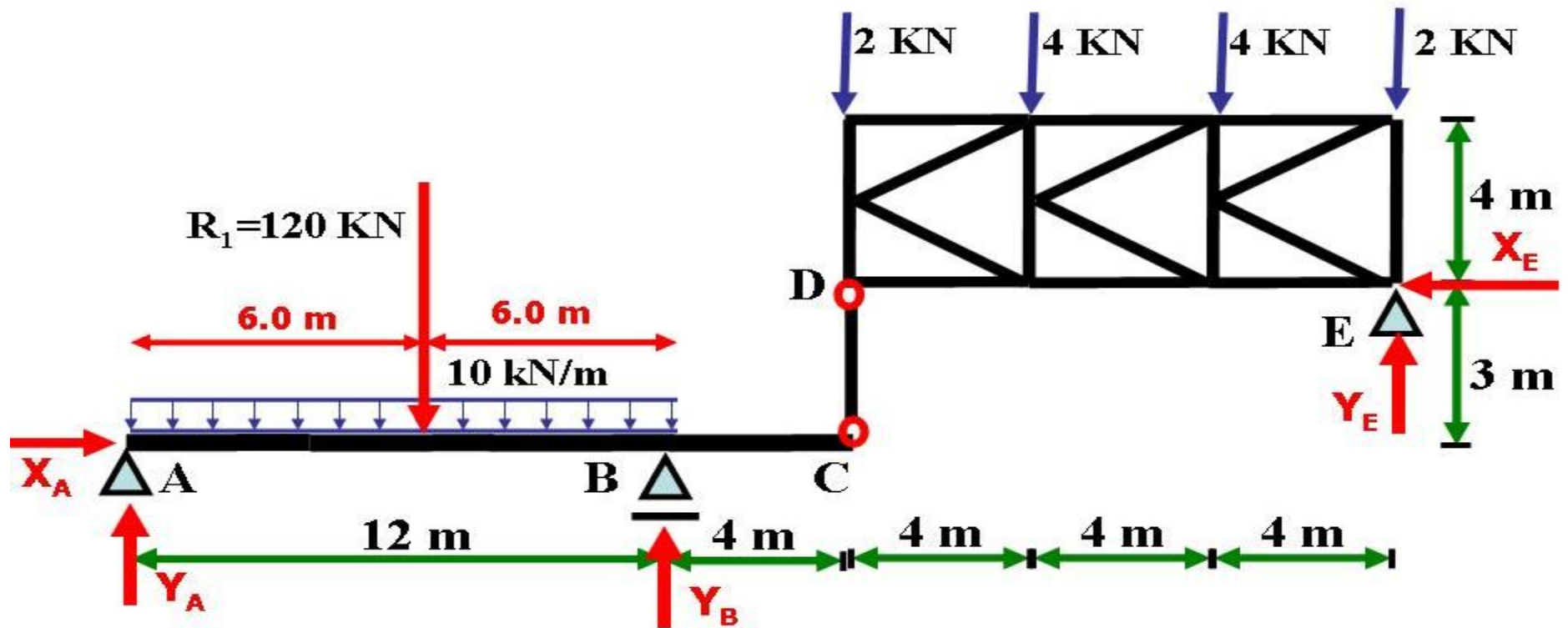
$$Y_E * 12.0 = 4 * 4.0 + 4 * 8.0 + 2 * 12.0$$

$$Y_E = 6.0 \text{ kN}$$

$$\sum \mathbf{M}_{CR} = 0.0$$

$$6.0 * 12.0 + X_E * 3 = 4 * 4.0 + 4 * 8.0 + 2 * 12.0$$

$$X_E = 0.0$$



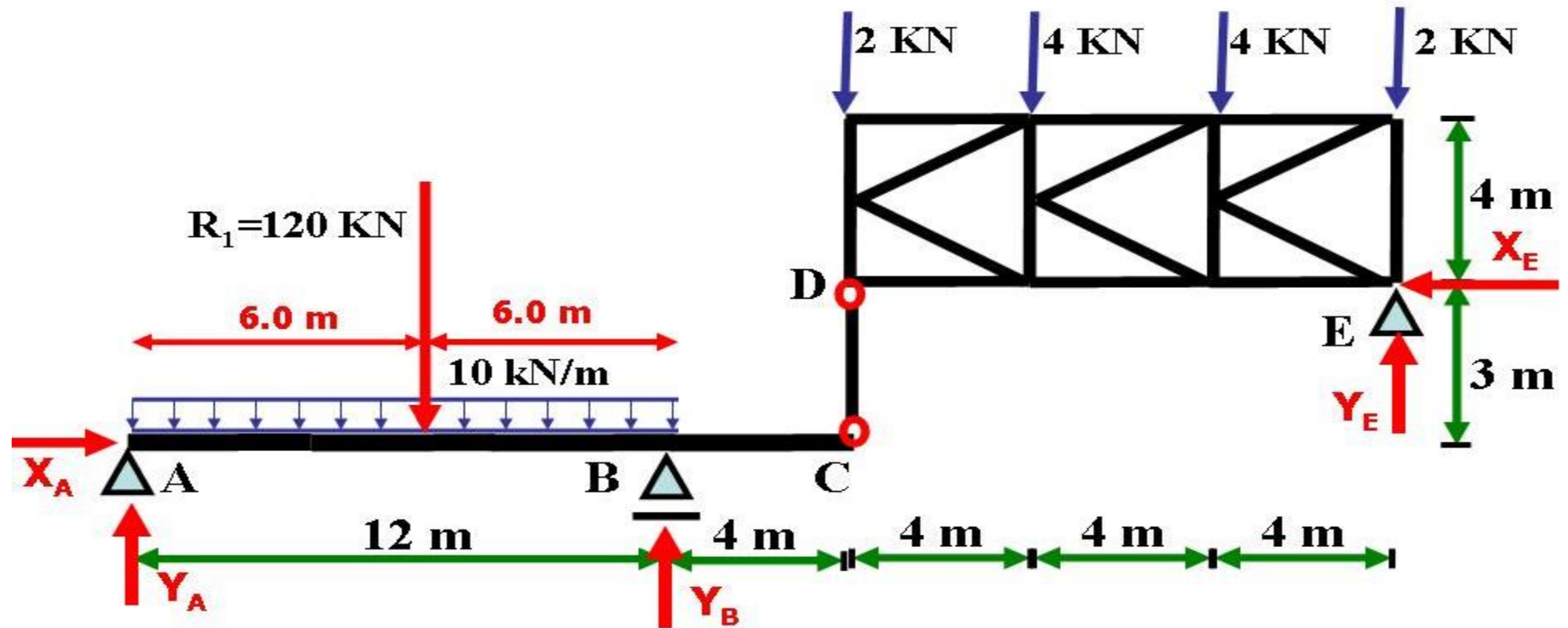
$$\sum \mathbf{F}_x = 0.0$$

$$X_A = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$

$$Y_B * 12.0 + 6 * 28 = 120 * 6 + 2 * 16 + 4 * 20.0 + 4 * 24.0 + 2 * 28.0$$

$$Y_B = 68.0 \text{ KN}$$

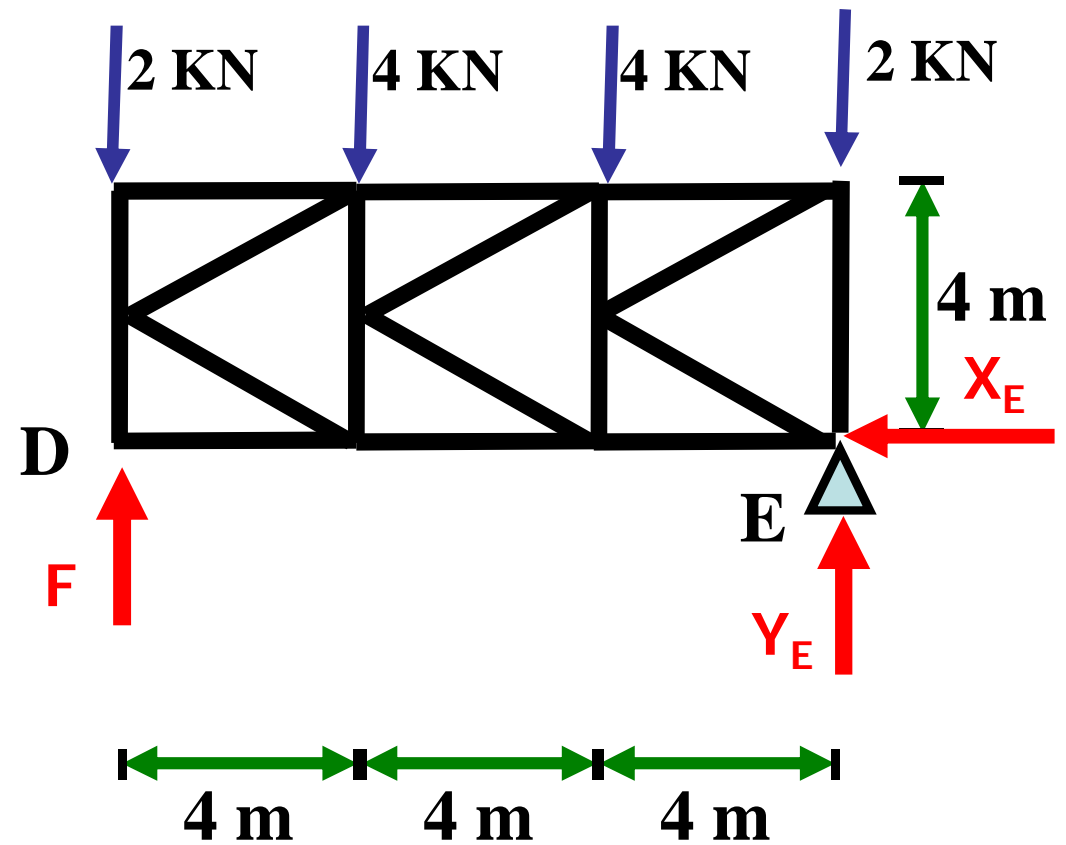
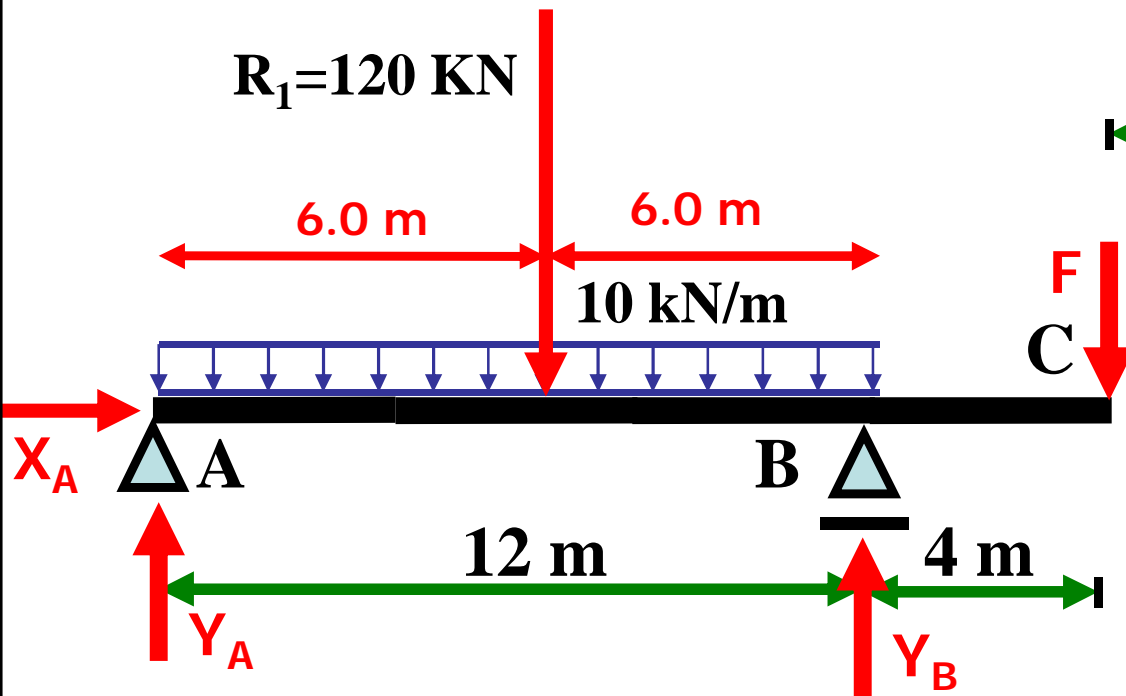


$$\sum \mathbf{F}_Y = 0.0$$

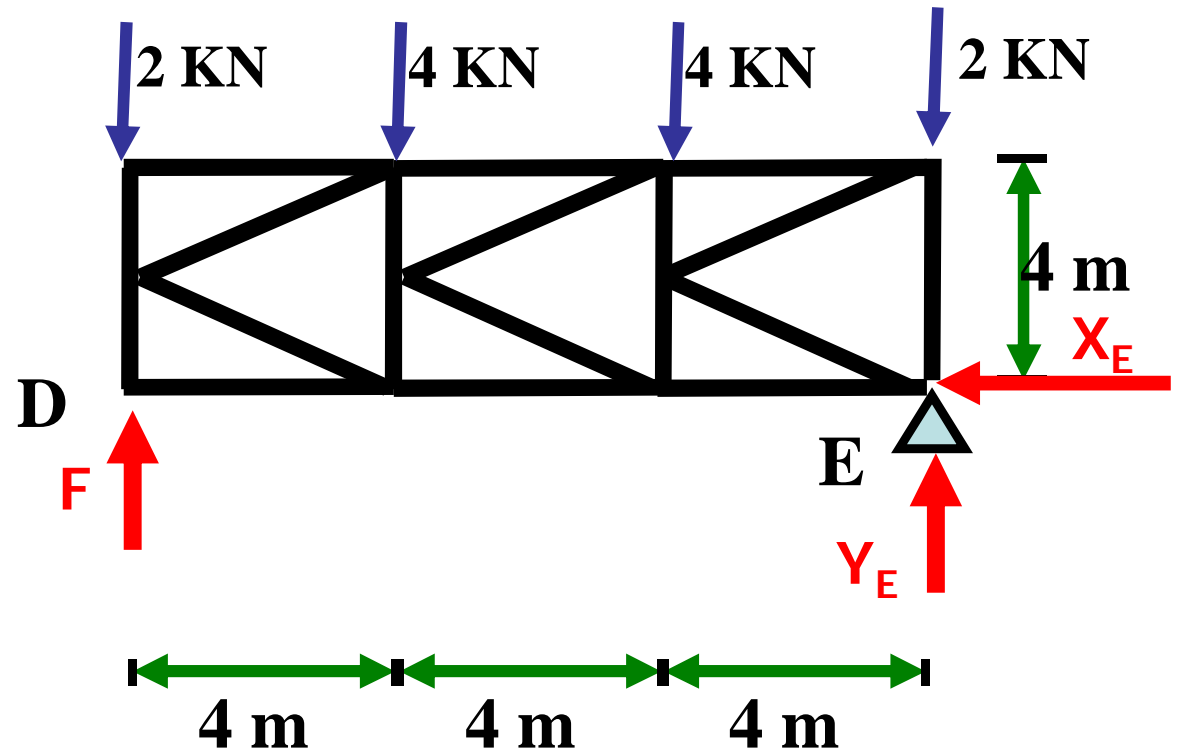
$$Y_A + 68 + 6 = 120 + 2 + 4 + 4 + 2$$

$$Y_A = 58 \text{ KN}$$

second method



Part DE



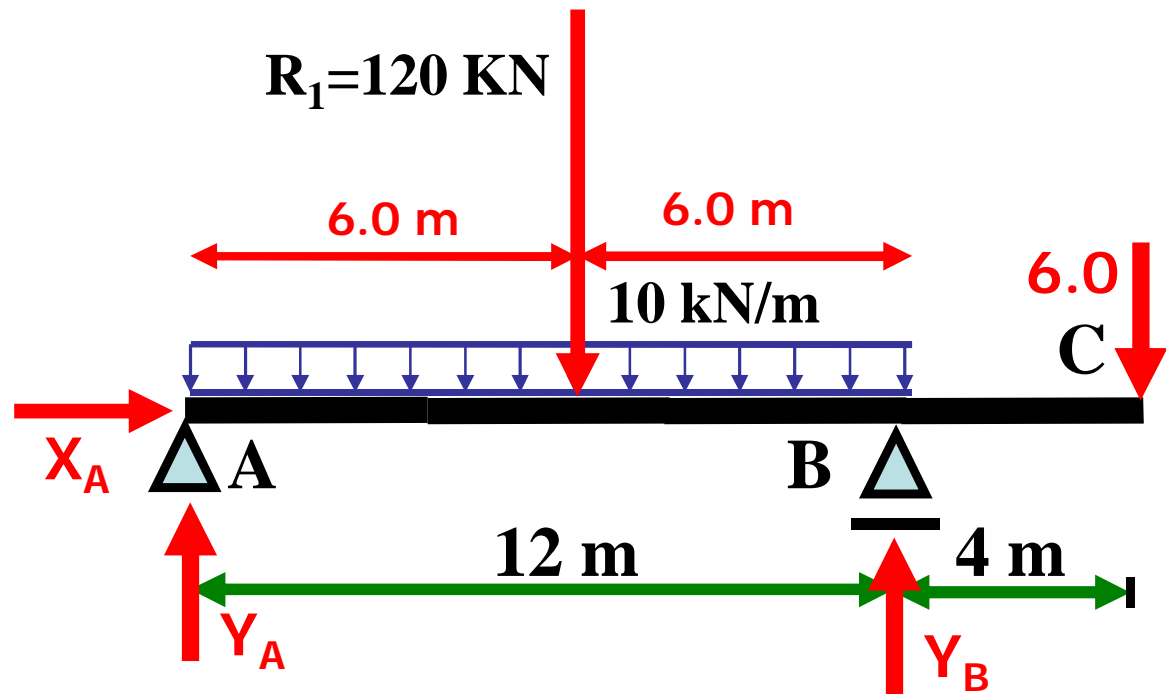
$$\sum \mathbf{F}_x = 0.0$$

$$X_E = 0.0$$

$$\sum \mathbf{F}_y = 0.0$$

$$F = Y_E = (2+4+4+2) / 2 = 6.0 \quad \text{From symmetry}$$

Part AD



$$\sum \mathbf{F}_x = 0.0$$

$$X_A = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$

$$Y_B * 12 = 120 * 6 + 6 * 16$$

$$Y_B = 68 \text{ KN}$$

$$\sum \mathbf{F}_y = 0.0$$

$$Y_A = 120 + 6 - 68 = 58 \text{ KN}$$

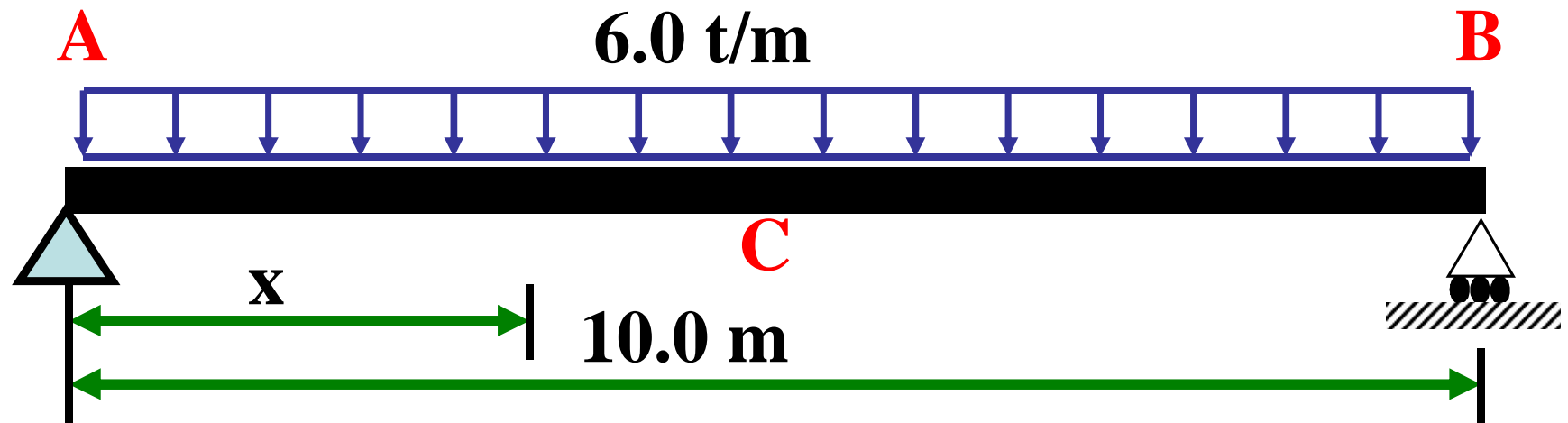
بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

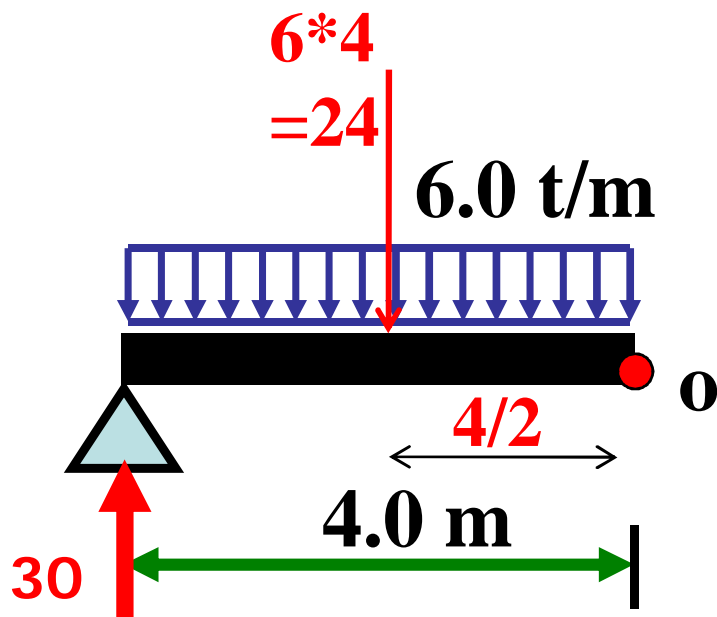
Double Integration

Eng : Aymman abdo

خطوات الحل:

1- يتم عمل معادلة للعزم بدلالة (x) حتى يتم استخدامها في التكاملات ويفضل ان يتم عمل المعادلة من الاتجاه الاسهل للكمرة.

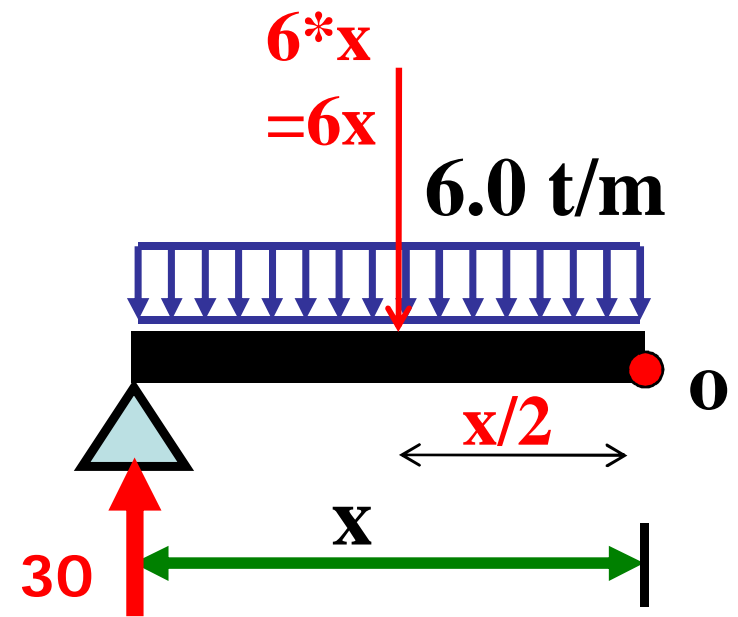




$$M_o = 30 * 4 - 6 * 4 * 4/2$$

$$= 72 \text{ t.m}$$

للتوضيح فقط



$$M_o = 30 * x - 6 * x * x/2$$

$$= 30 x - 3 x^2 \text{ t.m}$$

هذه الطريقة افضل فى الحسابات لاننا
نستطيع من خلالها حساب العزم فى اى مكان
بالتعويض بقيمة x للمكان

2- يتم التعويض بمعادلة العزم فى معادلة الـ double integration

$$y'' = -\frac{1}{EI}(M)$$

$$y'' = -\frac{1}{EI}(30x - 3x^2)$$

$$y' = \theta = -\frac{1}{EI}(30x^2 / 2 - 3x^3 / 3 + c_1) = -\frac{1}{EI}(15x^2 - x^3 + c_1)$$

$$y = -\frac{1}{EI}(5x^3 - x^4 / 4 + c_1x + c_2)$$

لاحظ اننا حصلنا على المعادلات المطلوبة
ولكن فقط بعد الحصول على الثوابت

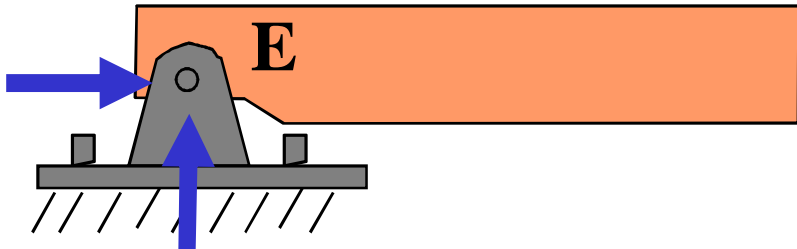
Boundary Conditions

ROLLER

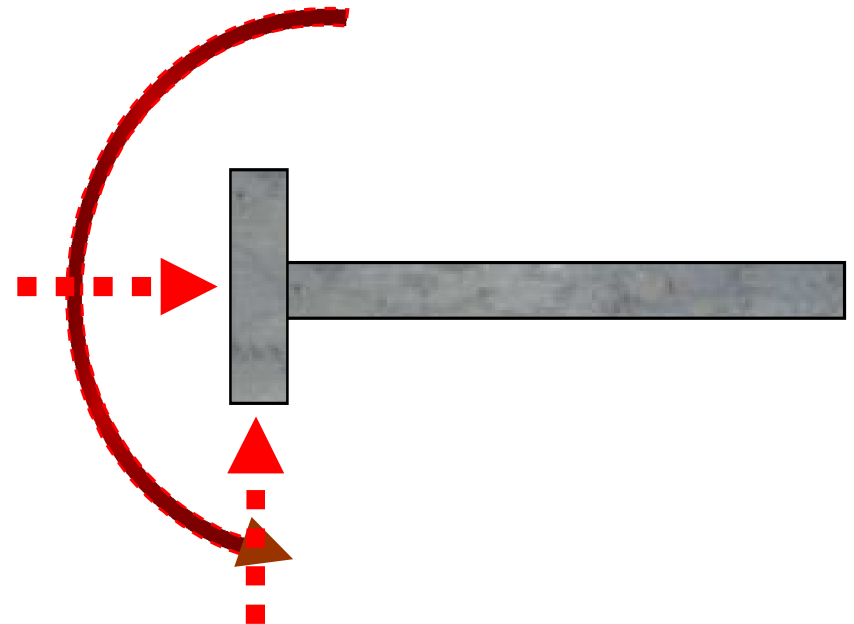


HING

E



$y = 0.0$ at roller
and hinged support



$y = 0.0$ and $\theta = 0.0$
at fixed support

3- يتم التعويض بـ boundary condition في معادلة الهبوط والدوران السابقة.

عند النقطة A الهبوط يساوى صفر **At $x = 0.0 \gggg y = 0.0$**

$$y = -\frac{1}{EI}(5x^3 - x^4 / 4 + c_1x + c_2) = 0.0$$

$$5 * (0)^3 - (0)^4 / 4 + c_1 * (0) + c_2 = 0.0$$

$$c_2 = 0.0$$

عند النقطة B الهبوط يساوى صفر **At $x = 10.0 \gggg y = 0.0$**

$$y = -\frac{1}{EI}(5x^3 - x^4 / 4 + c_1x + c_2) = 0.0$$

$$5 * (10)^3 - (10)^4 / 4 + c_1 * (10) + 0.0 = 0.0$$

$$c_1 = -250.0$$

$$>>>>> \quad \theta = -\frac{1}{EI}(15x^2 - x^3 - 250)$$

$$y = -\frac{1}{EI}(5x^3 - x^4/4 - 250x)$$

$$\text{at } x=0.0 \quad >>>>>$$

$$\theta = -\frac{1}{EI}(15*0^2 - 0^3 - 250) = \frac{250}{EI}$$

$$y = -\frac{1}{EI}(5*0^3 - 0^4/4 - 250*0) = 0$$

$$\text{at } x=5.0 \quad >>>>>$$

$$\theta = -\frac{1}{EI}(15*5^2 - 5^3 - 250) = \frac{0}{EI} = 0.0$$

$$y = -\frac{1}{EI}(5*5^3 - 5^4/4 - 250*5) = -\frac{781.25}{EI}$$

1- عند خط التماثل للكمرة

$$\Theta_c = 0.0$$

2- بسبب التماثل

$$\Theta_A = -\Theta_B$$

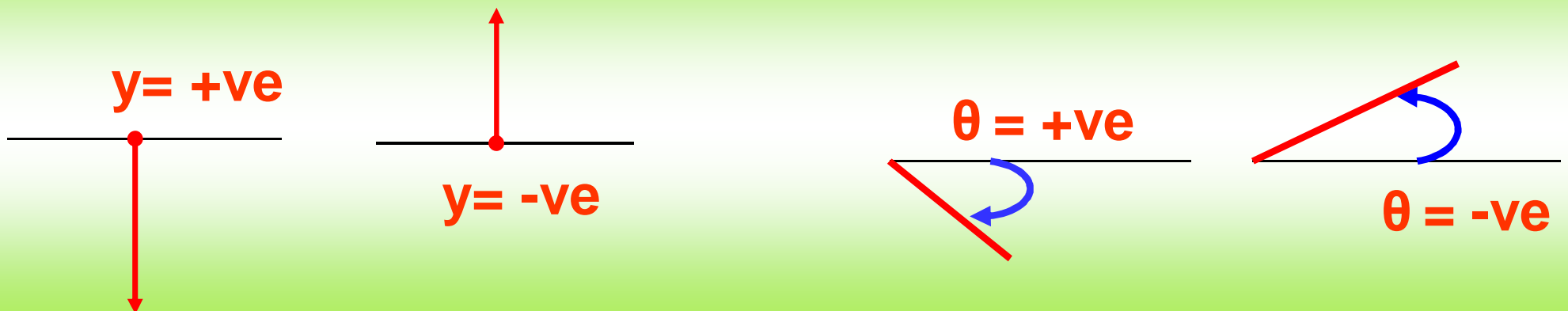
Position of max deflection = position of zero rotation

Y_{max} @ θ = 0.0

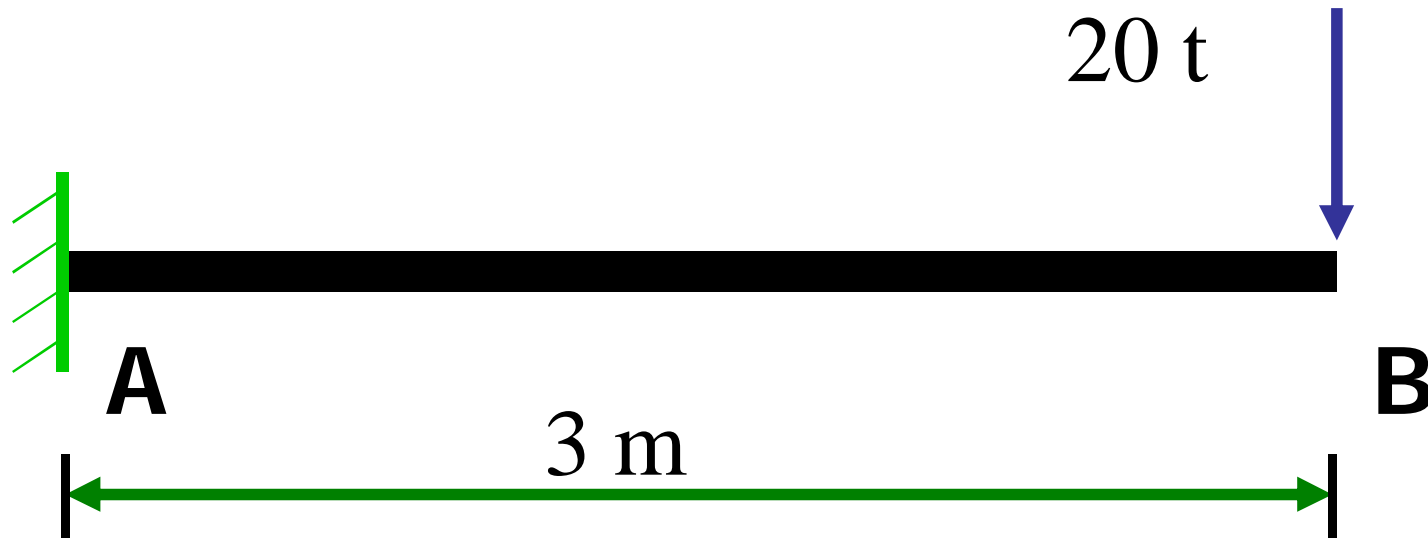
$$\theta = -\frac{1}{EI}(15 * x^2 - x^3 - 250) = 0.0$$

$$15 * x^2 - x^3 - 250 = 0.0 \quad \text{by using calculator} \quad x = 5.0$$

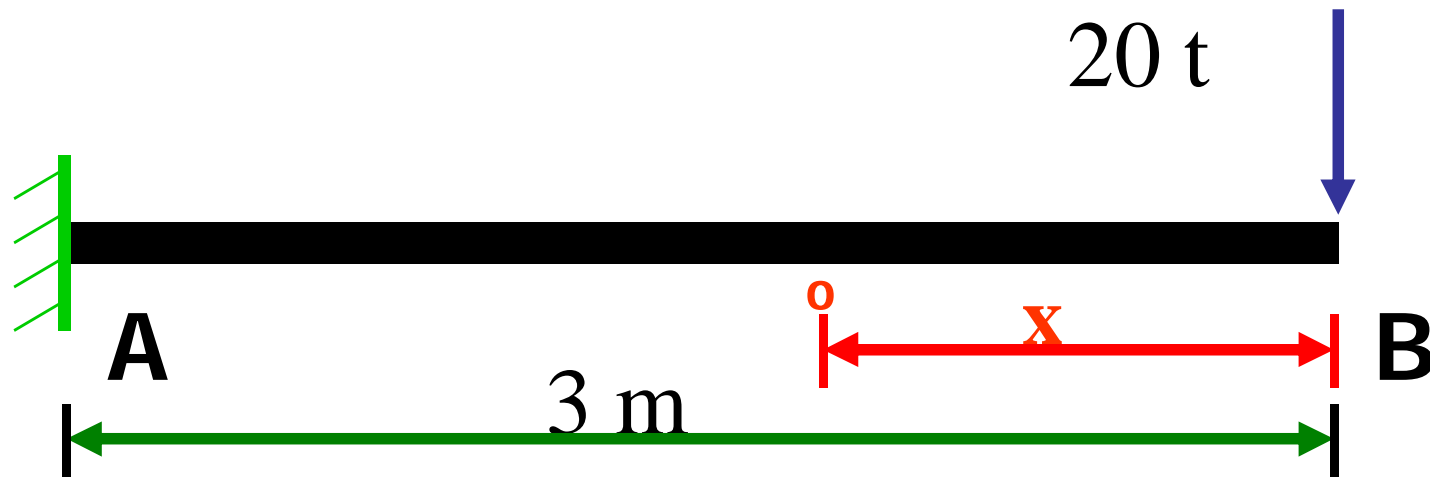
$$y_{\max} = -\frac{1}{EI}(5 * 5^3 - 5^4 / 4 - 250 * 5) = -\frac{781.25}{EI}$$



Example No. 2



Required the deflection and rotation at point b



بداية الـ x من B افضل حتى لو كان هناك خطأ في الـ reaction عند A لن يؤثر

$$M_o = - 20 x \quad \text{t.m}$$

$$y'' = - \frac{1}{EI} (M)$$

$$y'' = - \frac{1}{EI} (-20 x)$$

$$y' = \theta = - \frac{1}{EI} (-10 x^2 + c_1)$$

$$y = - \frac{1}{EI} (-10 x^3 / 3 + c_1 x + c_2)$$

عند النقطة A الدوران يساوى صفر

At $x = 3.0 \gggg \theta = 0.0$

$$y' = \theta = -\frac{1}{EI}(-10*3^2 + c_1) = 0.0$$

$$-10*3^2 + c_1 = 0.0$$

$$c_1 = 90.0$$

عند النقطة A الهبوط يساوى صفر

At $x = 3.0 \gggg y = 0.0$

$$y = -\frac{1}{EI}(-10*3^3 / 3 + 90*3 + c_2) = 0.0$$

$$-10*3^3 / 3 + 90*3 + c_2 = 0.0$$

$$c_2 = -180$$

Final equations

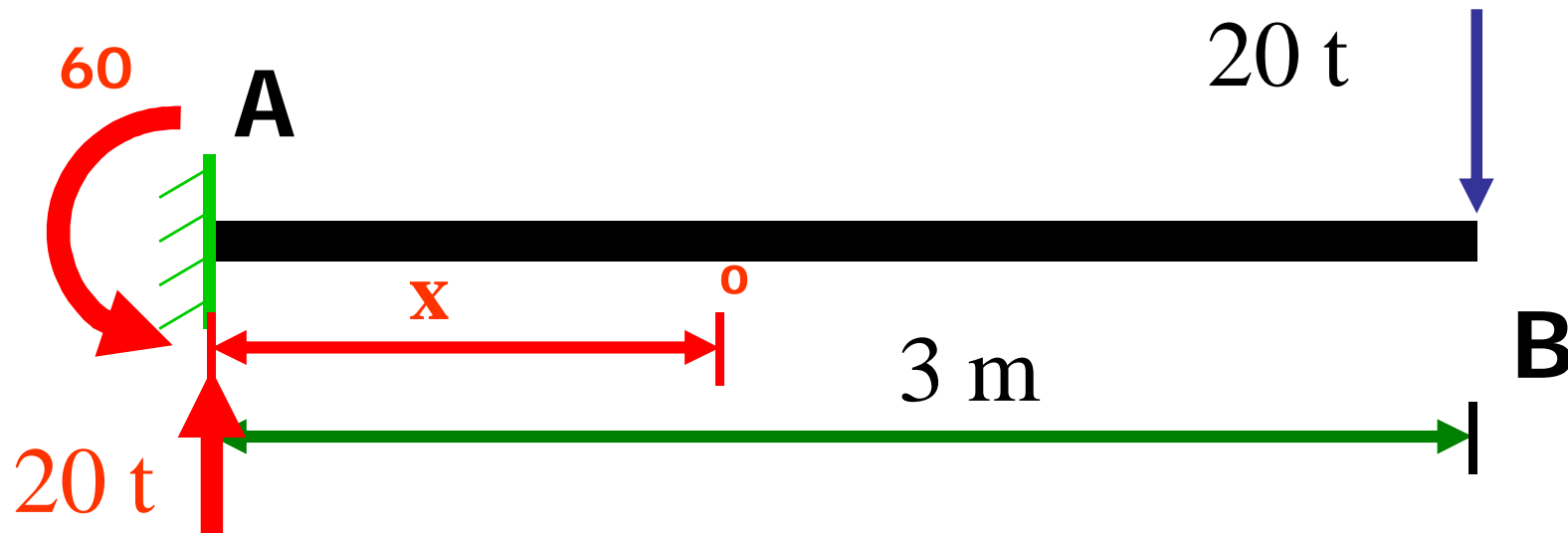
$$\theta = -\frac{1}{EI}(-10x^2 + 90)$$

$$y = -\frac{1}{EI}(-10x^3 / 3 + 90x - 180)$$

at $x=0.0$ >>>>

$$\theta = -\frac{1}{EI}(-10*0^2 + 90) = -\frac{90}{EI}$$

$$y = -\frac{1}{EI}(-10*0^3 / 3 + 90*0 - 180) = \frac{180}{EI}$$



$$M_o = 20 x - 60 \text{ t.m}$$

$$y'' = -\frac{1}{EI}(M)$$

$$y'' = -\frac{1}{EI}(20x - 60)$$

$$y' = \theta = -\frac{1}{EI}(10x^2 - 60x + c_1)$$

$$y = -\frac{1}{EI}(10x^3 / 3 - 30x^2 + c_1x + c_2)$$

At $x = 0.0 \gggg \theta = 0.0$

عند النقطة A الدوران يساوى صفر

$$y' = \theta = -\frac{1}{EI}(10*0^2 - 60*0 + c_1) = 0.0$$

$$10*0^2 - 60*0 + c_1 = 0.0$$

$$c_1 = 0.0$$

At $x = 0.0 \gggg y = 0.0$

عند النقطة A الهبوط يساوى صفر

$$y = -\frac{1}{EI}(10*0^3 / 3 - 30*0^2 + 0*0 + c_2) = 0.0$$

$$10*0^3 / 3 - 30*0^2 + 0*0 + c_2 = 0.0$$

$$c_2 = 0.0$$

Final equations

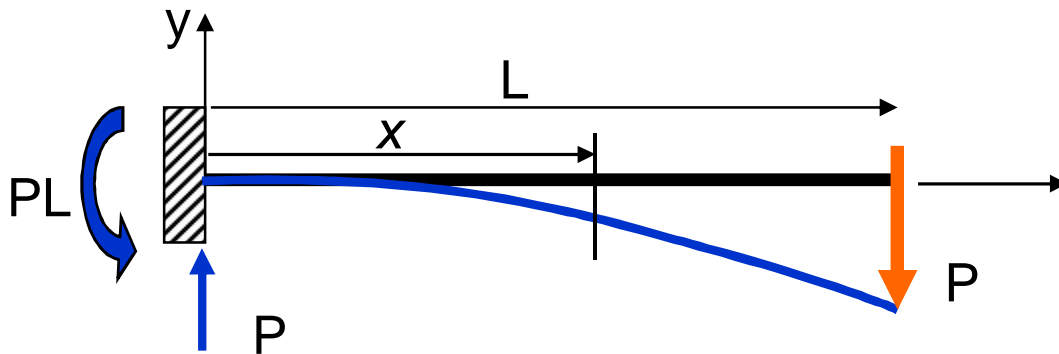
$$\theta = -\frac{1}{EI}(10x^2 - 60x)$$

$$y = -\frac{1}{EI}(10x^3/3 - 30x^2)$$

at $x=3.0$ >>>>

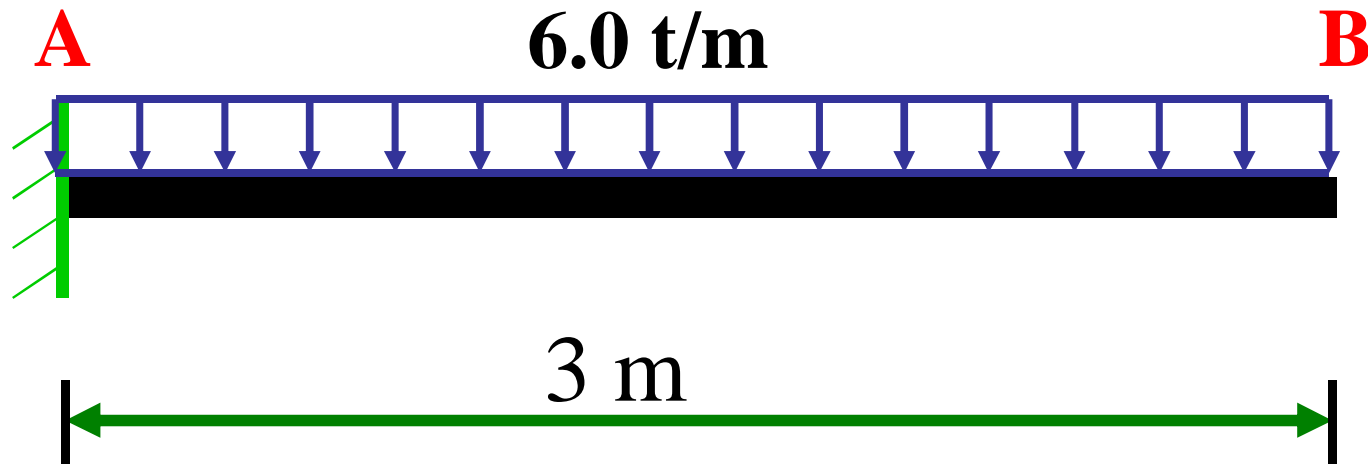
$$\theta = -\frac{1}{EI}(10*3^2 - 60*3) = -\frac{90}{EI}$$

$$y = -\frac{1}{EI}(10*3^3/3 - 30*3^2) = \frac{180}{EI}$$

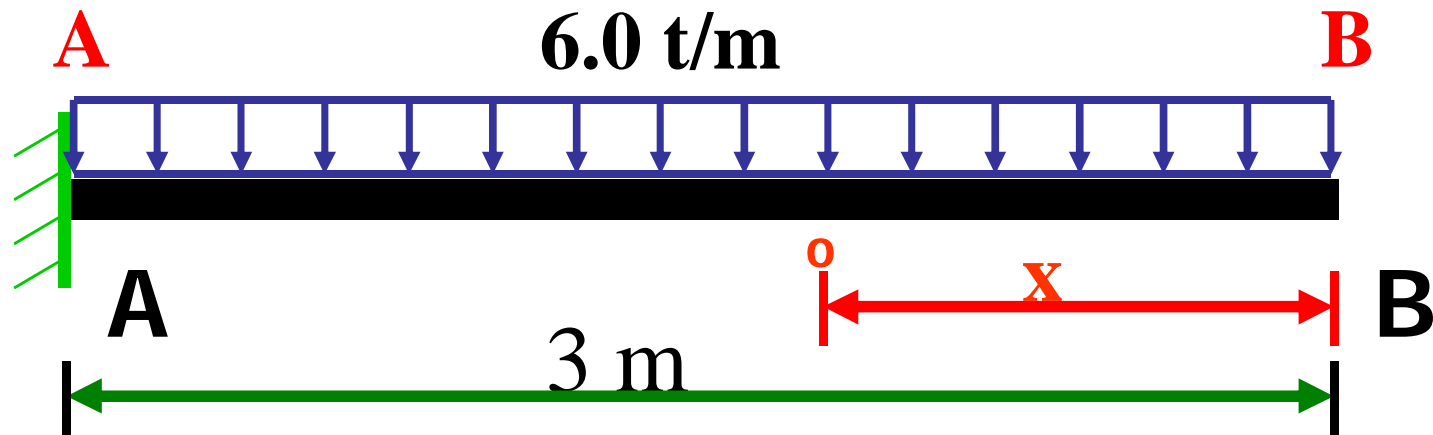


نفس الحل الاول وبالتالي
كان من الافضل اختيار
المعادلة الاولى في الحل

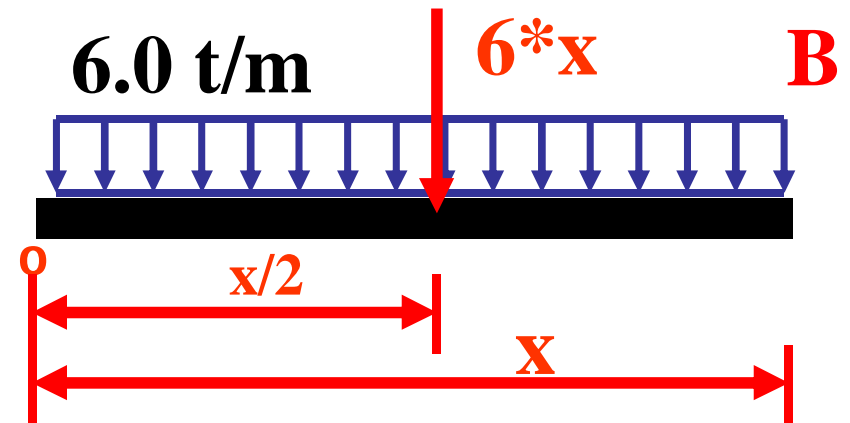
Problem 1



Required the deflection and rotation at point B



$$M_o = - 6 * x * x / 2 = -3x^2$$



Boundary Condition

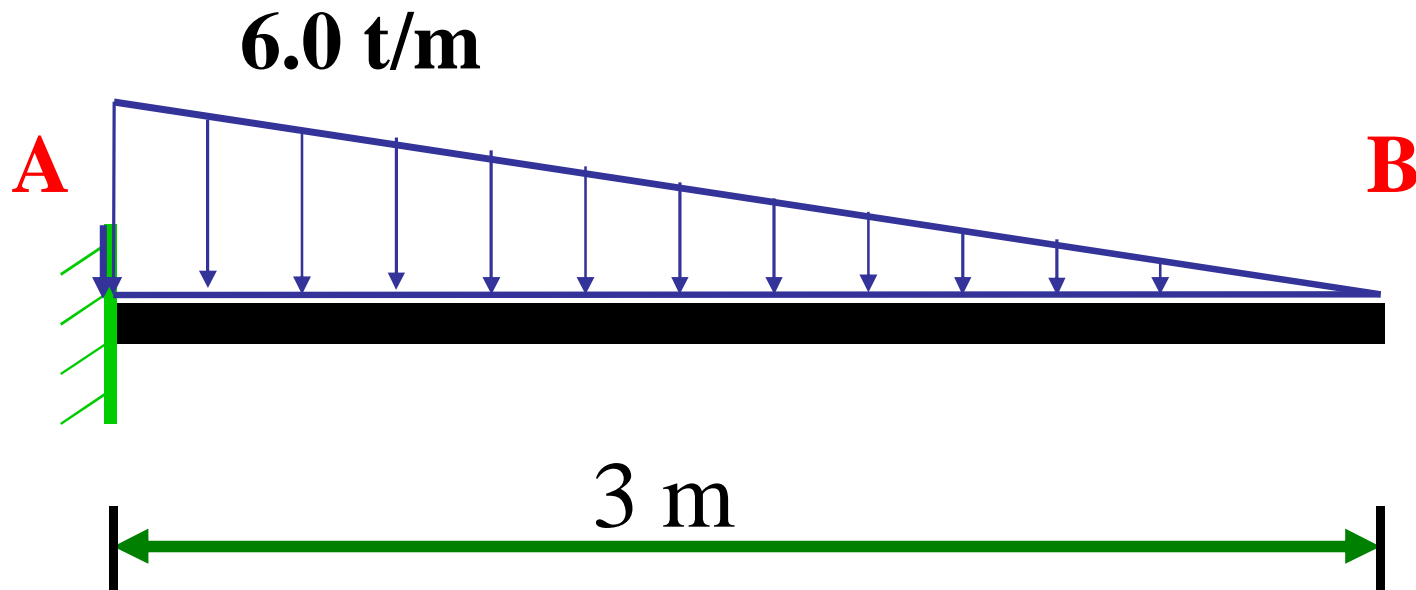
fixed support

at A >>> $x=3.0$ >>>> $\theta=0.0$

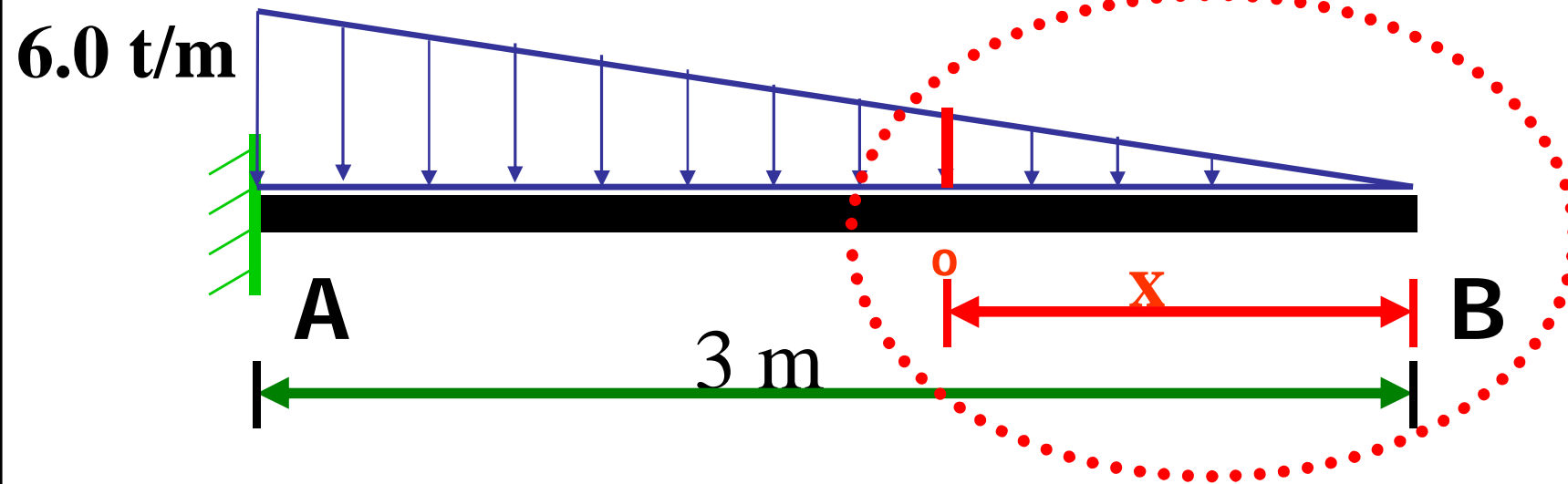
at A >>> $x=3.0$ >>>> $y=0.0$

..... باقي الحل بسيط

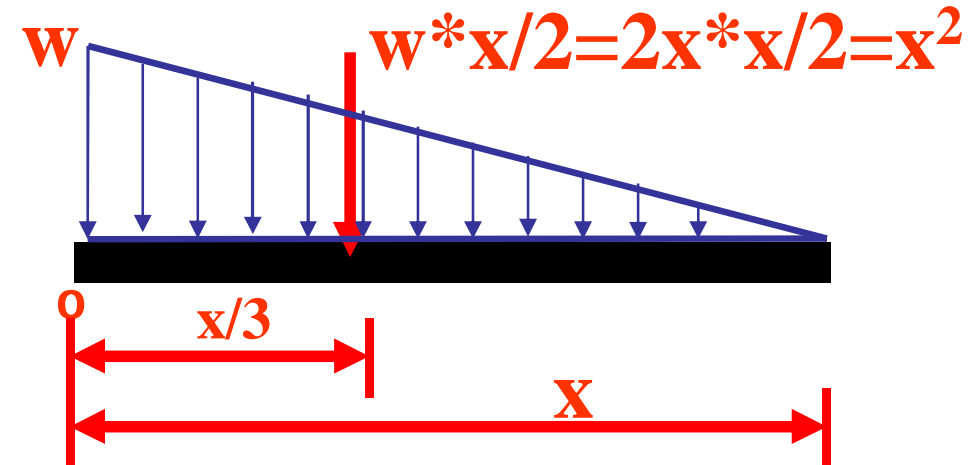
Problem 2



Required the deflection and rotation at point B



$$w/x = 6/3 \gg w = 2x$$



$$M_o = -x^2 * x/3 = -x^3/3$$

Boundary Condition

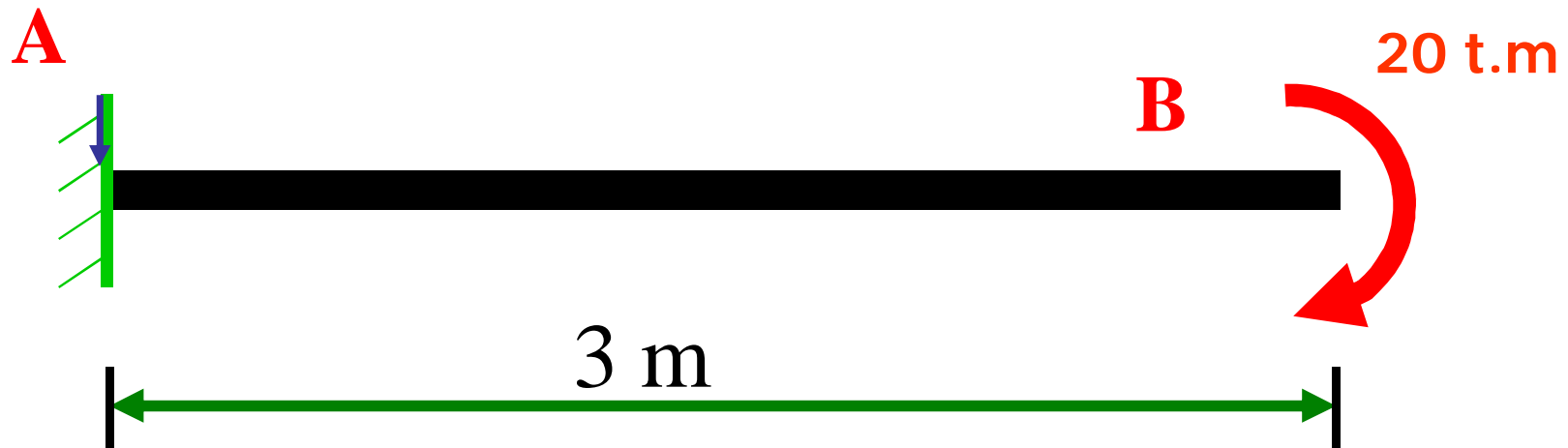
fixed support

at A >>> x=3.0 >>>> $\theta=0.0$

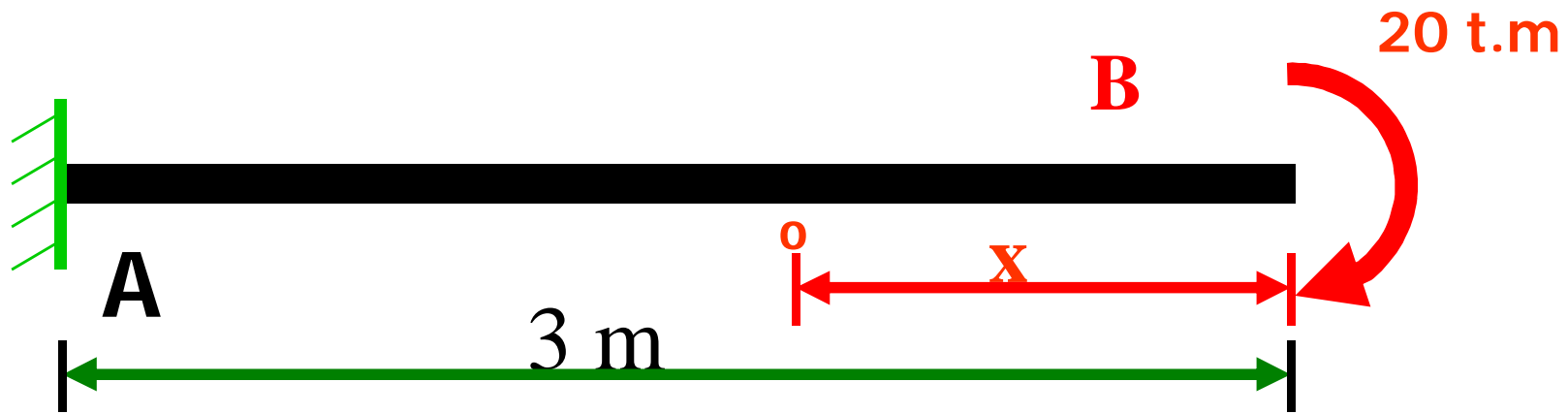
at A >>> x=3.0 >>>> $y=0.0$

..... باقي الحل بسيط

Problem 3



Required the deflection and rotation at point B



$$M_o = - 20$$



Boundary Condition

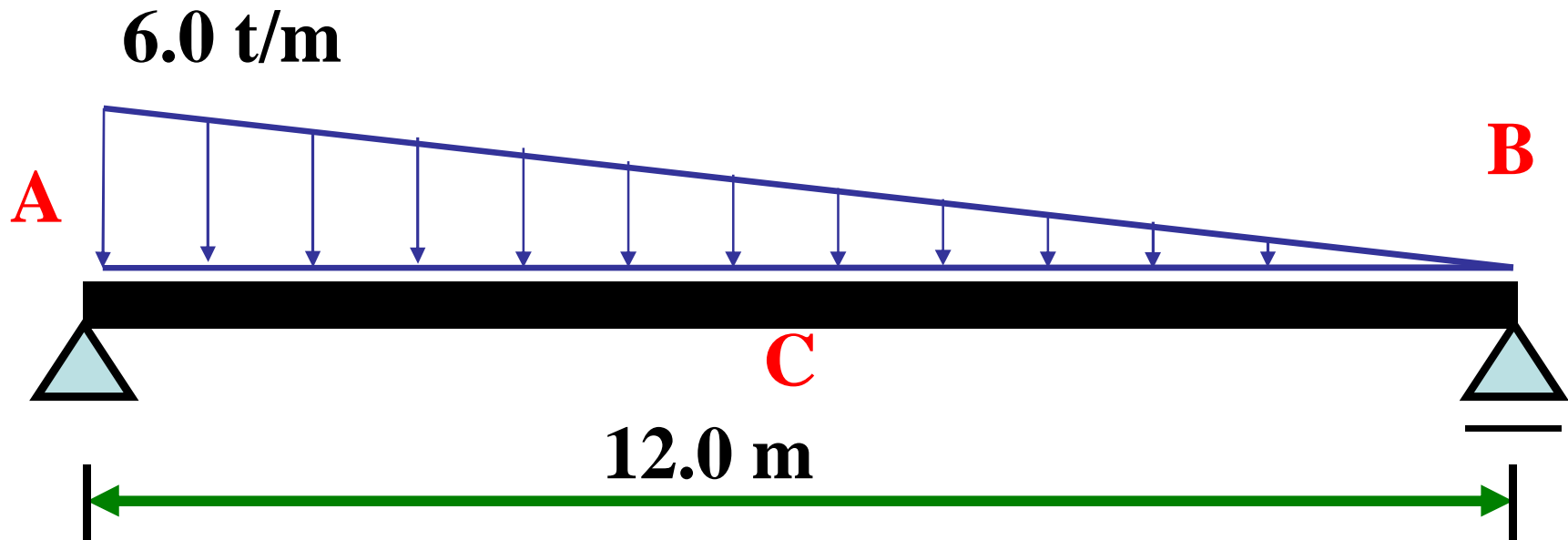
fixed support

at A >>> $x=3.0$ >>>> $\theta=0.0$

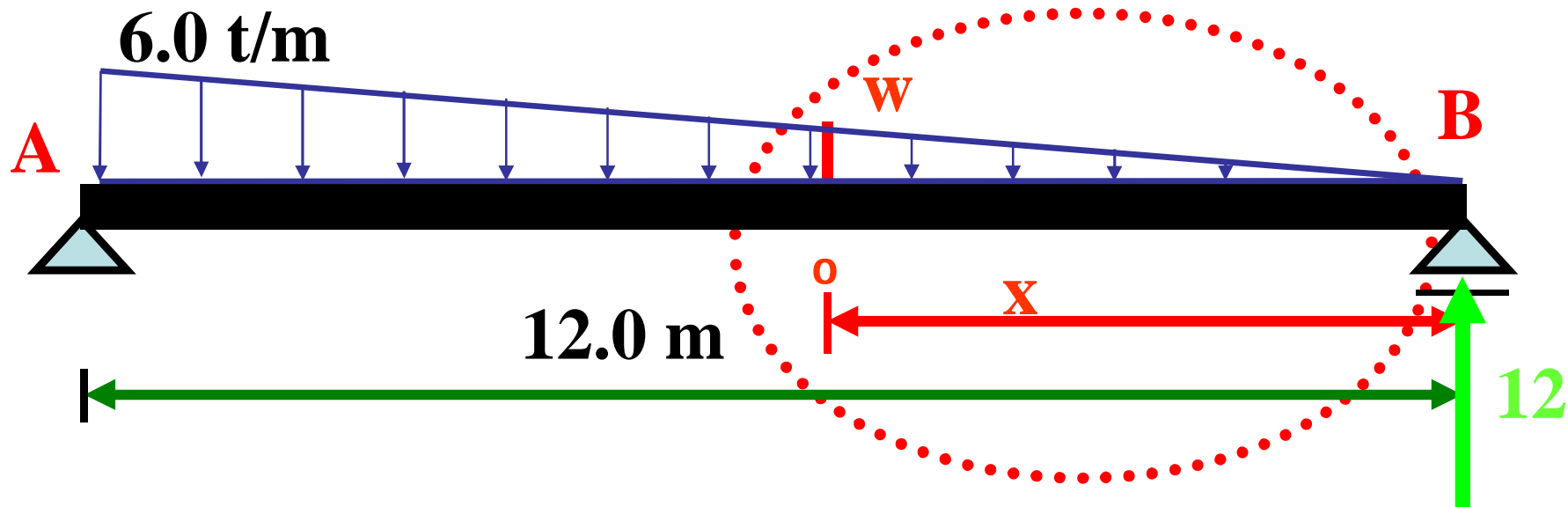
at A >>> $x=3.0$ >>>> $y=0.0$

..... باقي الحل بسيط

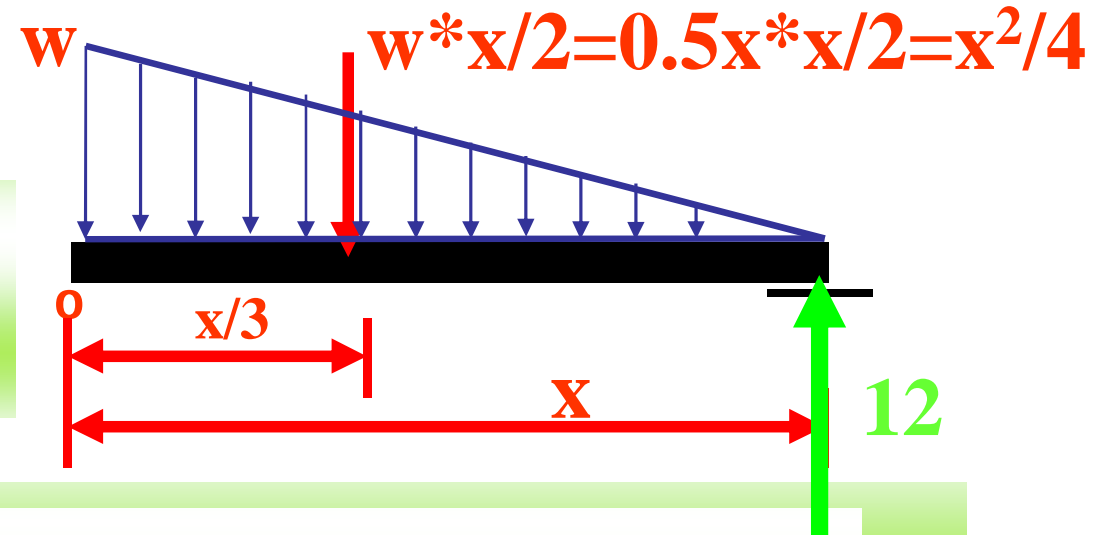
Problem 4



Required the deflection and rotation at point A, B and C



$$w/x = 6/12 \gg w = 0.5x$$



$$M_o = 12x - x^2/4 * x/3$$

$$= 12x - x^3/12$$

Boundary Condition

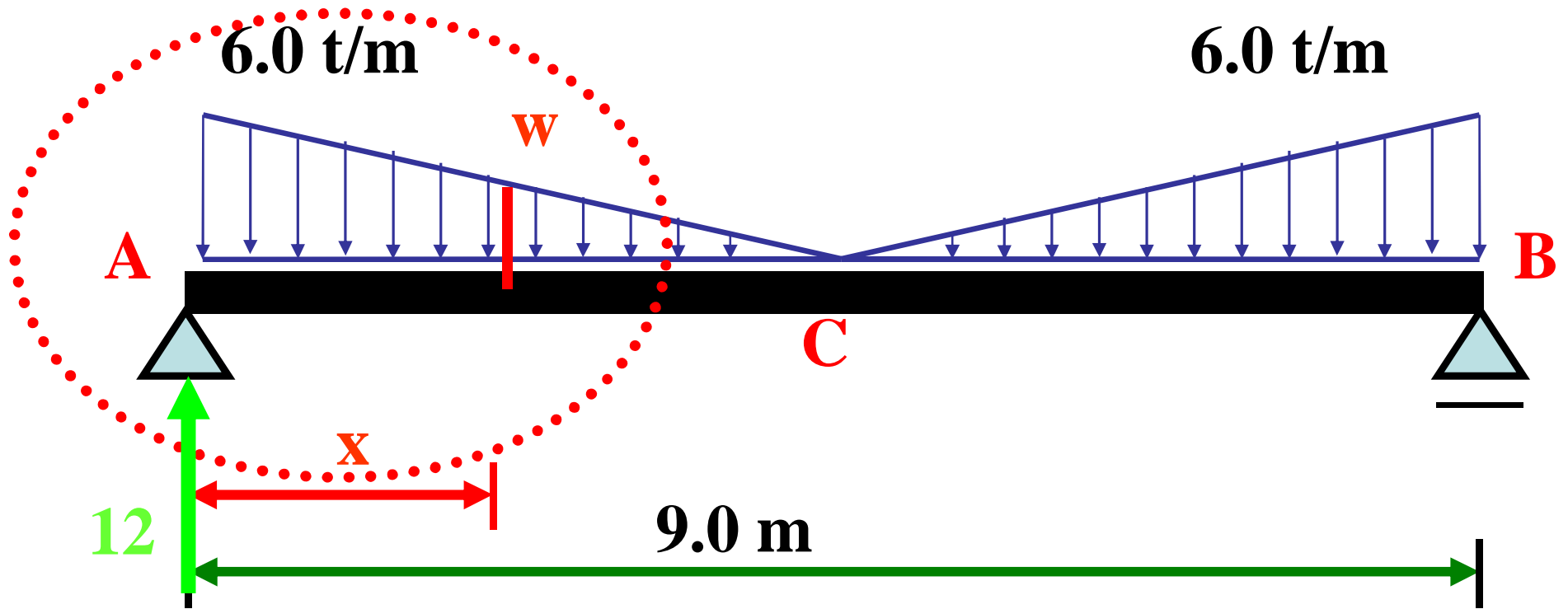
hinged support

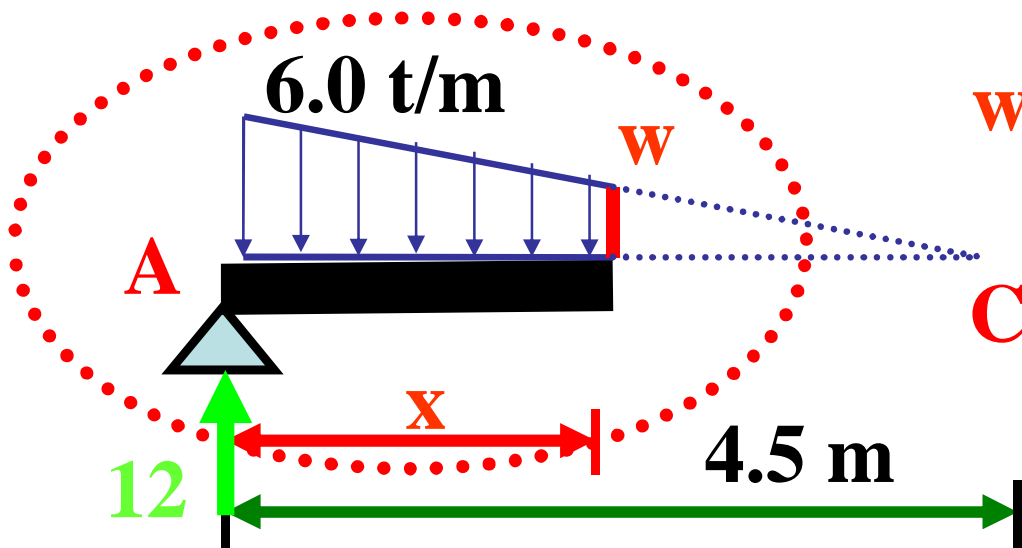
at B $\gg \gg x=0.0 \gg \gg y=0.0$

at A $\gg \gg x=12. \gg \gg y=0.0$

..... باقي الحل بسيط

Problem 5

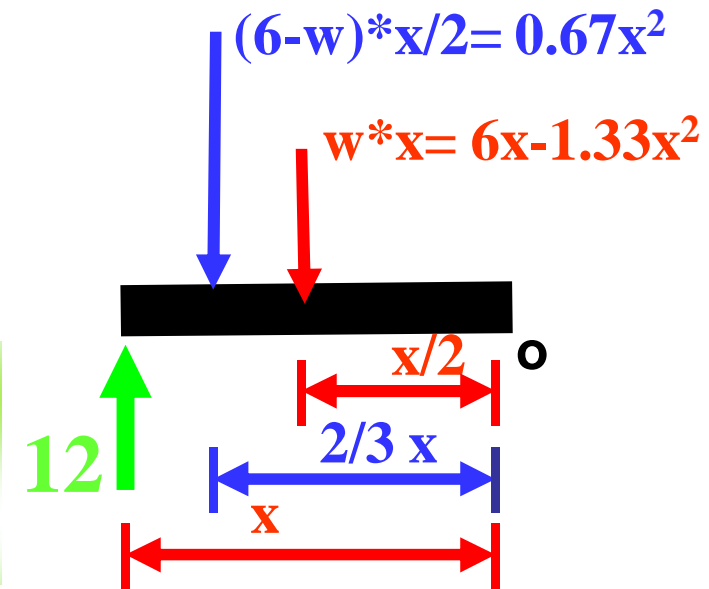




$$w/(4.5-x)=6/4.5 \gg w= 6-1.33x$$

$$M_o = 12x - 0.67x^2 * \frac{2}{3}x - (6x - 1.33x^2) * \frac{1}{2}x$$

$$= 12x - 3x^2 + 0.22x^3$$



Boundary Condition

hinged support

at A $\gg \gg x=0.0 \gg \gg y=0.0$

at C $\gg \gg x=4.5 \gg \gg \theta=0.0$ from symmetry

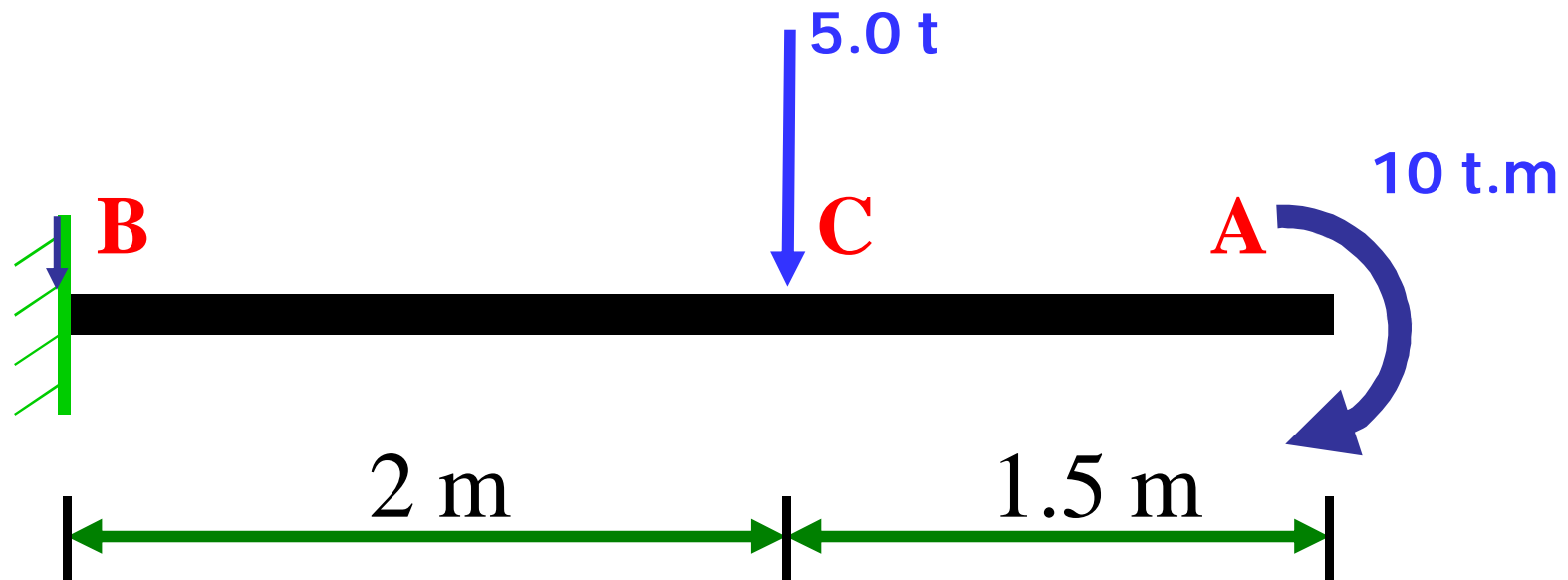
..... باقى الحل بسيط

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

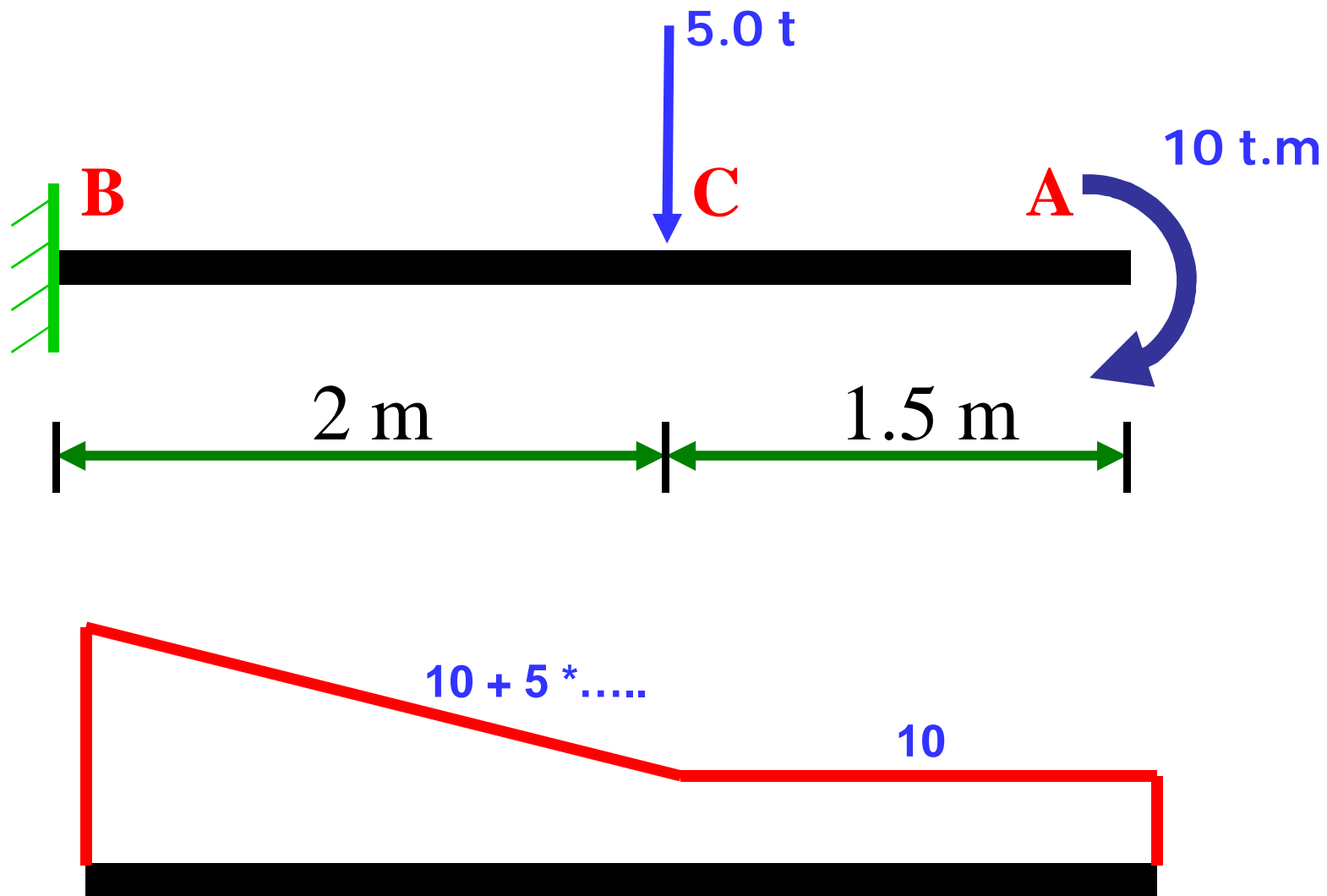
Double Integration With Variable Interval

Eng : Aymman abdo

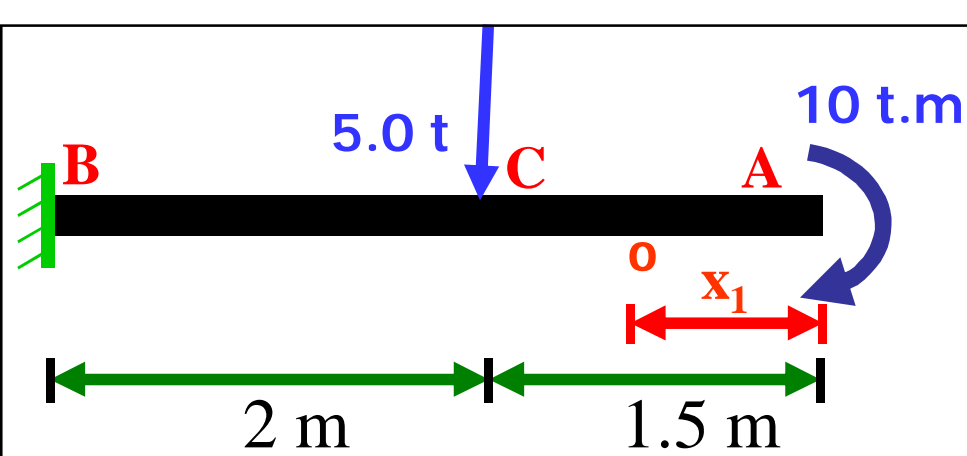
Problem 1



Required the deflection and rotation at point A and C



لو نظرنا الى العزم نجد انه مقسم الى جزئين كل جزء له معادله مختلفة وبالتالي عند التعويض في معادلة **double integration** بالتالي يتم التعويض مرتين مما سيتسبب في وجود **4 boundary conditions**



Part A >> C

$x_1 = 0 \gg x_1 = 1.5$

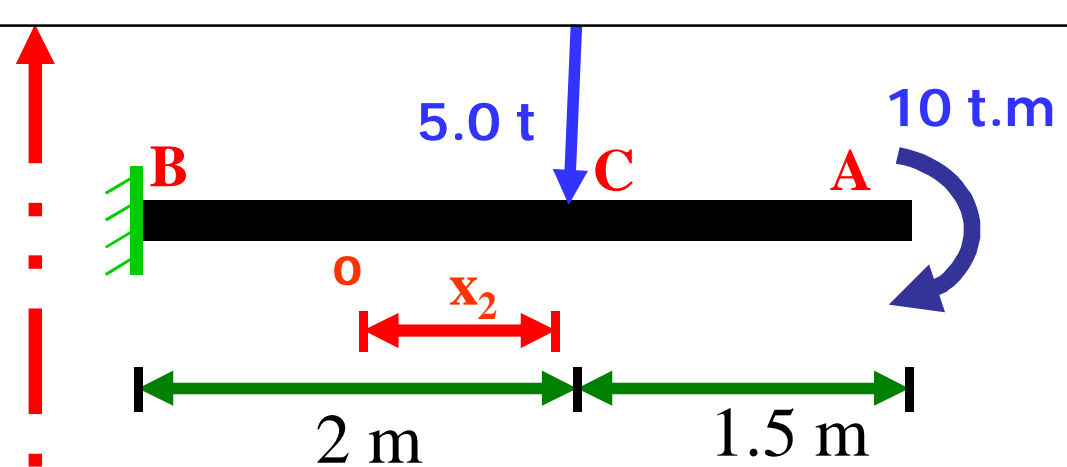
$$M_0 = -10$$

$$y_1'' = -\frac{1}{EI}(M)$$

$$y_1'' = -\frac{1}{EI}(-10)$$

$$y_1' = \theta_1 = -\frac{1}{EI}(-10x_1 + c_1)$$

$$y_1 = -\frac{1}{EI}(-5x_1^2 + c_1x_1 + c_2)$$



Part C >> B

$x_2 = 0 \gg x_2 = 2$

$$M_0 = -10 - 5x_2$$

$$y_2'' = -\frac{1}{EI}(M)$$

$$y_2'' = -\frac{1}{EI}(-10 - 5x_2)$$

$$y_2' = \theta_2 = -\frac{1}{EI}(-10x_2 - 2.5x_2^2 + c_3)$$

$$y_2 = -\frac{1}{EI}(-5x_2^2 - 0.83x_2^3 + c_3x_2 + c_4)$$



At $x_2 = 2.0 \gggg \theta_2 = 0.0$

عند النقطة B الدوران يساوى $\theta_2 =$ صفر

$$y'_2 = \theta_2 = -\frac{1}{EI}(-10x_2 - 2.5x_2^2 + c_3) = 0$$

$$-10*2 - 2.5*2^2 + c_3 = 0$$

$$c_3 = 30$$

At $x_2 = 2.0 \gggg y_2 = 0.0$

عند النقطة B الهبوط يساوى $y_2 =$ صفر

$$y_2 = -\frac{1}{EI}(-5x_2^2 - 0.83x_2^3 + c_3x_1 + c_4) = 0.0$$

$$-5*2^2 - 0.83*2^3 + 30*2 + c_4 = 0.0$$

$$c_4 = -33.33$$

عند النقطة C الدوران يساوى $\theta_1 = \theta_2$ At $x_1 = 1.5$ & $x_2 = 0.0 \gg \theta_1 = \theta_2$

$$\theta_1(x_1 = 1.5) = -\frac{1}{EI}(-10x_1 + c_1) = -\frac{1}{EI}(-10*1.5 + c_1) \gg 1$$

$$\theta_2(x_2 = 0) = -\frac{1}{EI}(-10x_2 - 2.5x_2^2 + 30) = -\frac{1}{EI}(+30) \gg 2$$

From 1 & 2

$$-\frac{1}{EI}(-10*1.5 + c_1) = -\frac{1}{EI}(+30) \gg \gg \gg c_1 = 45$$

عند النقطة C الدوران يساوى $y_1 = y_2$ At $x_1 = 1.5$ & $x_2 = 0.0 \gg y_1 = y_2$

$$y_1(x_1 = 1.5) = -\frac{1}{EI}(-5x_1^2 + c_1x_1 + c_2) = -\frac{1}{EI}(-5*1.5^2 + 45*1.5 + c_2) \gg \gg 1$$

$$y_2(x_2 = 0) = -\frac{1}{EI}(-5x_2^2 - 0.83x_2^3 + c_3x_2 + c_4) = -\frac{1}{EI}(-33.33) \gg \gg 2$$

From 1 & 2

$$-\frac{1}{EI}(-5*1.5^2 + 45*1.5 + c_2) = -\frac{1}{EI}(-33.33) \gg \gg \gg c_2 = -89.6$$

Final equations

Part A >> C

$x_1 = 0 \gg x_1 = 1.5$

$$y'_1 = \theta_1 = -\frac{1}{EI}(-10x_1 + 45)$$

$$y_1 = -\frac{1}{EI}(-5x_1^2 + 45x_1 - 89.6)$$

at A >> $x_1 = 0.0$

$$\theta_A = -\frac{1}{EI}(-10*0 + 45) = -\frac{45}{EI}$$

$$y_A = -\frac{1}{EI}(-5*0 + 45*0 - 89.6) = \frac{89.6}{EI}$$

at C >> $x_1 = 1.5$

$$\theta_c = -\frac{1}{EI}(-10*1.5 + 45) = -\frac{30}{EI}$$

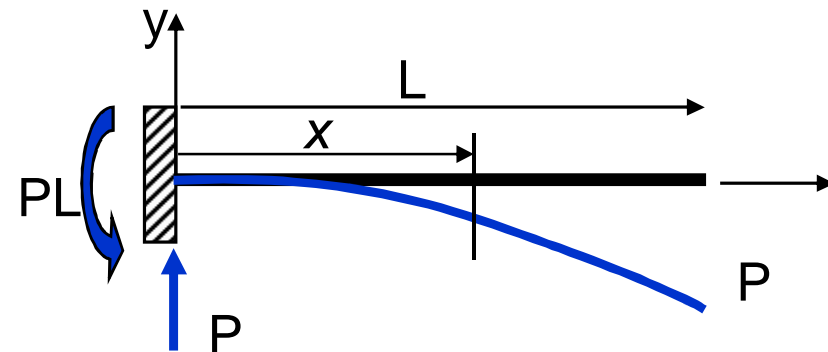
$$y_c = -\frac{1}{EI}(-5*1.5^2 + 45*1.5 - 89.6) = \frac{33.3}{EI}$$

Part C >> B

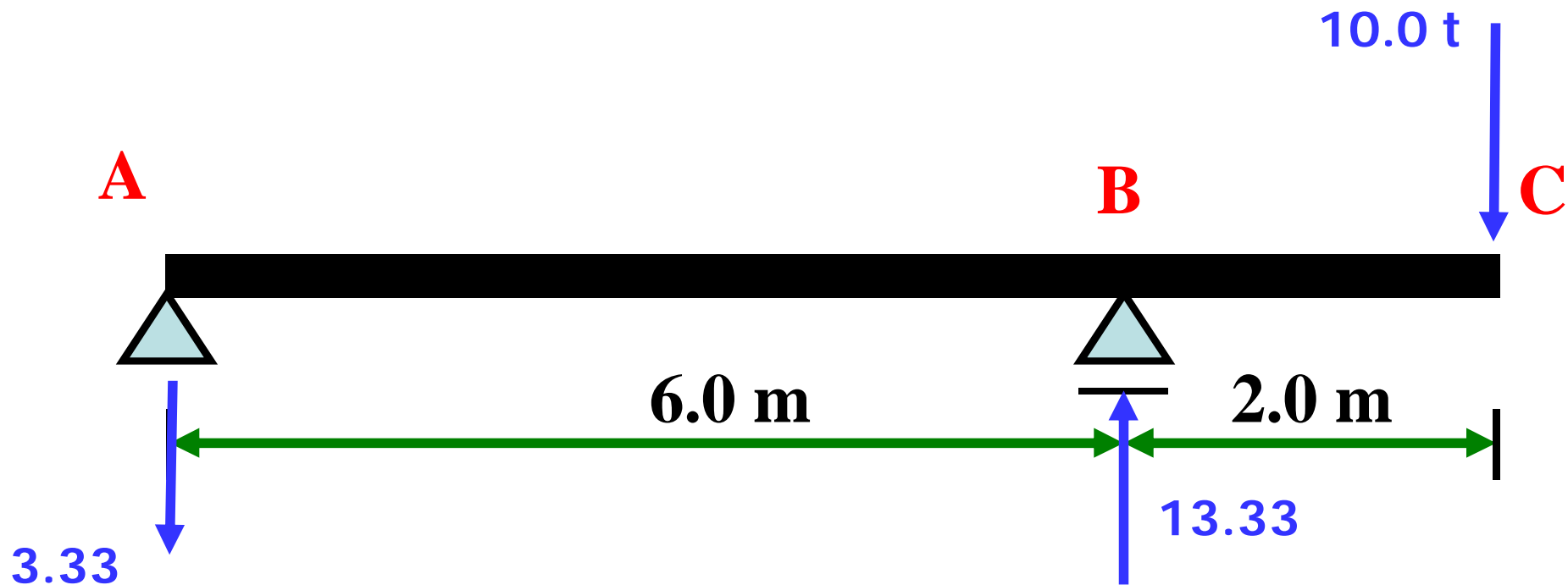
$x_2 = 0 \gg x_2 = 2$

$$y'_2 = \theta_2 = -\frac{1}{EI}(-10x_2 - 2.5x_2^2 + 30)$$

$$y_2 = -\frac{1}{EI}(-5x_2^2 - 0.83x_2^3 + 30x_1 - 33.3)$$



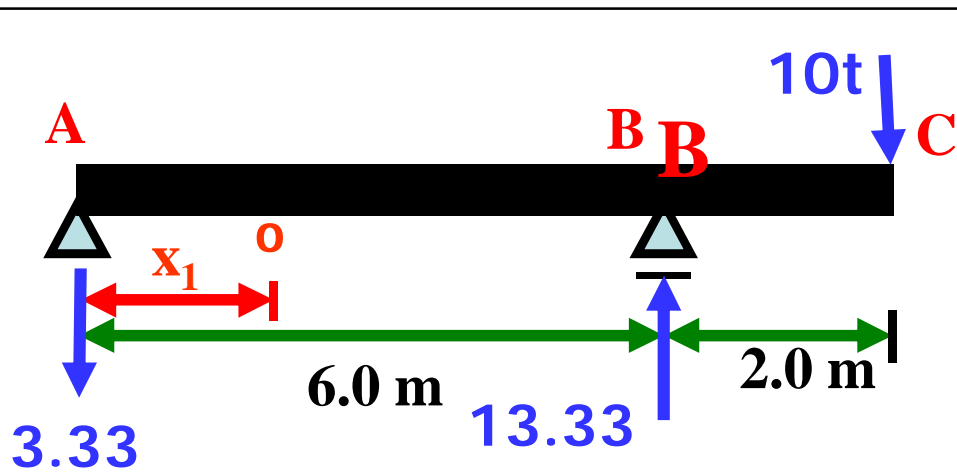
Problem 2



$$\sum M_A = 0.0 \quad \gg \quad Y_B * 6 = 10 * 8$$
$$Y_B = 13.33$$

$$\sum Y = 0.0 \quad \gg \quad Y_A = 3.33$$





Part A >> B

$x_1 = 0 \gg x_1 = 6.0$

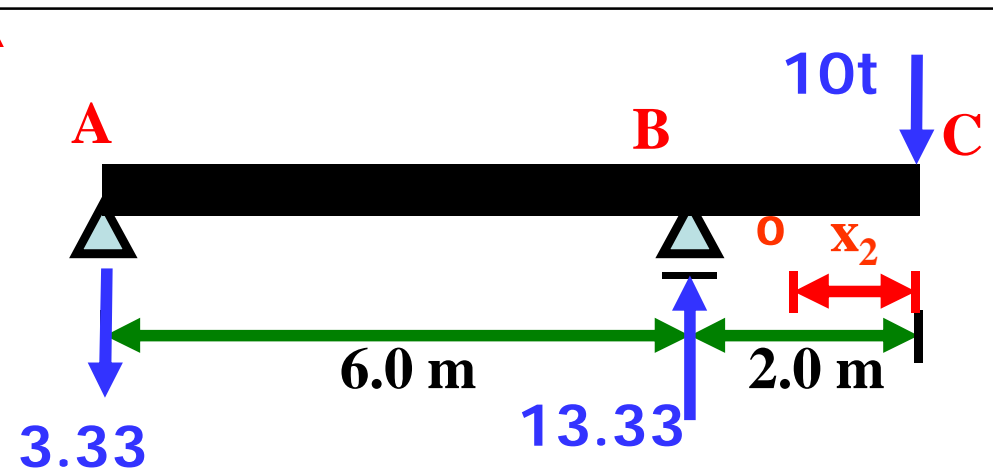
$$\mathbf{M_o = -3.3 \, x_1}$$

$$y_1'' = -\frac{1}{EI}(M)$$

$$y_1'' = -\frac{1}{EI}(-3.33x_1)$$

$$y_1' = \theta_1 = -\frac{1}{EI}(-1.67x_1^2 + c_1)$$

$$y_1 = -\frac{1}{EI}(-0.55x_1^3 + c_1x_1 + c_2)$$



Part C >> B

$x_2 = 0 \gg x_2 = 2$

$$\mathbf{M_o = -10 \, x_2}$$

$$y_2'' = -\frac{1}{EI}(M)$$

$$y_2'' = -\frac{1}{EI}(-10x_2)$$

$$y_2' = \theta_2 = -\frac{1}{EI}(-5x_2^2 + c_3)$$

$$y_2 = -\frac{1}{EI}(1.67x_2^3 + c_3x_2 + c_4)$$

At $x_1 = 0.0 \gggg y_1 = 0.0$

عند النقطة A الهبوط يساوى $y_1 = \text{صفر}$

$$y_1 = -\frac{1}{EI}(0+0+c_2)=0 \gggg c_2 = 0$$

At $x_1 = 6$ & $x_2 = 2 \gggg y_1 = y_2 = 0.0$

$\gggg \theta_1 = \theta_2$

At $x_1 = 6 \gggg y_1 = 0.0$

$$y_1 = -\frac{1}{EI}(-0.55x_1^3 + c_1x_1 + c_2) = 0$$

$$-0.55*6^3 + c_1*6 + 0 = 0 \gggg c_1 = 20$$

At $x_2 = 2 \gggg y_2 = 0.0$

$$y_2 = -\frac{1}{EI}(1.67x_2^3 + c_3x_2 + c_4) = 0$$

$$1.67*2^3 + c_3*2 + c_4 = 0 \ggg 2c_3 + c_4 = 13.33 \gg 1$$

At $x_1 = 6$ & $x_2 = 2 \gggg \theta_1 = \theta_2$

$$-\frac{1}{EI}(-1.67*6^2 + 20) = -\frac{1}{EI}(-5*2^2 + C_3) \gg C_3 = -20 \gg C_4 = 53.33$$

Final equations

Part A >> B

$x_1 = 0 \gg x_1 = 6.0$

$$y'_1 = \theta_1 = -\frac{1}{EI}(-1.67 x_1^2 + 20)$$

$$y_1 = -\frac{1}{EI}(-0.56 x_1^3 + 20x_1)$$

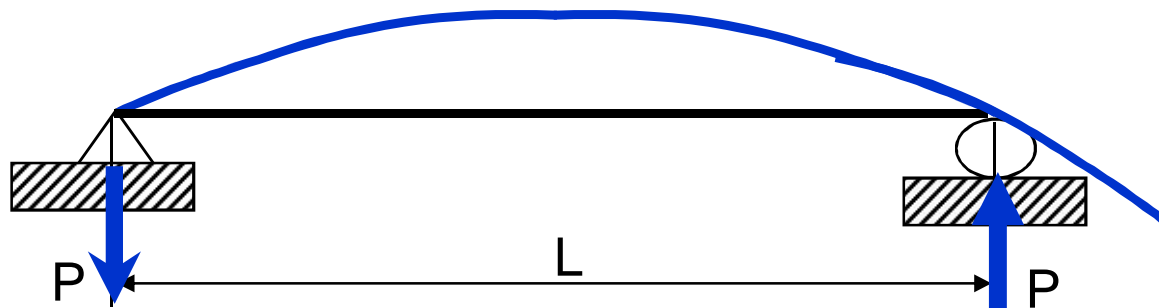
Part C >> B

$x_2 = 0 \gg x_2 = 2$

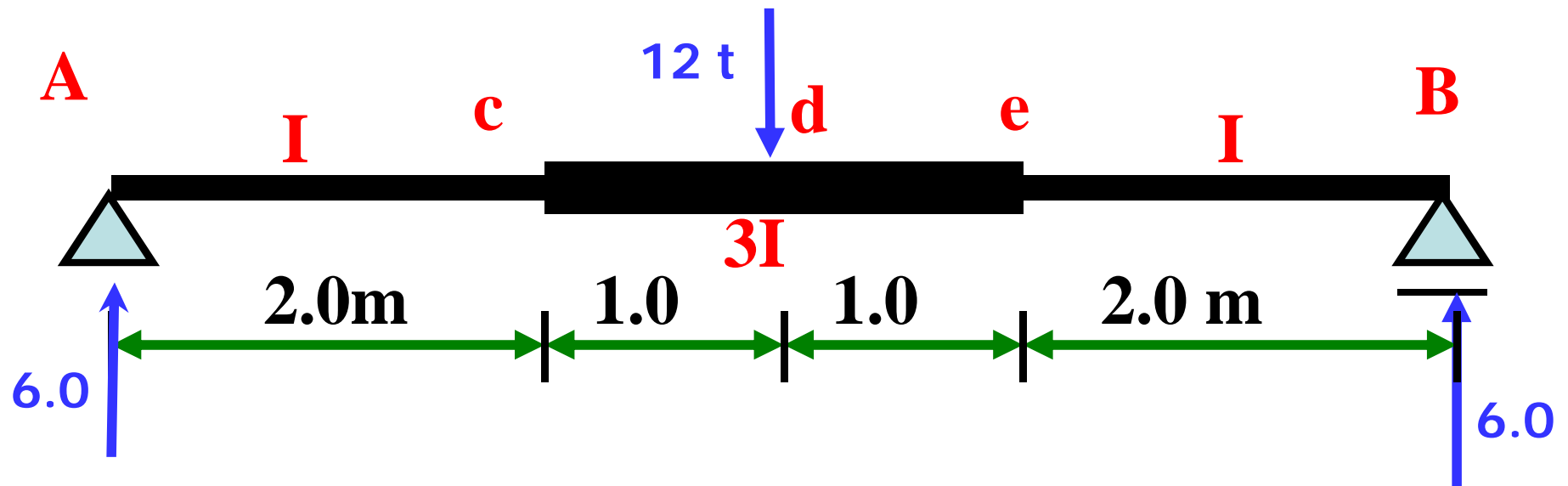
$$y'_2 = \theta_2 = -\frac{1}{EI}(-5 x_2^2 - 20)$$

$$y_2 = -\frac{1}{EI}(-1.67 x_2^3 - 20x_2 + 53.33)$$

بالتعويض باحداثى اى نقطه يمكن
الحصول على الهبوط والدوران عندها



Problem 3



معنى ان القطاع $3I$ يعنى الـ inertia بتضاعف ثلاث مرات

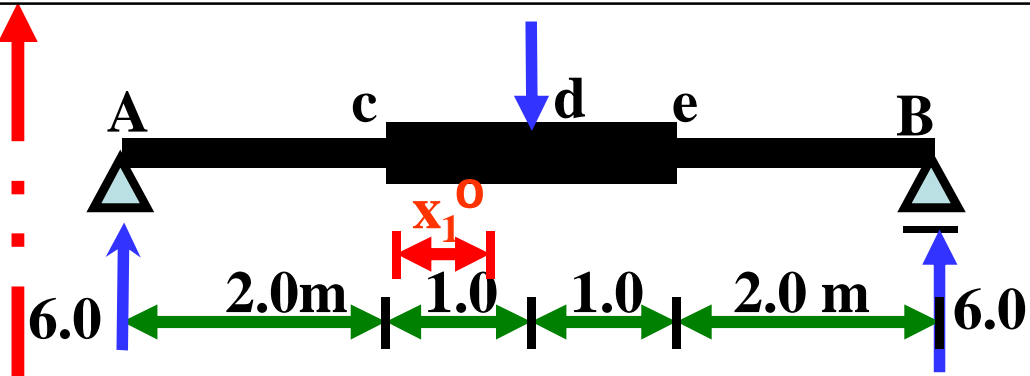
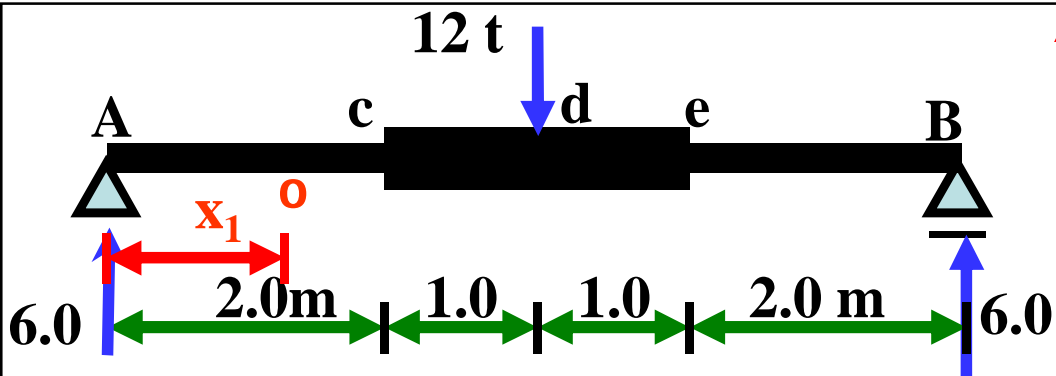


I

>>>>



$3I$



Part A >> C

$x_1 = 0 \gg x_1 = 2.0$

$$M_o = 6 x_1$$

$$y_1'' = -\frac{1}{EI}(M)$$

$$y_1'' = -\frac{1}{EI}(6x_1)$$

$$y_1' = \theta_1 = -\frac{1}{EI}(3x_1^2 + c_1)$$

$$y_1 = -\frac{1}{EI}(x_1^3 + c_1 x_1 + c_2)$$

Part C >> d

$x_2 = 0 \gg x_2 = 1$

$$M_o = 6*(2 + x_2) = 12 + 6x_2$$

$$y_2'' = -\frac{1}{3EI}(M)$$

$$y_2'' = -\frac{1}{3EI}(6x_2 + 12)$$

$$y_2' = \theta_2 = -\frac{1}{3EI}(12x_2 + 3x_2^2 + c_3)$$

$$y_2 = -\frac{1}{3EI}(6x_2^2 + x_2^3 + c_3 x_2 + c_4)$$

At $x_1 = 0.0 \gggg y_1 = 0.0$

عند النقطة A الهبوط يساوى $y_1 =$ صفر

$$y_1 = -\frac{1}{EI}(x_1^3 + c_1 x_1 + c_2) = 0.0 \ggg 0 + c_1 * 0 + c_2 = 0.0 \gg \boxed{c_2 = 0.0}$$

At $x_2 = 1.0 \gggg \theta_2 = 0.0$

عند النقطة d الدوران يساوى $\theta_2 =$ صفر
بسبب وجود تماثل عند هذه النقطة

$$y'_2 = \theta_2 = -\frac{1}{3EI}(12x_2 + 3x_2^2 + c_3) = 0 \ggg 12*1 + 3*1^2 + c_3 = 0.0 \gg \boxed{c_3 = -15}$$

عند النقطة C الدوران يساوى $\theta_1 = \theta_2$ At $x_1 = 2$ & $x_2 = 0.0 \gg \theta_1 = \theta_2$

$$\theta_1(x_1 = 2) = -\frac{1}{EI}(3*2^2 + c_1) = -\frac{1}{EI}(12 + c_1) \gg 1$$

$$\theta_2(x_2 = 0) = -\frac{1}{3EI}(12*0 + 3*0 - 15) = -\frac{1}{3EI}(-15) \gg 2$$

From 1 & 2

$$-\frac{1}{EI}(12 + c_1) = -\frac{1}{3EI}(-15) \gg \gg \gg \boxed{c_1 = -17}$$

عند النقطة C الدوران يساوى $y_1 = y_2$ At $x_1 = 1.5$ & $x_2 = 0.0 \gg y_1 = y_2$

$$y_1(x_1 = 2) = -\frac{1}{EI}(2^3 - 17*2 + 0) = -\frac{1}{EI}(-26) \gg \gg 1$$

$$y_2(x_2 = 0) = -\frac{1}{3EI}(0 + 0 + 0 + c_4) = -\frac{1}{3EI}(c_4) \gg \gg 2$$

From 1 & 2

$$c_4 = 3 * -26 = \boxed{-78}$$

Final equations

Part A >> c

$x_1 = 0 \gg x_1 = 2.0$

$$\theta_1 = -\frac{1}{EI}(3x_1^2 - 17)$$

$$y_1 = -\frac{1}{EI}(x_1^3 - 17x_1)$$

Part c >> d

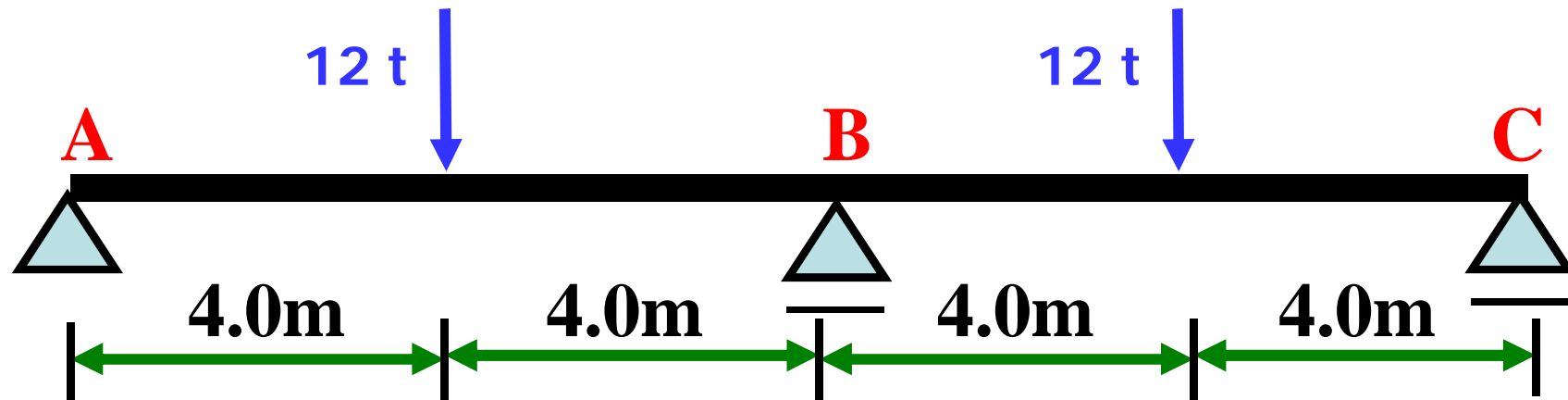
$x_2 = 0 \gg x_2 = 1$

$$\theta_2 = -\frac{1}{3EI}(12x_2 + 3x_2^2 - 15)$$

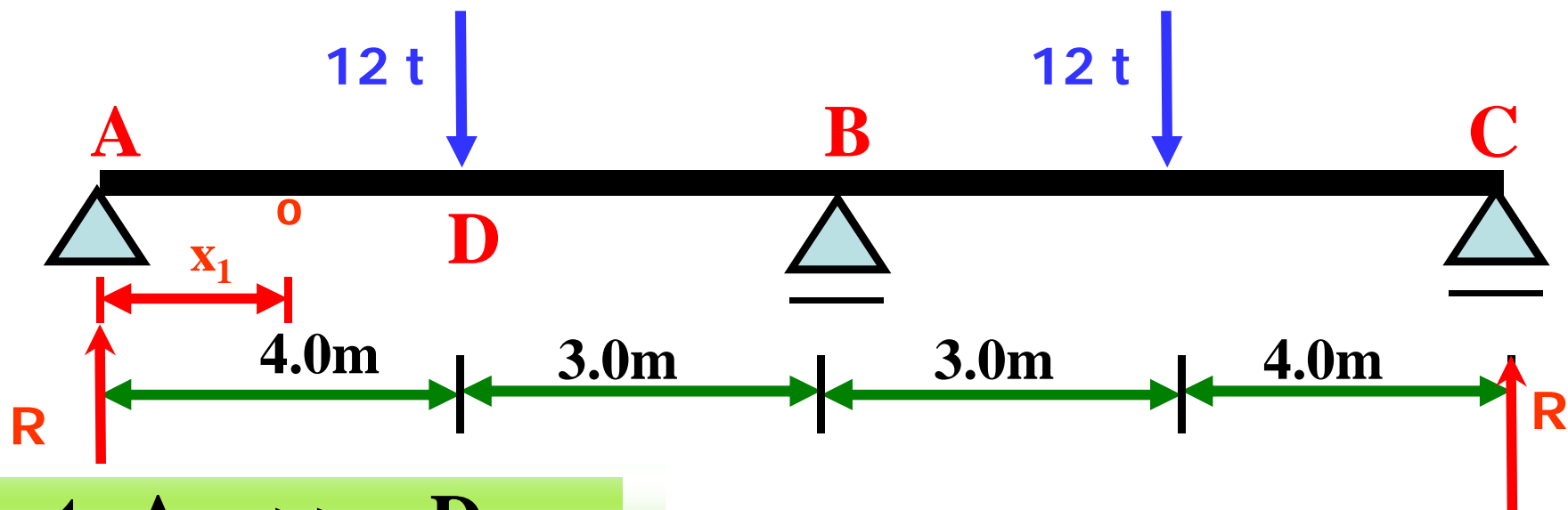
$$y_2 = -\frac{1}{3EI}(6x_2^2 + x_2^3 - 15x_2 - 78)$$

بالتعويض باحداثى اى نقطه يمكن
الحصول على الهبوط والدوران عندها

Problem 3



المطلوب رسم العزم لهذه الكمرة الغير محدده استاتيكيًا
مع العلم ان الهبوط عند النقطة B يساوى صفر



Part A >> D

$x_1 = 0 \gg x_1 = 4.0$

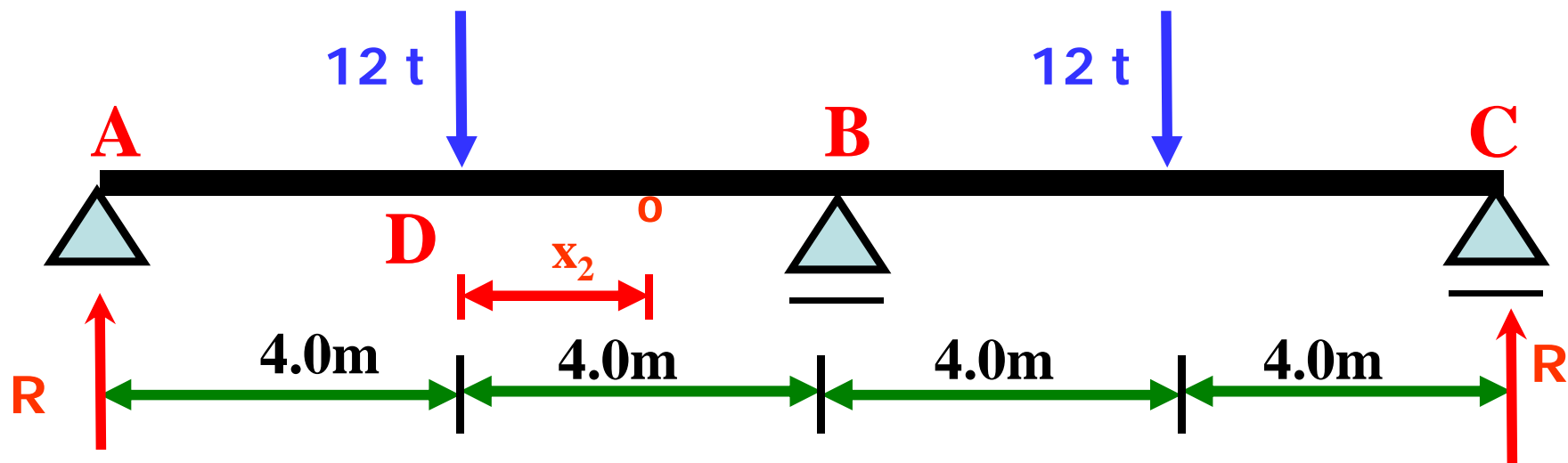
$$\mathbf{M_0 = R x_1}$$

$$y_1'' = -\frac{1}{EI}(M)$$

$$y_1'' = -\frac{1}{EI}(Rx_1)$$

$$y_1' = \theta_1 = -\frac{1}{EI}(0.5Rx_1^2 + c_1)$$

$$y_1 = -\frac{1}{EI}(0.167Rx_1^3 + c_1x_1 + c_2)$$



Part D >> B

$x_2 = 0 >> x_2 = 3$

$$M_o = R \cdot (4 + x_2) + 12 \cdot x_2 = 4R + Rx_2 + 12x_2$$

$$y_2'' = -\frac{1}{EI}(M)$$

$$y_2'' = -\frac{1}{EI}(4R + Rx_2 + 12x_2)$$

$$y_2' = \theta_2 = -\frac{1}{EI}(4Rx_2 + 0.5Rx_2^2 + 6x_2^2 + c_3)$$

$$y_2 = -\frac{1}{EI}(2Rx_2^2 + 0.167Rx_2^3 + 2x_2^3 + c_3x_2 + c_4)$$

At $x_1 = 0.0 \gg \gg \gg y_1 = 0.0$

عند النقطة A الهبوط يساوى $y_1 = 0$ صفر

$$y_1 = -\frac{1}{EI}(0.167R * 0 + c_1 * 0 + c_2) = 0 \gg c_2 = 0$$

At $x_1 = 1.0$

عند النقطة D الدوران يساوى $\theta_1 = \theta_2$
بسبب وجود تماثل عند هذه النقطة

$\gg \gg \gg \theta_1 = \theta_2$

At $x_2 = 1.0$

$$\theta_1[(x_1 = 4)] = -\frac{1}{EI}(0.5R * 4^2 + c_1) = \theta_2[x_2 = 0] = -\frac{1}{EI}(4R * 0 + 0.5R * 0 + 6 * 0 + c_3)$$

$$8R + c_1 = c_3 \gg 1$$

At $x_1 = 1.0$

عند النقطة D الدوران يساوى $y_1 = y_2$
بسبب وجود تماثل عند هذه النقطة

$\gg \gg \gg y_1 = y_2$

At $x_2 = 1.0$

$$y_1 = -\frac{1}{EI}(0.167R * 4^3 + c_1 * 4 + 0) = y_2 = -\frac{1}{EI}(2R * 0 + 0.167R * 0 + 2 * 0 + c_3 * 0 + c_4) = -\frac{1}{EI}(c_4)$$

$$10.67R + 4c_1 = c_4 \gg 2$$

At $x_2 = 4.0 \gggg \theta_2 = 0.0$

عند النقطة B الهبوط يساوى $\theta_2 =$ صفر

$$\theta_2 = -\frac{1}{3EI}(4R * 4 + 0.5R * 16 + 6 * 16 + c_3) = 0$$

$$4R * 4 + 0.5R * 16 + 6 * 16 + c_3 = 0$$

$$c_3 = -24R + 96 \gggg 3$$

At $x_2 = 4.0 \gggg y_2 = 0.0$

عند النقطة B الهبوط يساوى $y_2 =$ صفر

$$y_2 = -\frac{1}{EI}(2R * 16 + 0.167R * 64 + 2 * 64 + c_3 * 4 + c_4) = 0$$

$$2R * 16 + 0.167R * 64 + 2 * 64 + c_3 * 4 + c_4 = 0$$

$$42.66R + 128 + c_3 * 4 + c_4 = 0 \gggg 4$$

$$8R + c_1 = c_3 \gg 1$$

$$10.67R + 4c_1 = c_4 \gg 2$$

$$c_3 = -24R + 96 \ggg 3$$

$$42.66R + 128 + c_3 * 4 + c_4 = 0 \ggg 4$$

From 3 in 1

$$8R + c_1 = -24R + 96$$

$$\ggg c_1 = -32R + 96 \gg 5$$

From 5 in 2

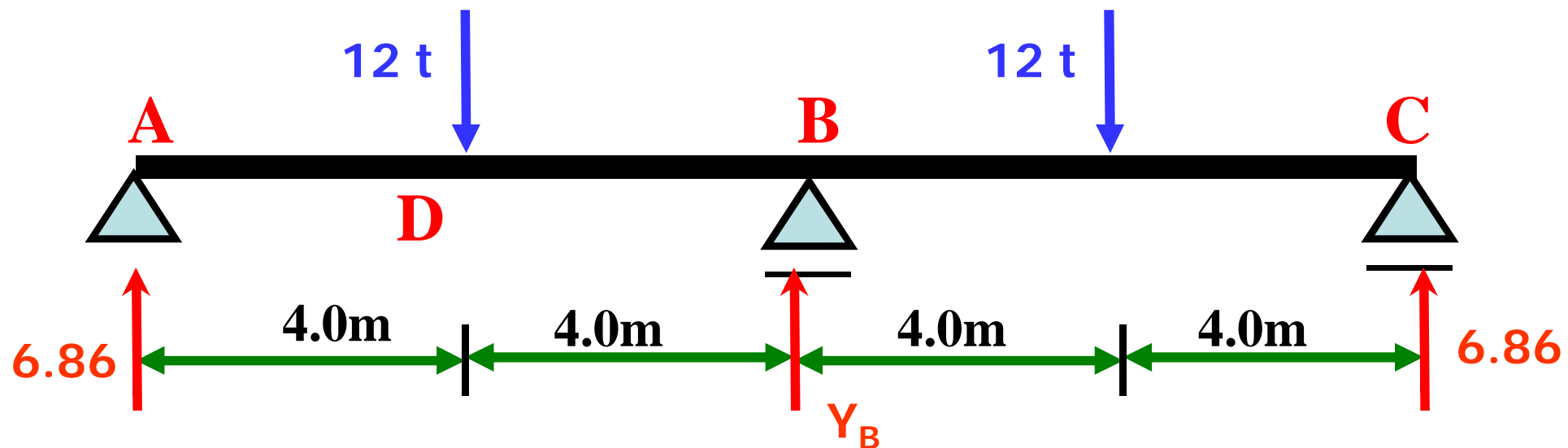
$$10.67R + 4(-32R + 96) = c_4$$

$$\ggg c_4 = -117.33R + 384 \gg 6$$

From 5, 6 in 4

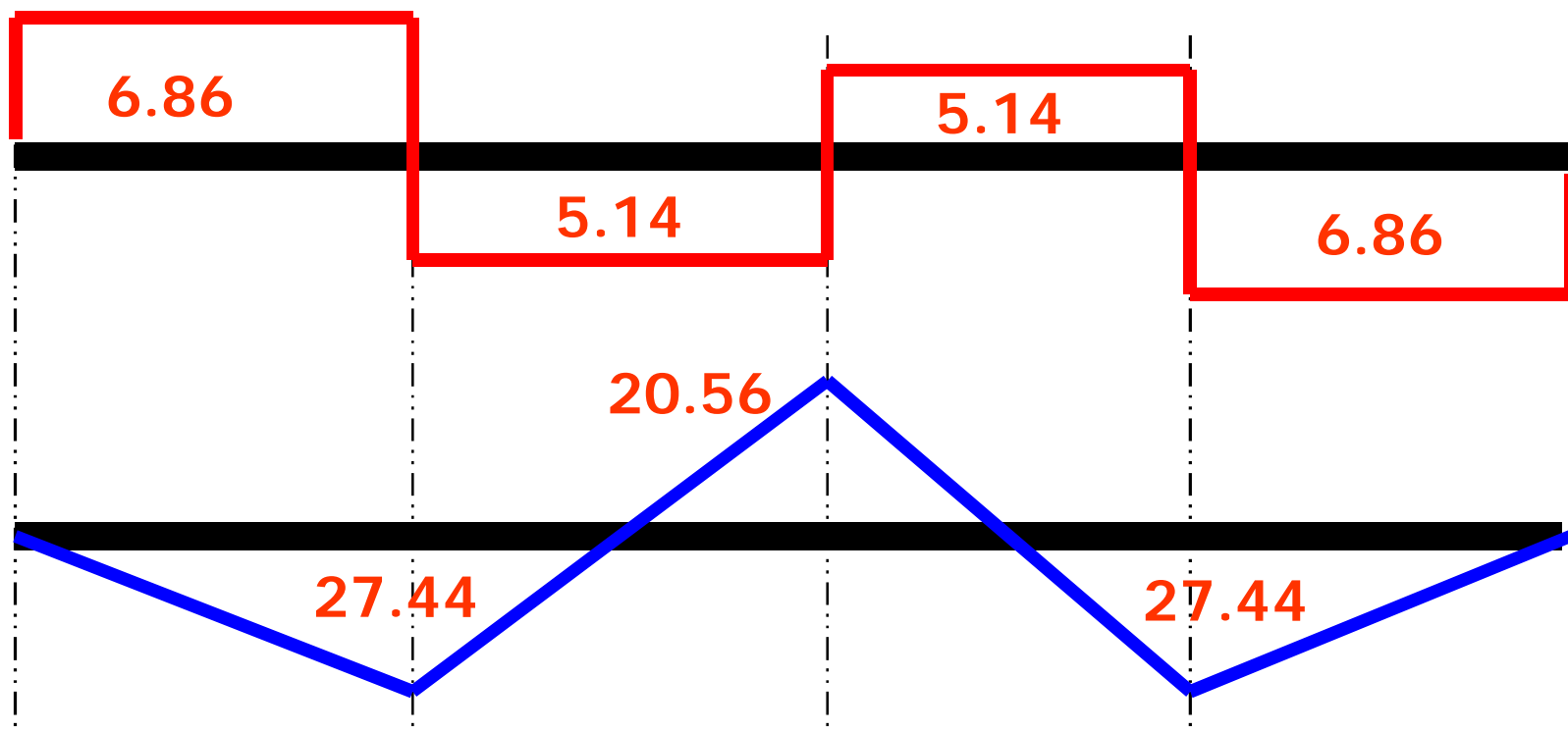
$$42.67R + 128 + 4(-14R + 96) - 117.33R + 384 = 0$$

$$\ggg R = 6.86 \text{ ton}$$

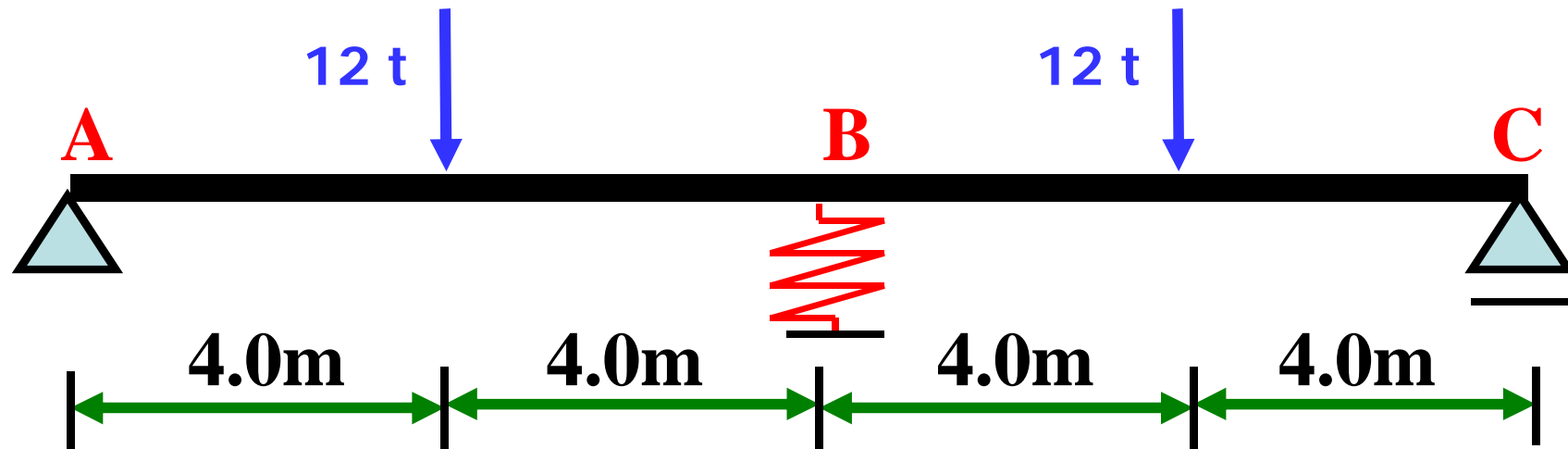


$$\sum Y = 0$$

$$Y_B = 24 - 6.86 \times 2 = 10.28$$



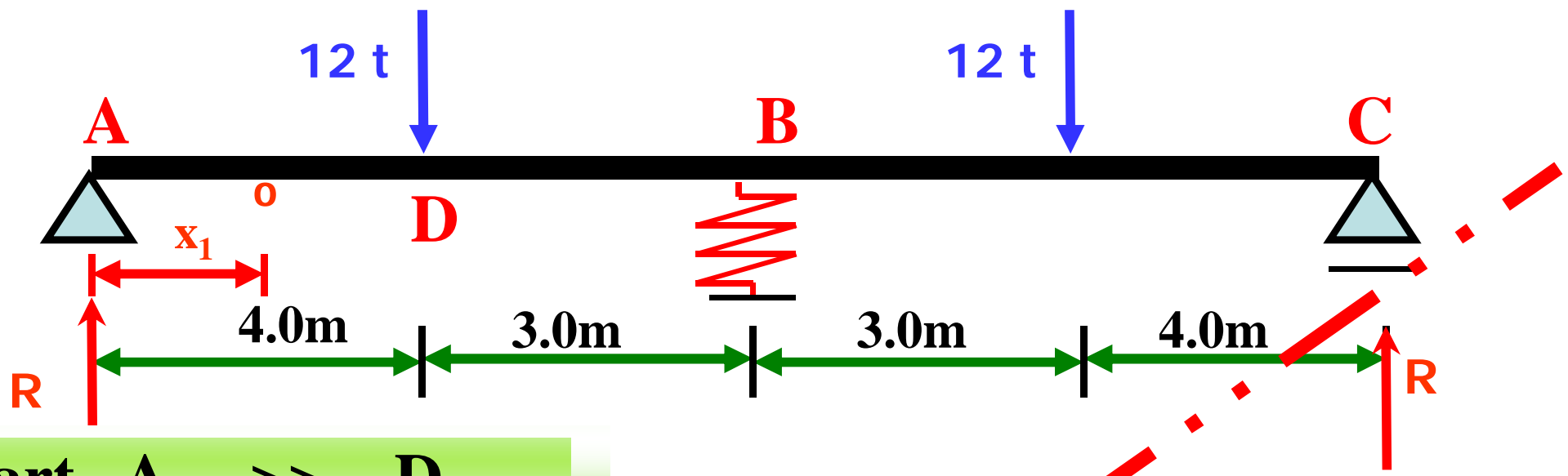
Problem 3



المطلوب رسم العزم لهذه الكمرة الغير محدده استاتيكيًا

مع العلم ان الهبوط عند النقطة B يساوي 2cm

$$EI=4000 \text{ t/m}^2$$



Part A >> D
 $x_1 = 0 >> x_1 = 4.0$

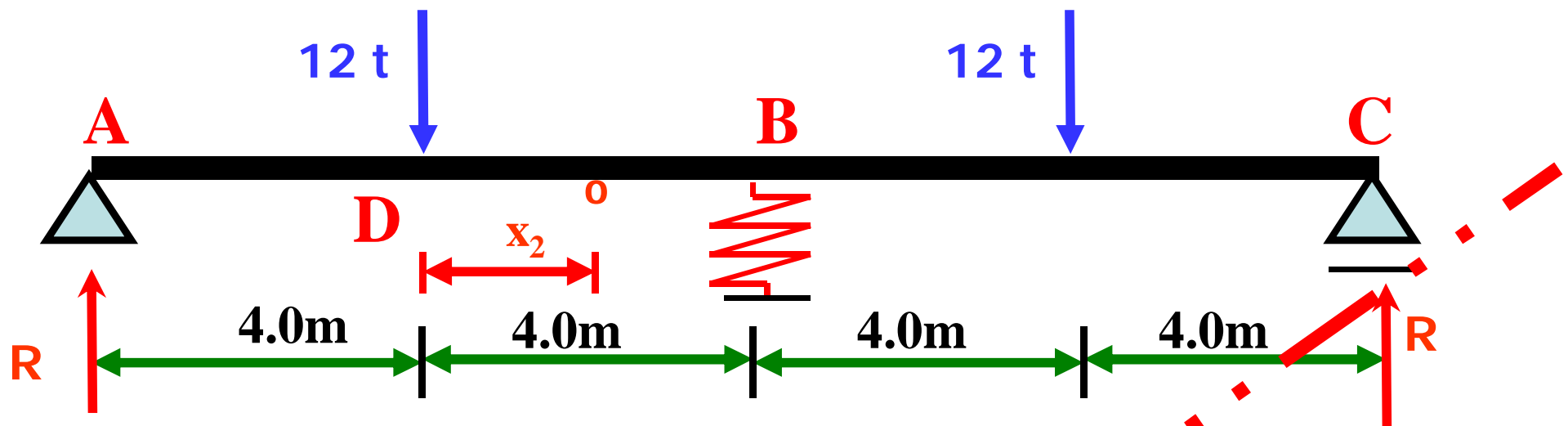
$$M_0 = R x_1$$

$$y_1'' = -\frac{1}{EI}(M)$$

$$y_1'' = -\frac{1}{EI}(R x_1)$$

$$y_1' = \theta_1 = -\frac{1}{EI}(0.5 R x_1^2 + c_1)$$

$$y_1 = -\frac{1}{EI}(0.167 R x_1^3 + c_1 x_1 + c_2)$$



Part D >> B

$x_2 = 0 >> x_2 = 3$

$$M_0 = R \cdot (4 + x_2) + 12 \cdot x_2 = 4R + Rx_2 + 12x_2$$

$$y_2'' = -\frac{1}{EI}(M)$$

$$y_2'' = -\frac{1}{EI}(4R + Rx_2 + 12x_2)$$

$$y_2' = \theta_2 = -\frac{1}{EI}(4Rx_2 + 0.5Rx_2^2 + 6x_2^2 + c_3)$$

$$y_2 = -\frac{1}{EI}(2Rx_2^2 + 0.167Rx_2^3 + 2x_2^3 + c_3x_2 + c_4)$$

At $x_1 = 0.0 \gg \gg \gg y_1 = 0.0$

عند النقطة A الهبوط يساوى $y_1 = 0$ صفر

$$y_1 = -\frac{1}{EI}(0.167R * 0 + c_1 * 0 + c_2) = 0 \gg c_2 = 0$$

At $x_1 = 1.0$

$\gg \gg \gg \theta_1 = \theta_2$

عند النقطة D الدوران يساوى $\theta_1 = \theta_2$
بسبب وجود تماثل عند هذه النقطة

At $x_2 = 1.0$

$$\theta_1[(x_1 = 4)] = -\frac{1}{EI}(0.5R * 4^2 + c_1) = \theta_2[x_2 = 0] = -\frac{1}{EI}(4R * 0 + 0.5R * 0 + 6 * 0 + c_3)$$

$$8R + c_1 = c_3 \gg 1$$

At $x_1 = 1.0$

$\gg \gg \gg y_1 = y_2$

عند النقطة D الدوران يساوى $y_1 = y_2$
بسبب وجود تماثل عند هذه النقطة

At $x_2 = 1.0$

$$y_1 = -\frac{1}{EI}(0.167R * 4^3 + c_1 * 4 + 0) = y_2 = -\frac{1}{EI}(2R * 0 + 0.167R * 0 + 2 * 0 + c_3 * 0 + c_4) = -\frac{1}{EI}(c_4)$$

$$10.67R + 4c_1 = c_4 \gg 2$$

At $x_2 = 4.0 \gggg \theta_2 = 0.0$

عند النقطة B الهبوط يساوى $\theta_2 =$ صفر

$$\theta_2 = -\frac{1}{3EI}(4R * 4 + 0.5R * 16 + 6 * 16 + c_3) = 0$$

$$4R * 4 + 0.5R * 16 + 6 * 16 + c_3 = 0$$

$$c_3 = -24R + 96 \gggg 3$$

At $x_2 = 4.0 \gggg y_2 = 0.02$

عند النقطة B الهبوط يساوى $0.02 = y_2$

$$y_2 = -\frac{1}{4000}(2R * 16 + 0.167R * 64 + 2 * 64 + c_3 * 4 + c_4) = 0.02$$

$$2R * 16 + 0.167R * 64 + 2 * 64 + c_3 * 4 + c_4 = 80$$

$$42.66R + c_3 * 4 + c_4 = 48 \gggg 4$$

$$8R + c_1 = c_3 \quad \gg 1$$

$$10.67R + 4c_1 = c_4 \quad \gg 2$$

$$c_3 = -24R + 96 \quad \ggg 3$$

$$42.66R + c_3 * 4 + c_4 = 48 \quad \ggg 4$$

From 3 in 1

$$8R + c_1 = -24R + 96$$

$$\ggg c_1 = -32R + 96 \quad \gg 5$$

From 5 in 2

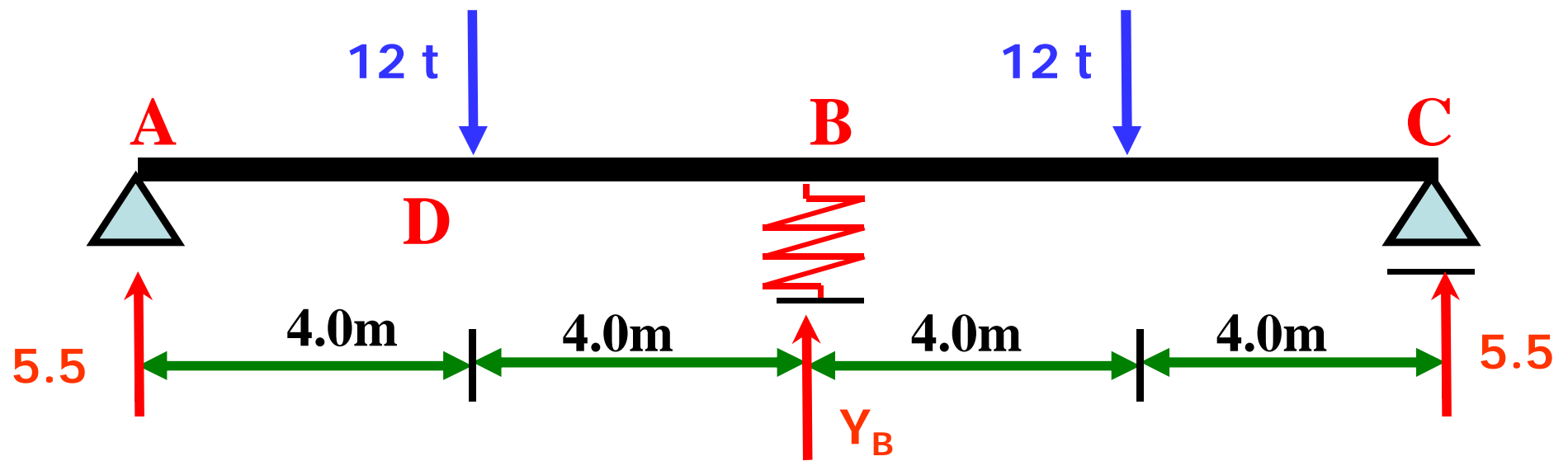
$$10.67R + 4(-32R + 96) = c_4$$

$$\ggg c_4 = -117.33R + 384 \quad \gg 6$$

From 5, 6 in 4

$$42.67R + 4(-14R + 96) - 117.33R + 384 = 48$$

$$\ggg \quad R = 5.5 \text{ ton}$$



$$\sum Y = 0$$

$$Y_B = 24 - 5.5 \times 2 = 13$$

بالتالى يمكن الرسم

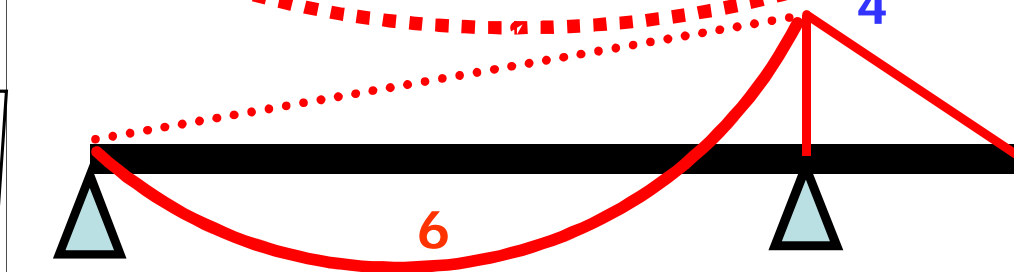
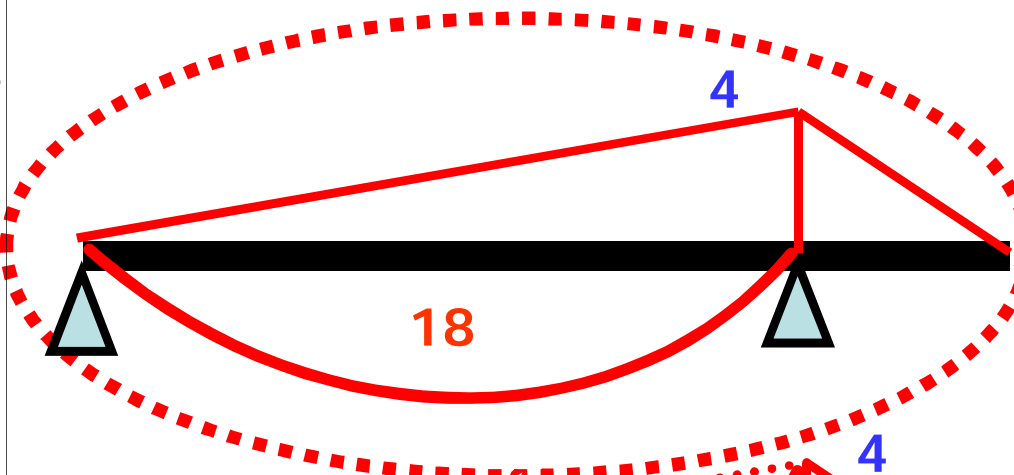
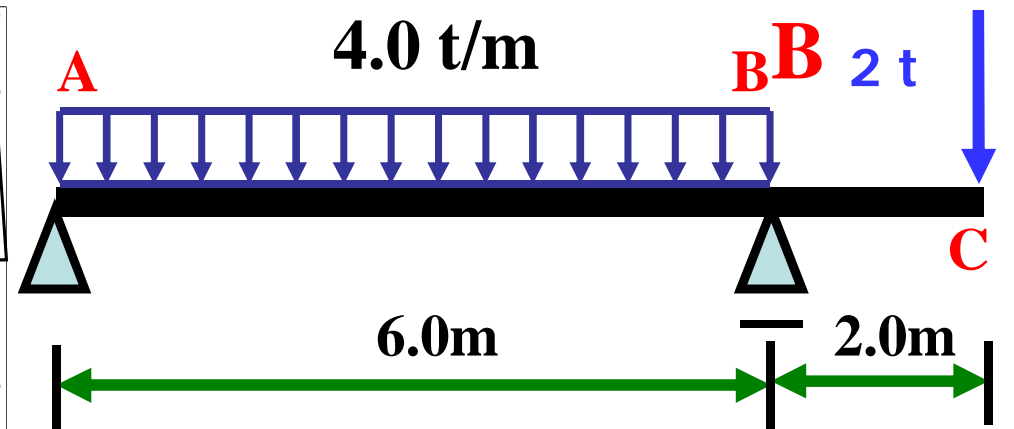
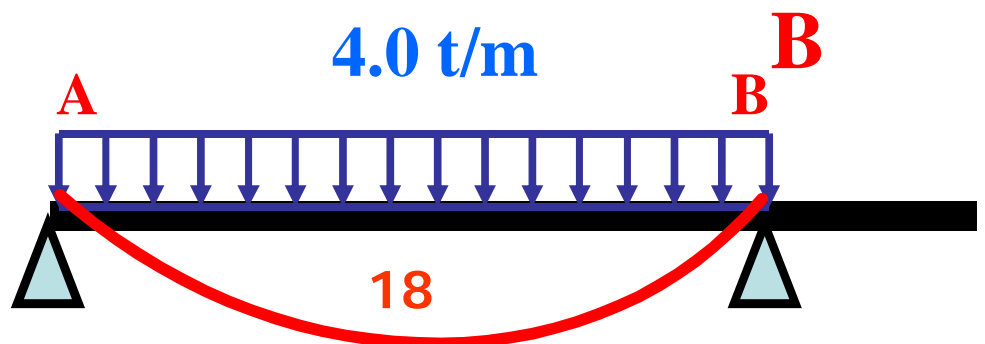
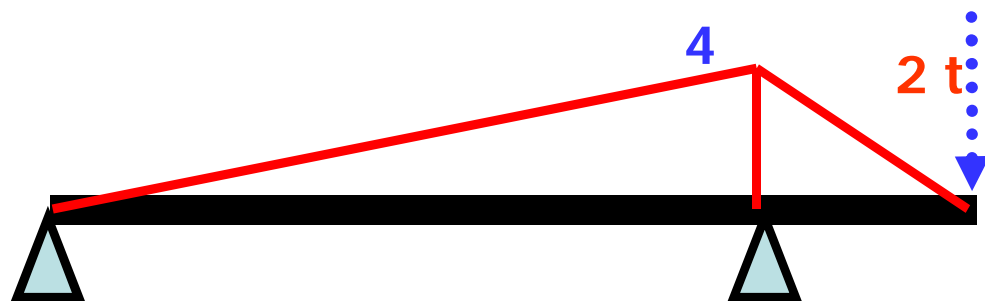
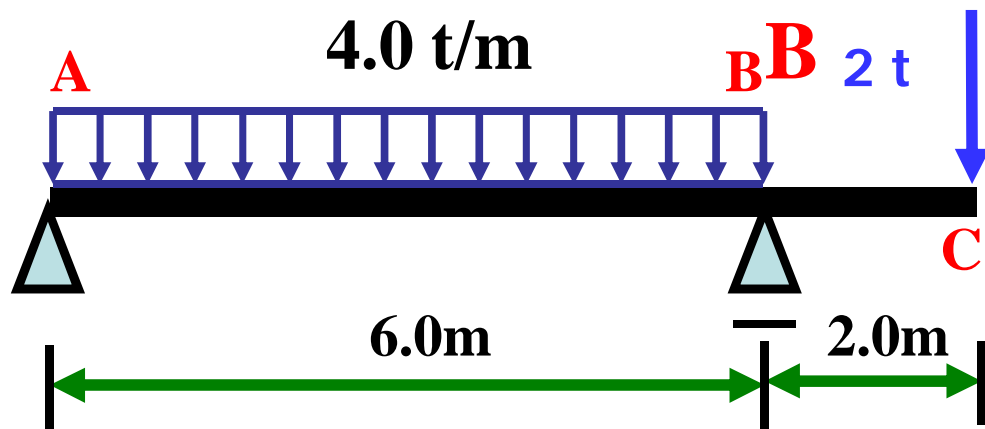
بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

CONJEGATE BEAM METHOD

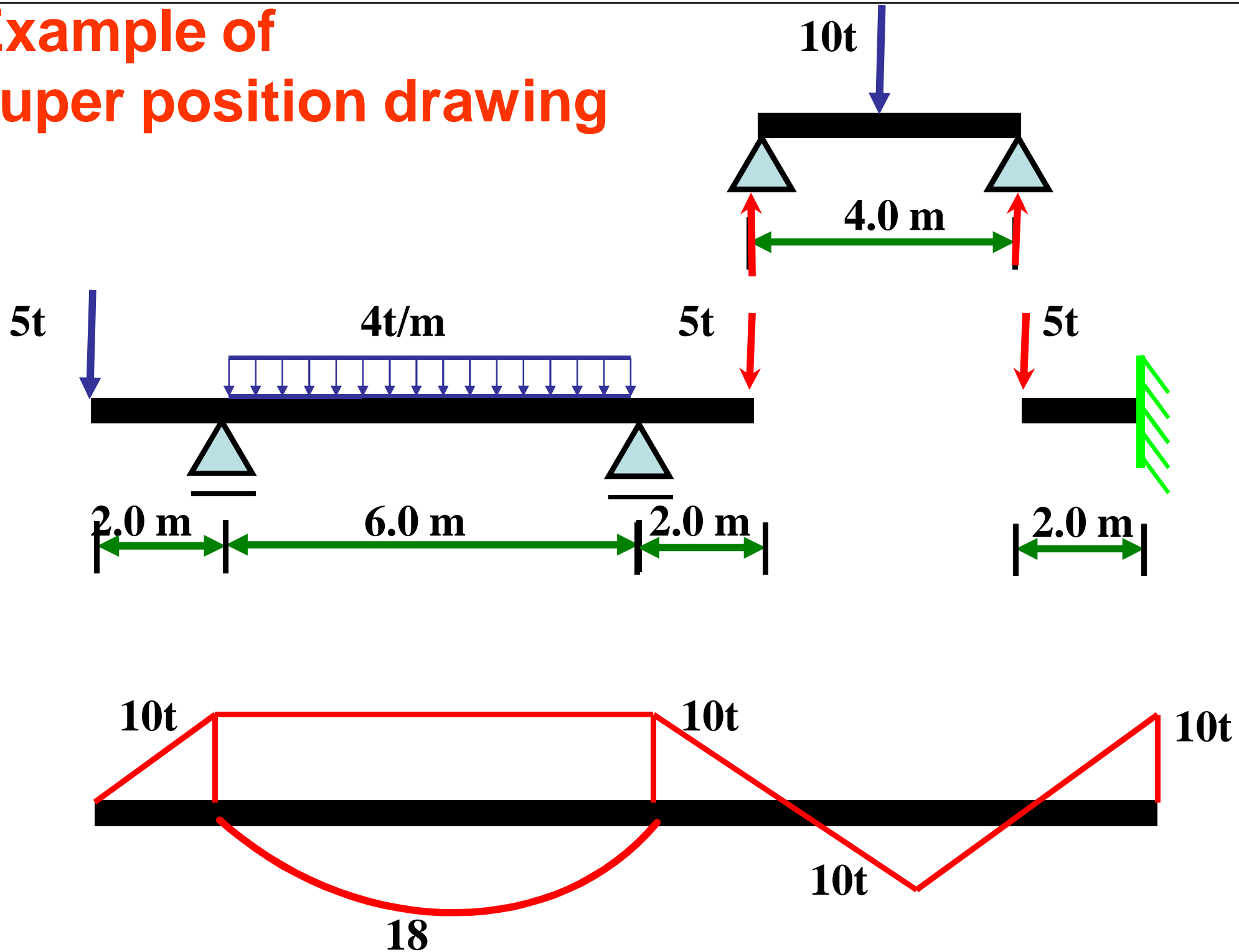
Eng : Aymman abdo

خطوات الحل:



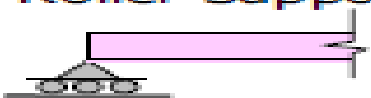
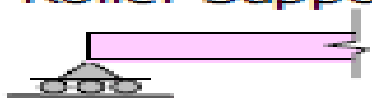
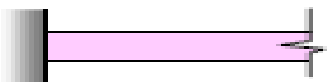

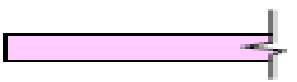
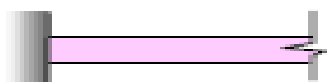
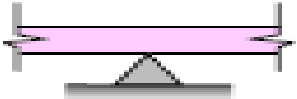
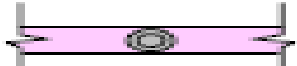

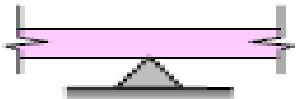
1- يتم رسم العزم على الكمرة المعطاه ويفضل بطريقة الـ super position.



Example of super position drawing



2- يتم تحويل الكمرة الى conjugate beam.

Real Beam Support	Conjugate Beam Support
<p>Hinged Support</p> 	<p>Hinged Support</p> 
<p>Roller Support</p> 	<p>Roller Support</p> 
<p>Fixed Support</p> 	<p>Free End</p> 
<p>Free End</p> 	<p>Fixed Support</p> 
<p>Interior Support</p> 	<p>Internal Hinge</p> 
<p>Internal Hinge</p> 	<p>Interior Support</p> 

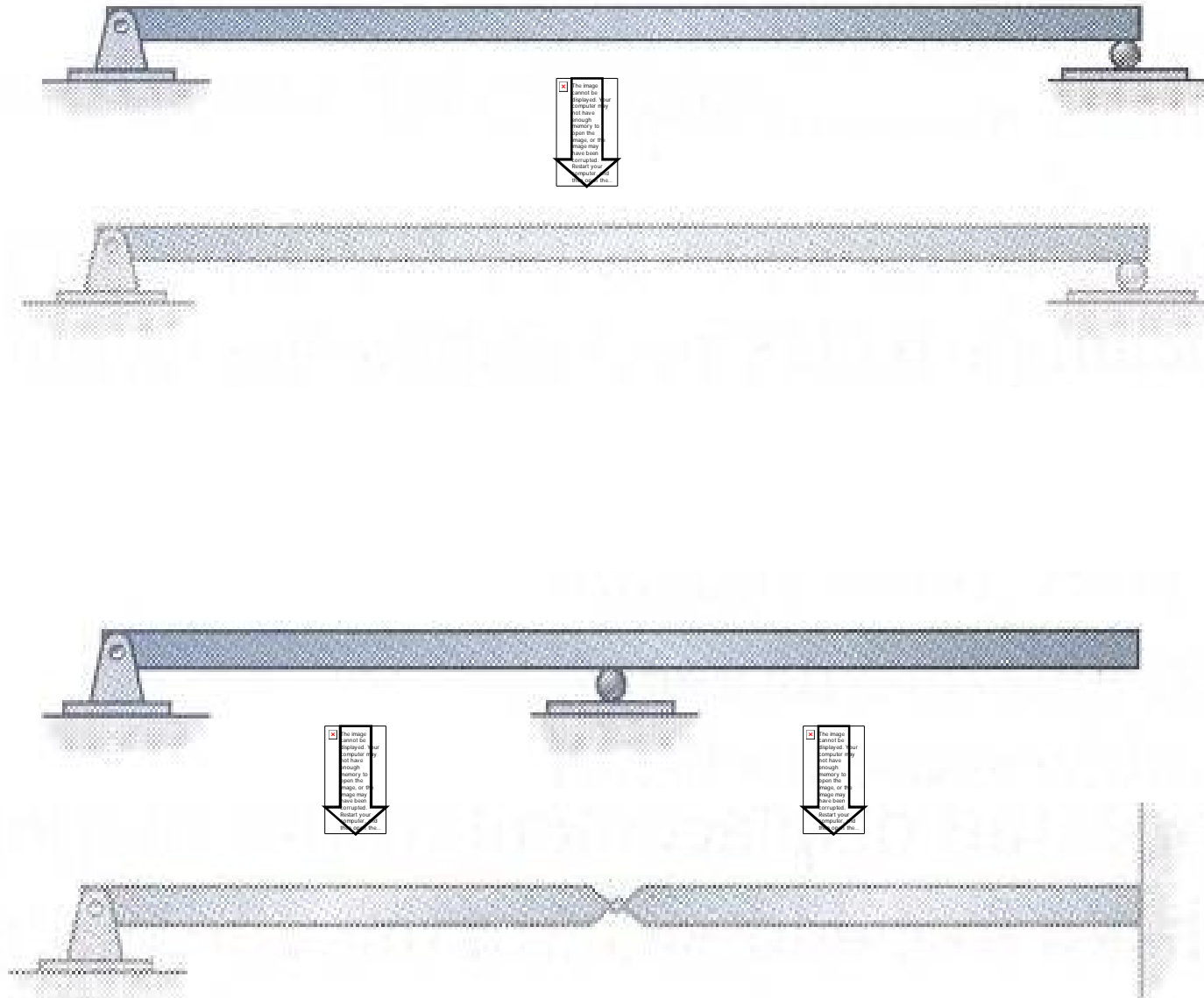
Examples of Conjugate Beam Supports

Real Beam

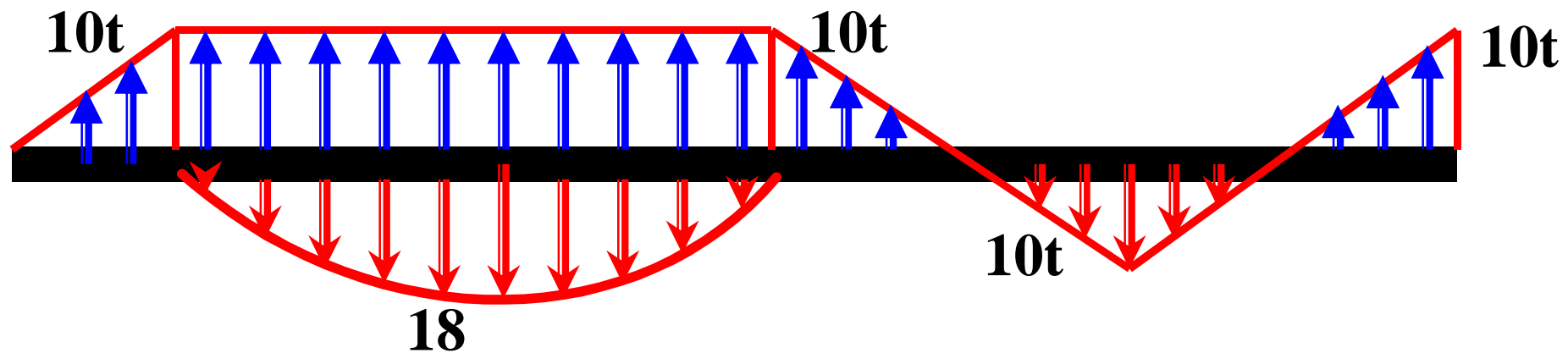
Conjugate Beam

Examples of Conjugate Beam Supports



3- يتم وضع رسمة العزم الموجودة بالخطوه (1) على الكمرة الـ conjugate على انها حمل بنفس شكلها ويكون اتجاه اسهم الاحمال مرتبط بوجوده على الكمره. مثلا لو كان الحمل اعلى الكمرة يتم توجيهه لاعلى والعكس.



4- من الكمرة الجديده يتم حساب قيمة
 الـ moment عند النقط المطلوب عندها الـ deflection
 وايضا حساب الـ shear عند النقط المطلوب عندها الـ rotation.

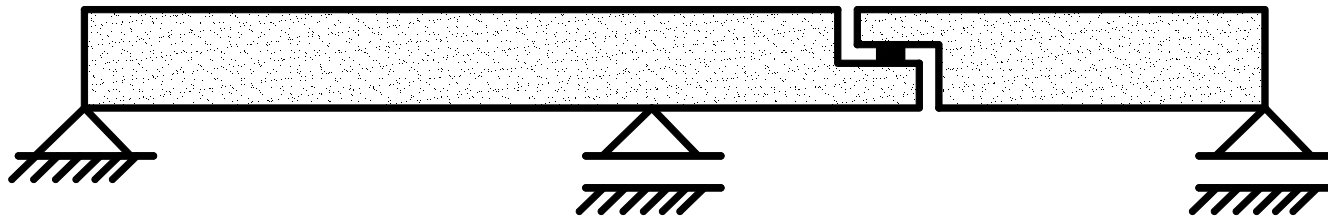
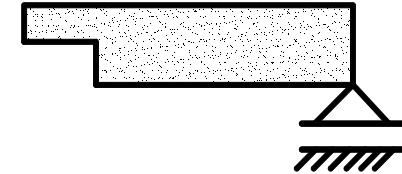
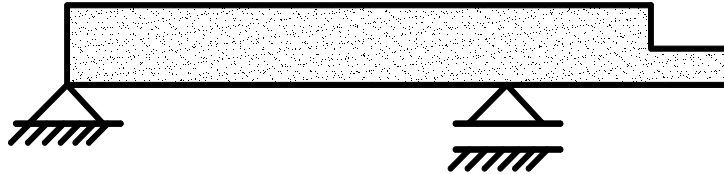
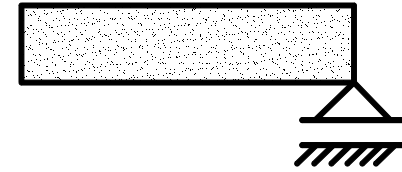
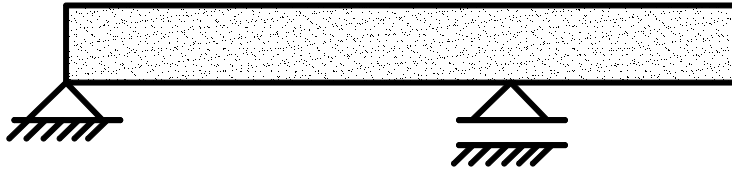
$$\theta = \frac{(shear)}{EI}$$

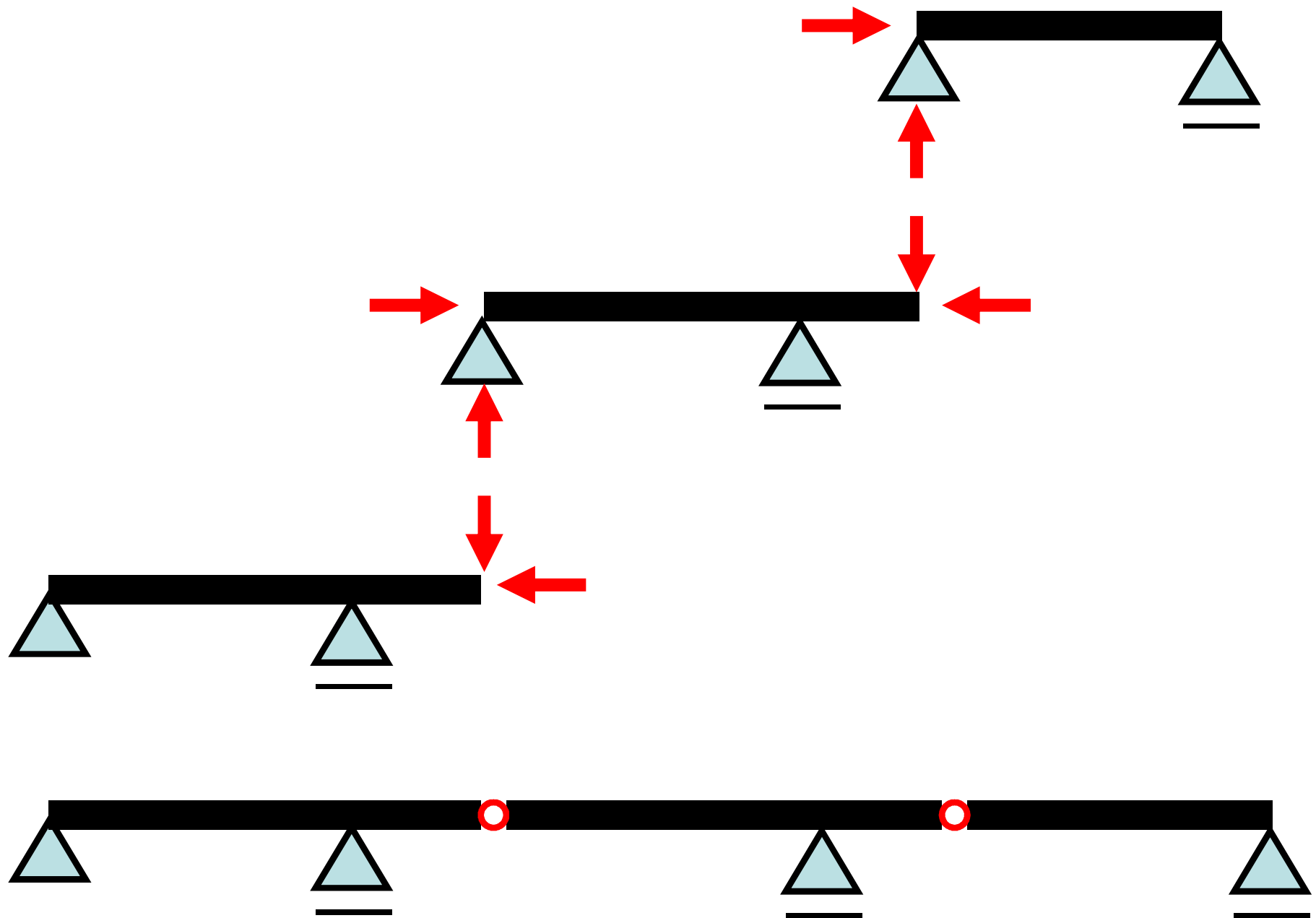
from the new beam

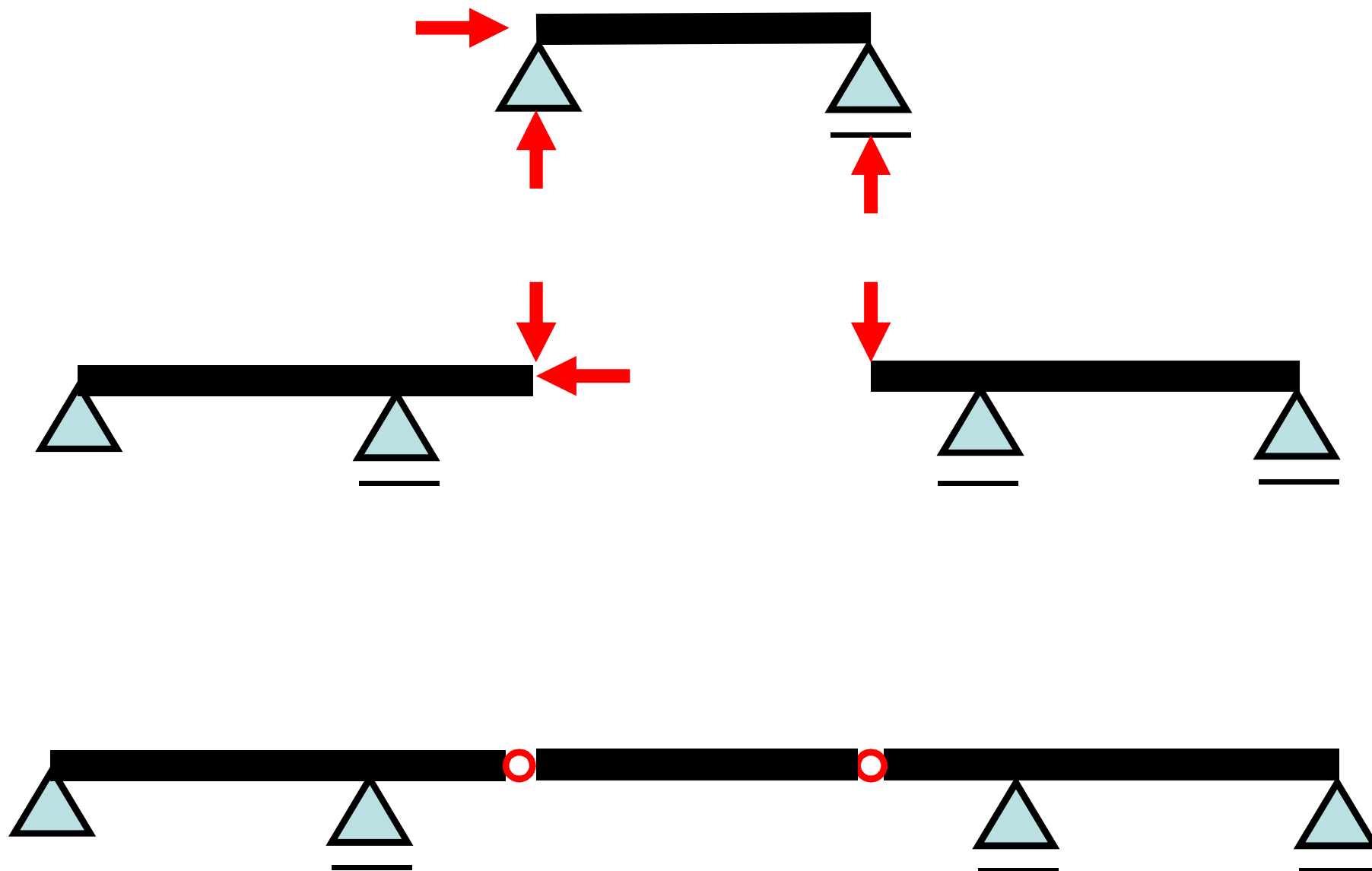
$$y = \frac{(moment)}{EI}$$

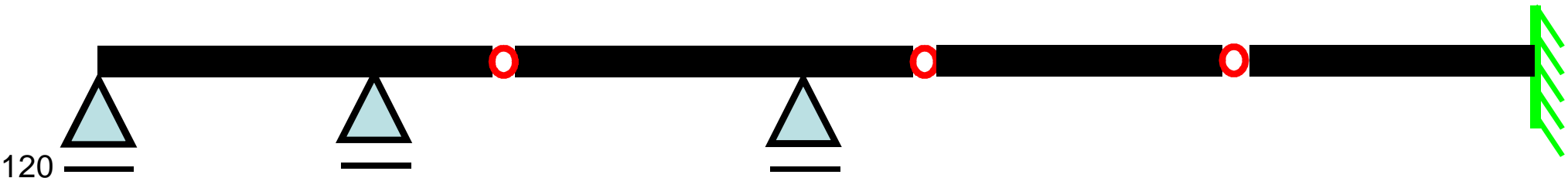
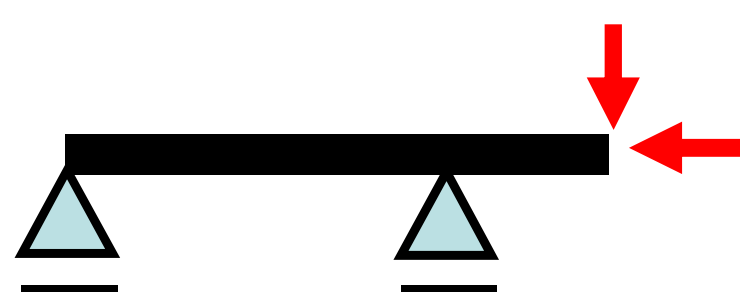
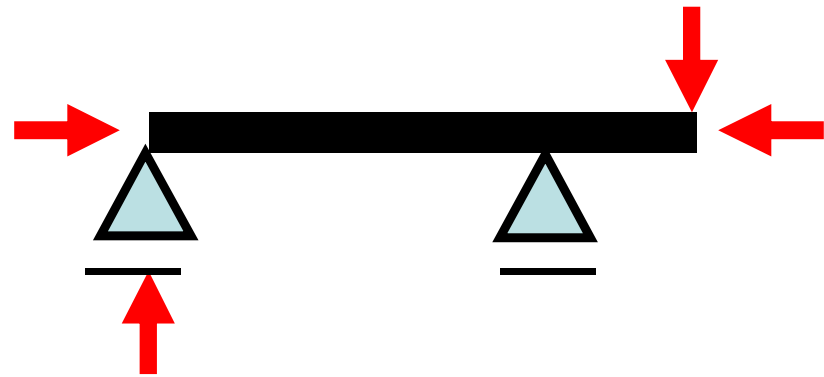
from the new beam

بعض انواع الفصل





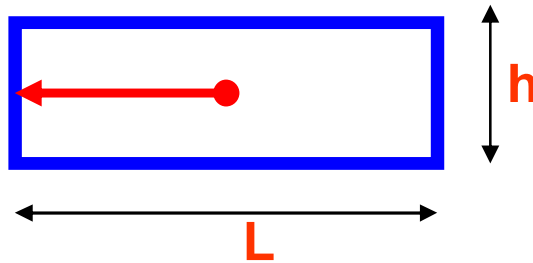




بعض المساحات المطلوب معرفة قيمتها واماكن تركيزها

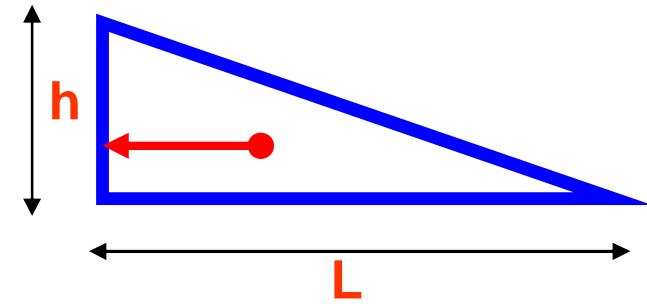
$$A = L.h$$

$$\zeta @ L/2$$



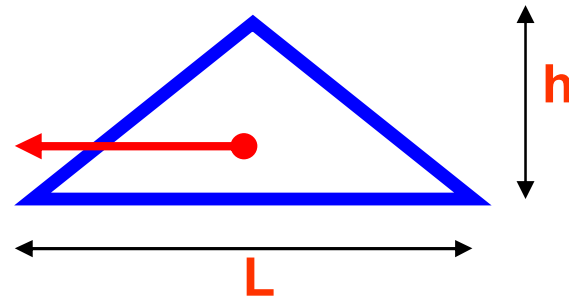
$$A = L.h/2$$

$$\zeta @ L/3$$



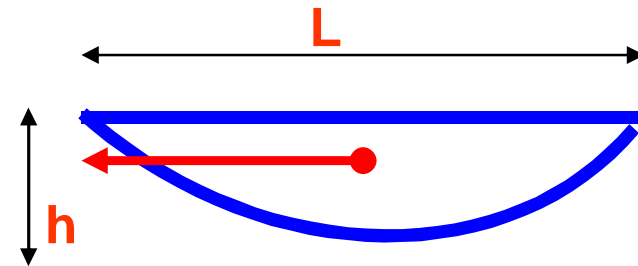
$$A = L.h/2$$

$$\zeta @ L/2$$



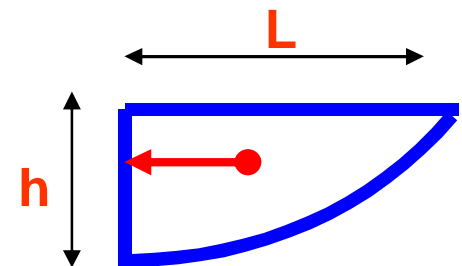
$$A = 2/3 L.h$$

$$\zeta @ L/2$$



$$A = 2/3 L.h$$

$$\zeta @ 3/8 L$$



Problem 1

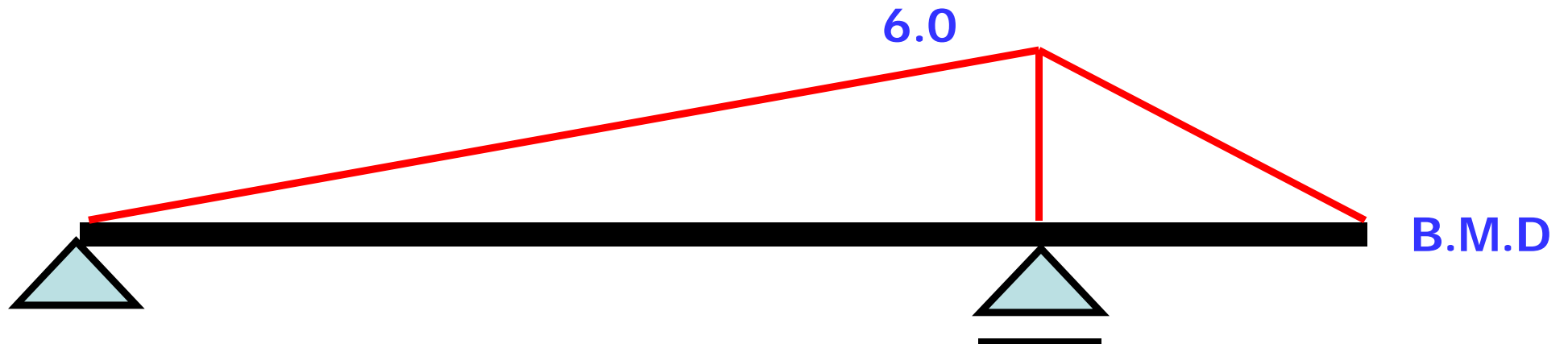
اذكر الله



Required the deflection and rotation at point **a**, **b**, **c** and **d**.

And also draw elastic curve

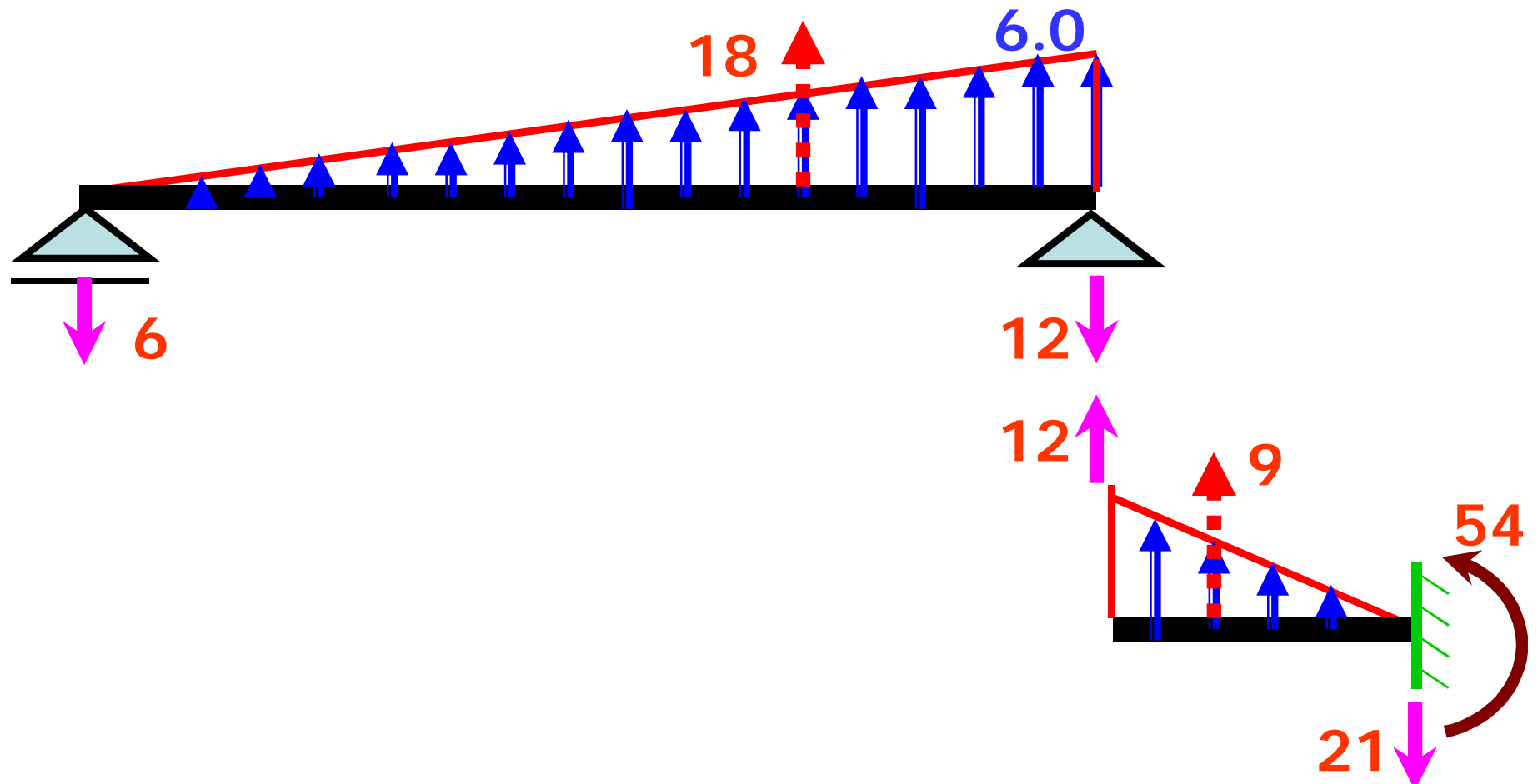
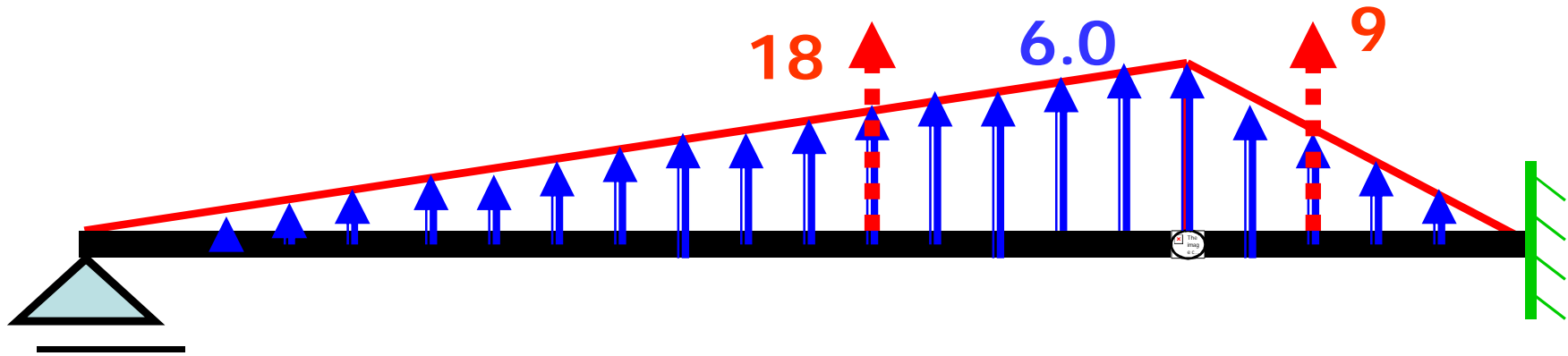
1-Draw B.M.d



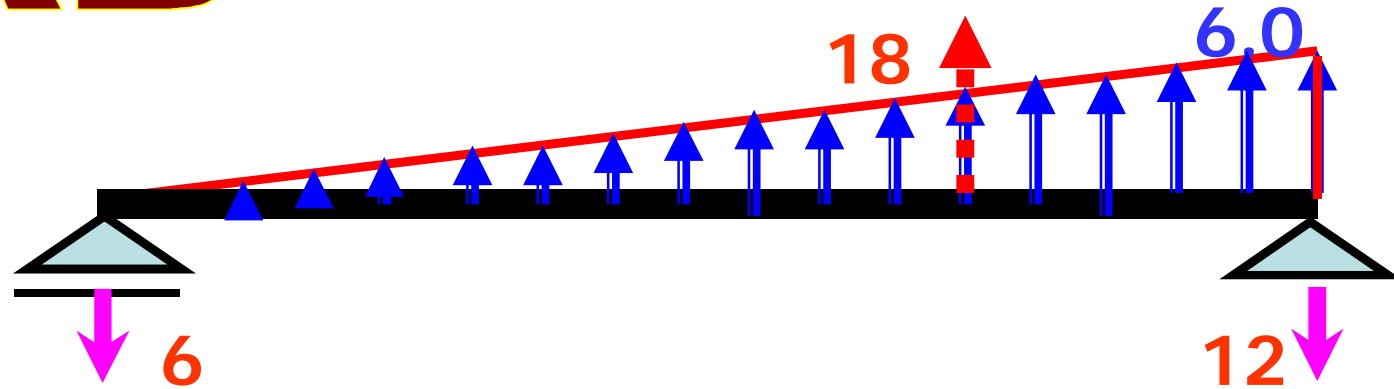
2- convert to conjugate



3-Final Beam



Part AB



$$sh_a = -6 \quad >>>> \quad \theta_a = \frac{(-6)}{EI}$$

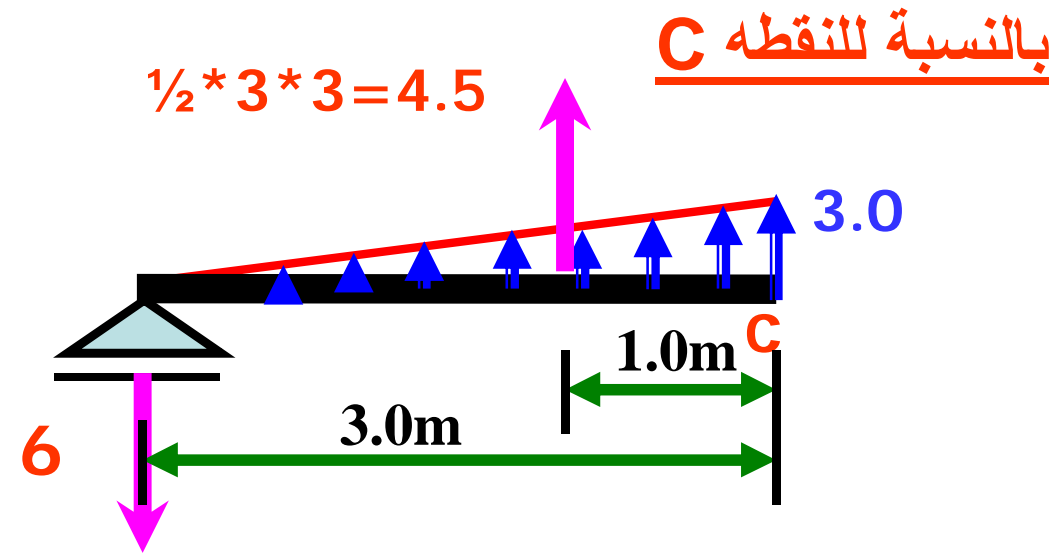
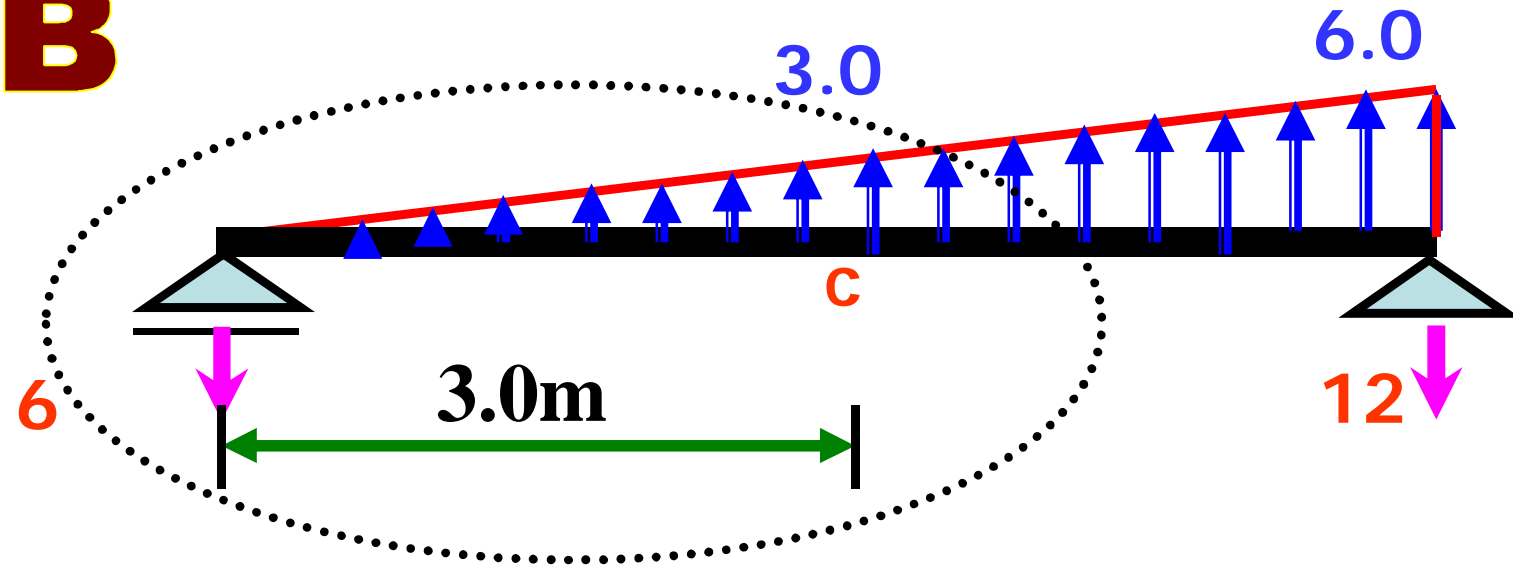
$$M_a = 0.0 \quad >>>> \quad y_a = \frac{(0.0)}{EI} = 0$$

$$sh_b = +12 \quad >>>> \quad \theta_b = \frac{(+12)}{EI}$$

$$M_b = 0.0 \quad >>>> \quad y_b = \frac{(0.0)}{EI} = 0$$

الـ shear من الشمال وطالع موجب ومن
اليمن ونازل برضه موجب والعكس

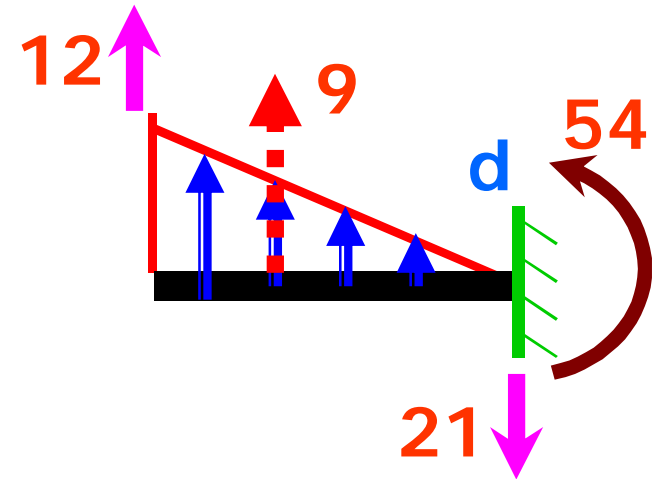
Part AB



$$sh_c = -6 + 4.5 = -1.5 \quad \gggg \quad \theta_a = \frac{(-1.5)}{EI}$$

$$M_c = -6 * 3 + 4.5 * 1 = -13.5 \quad \gggg \quad y_a = \frac{(-13.5)}{EI}$$

Part BD



$$sh_a = +21 \quad >>>> \quad \theta_a = \frac{(+21)}{EI}$$

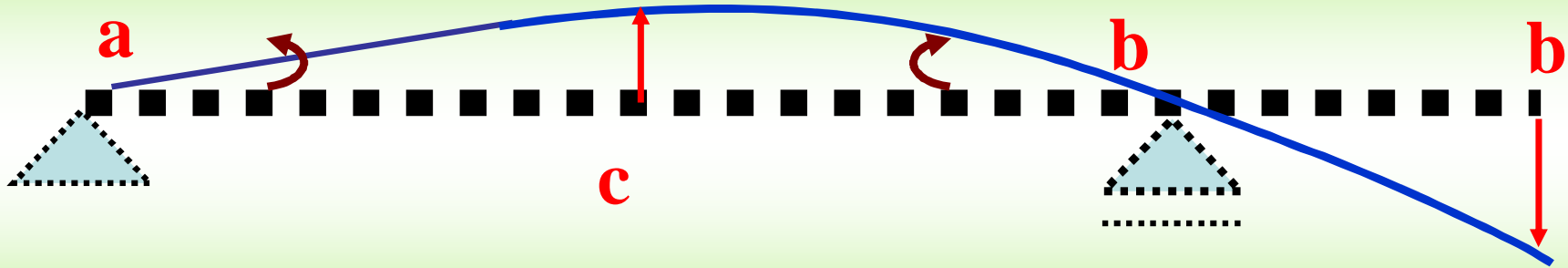
$$M_a = -54 \quad >>>> \quad y_a = \frac{(+54)}{EI}$$

Draw Elastic Curve

$$\theta_a = -ve$$



$$\theta_b = +ve$$



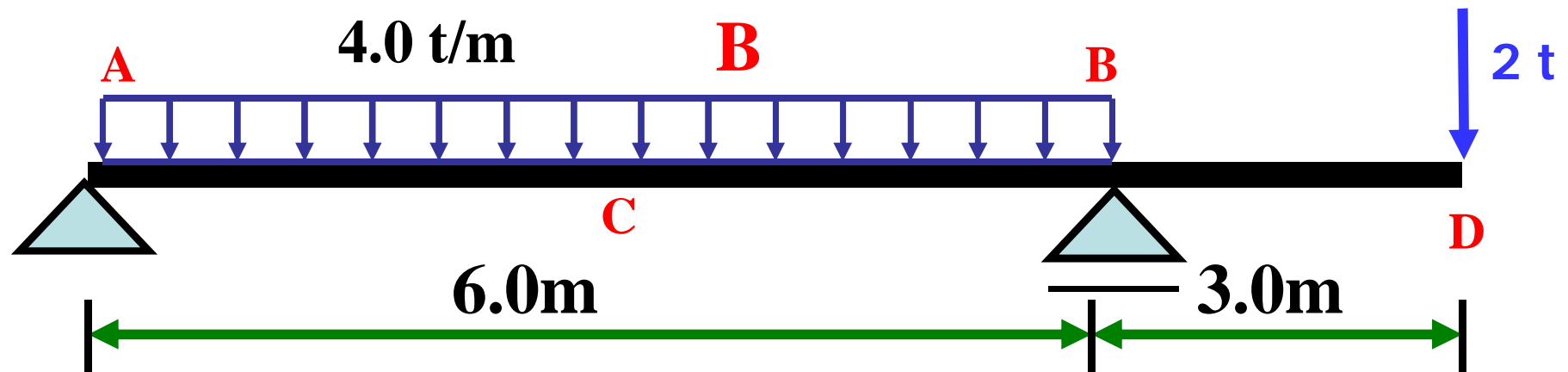
$$y_c = -ve$$



$$y_d = +ve$$



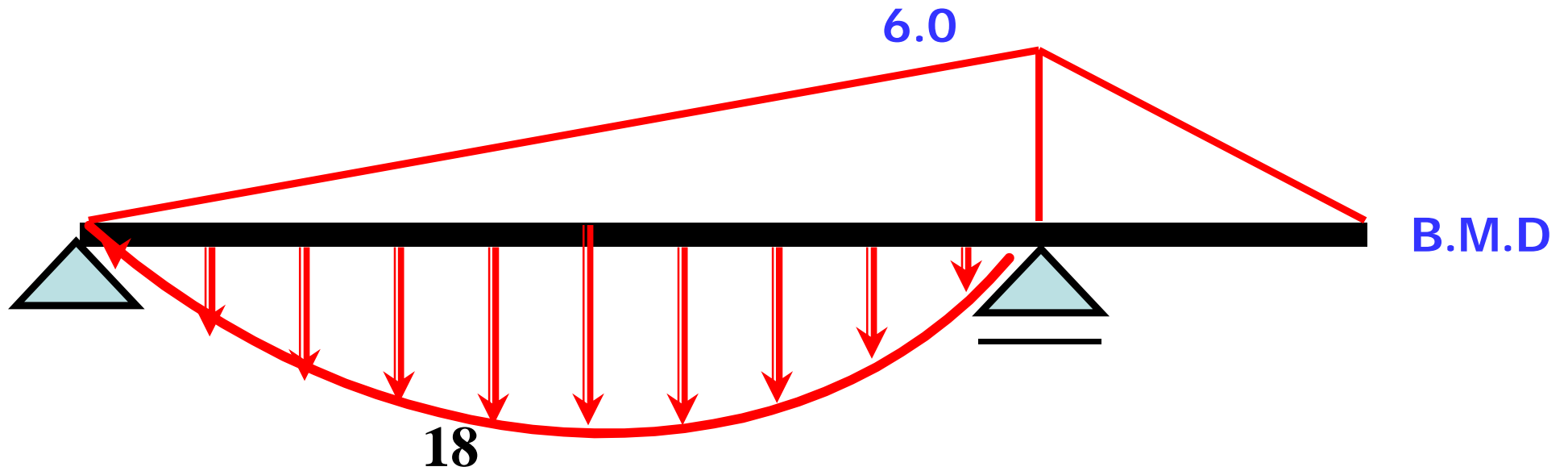
Problem 2



Required the deflection and rotation at point a, b, c and d.

And also draw elastic curve

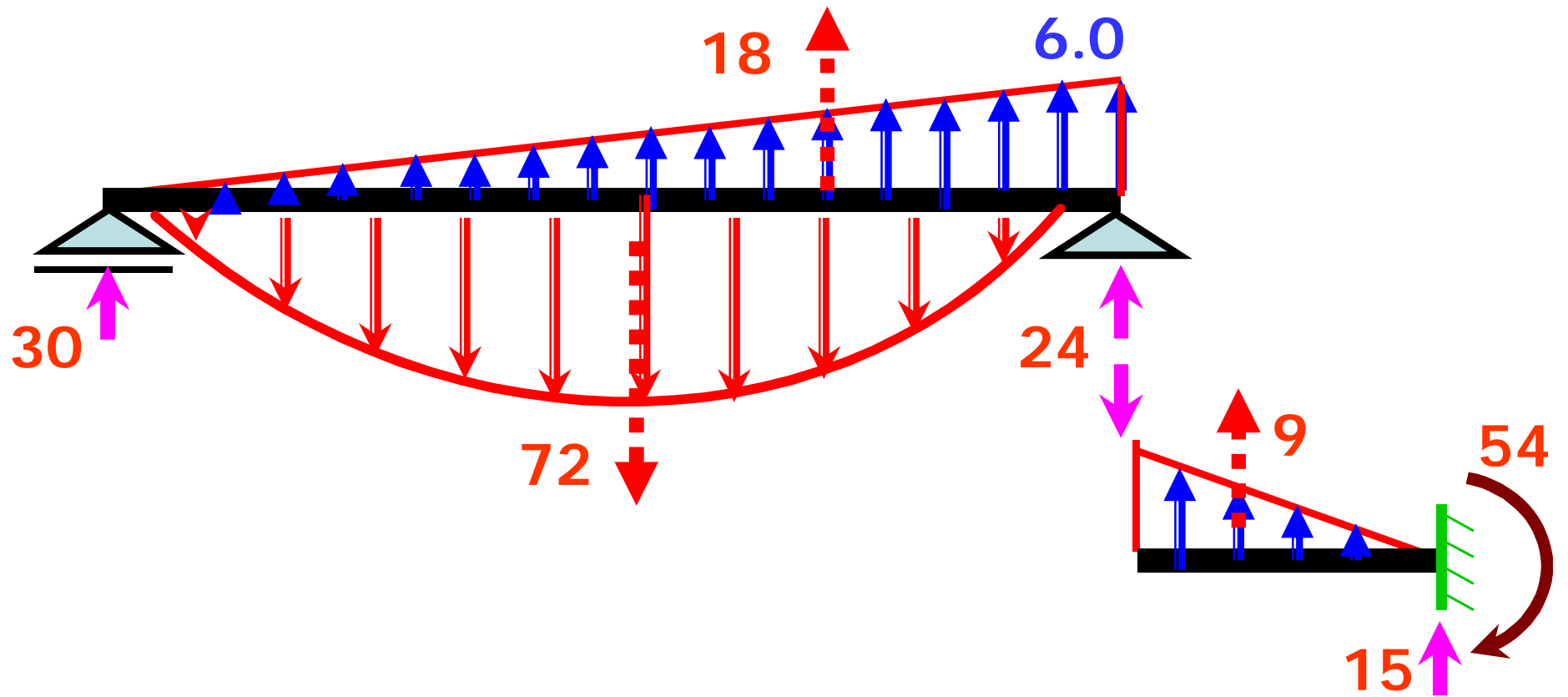
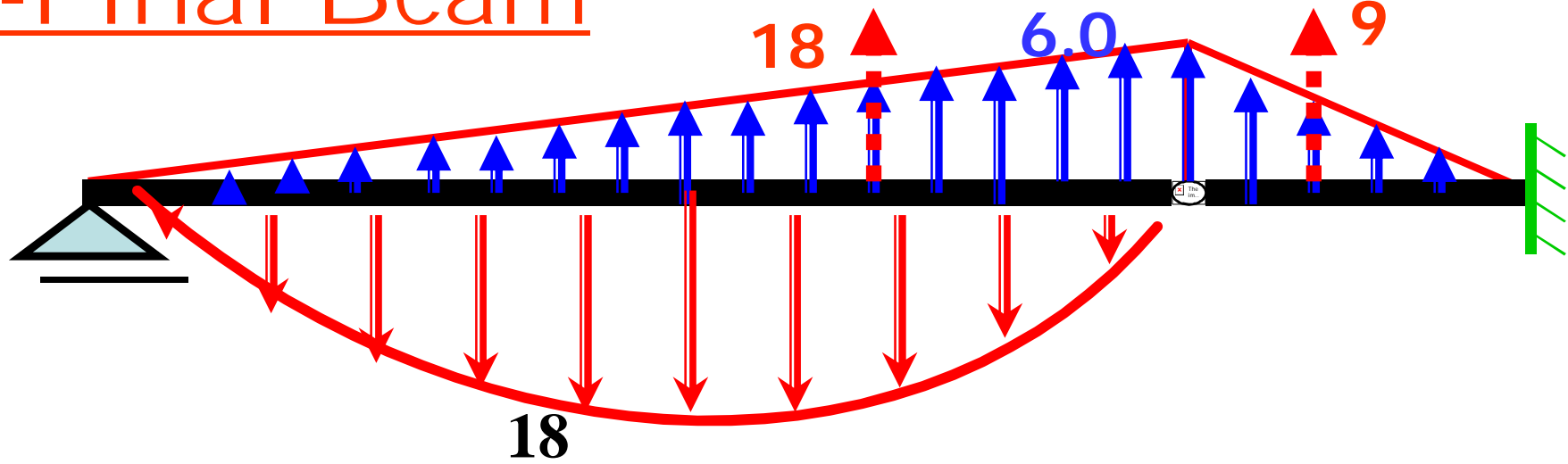
1-Draw B.M.d



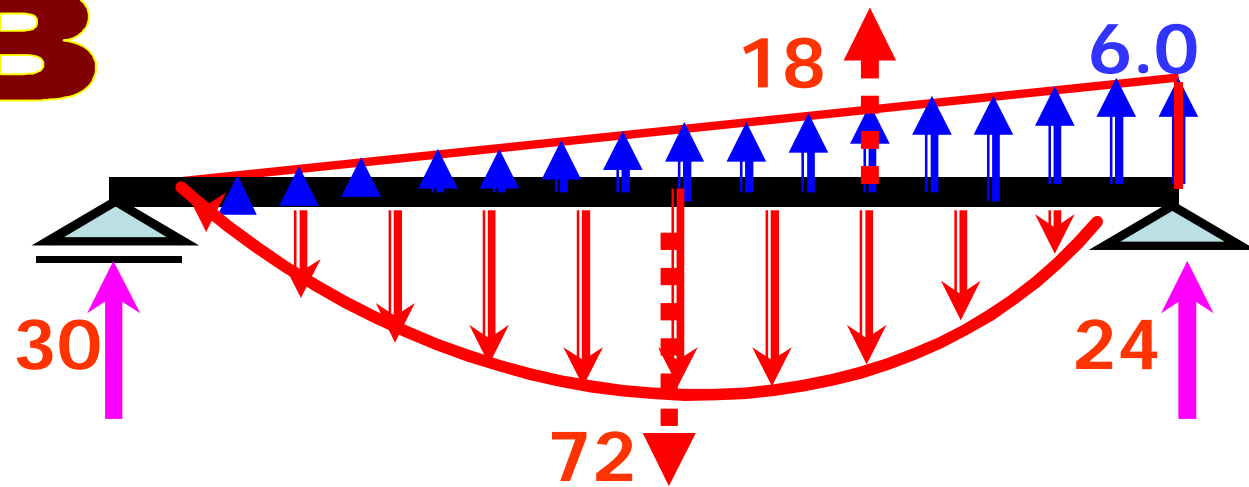
2- convert to conjugate



3-Final Beam



Part AB



$$sh_a = +30 \quad >>>> \quad \theta_a = \frac{(+30)}{EI}$$

$$M_a = 0.0 \quad >>>> \quad y_a = \frac{(0.0)}{EI} = 0$$

$$sh_b = -24 \quad >>>> \quad \theta_b = \frac{(-24)}{EI}$$

$$M_b = 0.0 \quad >>>> \quad y_b = \frac{(0.0)}{EI} = 0$$

الـ shear من الشمال وطالع موجب ومن
اليمن ونازل برضه موجب والعكس

Part AB

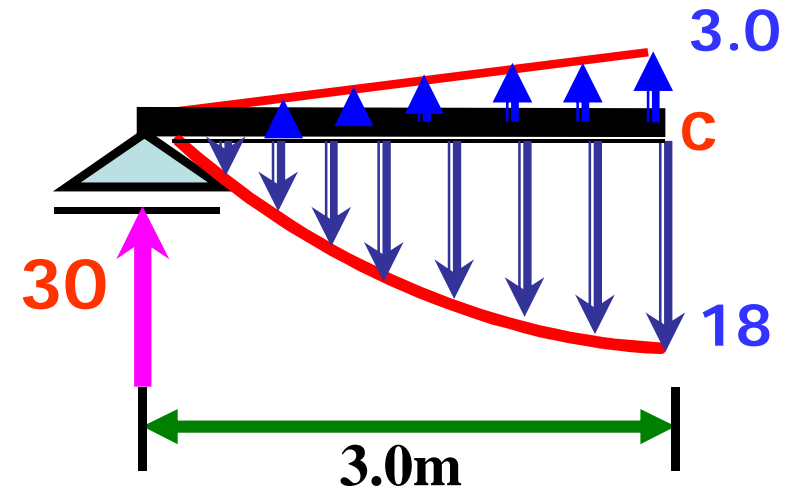
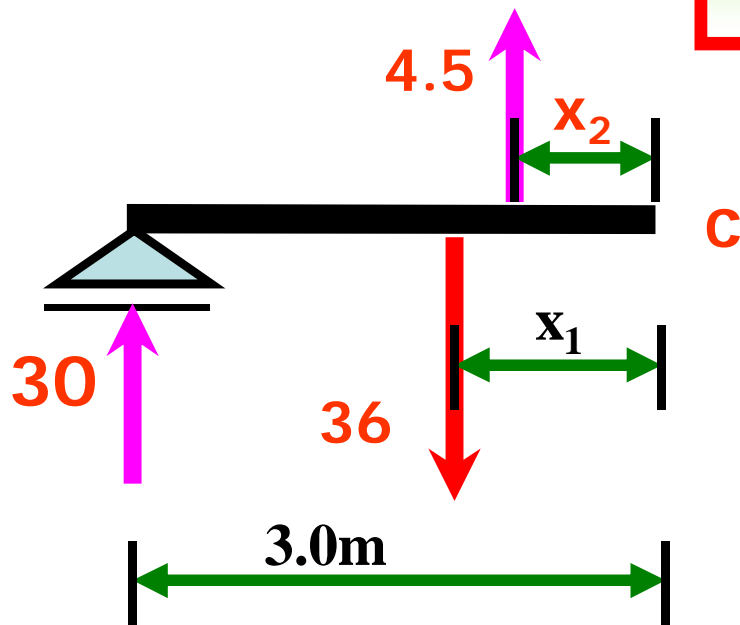
بالنسبة للنقطة C

$$R_1 = 2/3 * 3 * 18 = 36$$

$$X_1 = 3/8 * 3 = 1.125$$

$$R_2 = 1/2 * 3 * 3 = 4.5$$

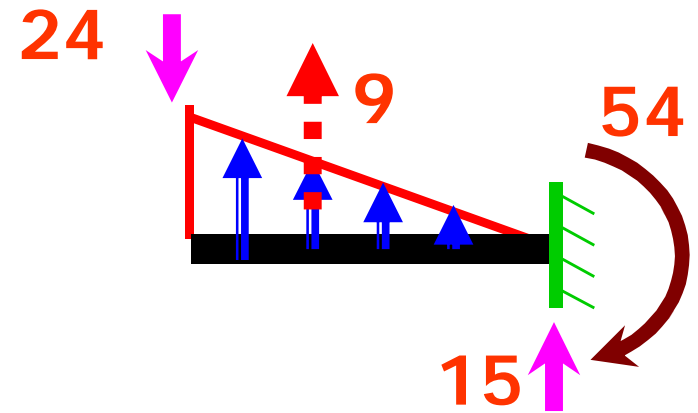
$$X_2 = 1/3 * 3 = 1$$



$$sh_a = 30 - 36 + 4.5 = -1.5 \gggg \theta_a = \frac{(-1.5)}{EI}$$

$$M_a = 30 * 3 + 4.5 * 1 - 36 * 1.125 = 54 \gggg y_a = \frac{(54)}{EI}$$

Part BD



$$sh_a = 15 \ggggg \theta_a = \frac{(-15)}{EI}$$

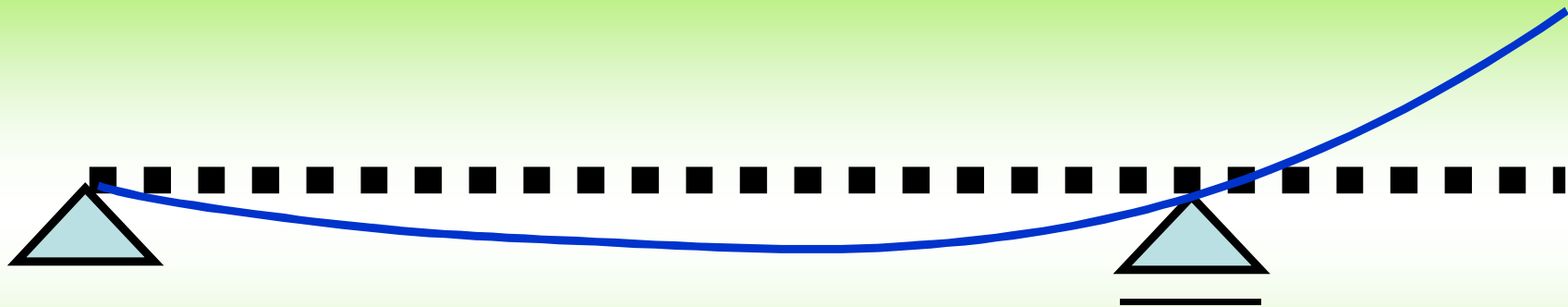
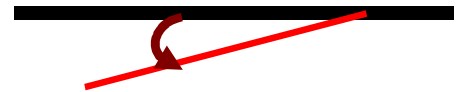
$$M_a = -54 \ggggg y_a = \frac{(-54)}{EI}$$

Draw Elastic Curve

$$\theta_a = -ve$$



$$\theta_b = +ve$$



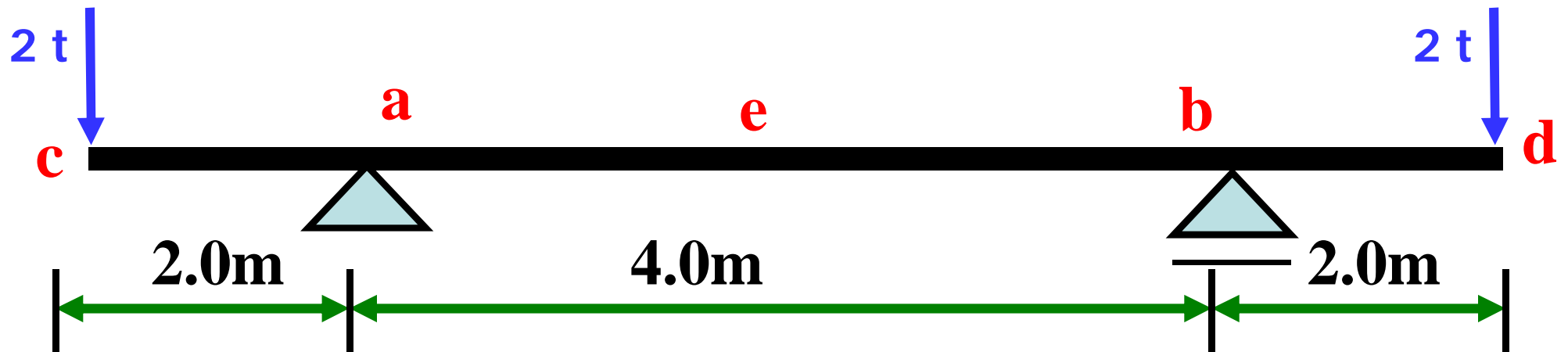
$$y_c = +ve$$



$$y_d = -ve$$



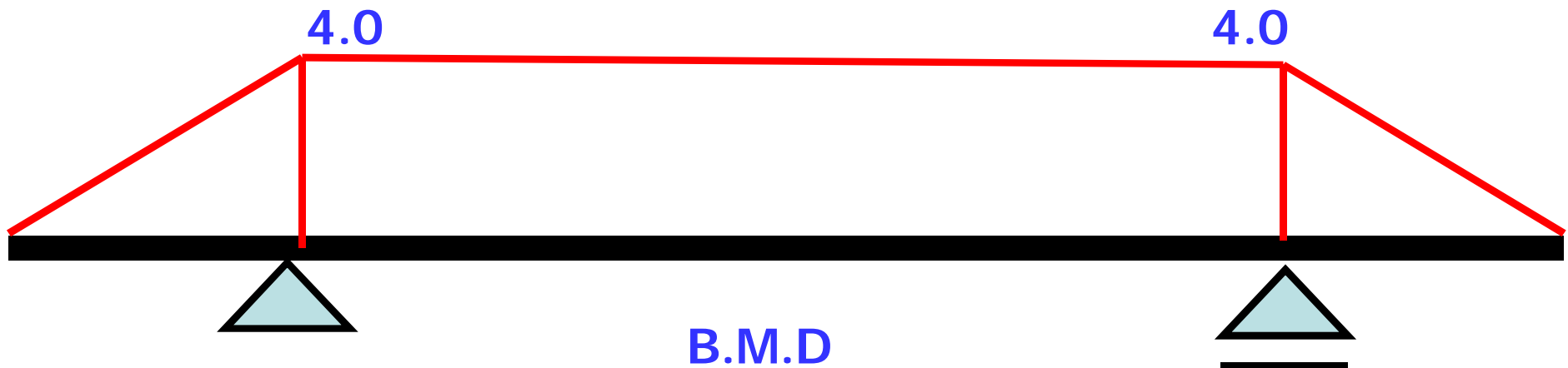
Problem 3



Required the deflection and rotation at point a, b, c, d and e.

And also draw elastic curve

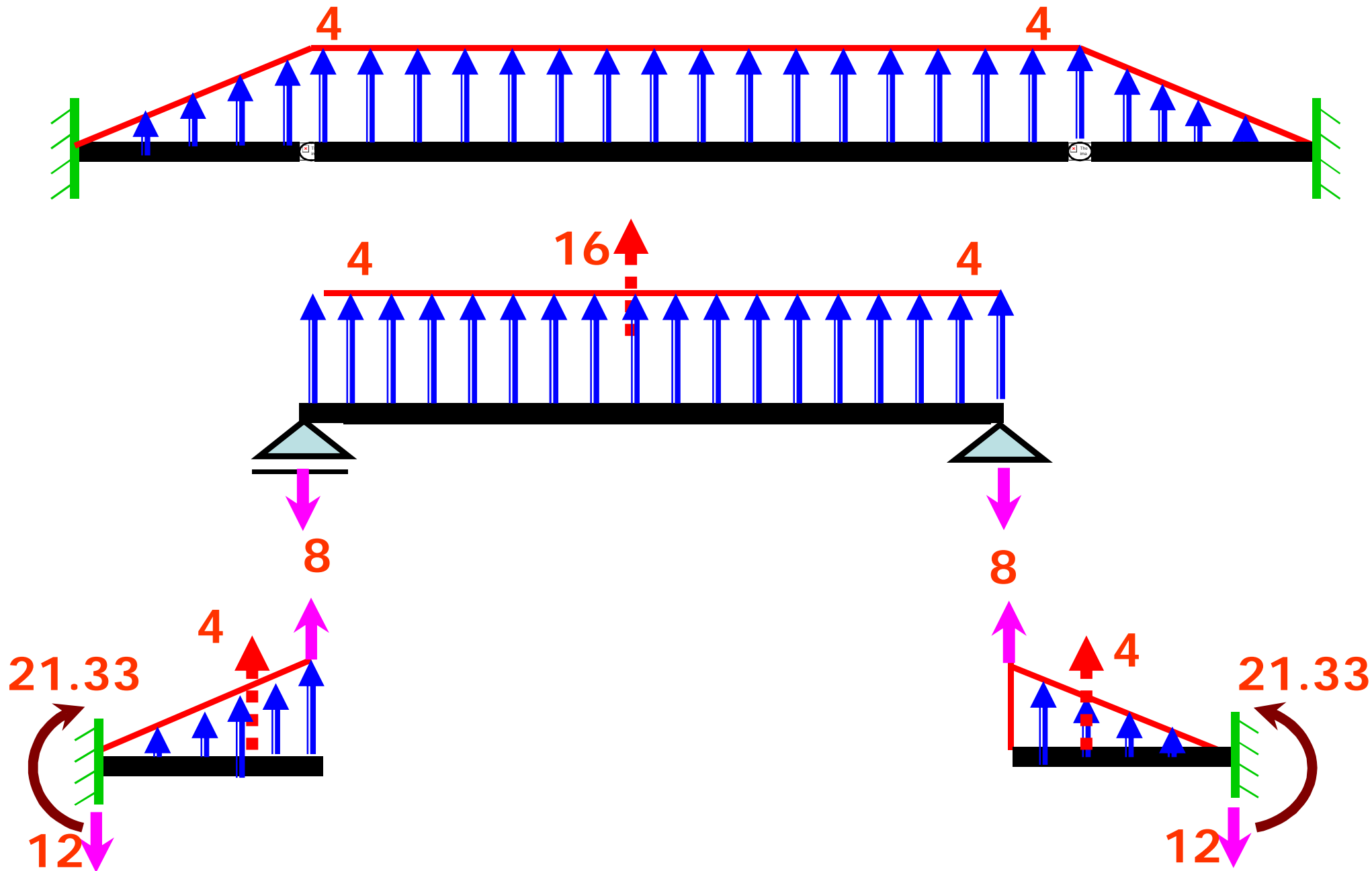
1-Draw B.M.d



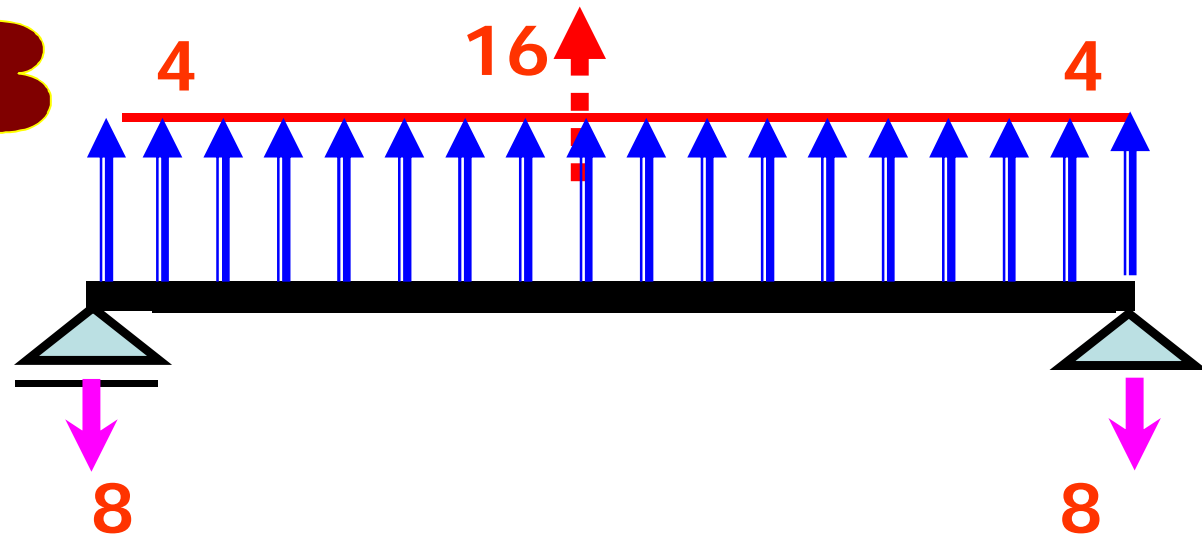
2- convert to conjugate



3-Final Beam



Part AB



$$sh_a = -8 \quad >>>> \quad \theta_a = \frac{(-8)}{EI}$$

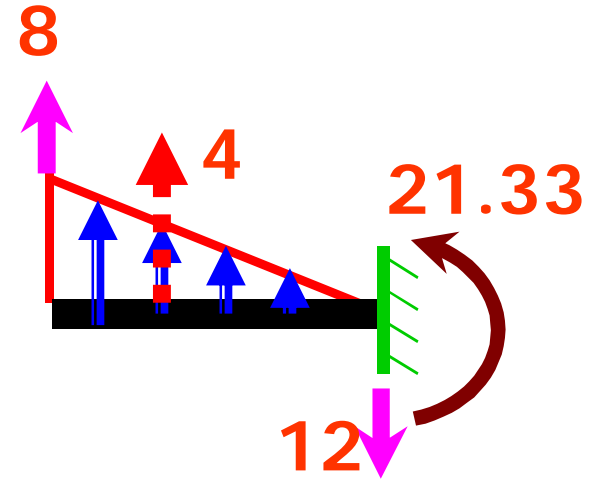
$$M_a = 0.0 \quad >>>> \quad y_a = \frac{(0.0)}{EI} = 0$$

$$sh_b = +8 \quad >>>> \quad \theta_b = \frac{(+8)}{EI}$$

$$M_b = 0.0 \quad >>>> \quad y_b = \frac{(0.0)}{EI} = 0$$

الـ shear من الشمال وطالع موجب ومن
اليمن ونازل برضه موجب والعكس

Part BD

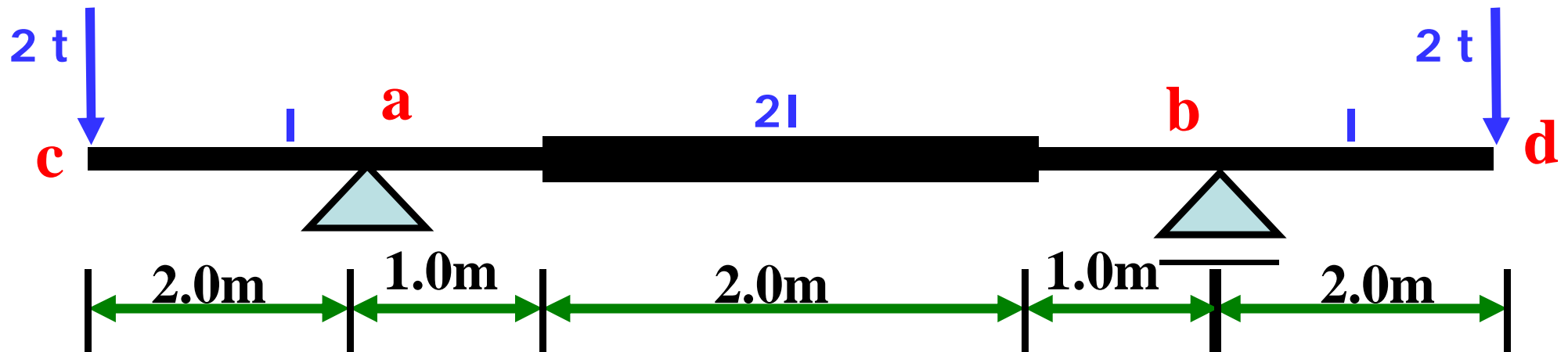


$$sh_d = +12 \gggg \theta_d = \frac{(+12)}{EI}$$

$$M_d = +21.33 \gggg y_d = \frac{(+21.33)}{EI}$$

بعض الأفكار المهمة جداً

1- حالة وجود تغير فى الـ inertia

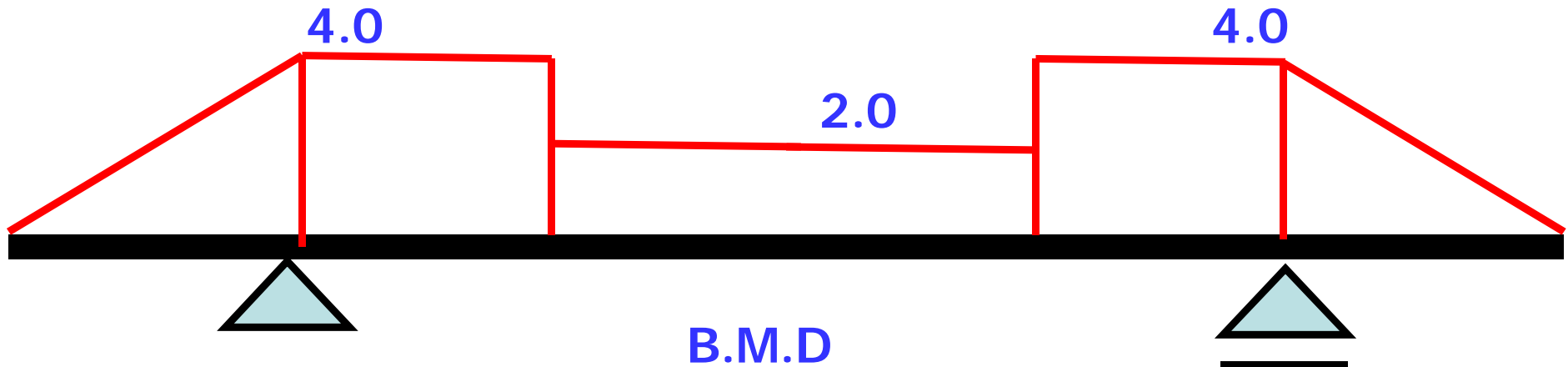


Required the deflection and rotation at point a, b, c and d.

And also draw elastic curve

1-Draw B.M.d

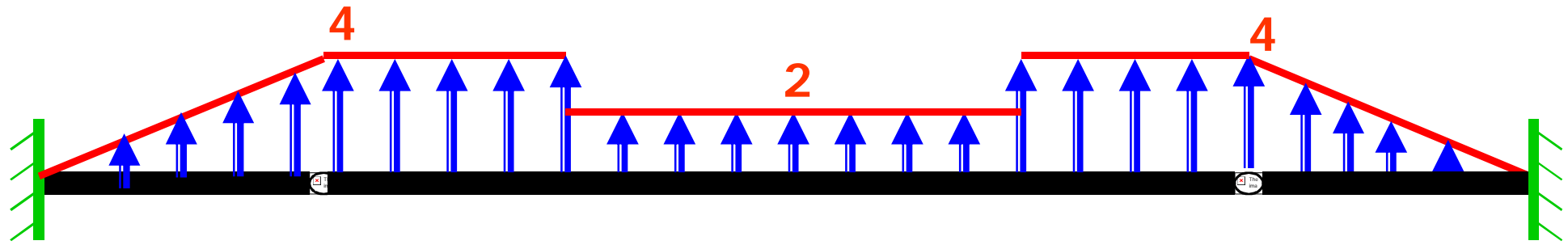
يتم قسمة المساحة على المعامل
الموجود بجوار كل inertia



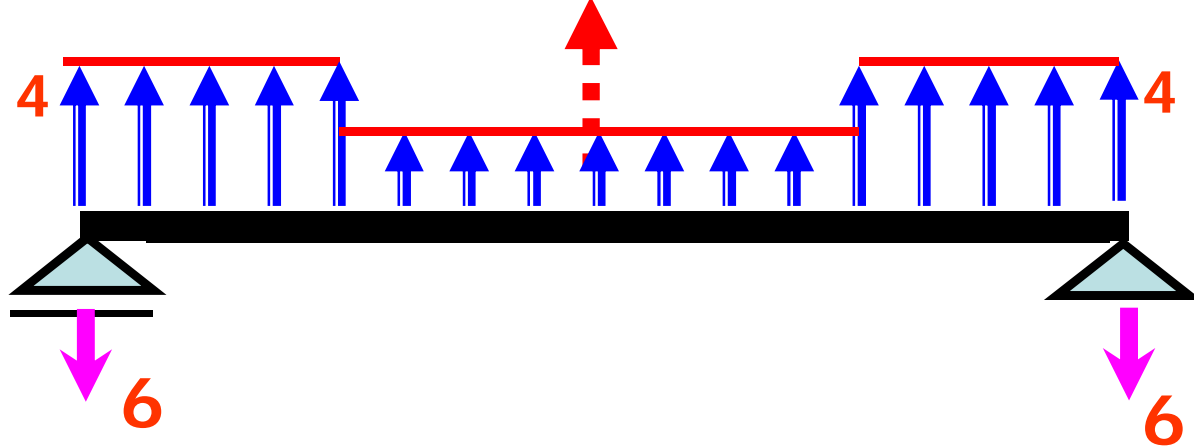
2- convert to conjugate



3-Final Beam



$$4 * 1 + 4 * 1 + 2 * 2 = 12$$



كامل الحل عادى

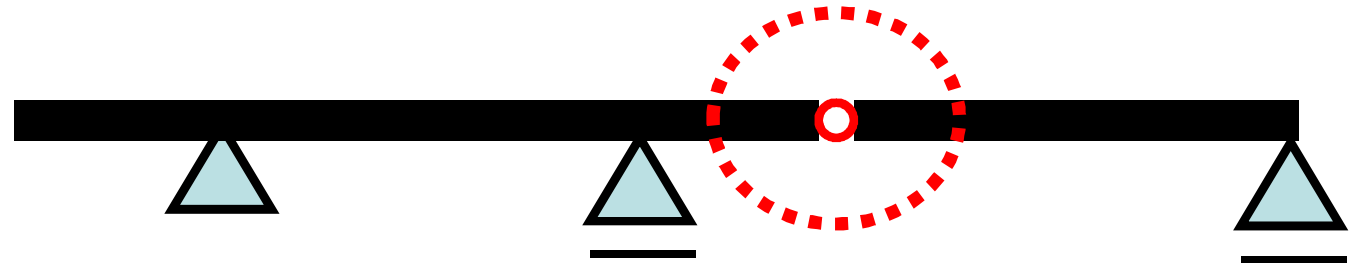
$$\theta = \frac{(\text{shear})}{EI}$$

$$y = \frac{(\text{moment})}{EI}$$

1- حالة وجود intermediate hinge فى الكمرة الرئيسية

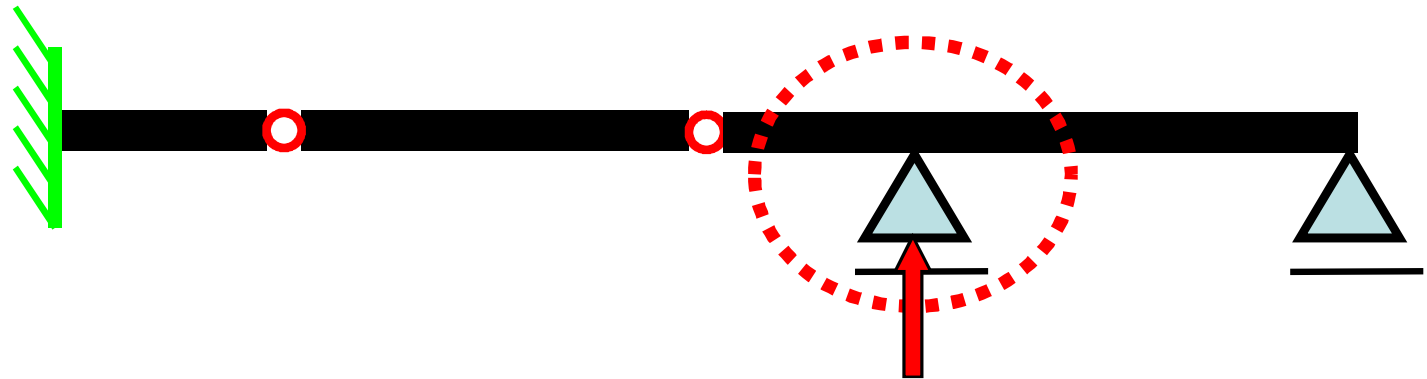
This image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

Real beam



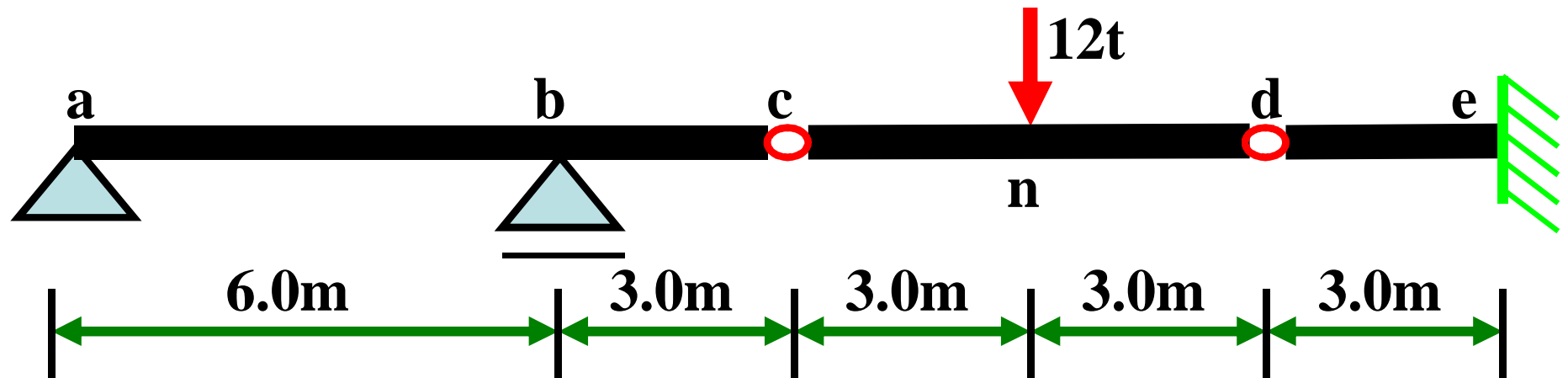
This image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been corrupted. Restart your computer, and then open the file again. If the red x still appears, you may have to delete the image and then insert it again.

conjugate beam



نلاحظ عند تغيير intermediate hinge ال hinge وسطى يكون هناك reaction عند هذه النقطة وبالتالي يكون هناك قيمتين لل shear عند هذه النقطة وبالتالي وجود قيمتين للدوران يتم طلبهم فى المسألة بالدوران يمين وشمال النقطة θ_R and θ_L

final 2006

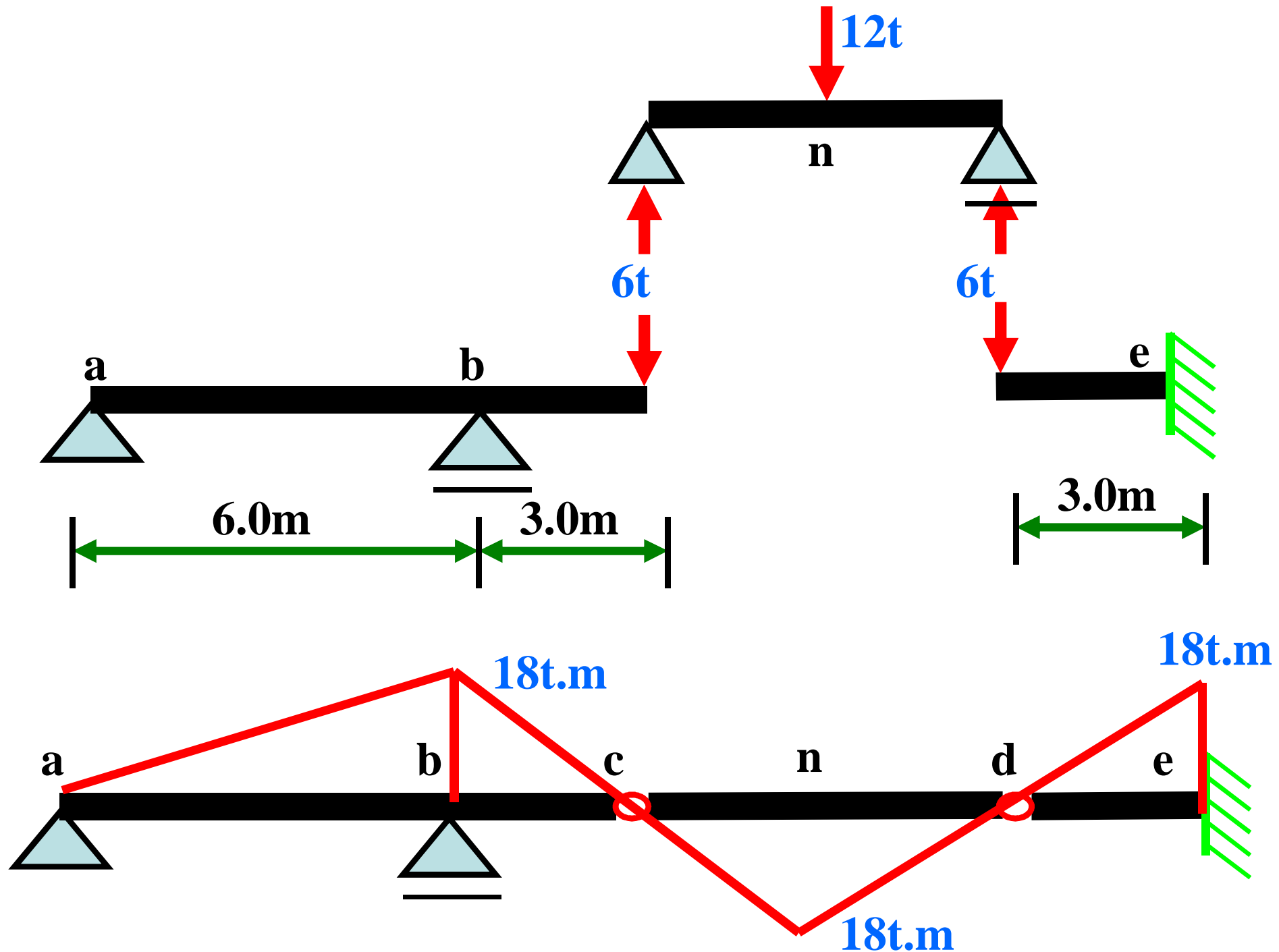


Required the deflection at c, d, n and slope at point n, b.

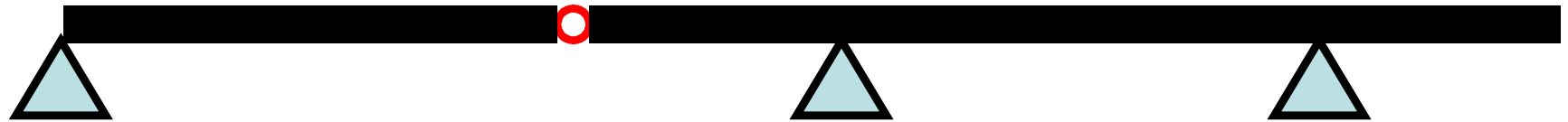
And draw elastic curve

$$EI = 5400 \text{ t.m}^2$$

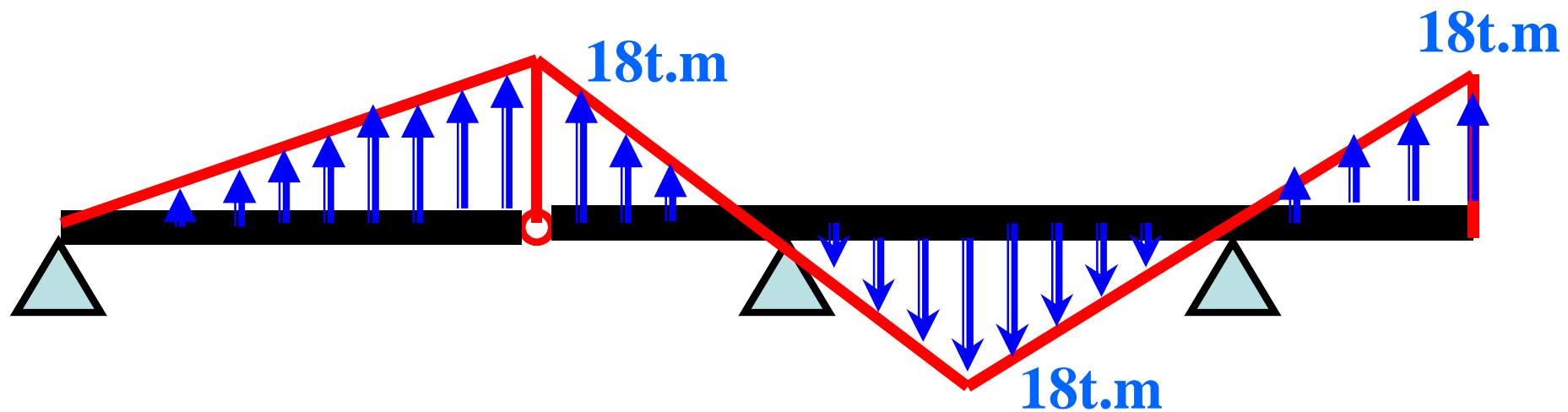
draw B.M.D

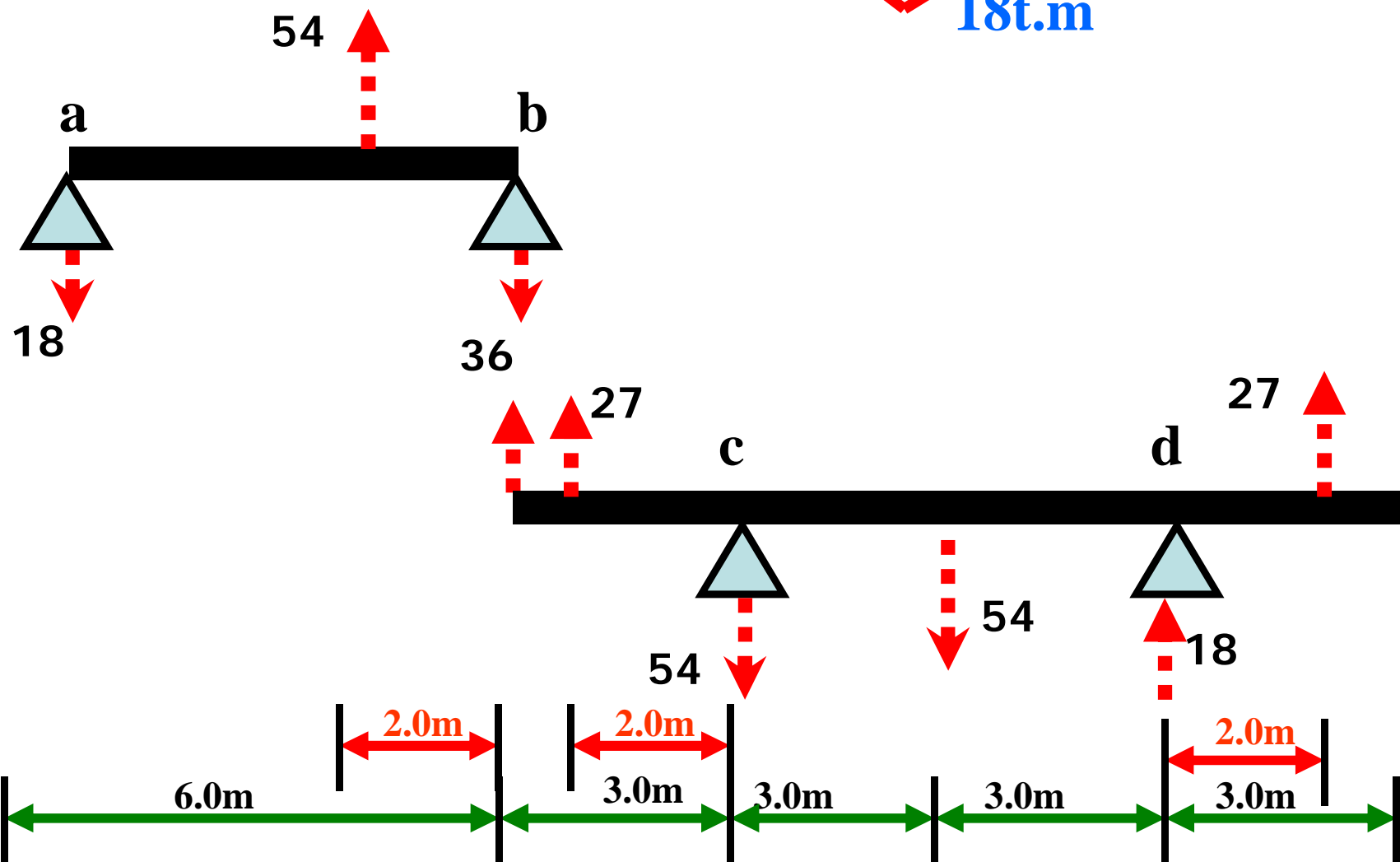
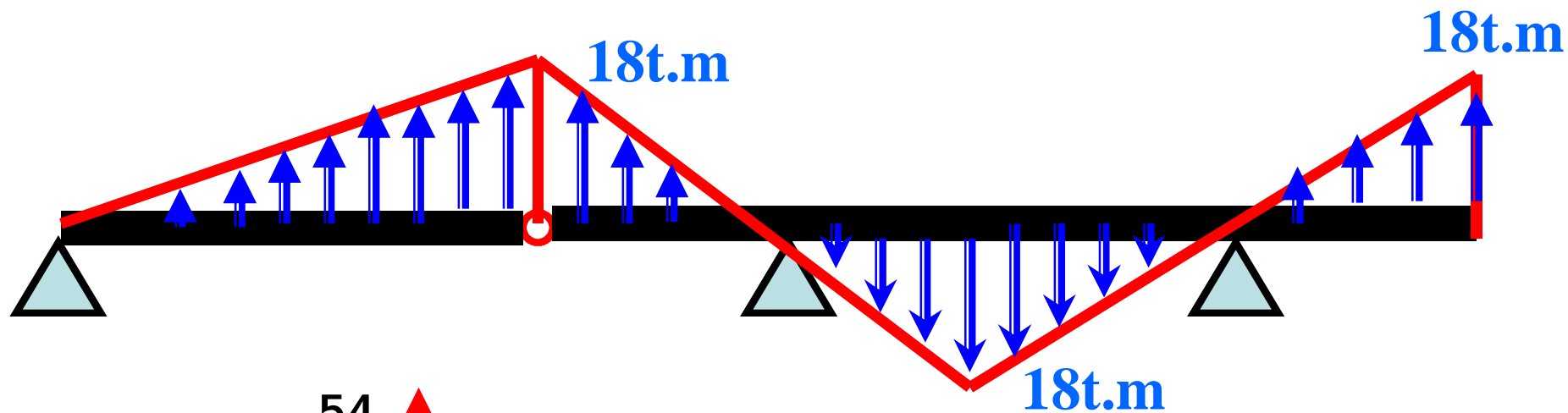


convert to conjugate beam



the final beam with the new load

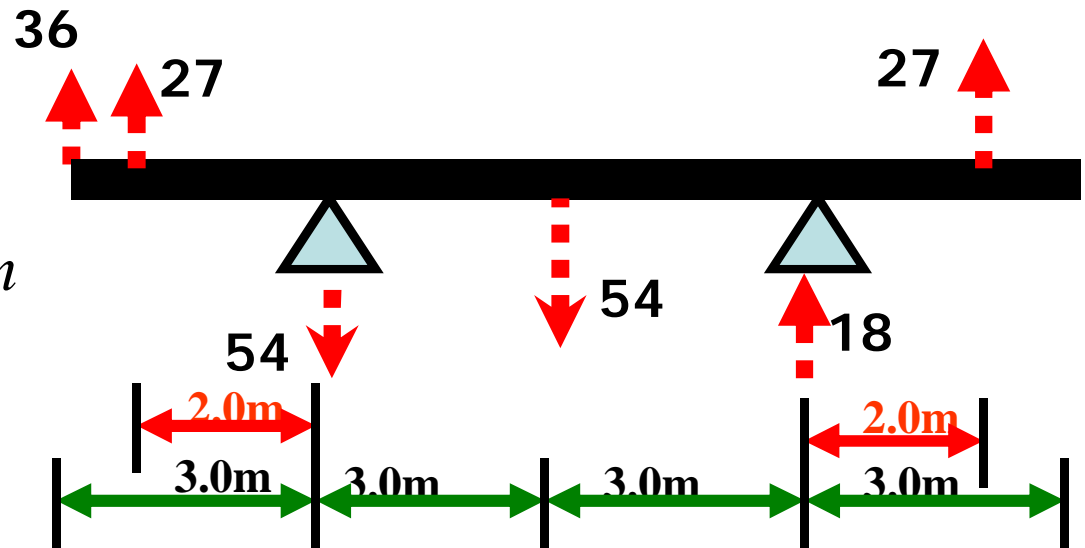




Part B-D

$$y_c = \frac{(36 * 3 + 27 * 2)}{5400} = 0.03 \text{ m}$$

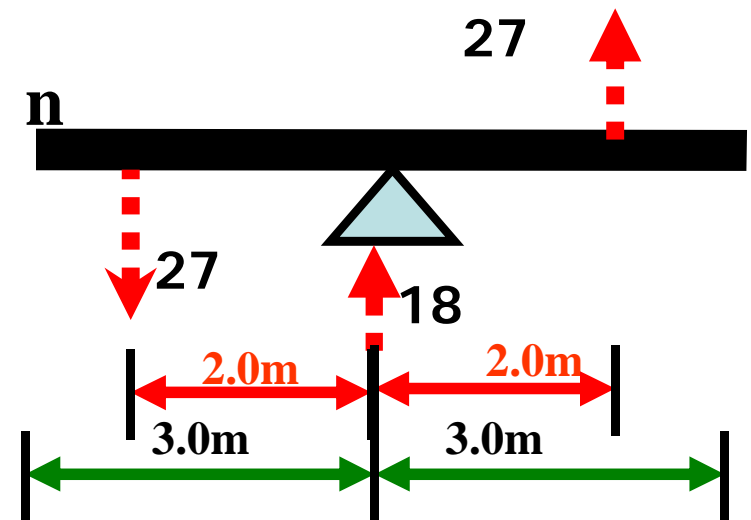
$$y_d = \frac{(27 * 2)}{5400} = 0.01 \text{ m}$$



point n

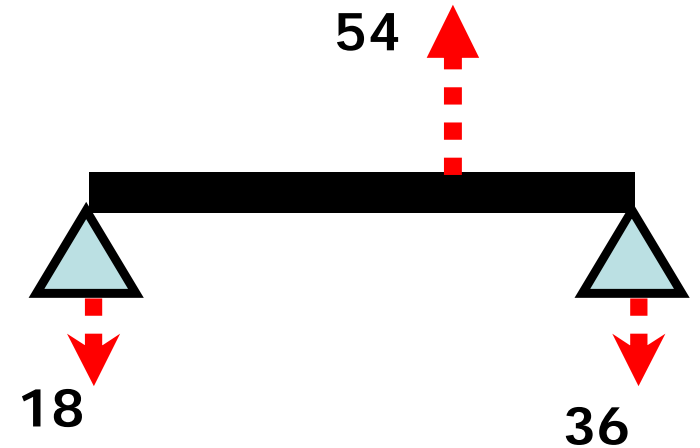
$$\theta_n = \frac{-27 - 18 + 18}{5400} = -0.0033$$

$$y_d = \frac{(-27 * 1 + 18 * 3 + 27 * 5)}{5400} = 0.03 \text{ m}$$

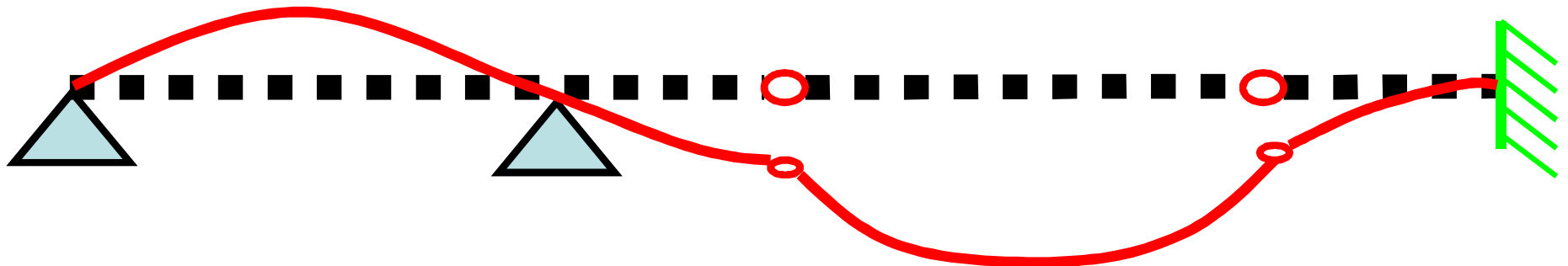


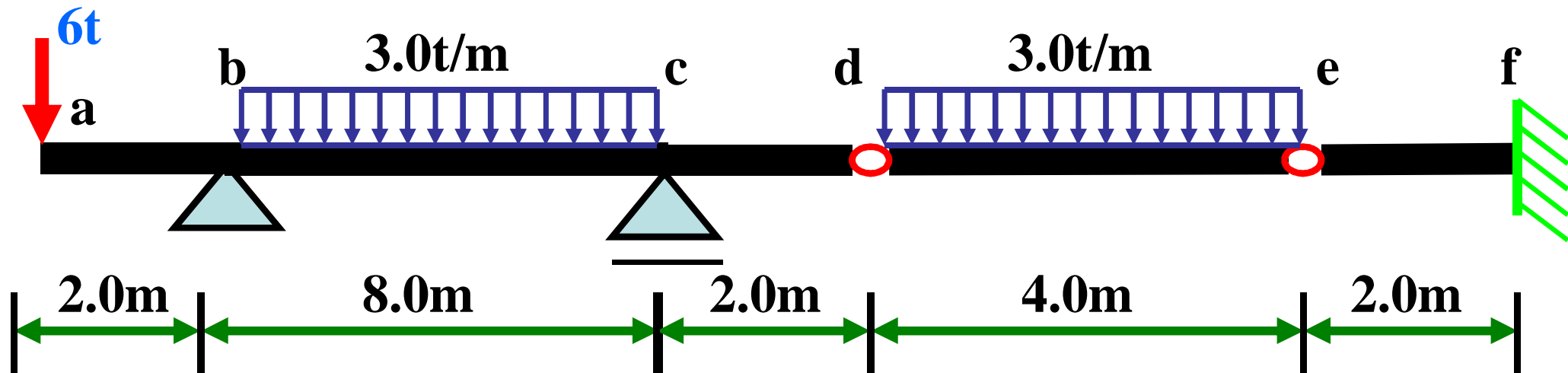
Part A-B

$$\theta_b = \frac{(36)}{5400} = 0.0066 \text{ rad}$$



Elastic Curve

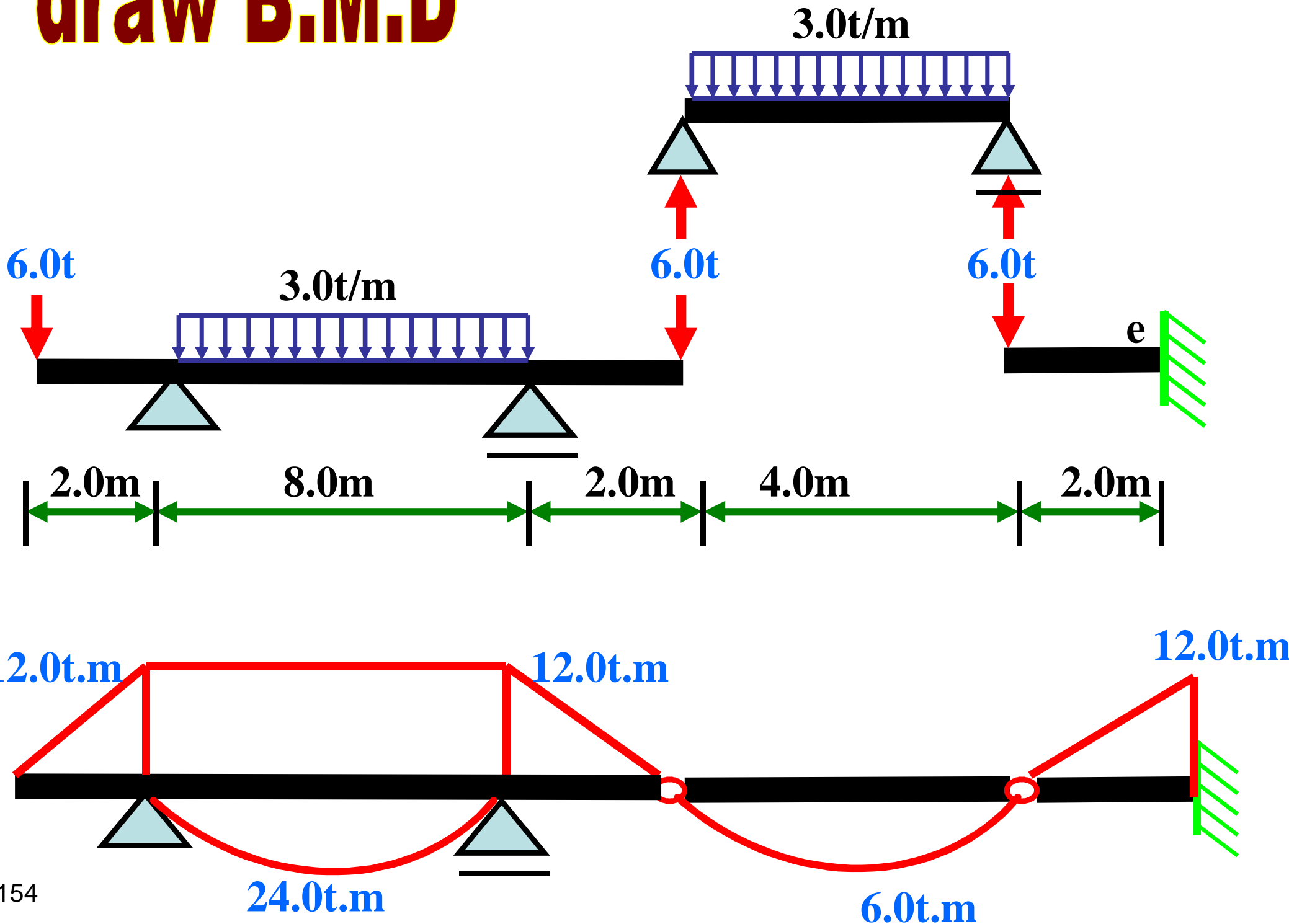




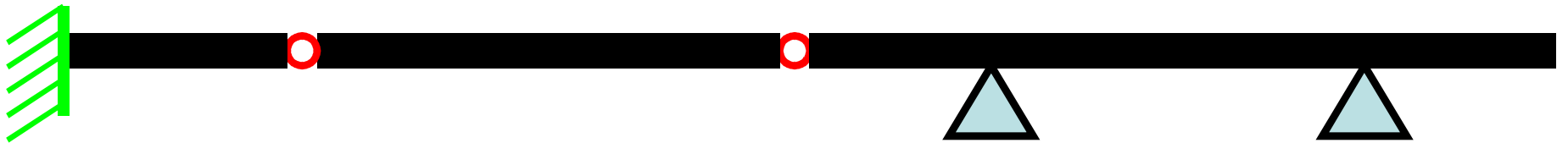
Required the deflection at a, d, e and slope at point b and (θ_{dR}) and (θ_{dL}) and $(\theta_{e\text{rel}})$. And draw elastic curve.

$$EI = 8000 \text{ t.m}^2$$

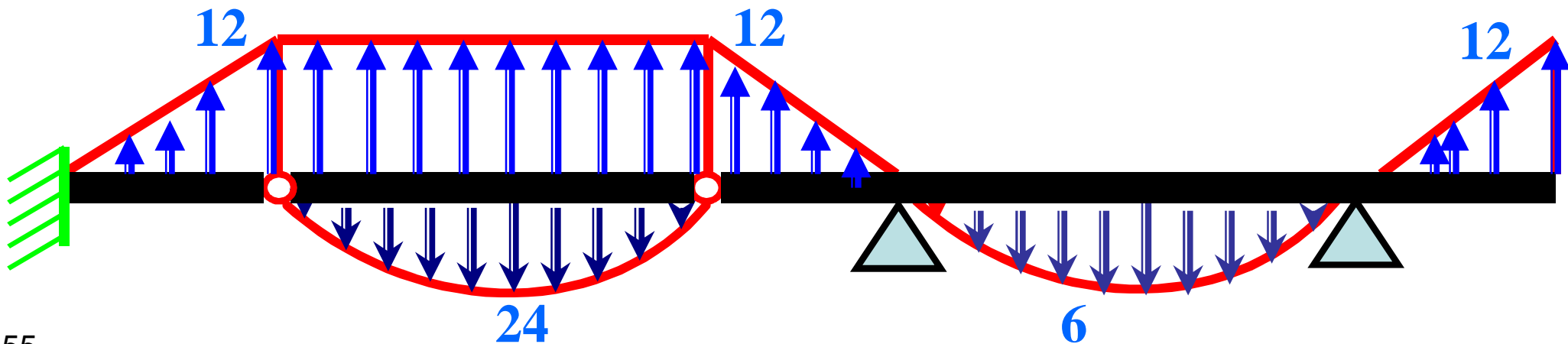
draw B.M.D

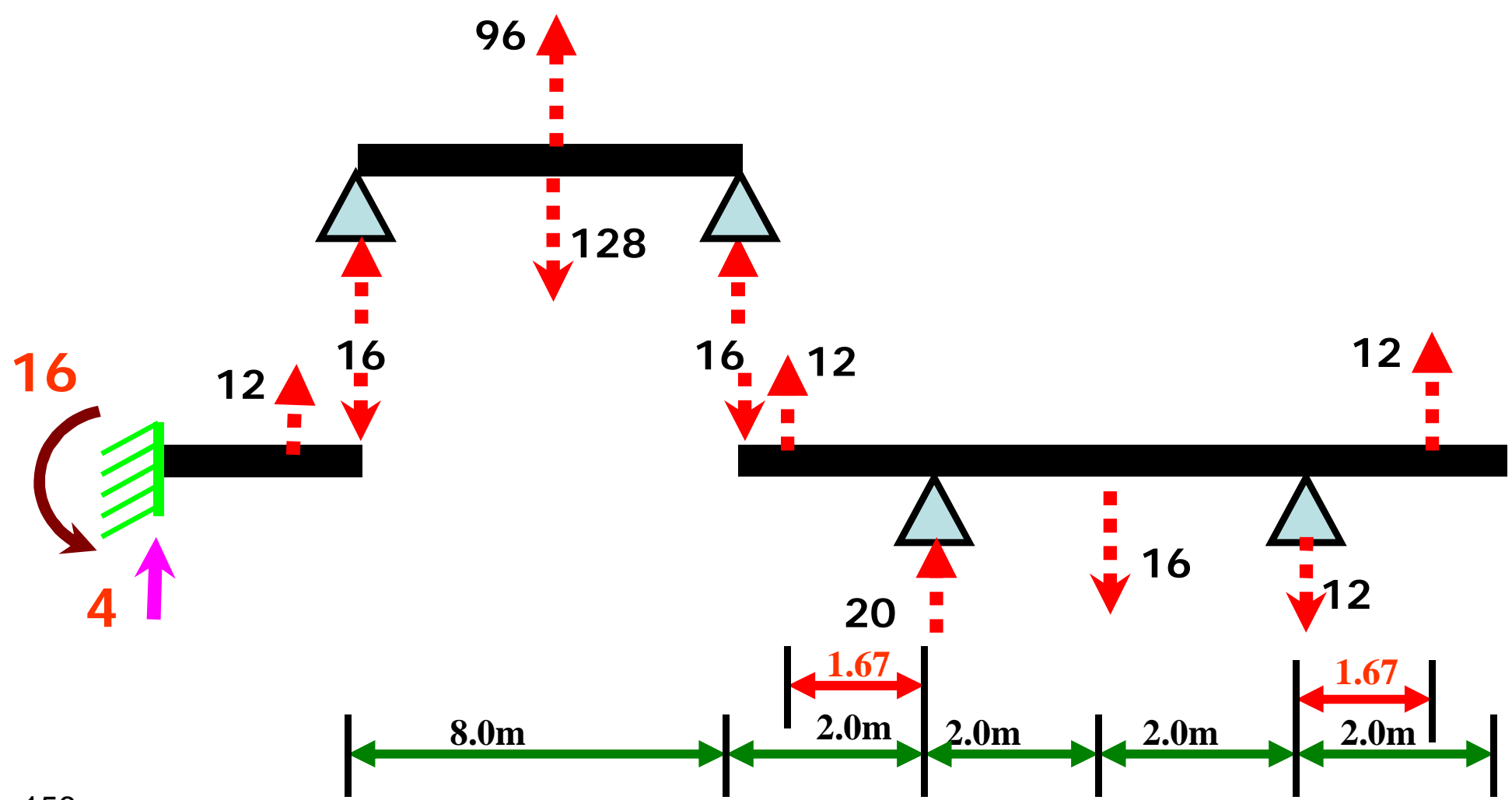
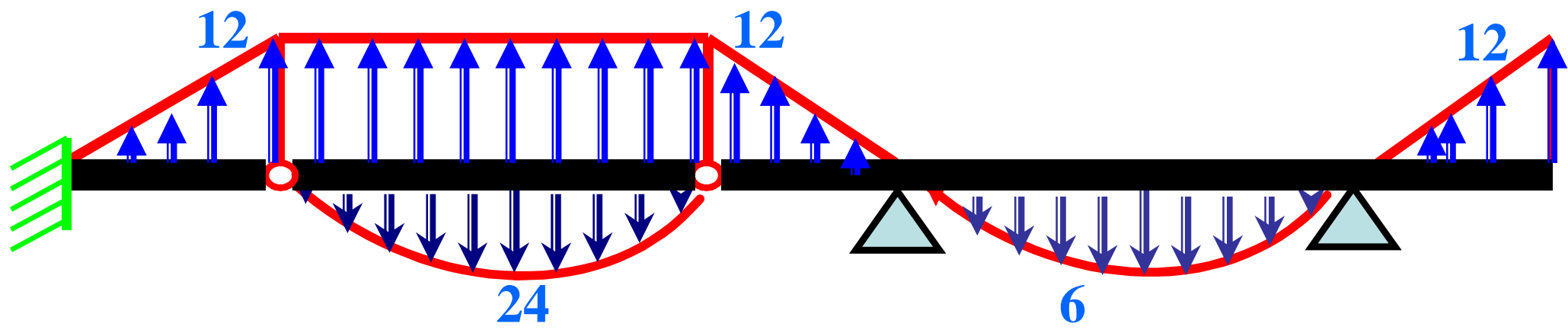


convert to conjugate beam

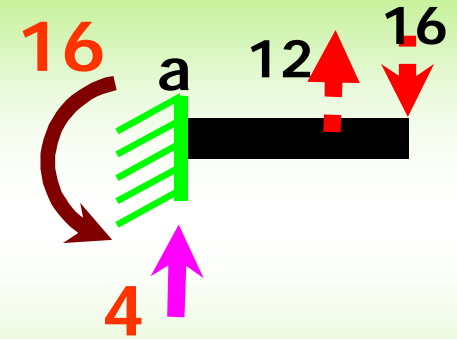


the final beam with the new load



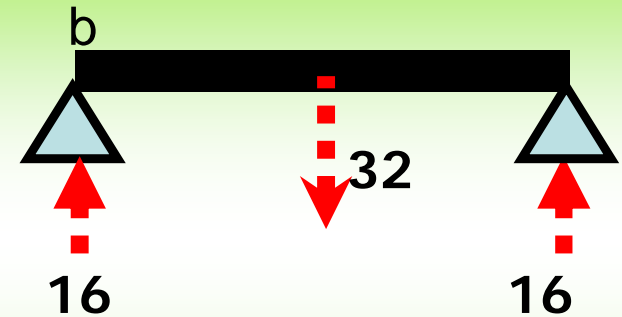


Part A-B



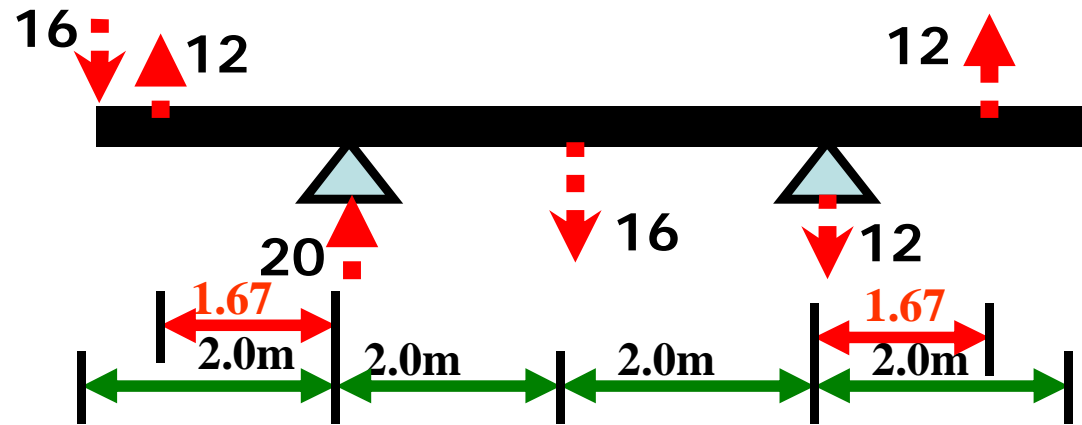
$$y_a = \frac{(-16)}{8000} = -0.002 \text{ m}$$

Part B-C



$$y_b = \frac{(+16)}{8000} = 0.002 \text{ rad}$$

Part C-F



$$y_d = \frac{(-16 * 2 + 12 * 1.67)}{8000} = -0.002 \text{ m}$$

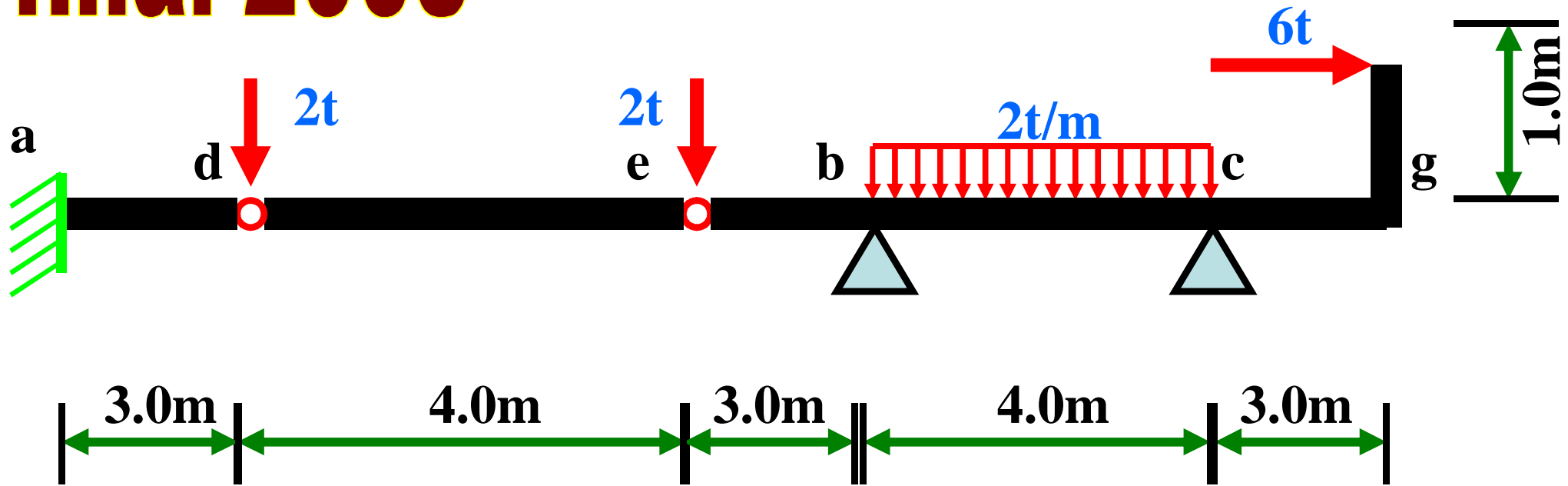
$$y_e = \frac{(12 * 1.67)}{8000} = 0.002 \text{ m}$$

$$\theta_{d \text{ rel}} = \frac{(\text{reaction @ } d)}{8000} = \frac{(20)}{8000} = 0.0025 \text{ rad}$$

$$\theta_{d \text{ L}} = \frac{(\text{shear @ } d \text{ left})}{8000} = \frac{(-16 + 12)}{8000} = -0.0005 \text{ rad}$$

$$\theta_{d \text{ R}} = \frac{(\text{shear @ } d \text{ right})}{8000} = \frac{(-16 + 12 + 20)}{8000} = 0.002 \text{ rad}$$

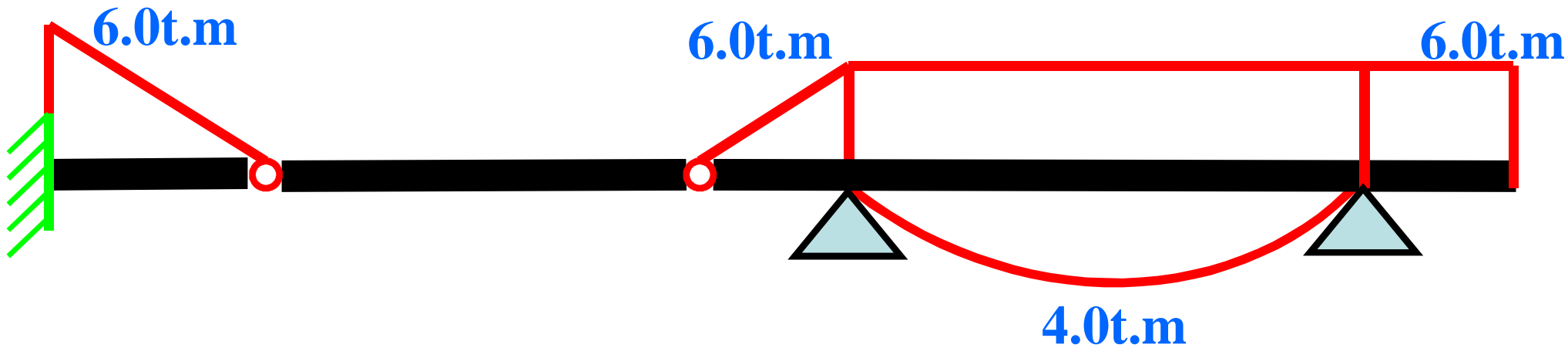
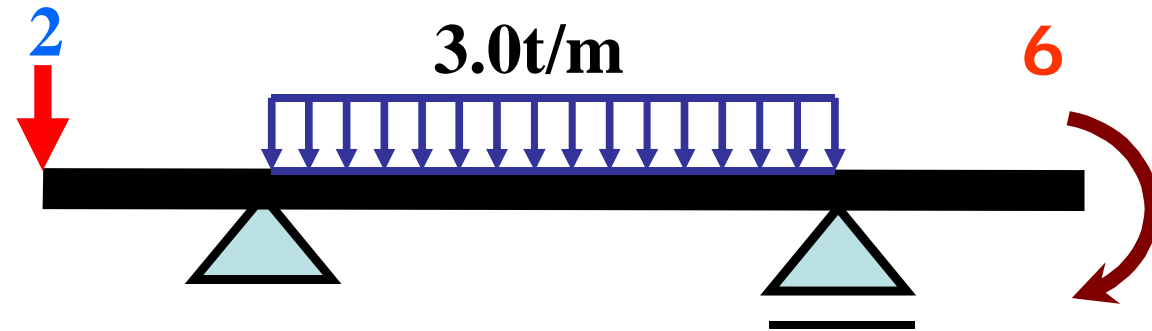
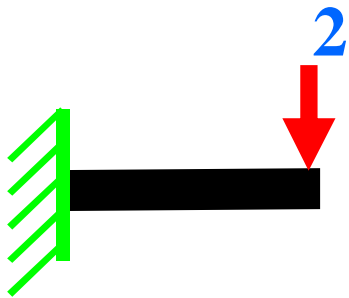
final 2008



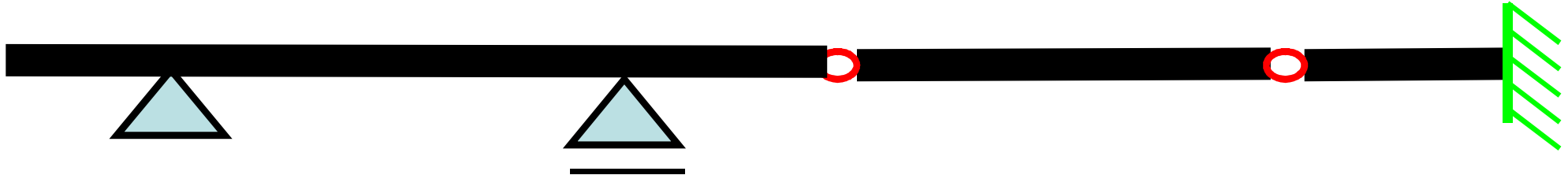
Required the deflection at **g**, **d**, **e** and slope at point **b** and (θ_{dR}) and $(\theta_{e\text{rel}})$.

$$EI = 8000 \text{ t.m}^2$$

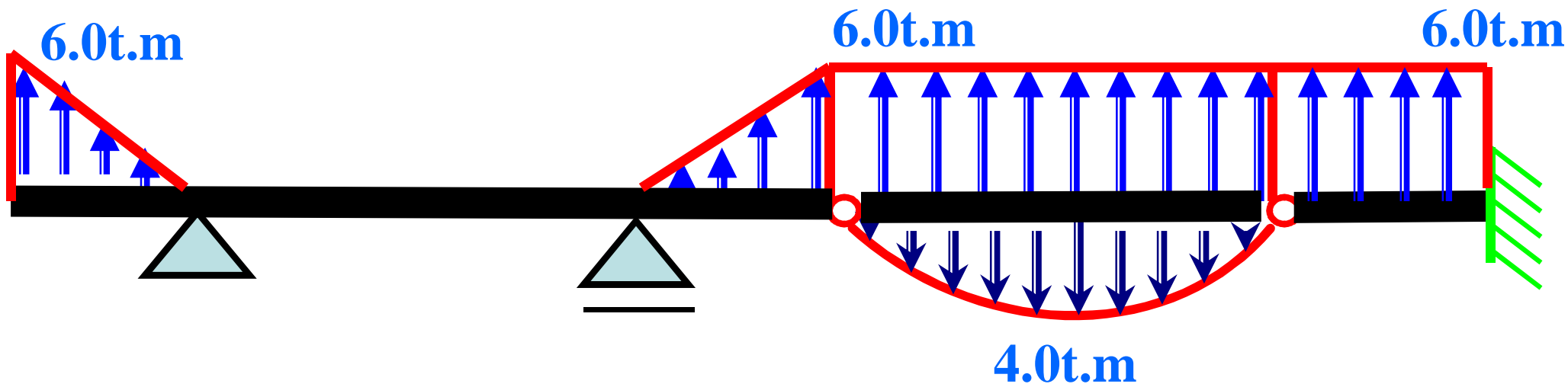
draw B.M.D

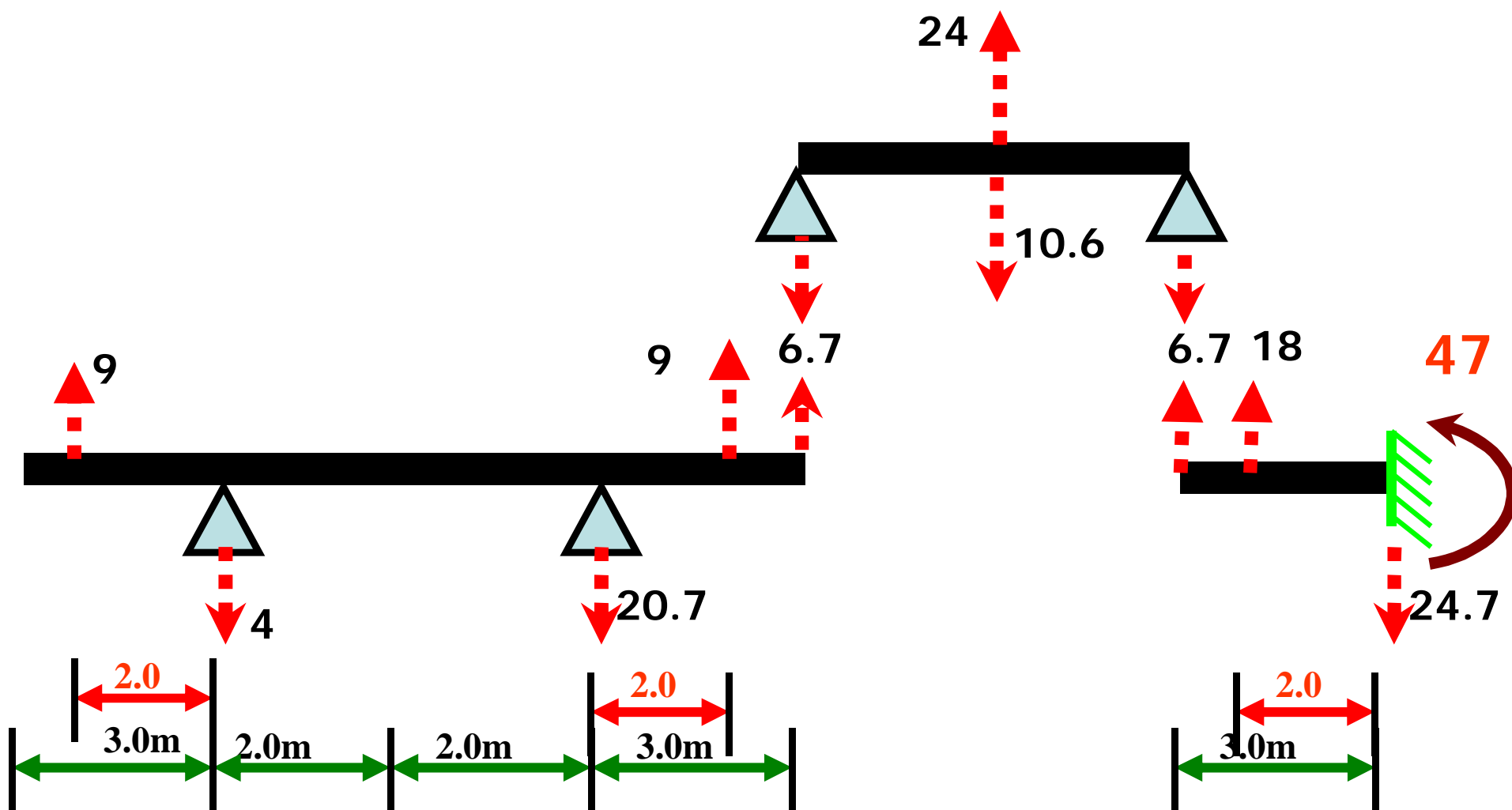
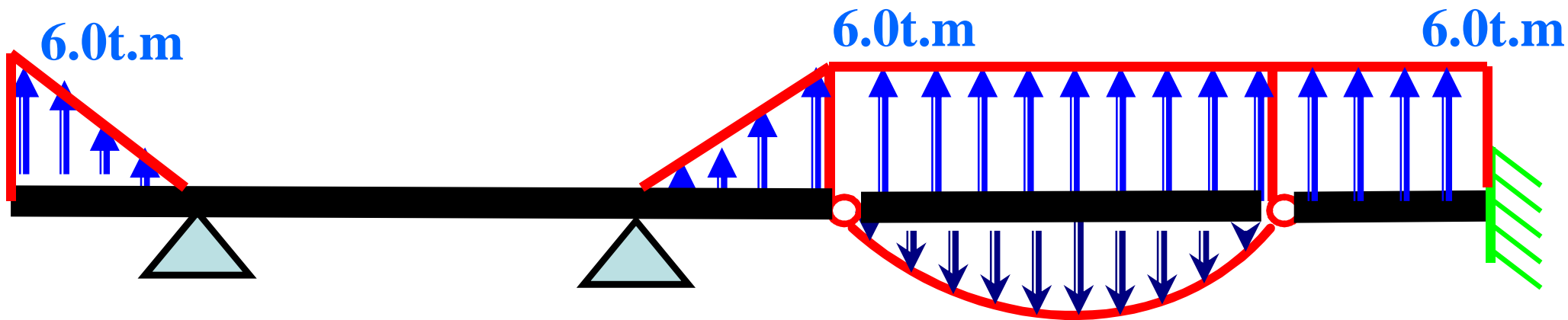


convert to conjugate beam



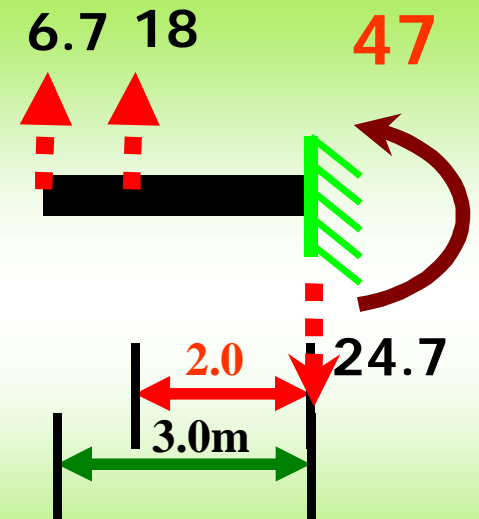
the final beam with the new load





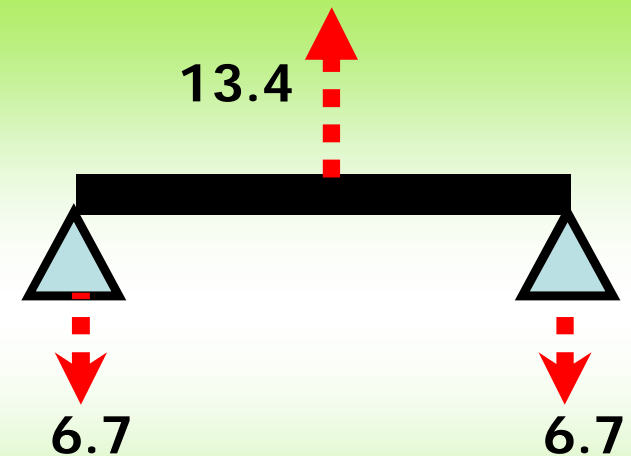
Part A-B

$$y_g = \frac{(47)}{8000} = 0.005875 \text{ m}$$

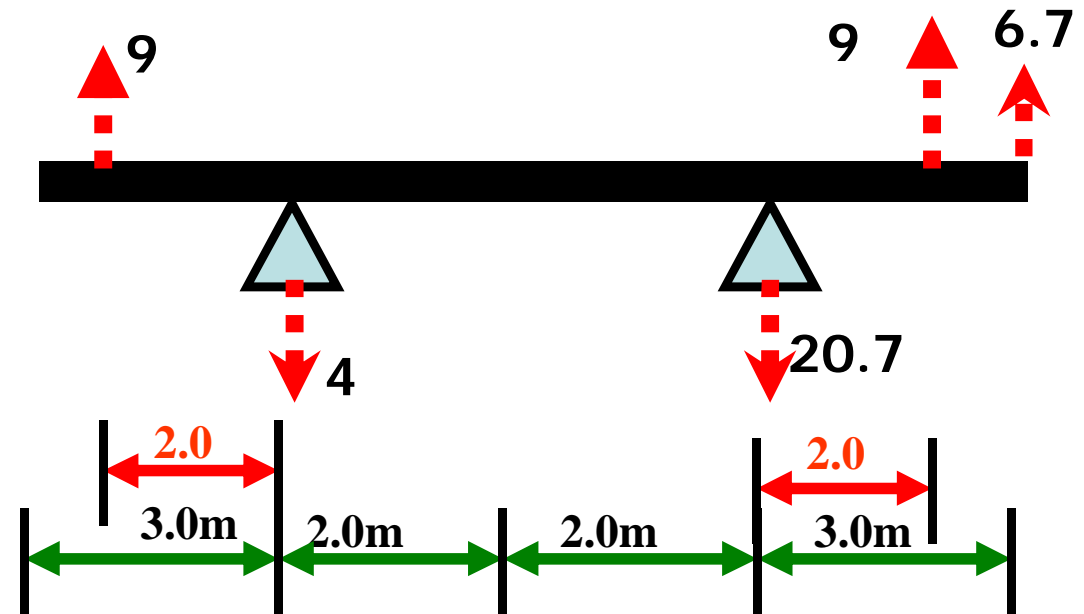


Part B-C

$$\theta_b = \frac{(-6.7)}{8000} = -0.0083 \text{ rad}$$



Part C-F



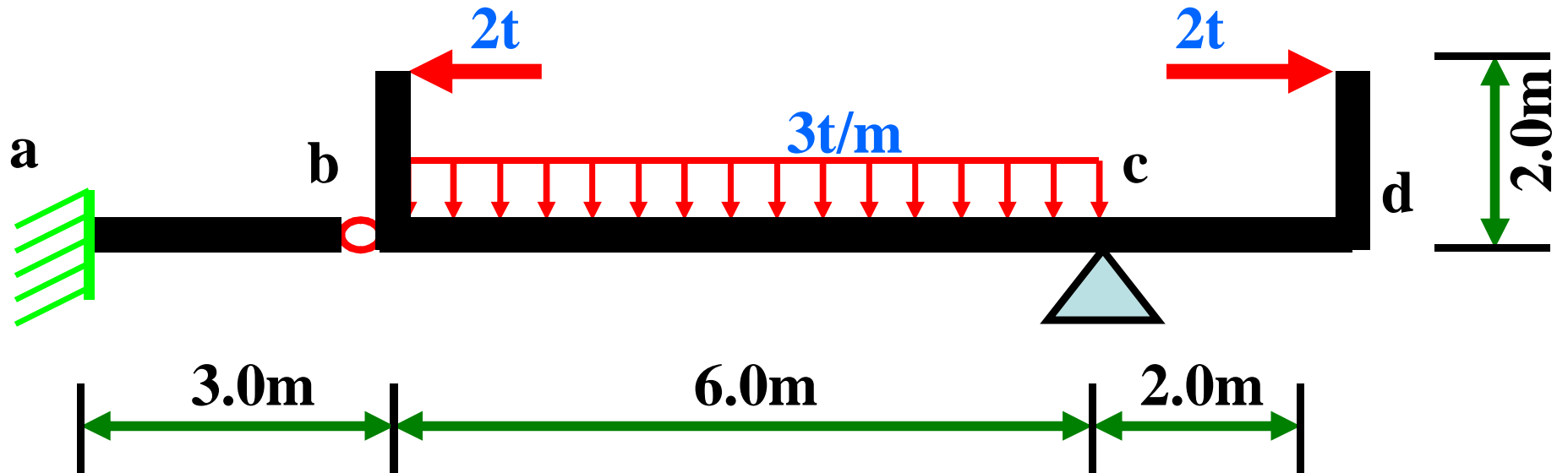
$$y_d = \frac{(9 * 2)}{8000} = 0.00225 \text{ m}$$

$$y_e = \frac{(9 * 2 + 6.7 * 3)}{8000} = 0.00475 \text{ m}$$

$$\theta_{e \text{ rel}} = \frac{(\text{reaction @ } e)}{8000} = \frac{(-20.7)}{8000} = -0.00257 \text{ rad}$$

$$\theta_{d \text{ R}} = \frac{(\text{shear @ } d \text{ right})}{8000} = \frac{(9 - 4)}{8000} = 0.000625 \text{ rad}$$

Mid term 2008

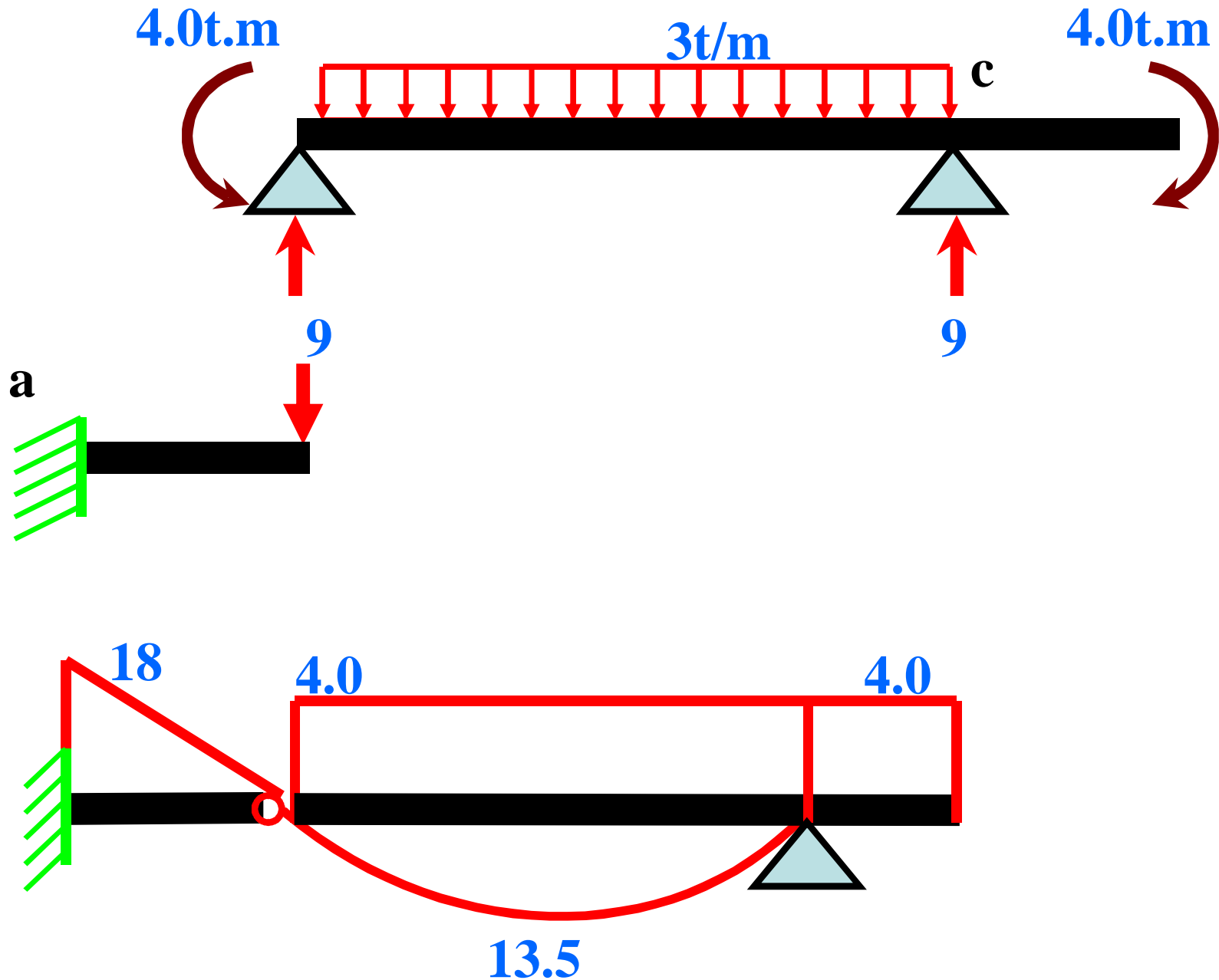


Required:

- 1- the left rotation at b.
- 2- slope at d.
- 3- deflection at b and d.
- 4- draw elastic curve.

$$EI = 8000 \text{ t.m}^2$$

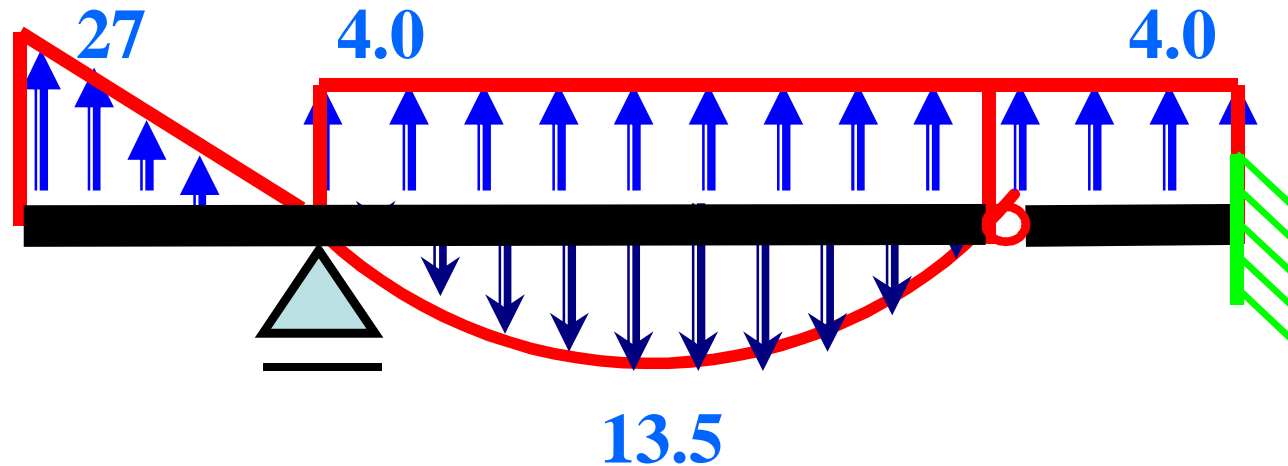
draw B.M.D

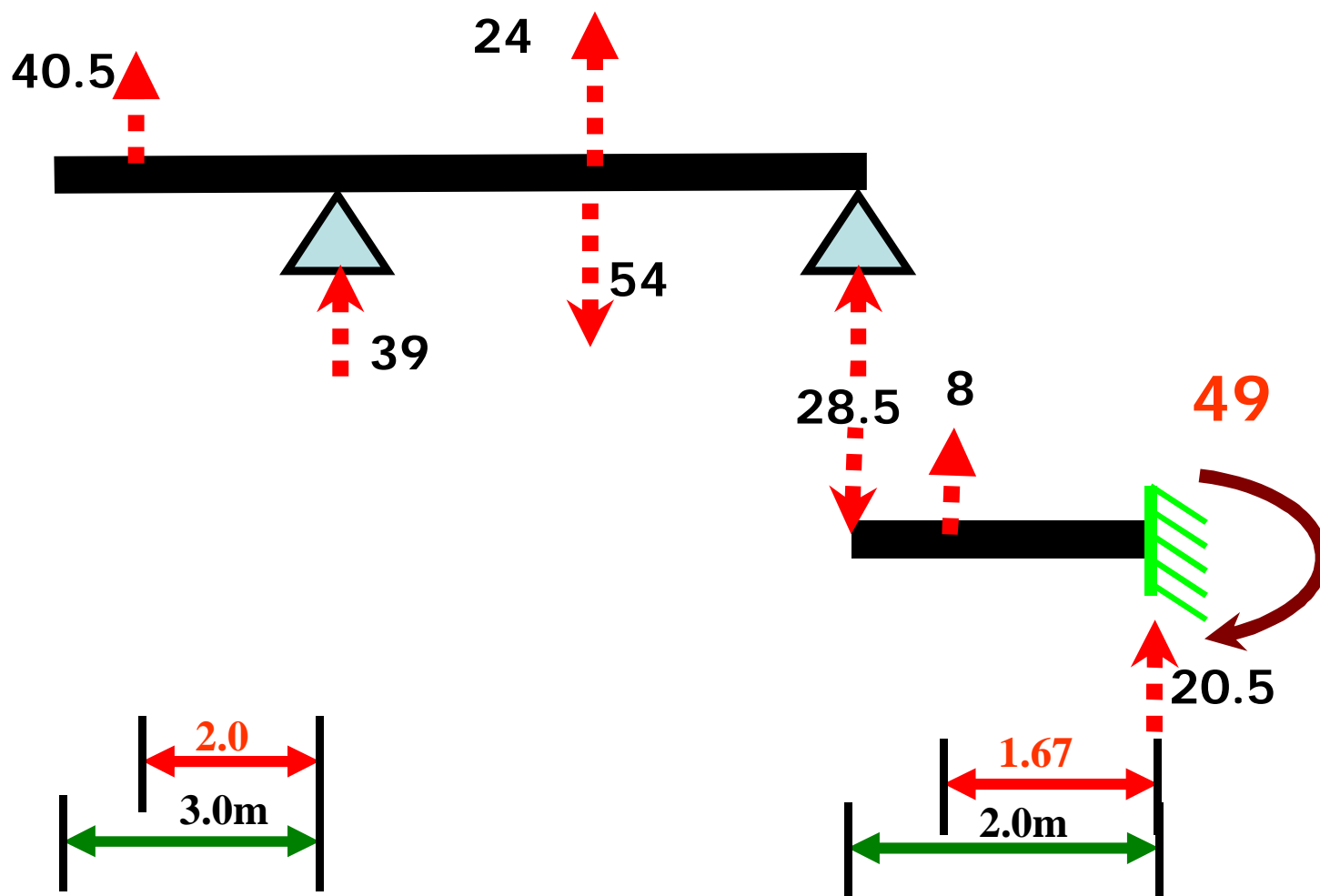
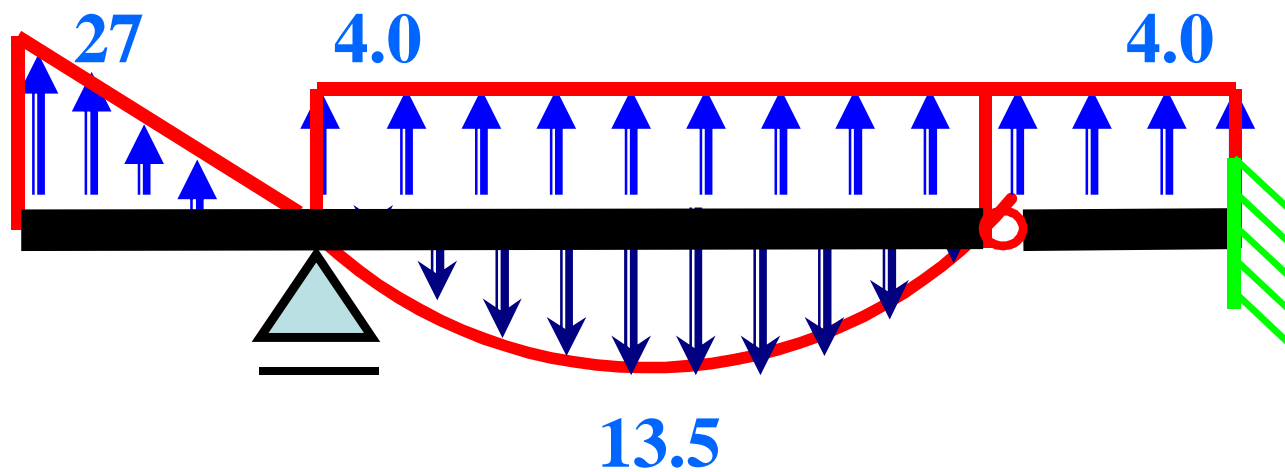


convert to conjugate beam



the final beam with the new load



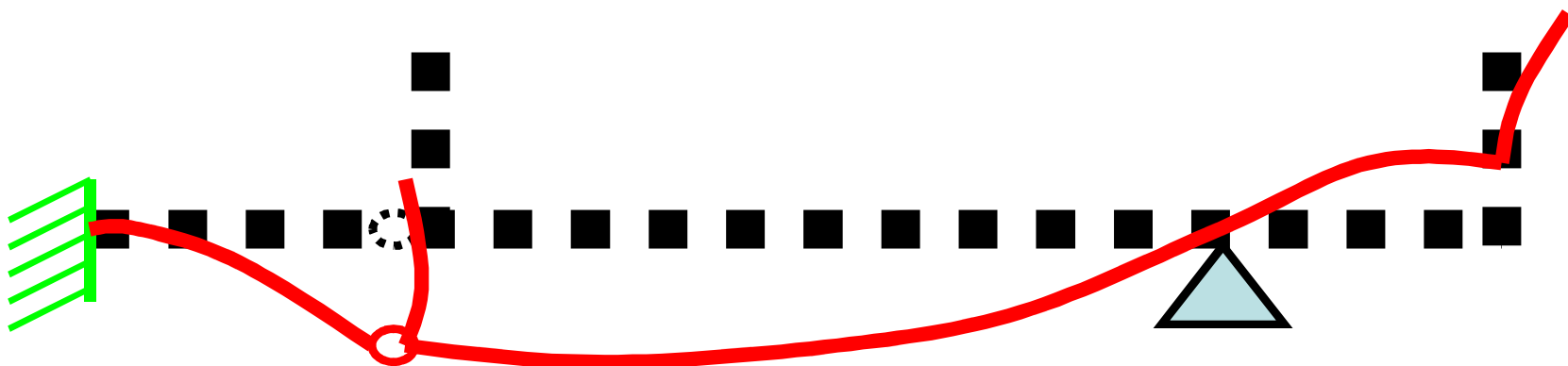


$$\theta_{b \text{ left}} = \frac{(40.5)}{8000} = 0.005 \text{ rad}$$

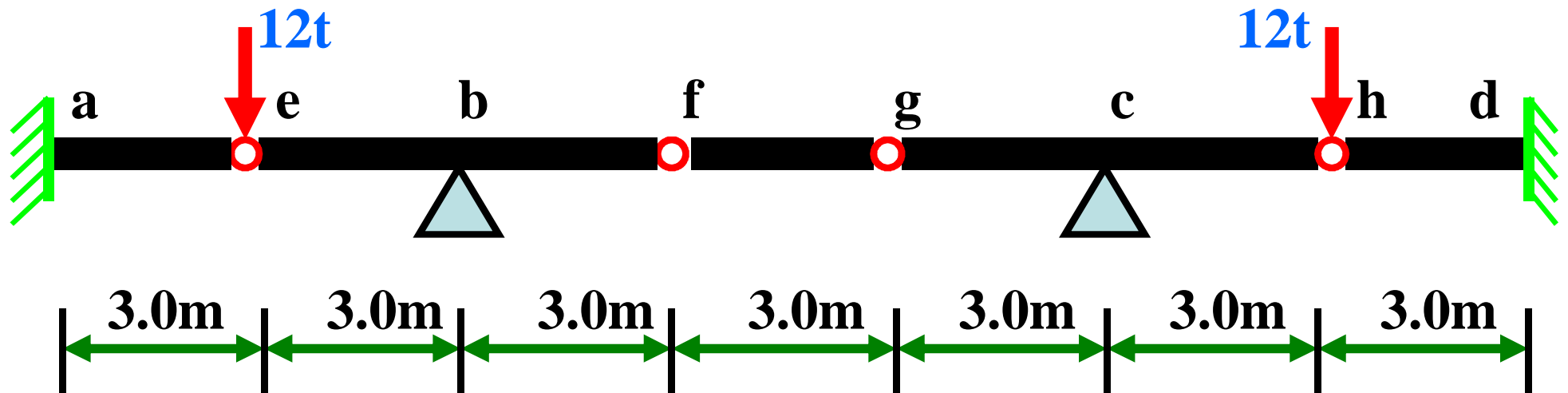
$$\theta_d = \frac{(-20.5)}{8000} = -0.0025 \text{ rad}$$

$$y_b = \frac{(40.5 * 2)}{8000} = 0.01 \text{ m}$$

$$y_d = \frac{(-49)}{8000} = -0.006 \text{ m}$$



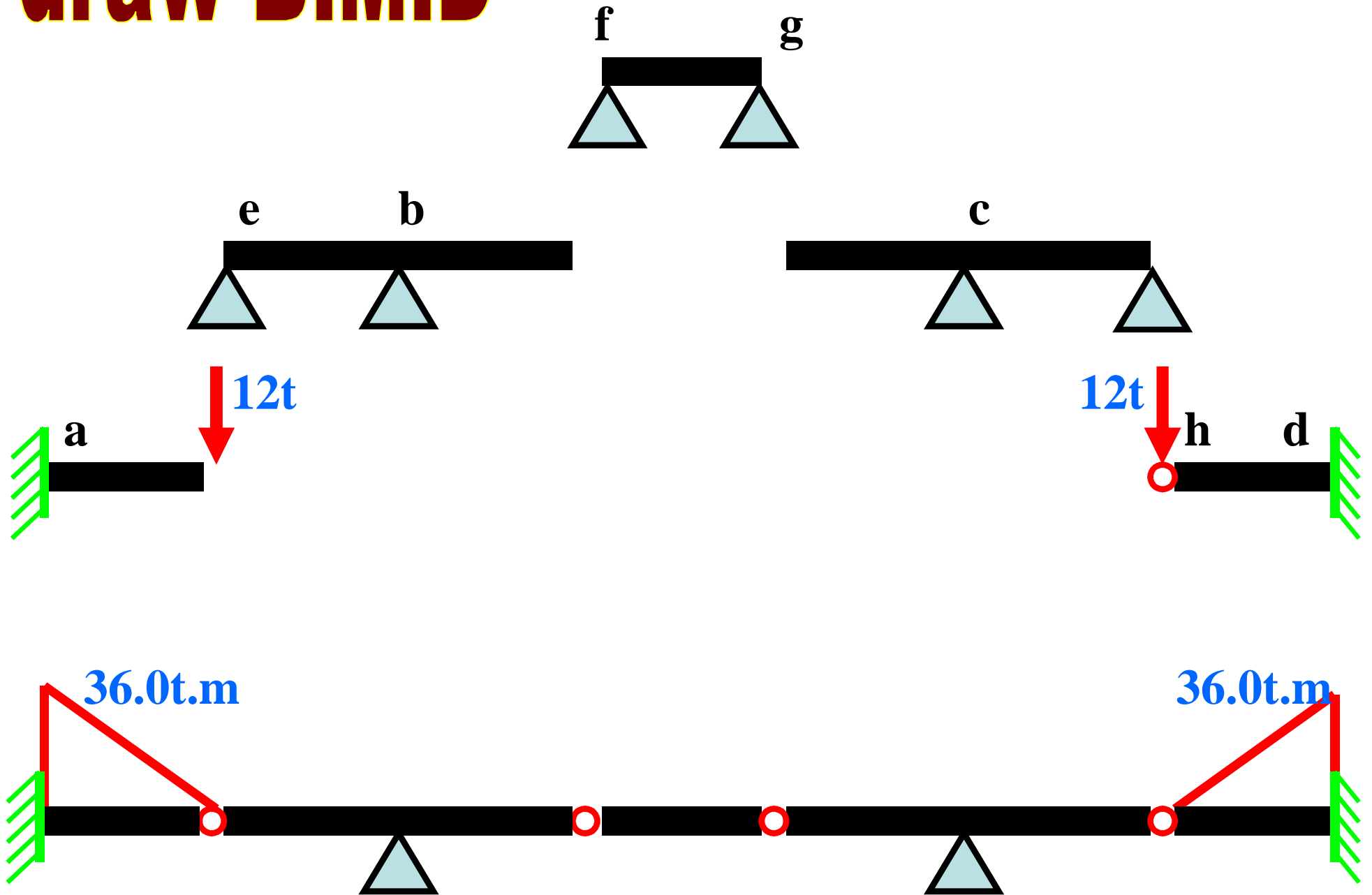
final 2009



Required slope and deflection at point **b**, **e**, **f** and **g**.

$$EI = 3600 \text{ t.m}^2$$

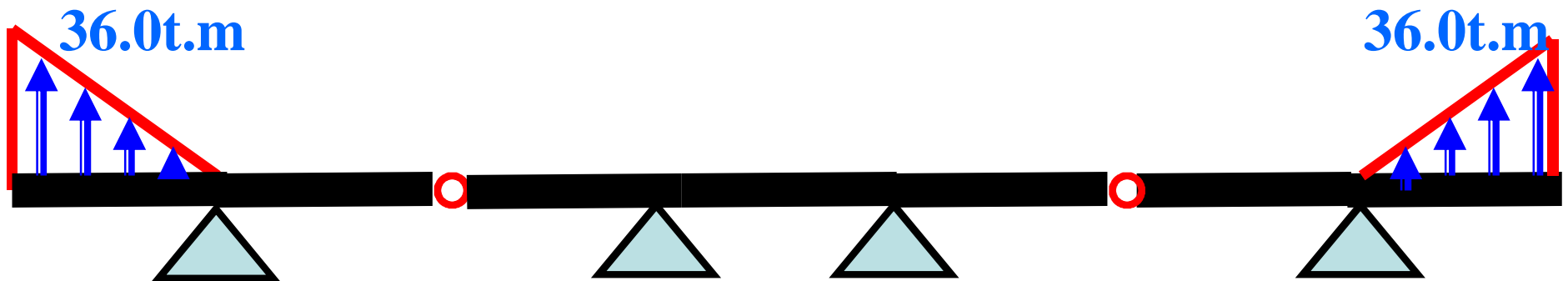
draw B.M.D

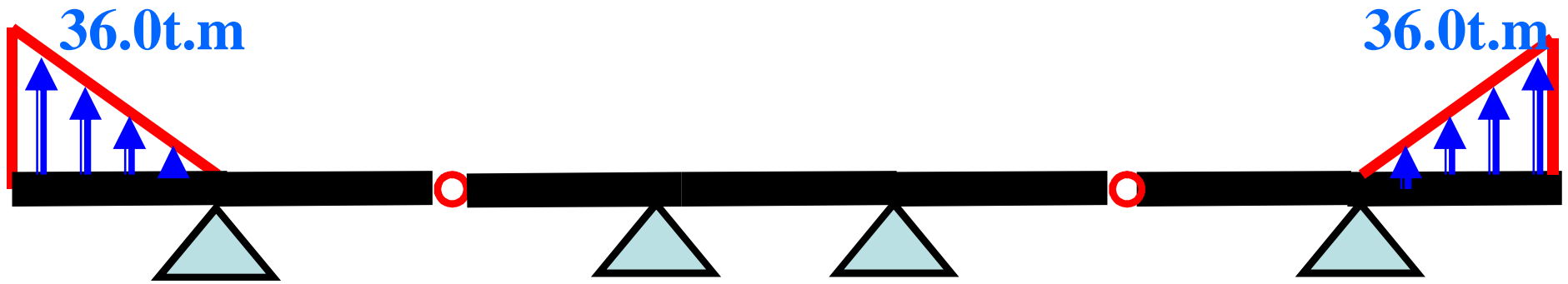


convert to conjugate beam



the final beam with the new load





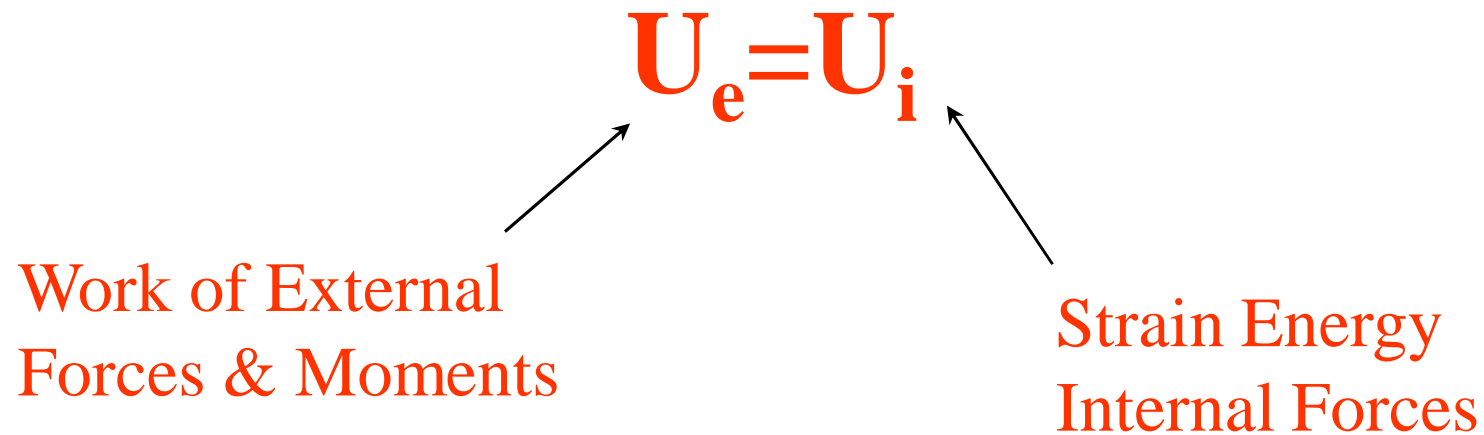
try your self

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

VERTUAL WORK

Eng : Aymman abdo

The principle of work and energy is:



$$\omega_{\text{internal}} = \int \frac{M_0 \cdot M_1}{EI} \cdot dl + \int \frac{Q_0 \cdot Q_1}{GA_r} \cdot dl + \int \frac{N_0 \cdot N_1}{EA} \cdot dl$$

M >>> Moment.

Q >>> Shear.

N >>> Normal.

$$\omega_{\text{internal}} = \int \frac{M_0 \cdot M_1}{EI} \cdot dl$$

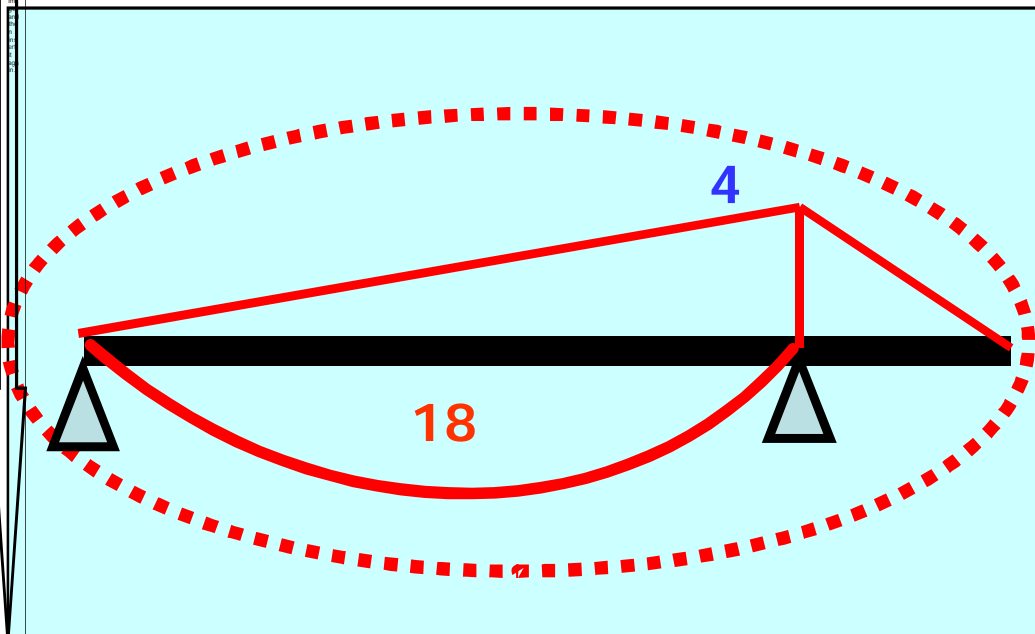
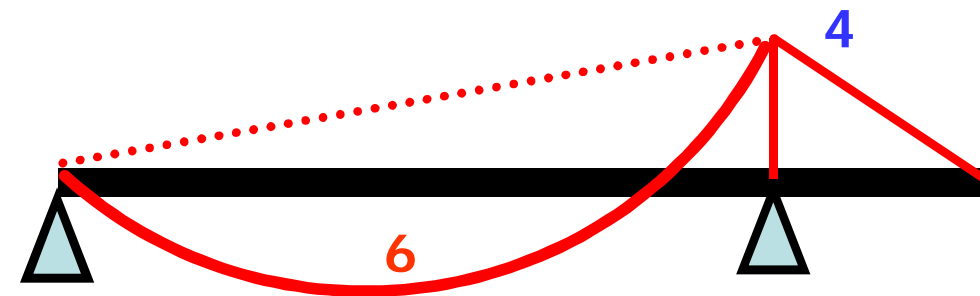
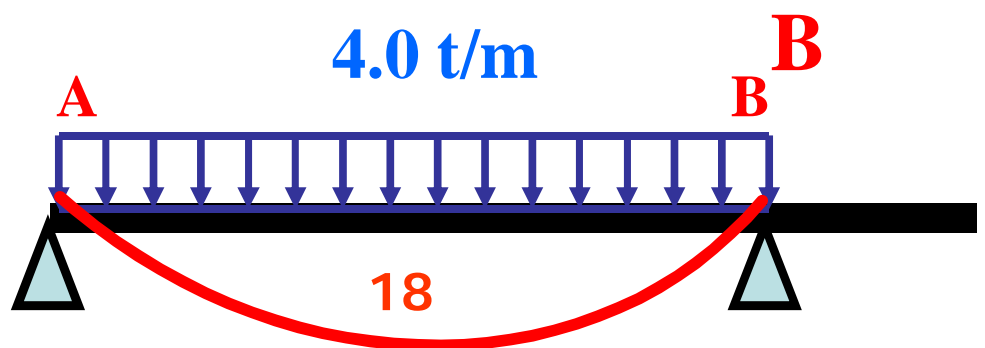
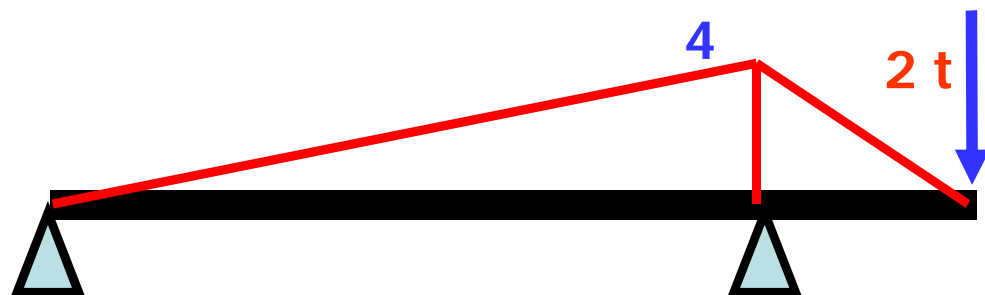
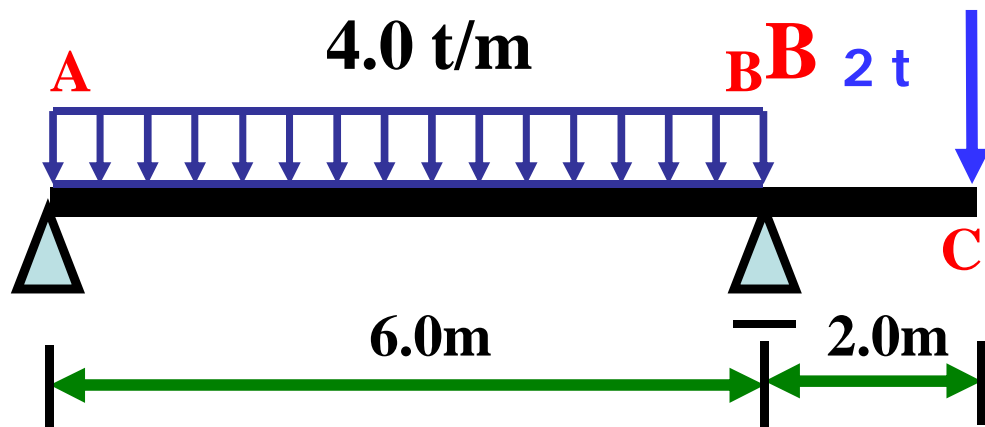
For Beams

$$\omega_{\text{internal}} = \int \frac{N_0 \cdot N_1}{EA} \cdot dl$$

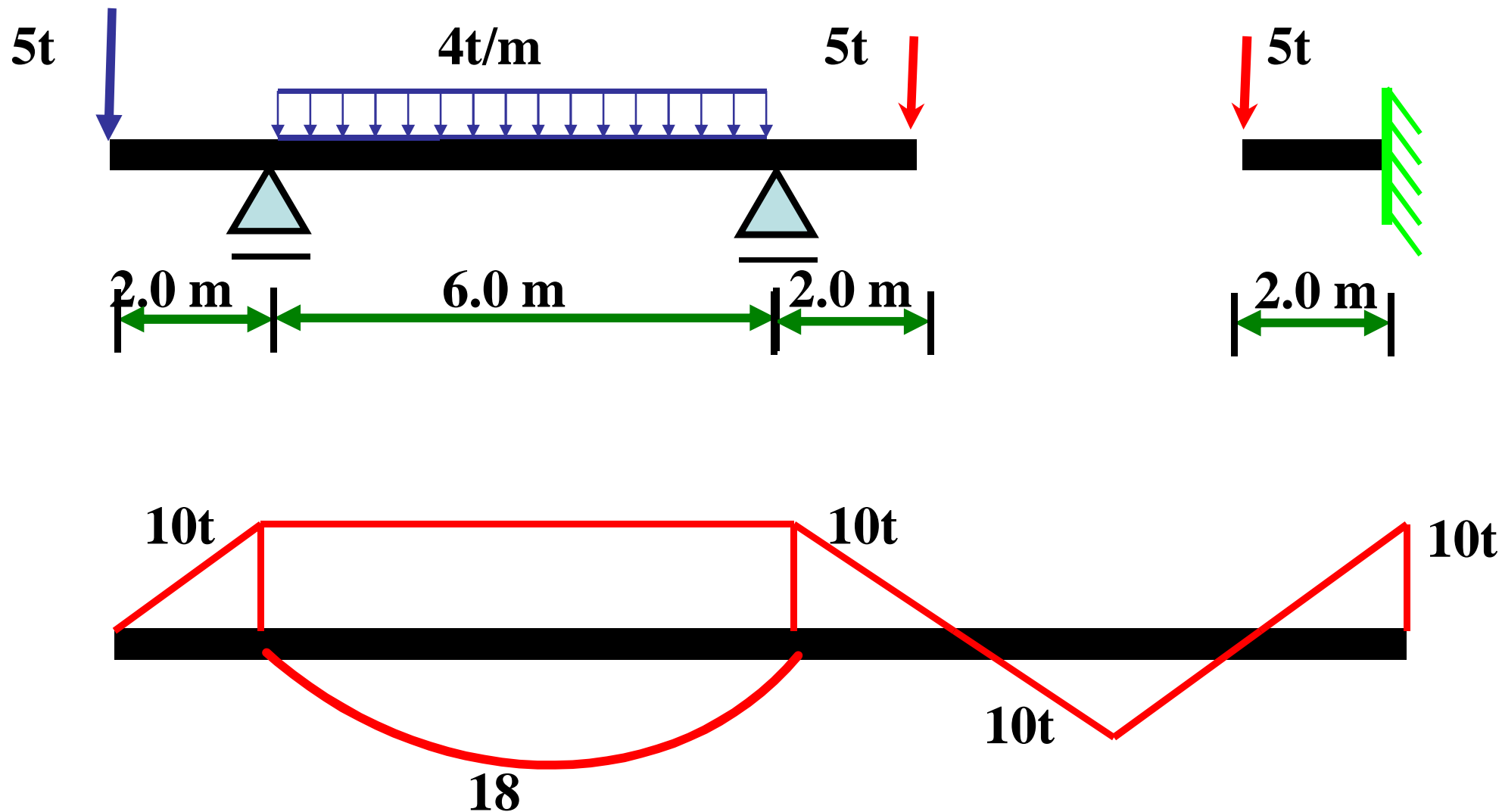
For truss

خطوات الحل:

1- يتم رسم العزم على الكمرة المعطاه بطريقة الـ super position.

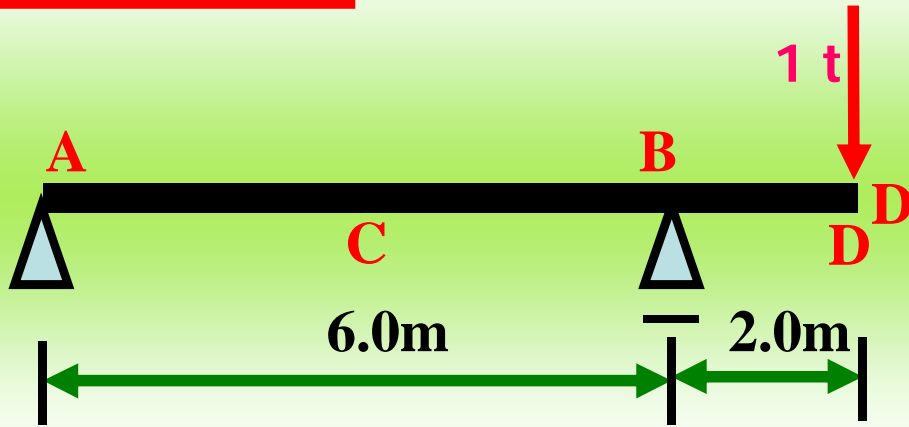


Example of super position drawing

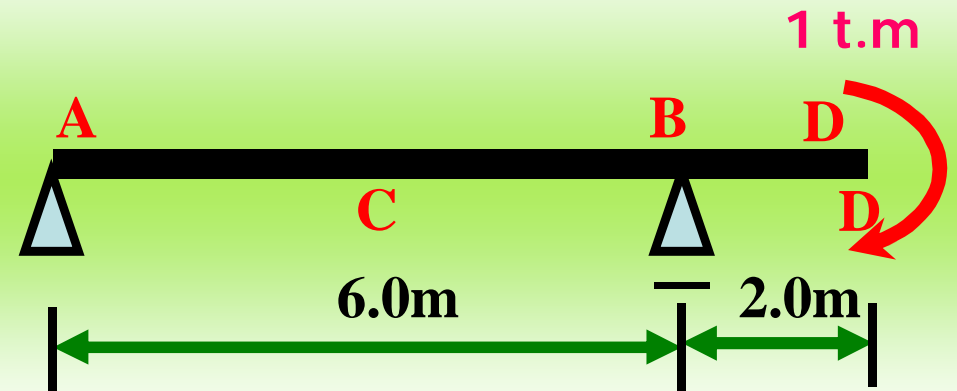


2- يتم ازالة الاحمال كلها من على الكمرة ووضع **1 ton** عند النقطة المطلوب حساب الهبوط عندها في اتجاه الحركة او **1t.m** عند طلب الدوران عند نقطة، ورسم العزم عليها.

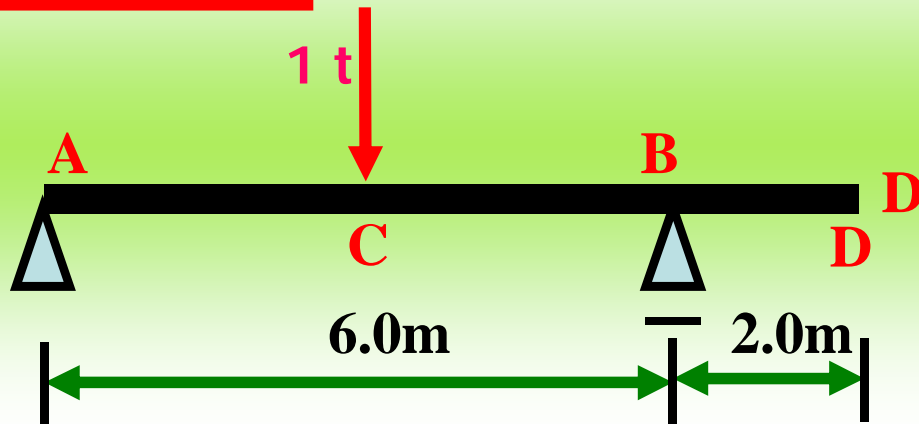
For Y_D



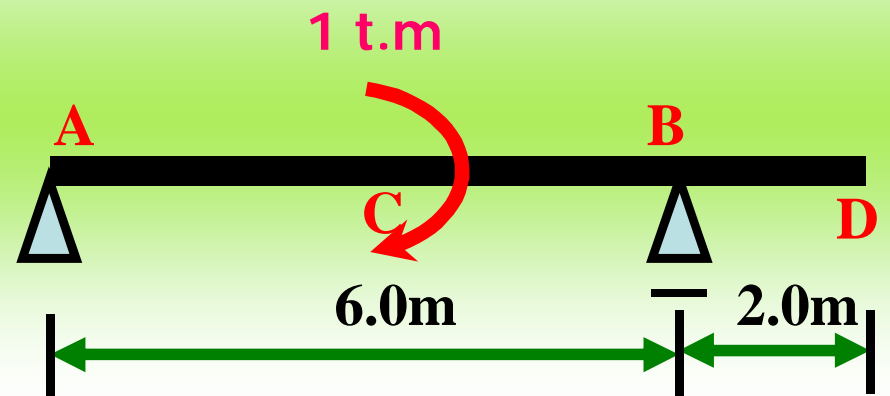
For θ_D



For Y_C



For θ_C

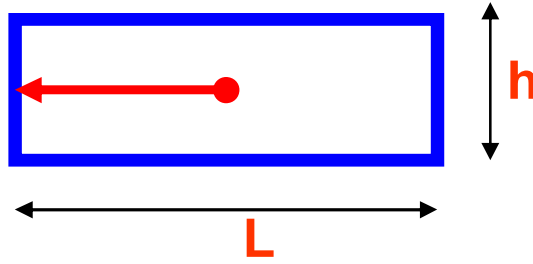


3- يتم تكامل الشكل الاساسى بالخطوه 1 والشكل المرسوم للمطلوب الاول بالخطوه 2 والتكامل هنا سيكون تكامل مساحات:

بعض المساحات المطلوب معرفة قيمتها واماكن تركيزها

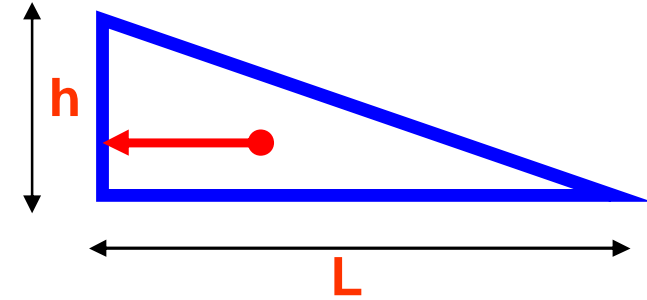
$$A=L.h$$

$$\zeta @ L/2$$



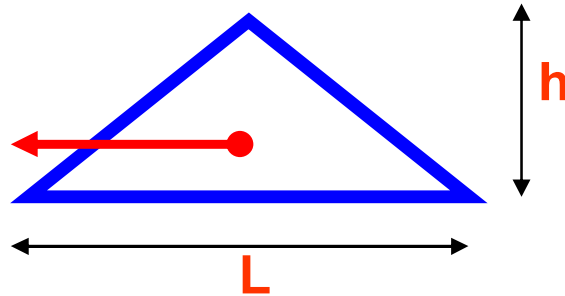
$$A=L.h/2$$

$$\zeta @ L/3$$



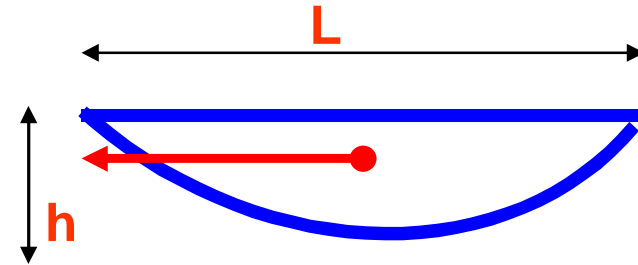
$$A=L.h/2$$

$$\zeta @ L/2$$



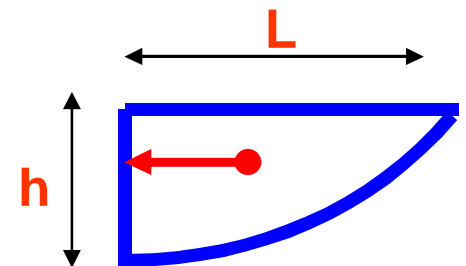
$$A=2/3L.h$$

$$\zeta @ L/2$$



$$A=2/3L.h$$

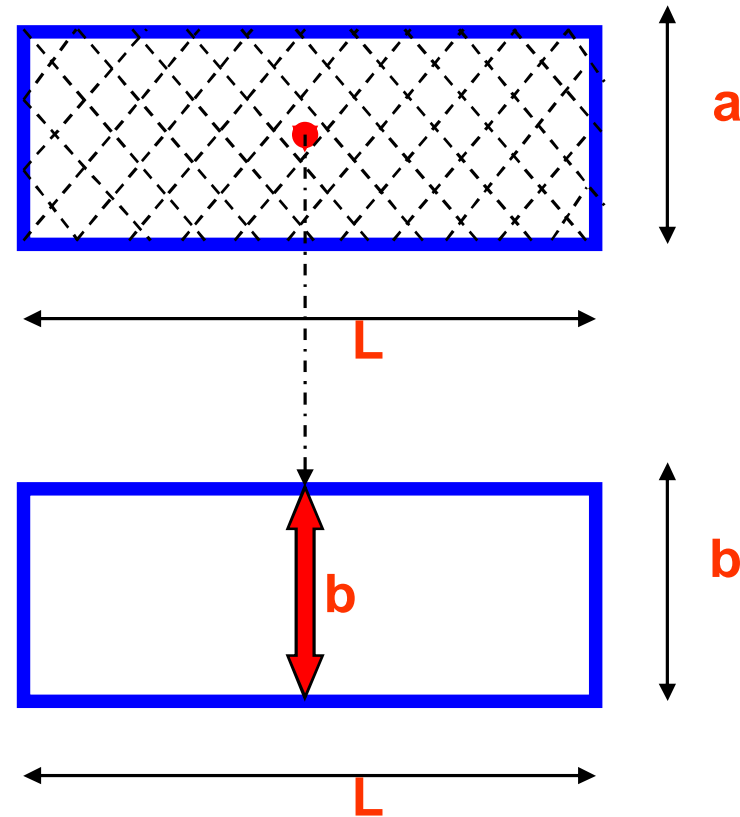
$$\zeta @ 3/8L$$



تكامل المساحتين هو عبارة عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

$$A = a.L$$

Ç @ L/2

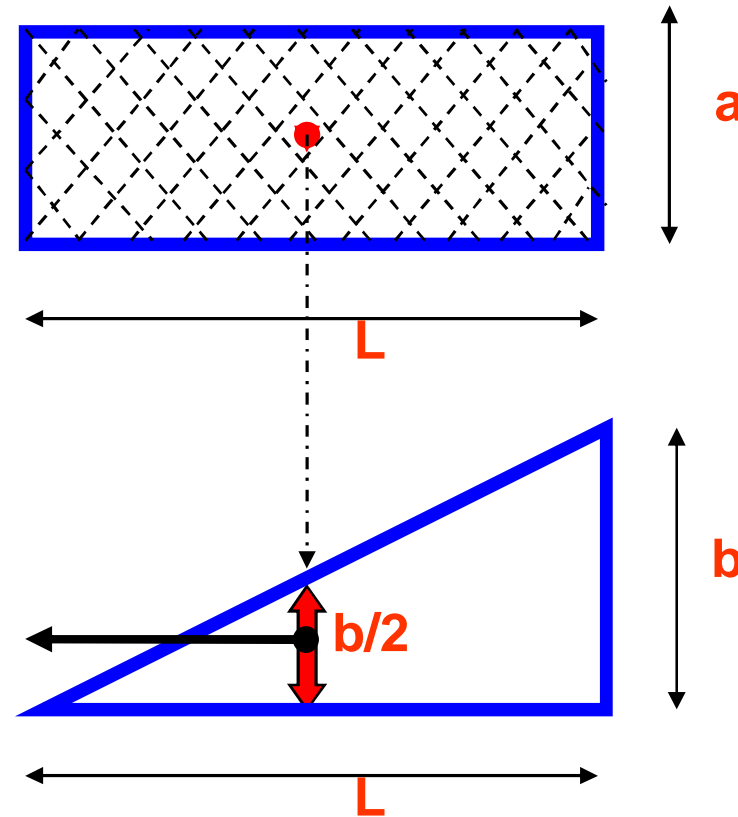


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L) * b = a.b.L$$

تكامل المساحتين هو عبارة عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

$$A = a.L$$

Ç @ L/2

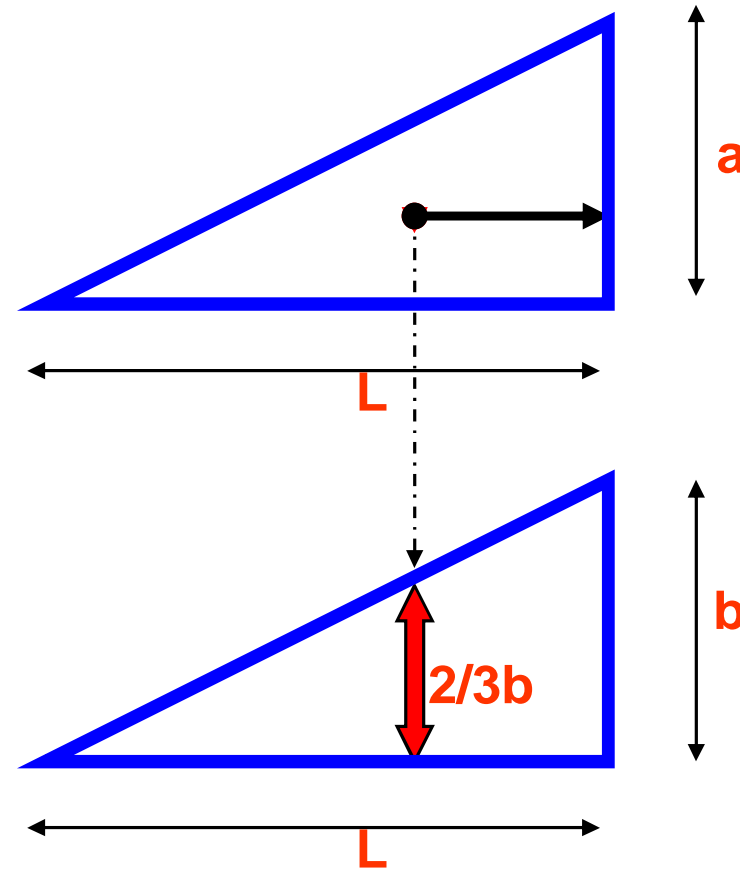


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L) * b/2 = a.b.L/2$$

تكامل المساحتين هو عبارة عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

$$A = a.L/2$$

Ç @ L/3

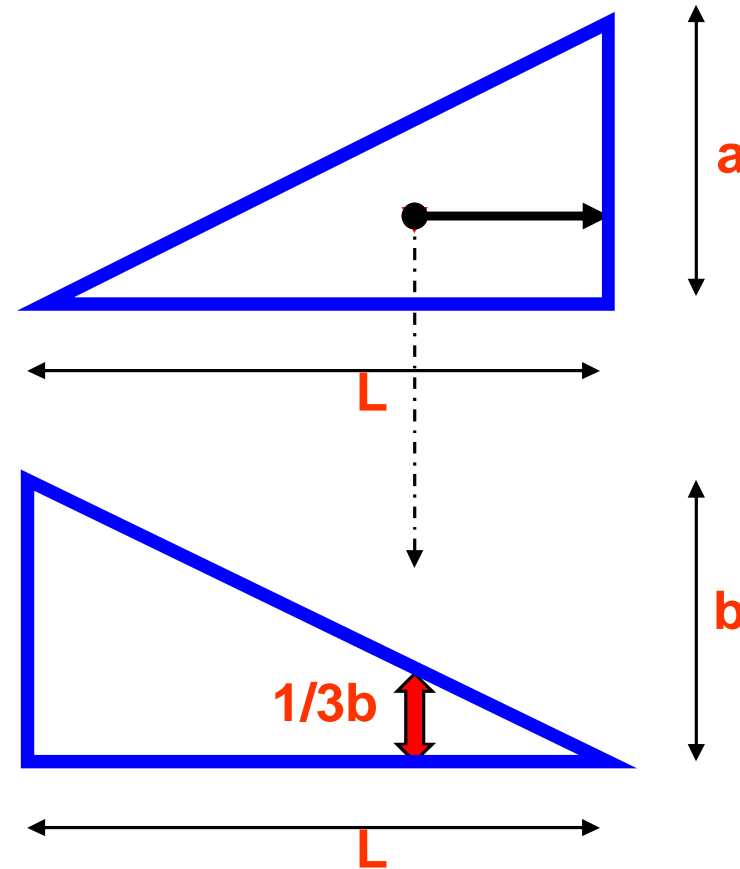


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L/2) * 2/3b = a.b.L/3$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

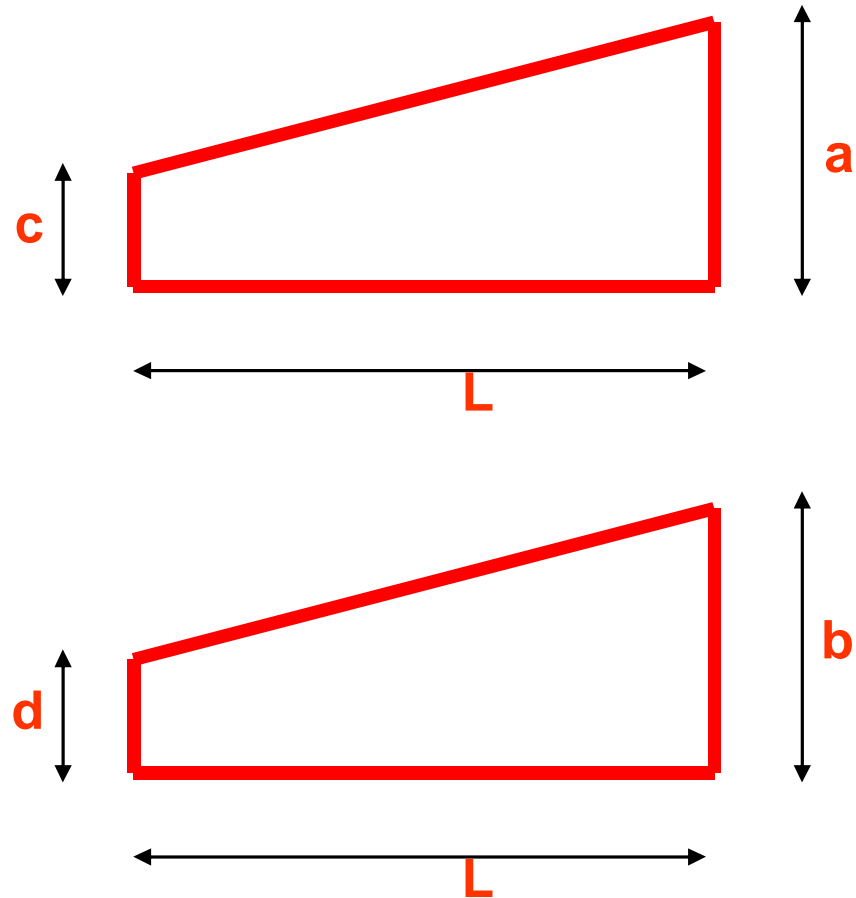
$$A = a.L/2$$

Ç @ L/3



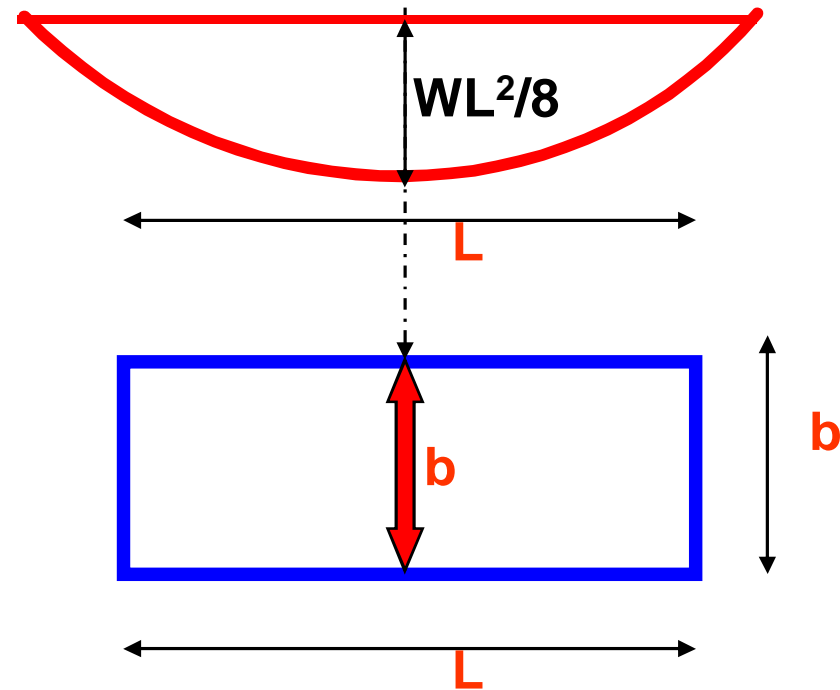
$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L/2) * 1/3b = a.b.L/6$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى



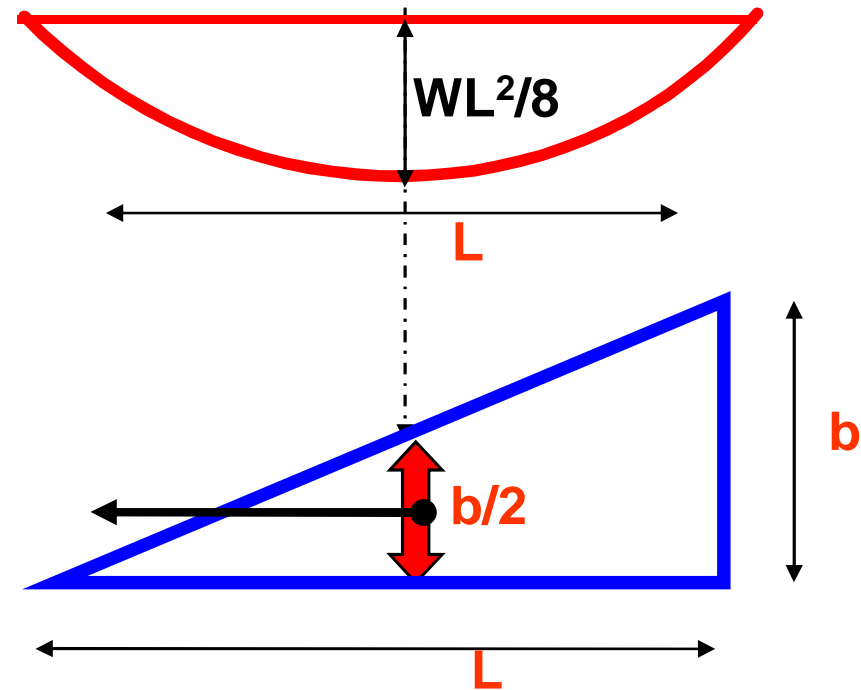
$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (L/3)[ab+dc+0.5*(ad+bc)]$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى



$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = 2/3 * wL^2 * L/8 * b = wL^3/12 * b$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

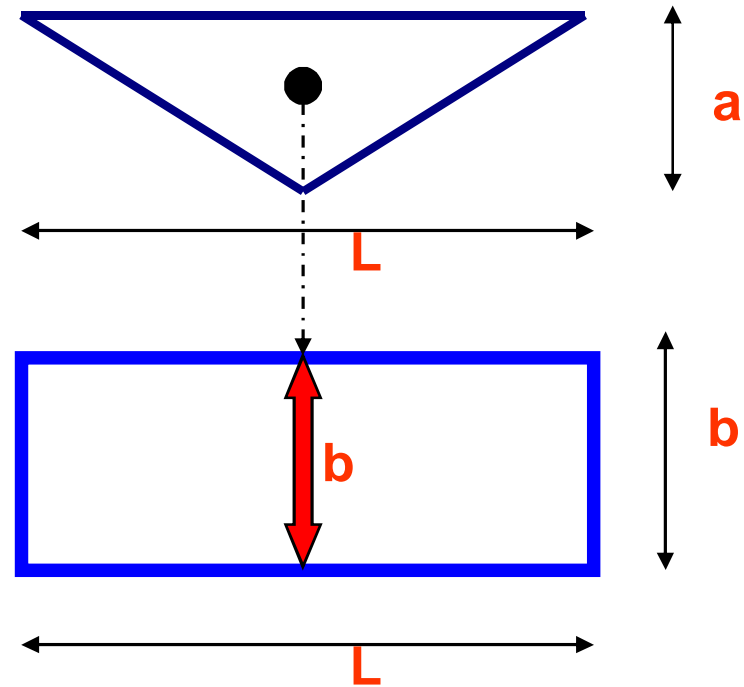


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = 2/3 \cdot wL^2 \cdot L/8 \cdot b/2 = wL^3/24 \cdot b$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثاني

$$A = a.L/2$$

Ç @ L/2

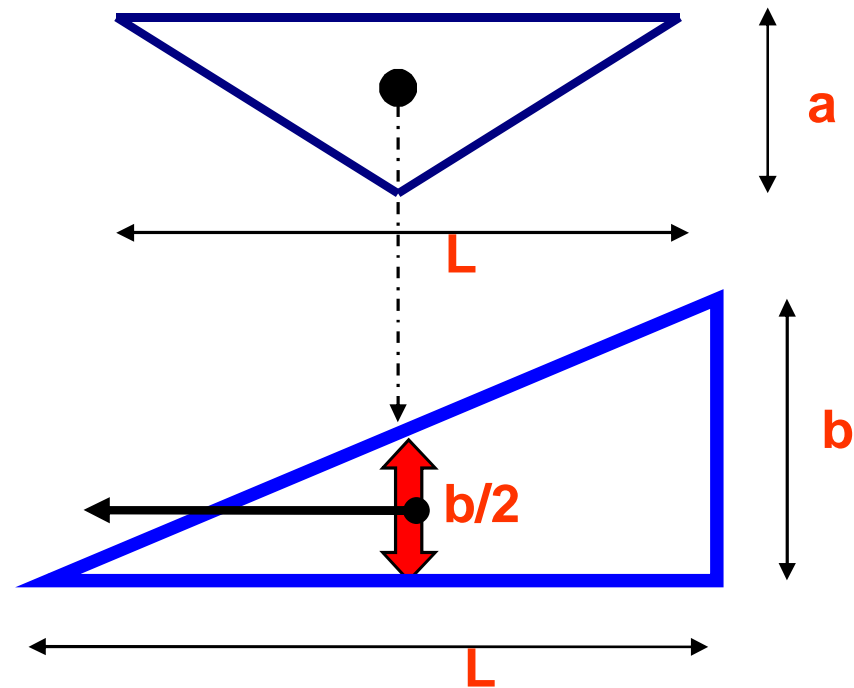


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L/2) * b = a.b.L/2$$

تكامل المساحتين هو عبارته عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثاني

$$A = a.L/2$$

Ç @ L/2

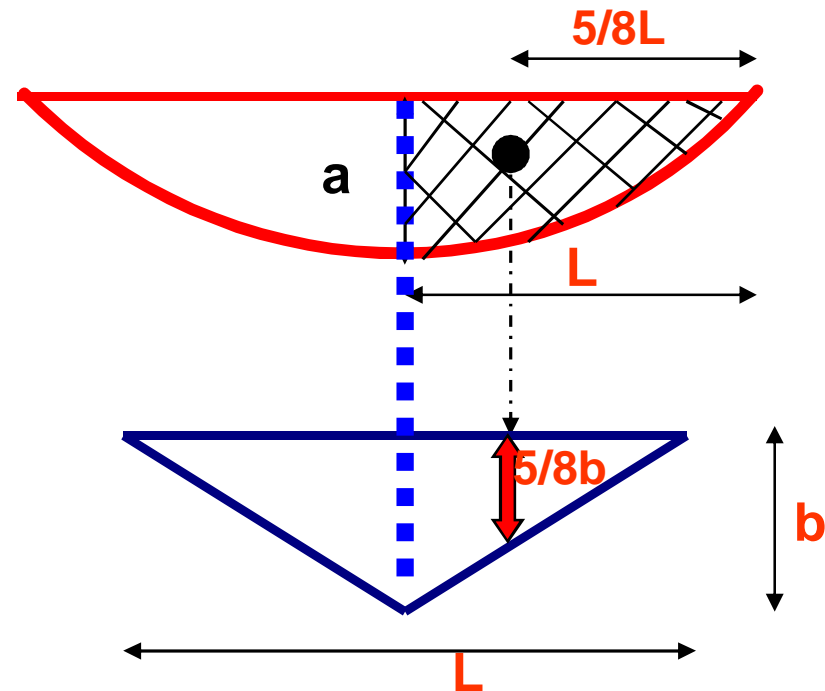


$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = (a.L/2) * b/2 = a.b.L/4$$

تكامل المساحتين هو عبارة عن مساحة احدهم * الارتفاع
المقابل لمركز هذه المساحة على الشكل الثانى

$$A = \frac{2}{3} \cdot a \cdot L$$

Ç @ 5/8L



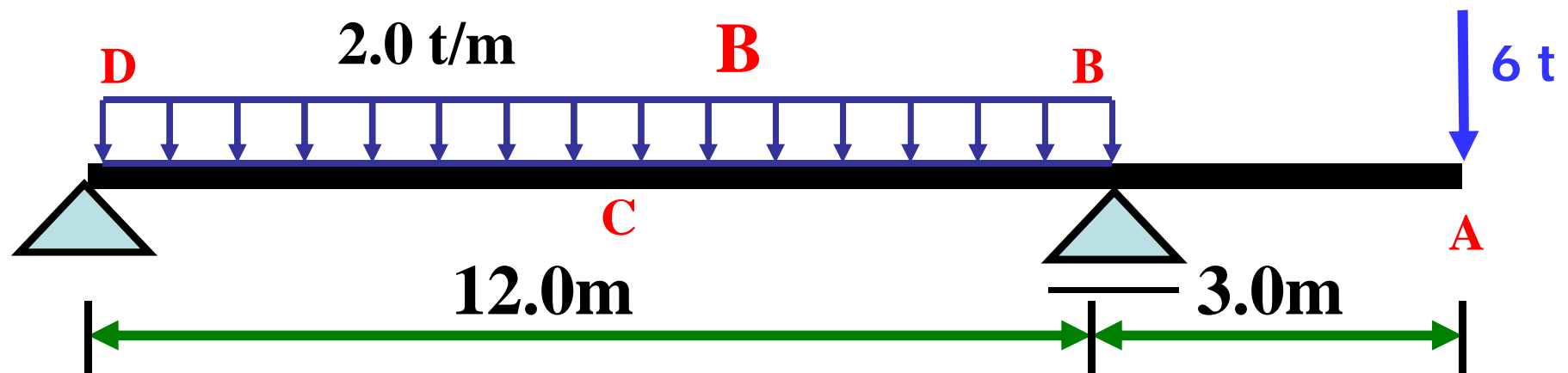
$$\int \frac{M_0 \cdot M_1}{EI} \cdot dl = 2 * [(\frac{2}{3} \cdot a \cdot L) * \frac{5}{8}b]$$

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

VERTUAL WORK FOR BEAMS

Eng : Aymman abdo

Problem 1



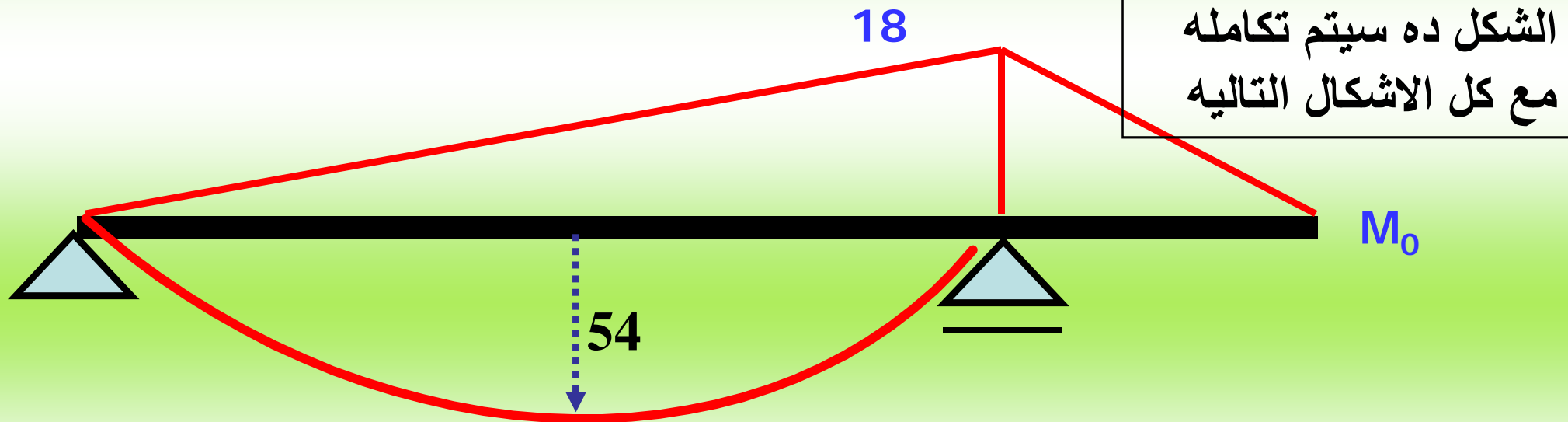
Required (1) the deflection at a, c.

(2) The rotation at point a, b, c, d.

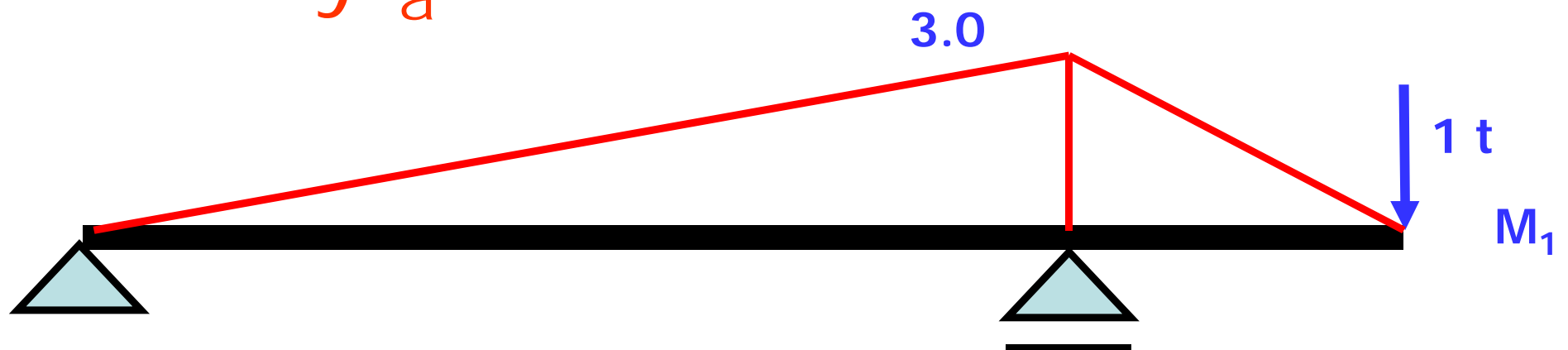
And also draw elastic curve

$$EI = 8000 \text{ t.m}^2$$

1-Draw B.M.d



2- For y_a

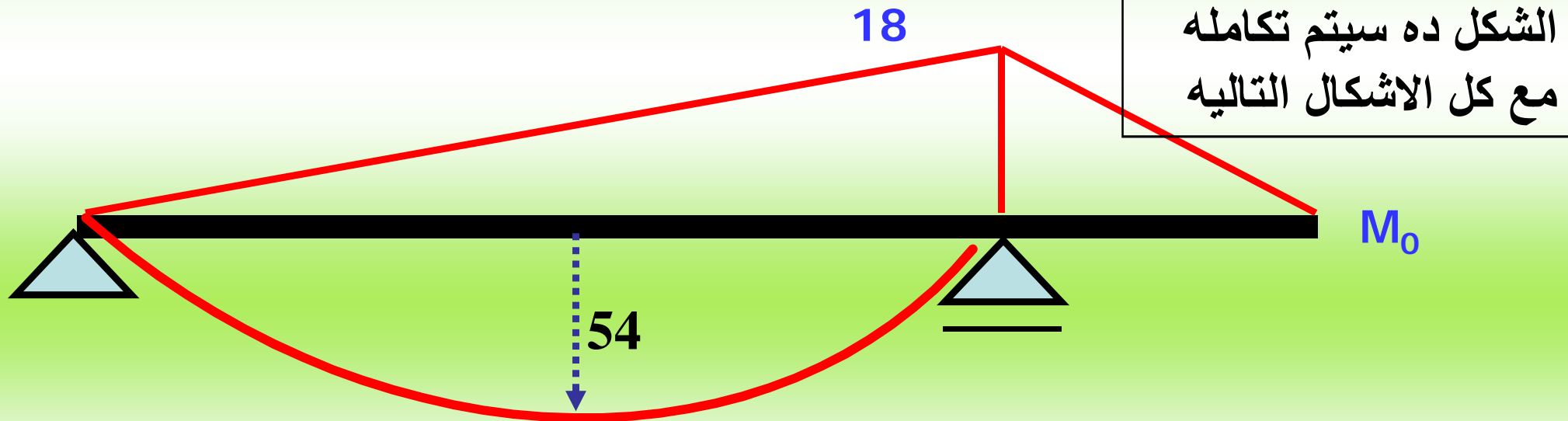


$$y_a = \int \frac{M_0 \cdot M_1}{EI} \cdot dl = \frac{1}{8000} \left[\frac{18 \cdot 3 \cdot 3}{3} + \frac{18 \cdot 3 \cdot 12}{3} - \left(\frac{2}{3} \cdot 54 \cdot 12 \cdot \frac{1}{2} \cdot 3 \right) \right]$$

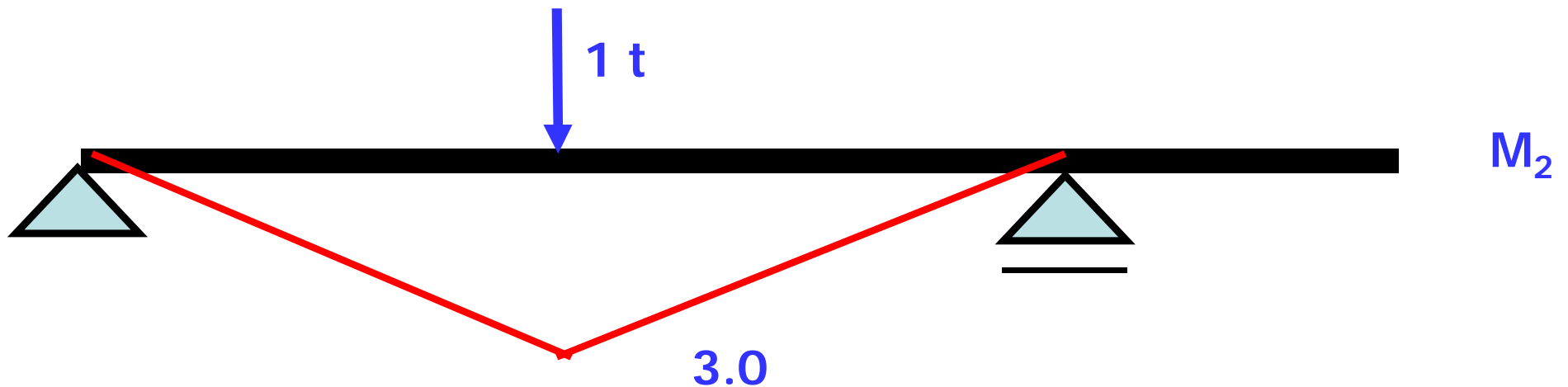
$$= -0.04725 \text{ m}$$

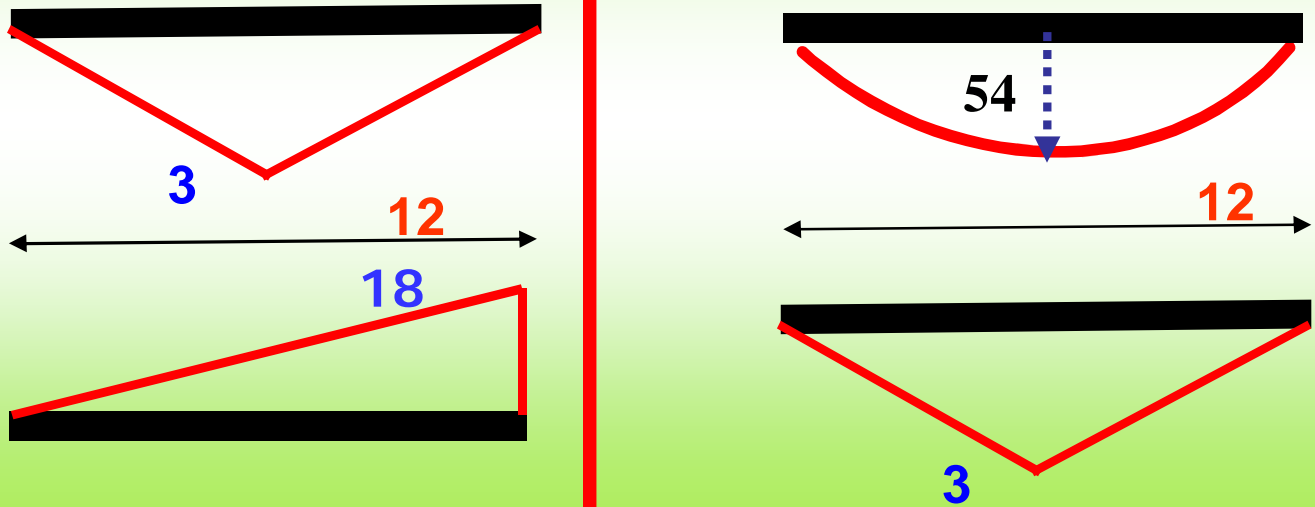
عندما فرضنا الـ (y_a) فرضناها لاسفل ومن الحسابات
سالبه معناها ان الحركة عكس الاتجاه.

1-Draw B.M.d



2- For y_c



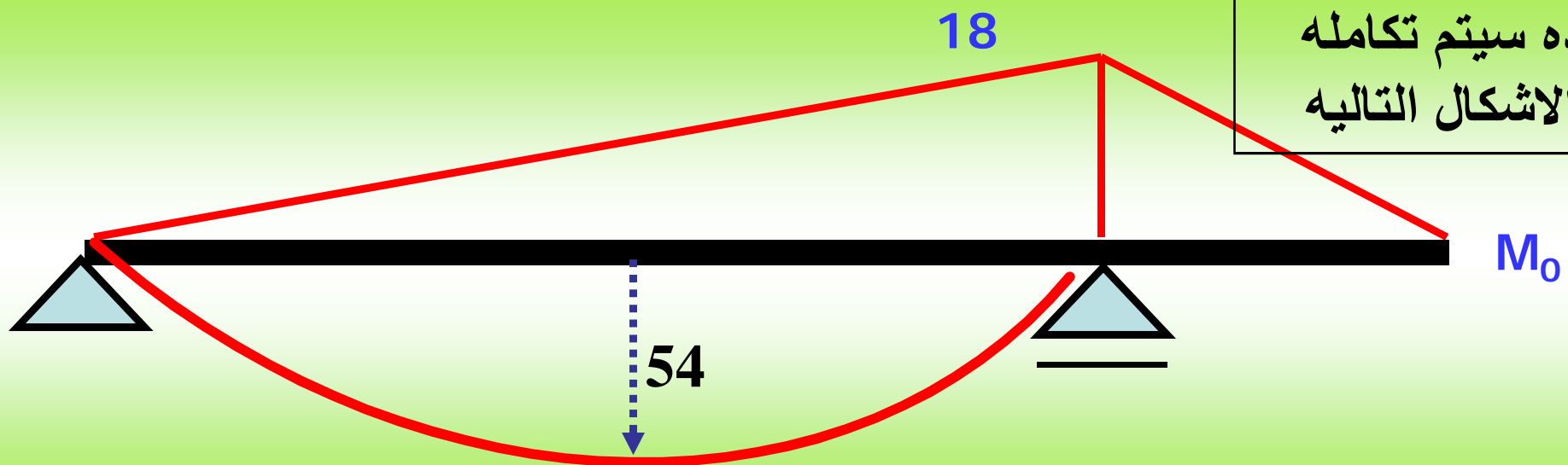


$$y_c = \int \frac{M_0 \cdot M_1}{EI} \cdot dl = \frac{1}{8000} \left[-1/2 * 3 * 12 * 9 + 2 * \left(\frac{2}{3} * 54 * 6 * 5/8 * 3 \right) \right]$$

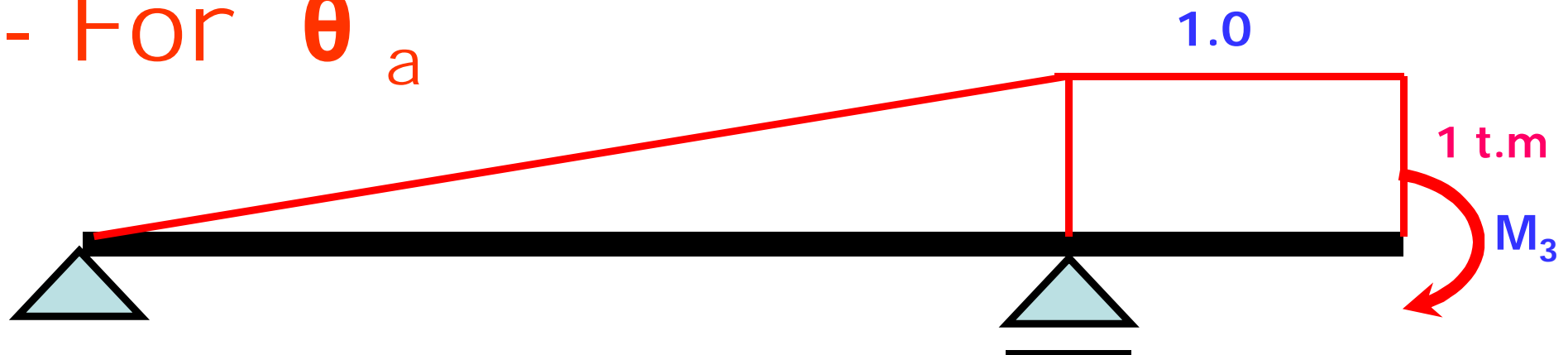
$$= 0.081 \text{ m}$$

عندما فرضنا الـ (y_c) فرضناها لاسفل ومن الحسابات
موجبه معناها ان الحركه بنفس الاتجاه.

الشكل ده سيتم تكامله
مع كل الاشكال التاليه



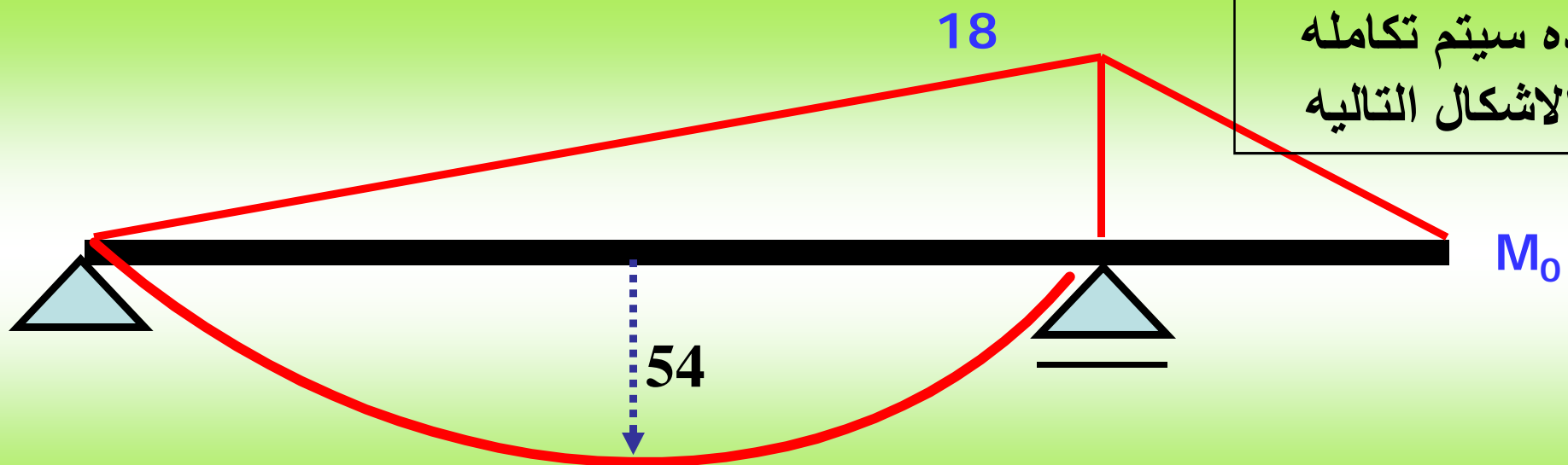
2- For θ_a



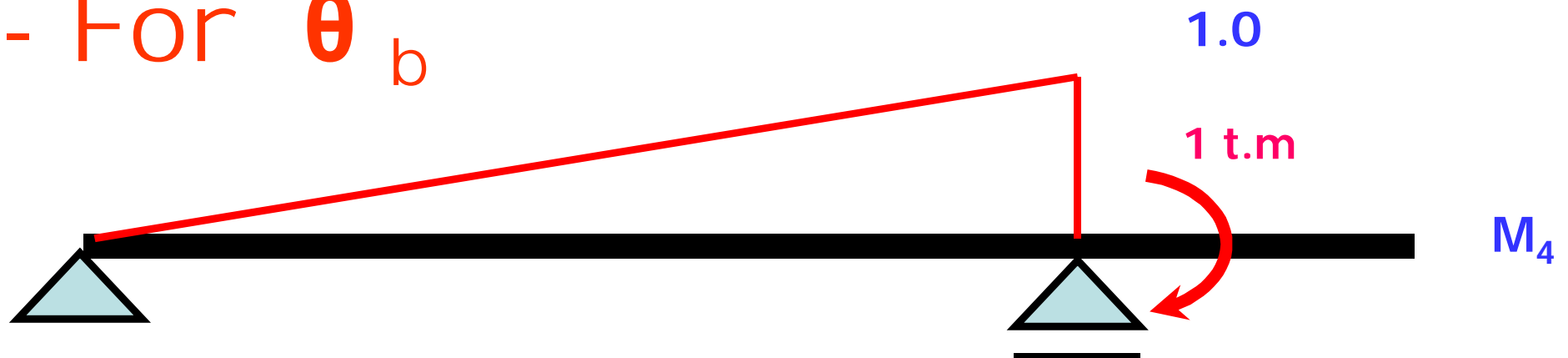
$$\theta_a = \int \frac{M_0 \cdot M_1}{EI} \cdot dl = \frac{1}{8000} \left[\frac{1 \cdot 3 \cdot 18}{2} + \frac{1 \cdot 18 \cdot 12}{3} - \frac{2}{3} \cdot 54 \cdot 12 \cdot 0.5 \right]$$

$$= -0.0146 \text{ rad}$$

الشكل ده سيتم تكامله
مع كل الاشكال التاليه



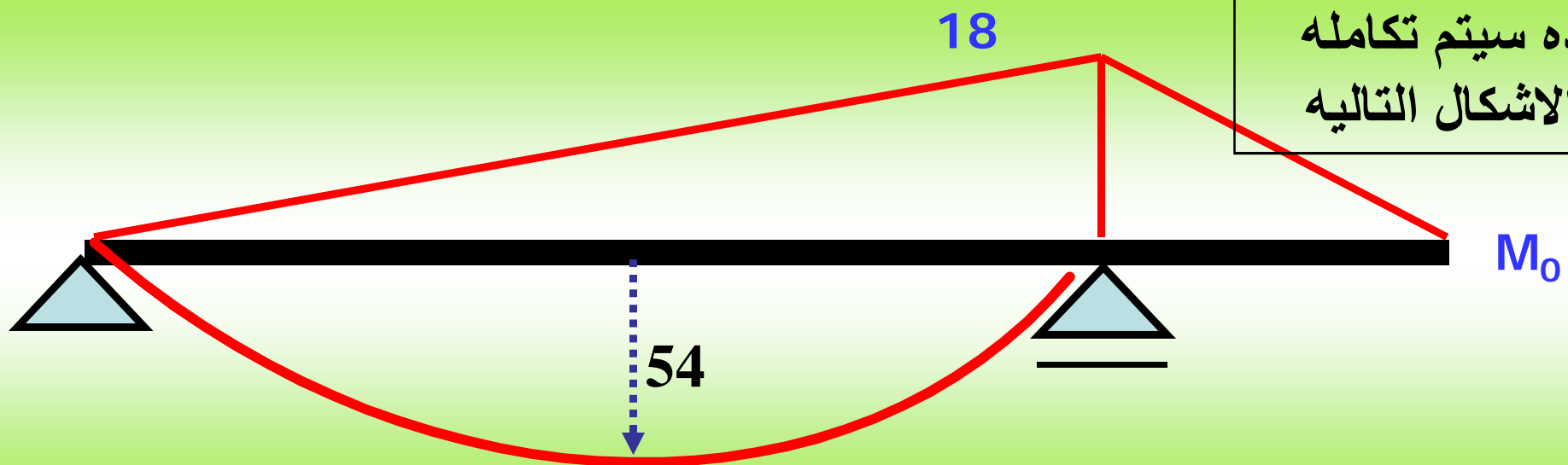
2- For θ_b



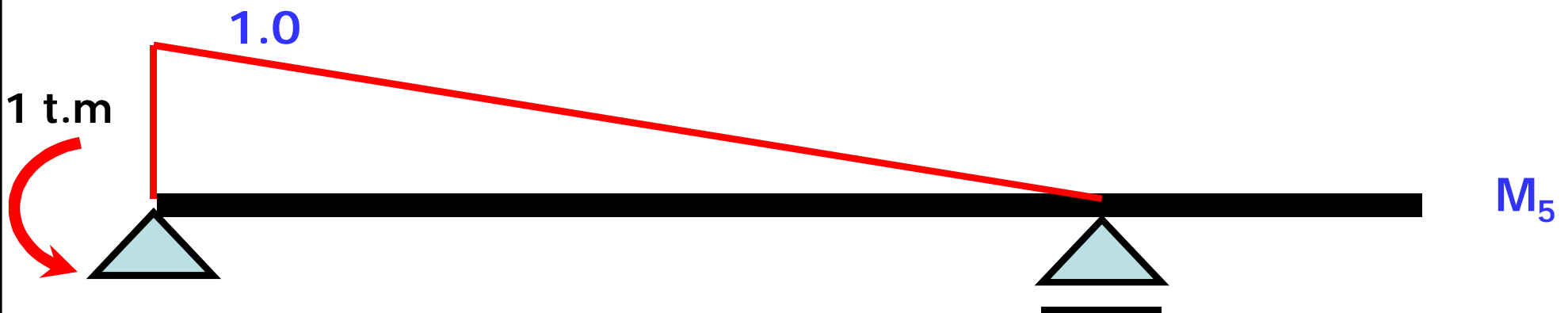
$$\theta_a = \int \frac{M_0 \cdot M_1}{EI} \cdot dl = \frac{1}{8000} \left[\frac{1 \cdot 18 \cdot 12}{3} - \frac{2}{3} \cdot 54 \cdot 12 \cdot 0.5 \right]$$

$$= -0.018 \text{ rad}$$

الشكل ده سيتم تكامله
مع كل الاشكال التاليه



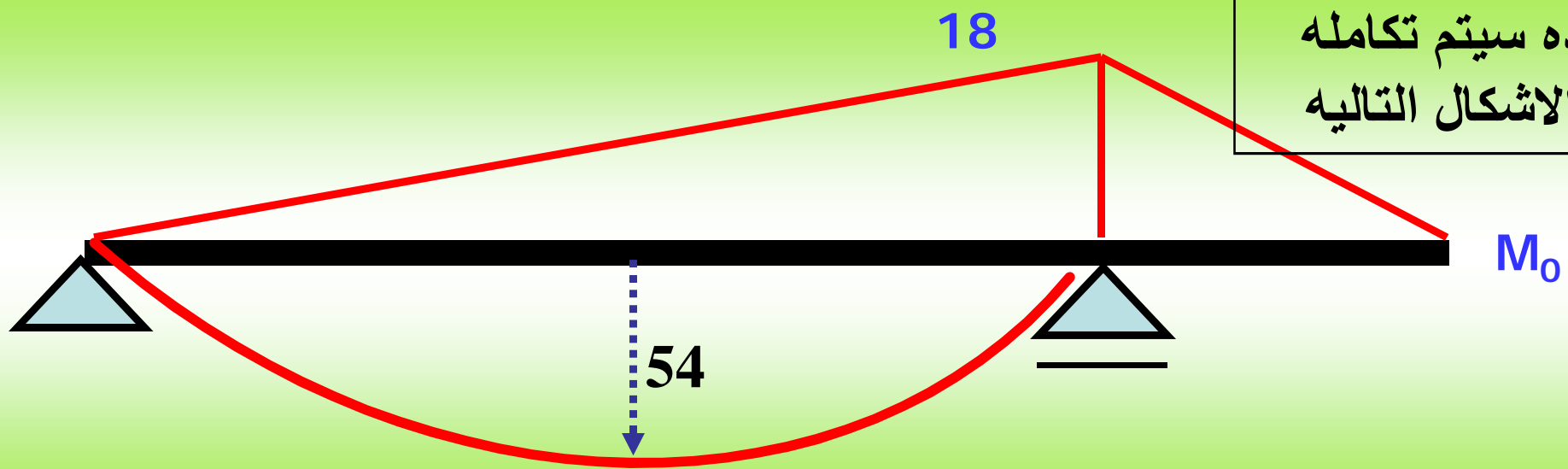
2- For θ_d



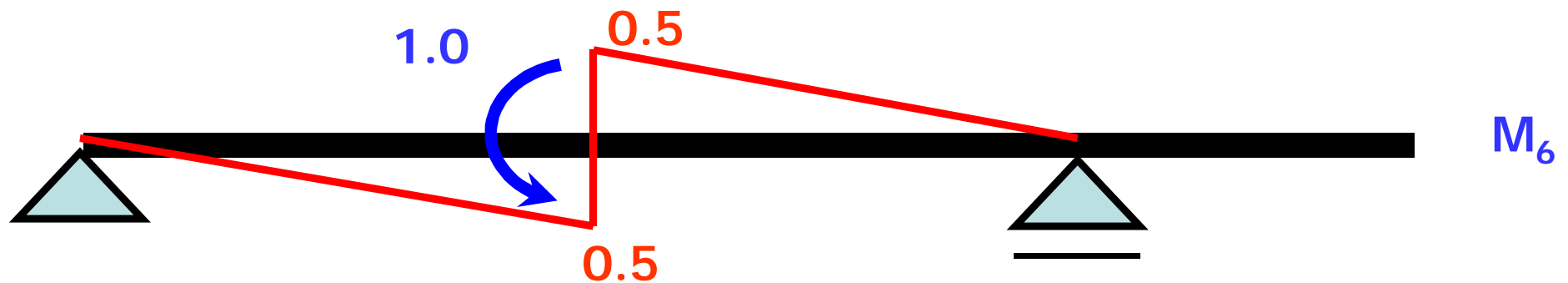
$$\theta_a = \int \frac{M_0 \cdot M_1}{EI} \cdot dl = \frac{1}{8000} \left[\frac{1 \cdot 18 \cdot 12}{6} - \frac{2}{3} \cdot 54 \cdot 12 \cdot 0.5 \right]$$

$$= -0.0225 \text{ rad}$$

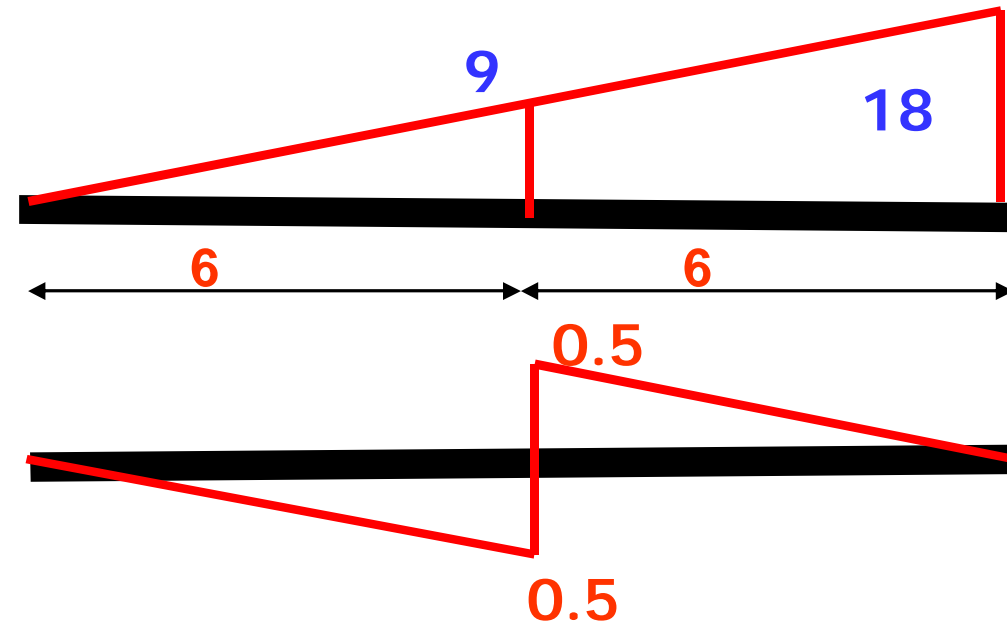
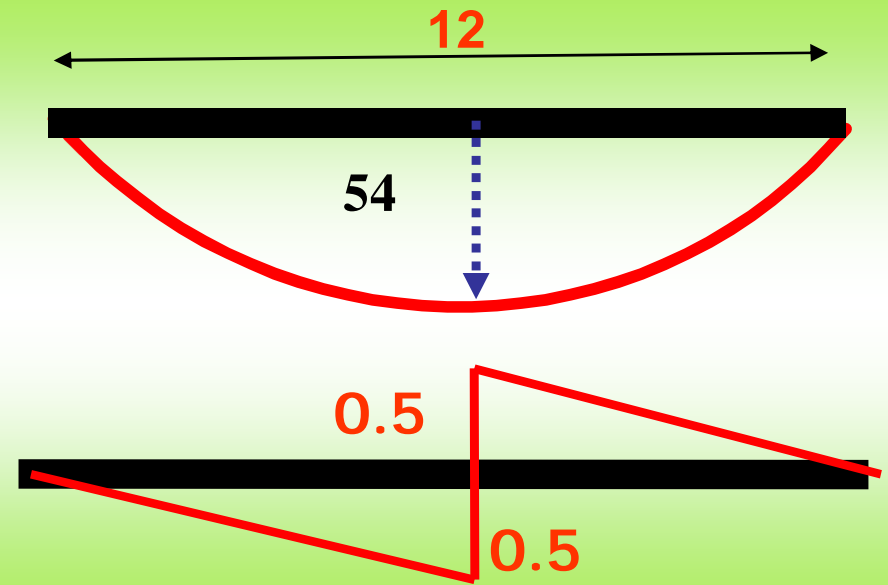
الشكل ده سيتم تكامله
مع كل الاشكال التاليه



2- For θ_c



تكامل الشكليين يعطى صفر لان النصف الشمال يعطى اشارته موجب والجزء اليمين يعطى اشارته سالب بنفس القيمة وبالتالي المحصله صفر

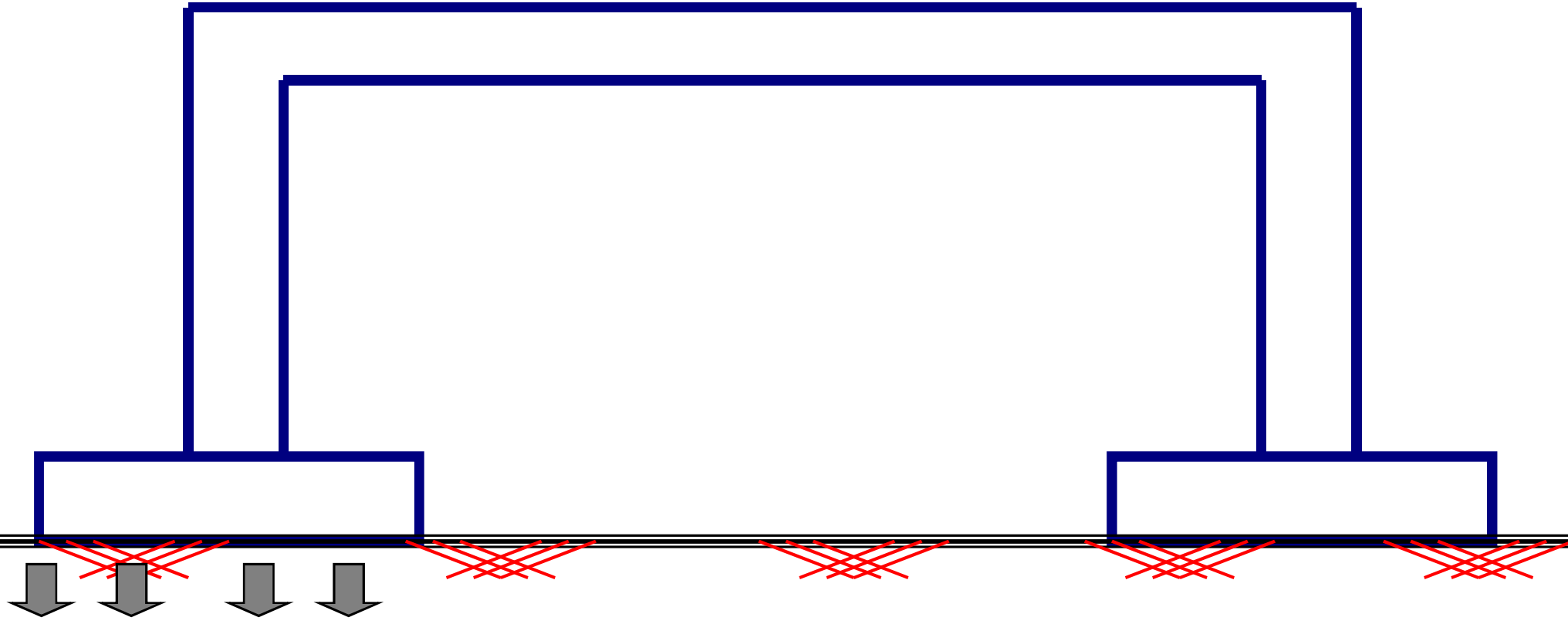


$$\theta_c = \frac{1}{8000} \left[\frac{0.5 * 9 * 6}{3} - \frac{6}{3} * (0.5 * 9 + 0 + 0.5 * (0.5 * 18)) \right]$$

$$= -0.001125 \text{ rad}$$

CASE OF SETTELMENT

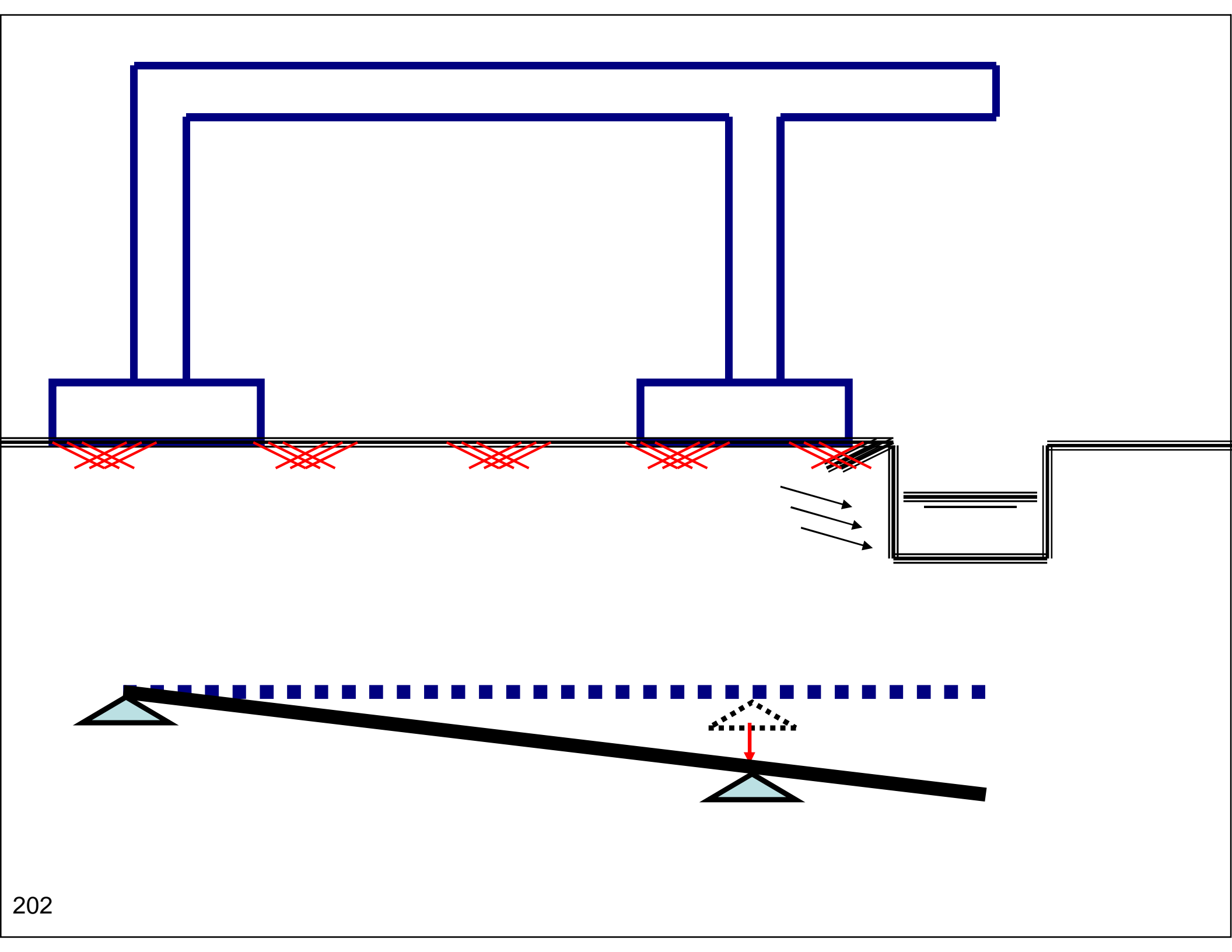
الهبوط



هبوط الركائز:

تربه انتفاشيه
Swelling soil

* يحدث هبوط الاعمده بسبب مشاكل التربه تحت الاعمده.
* قد يحدث ايضا بسبب الحفر بجوار المنشأ والذي يسبب هروب الماء من اسفل الاعمده.

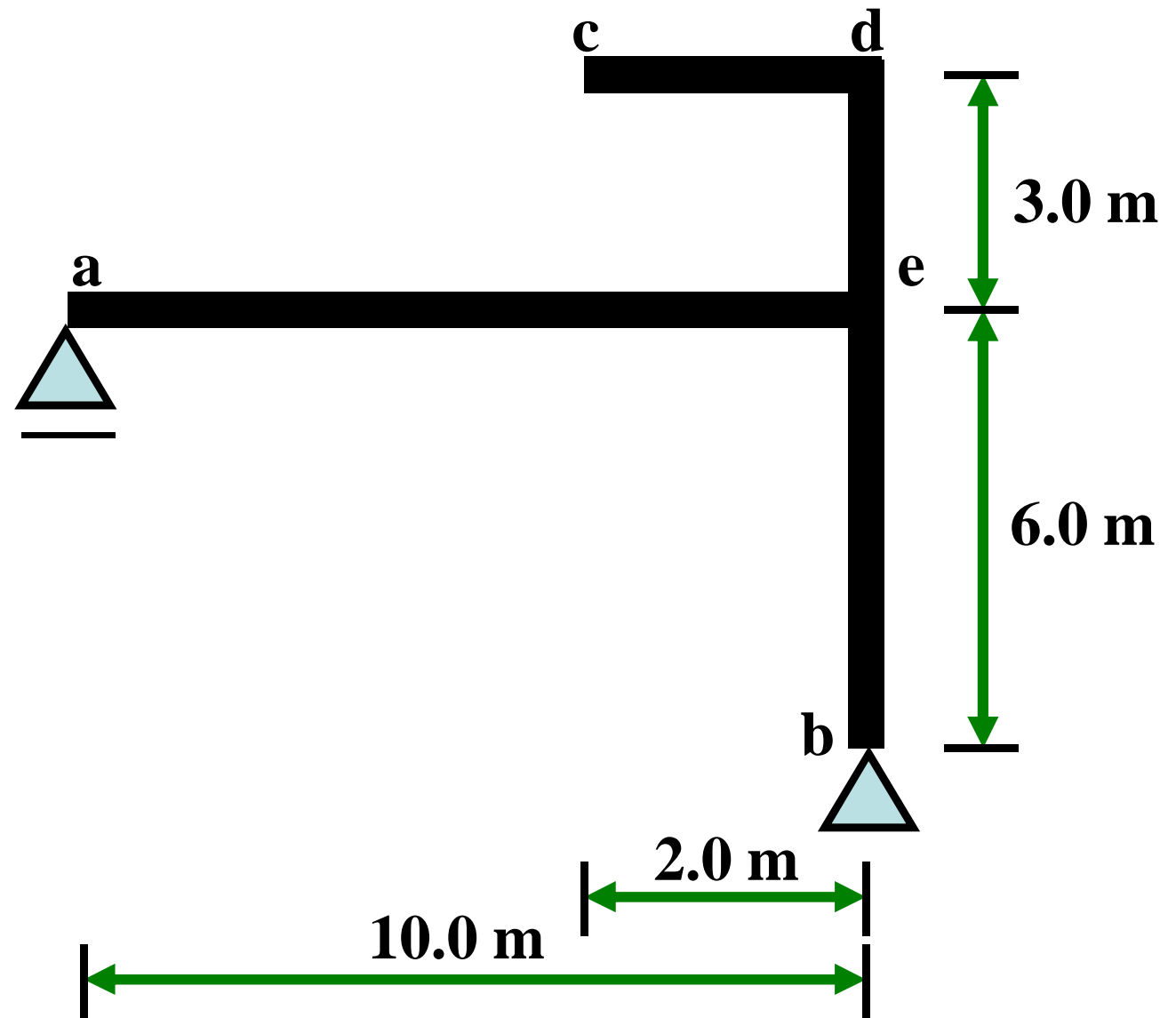


طريقة الحل

- 1- يتم ازالة كل الاحمال من الكمرة.
- 2- يتم وضع 1ton عند النقطة المطلوب حساب الهبوط عندها.
- 3- يتم حساب ال-reaction الناتج عن ال-1ton.

$$\sum (F * y + M * \theta) = 0.0$$

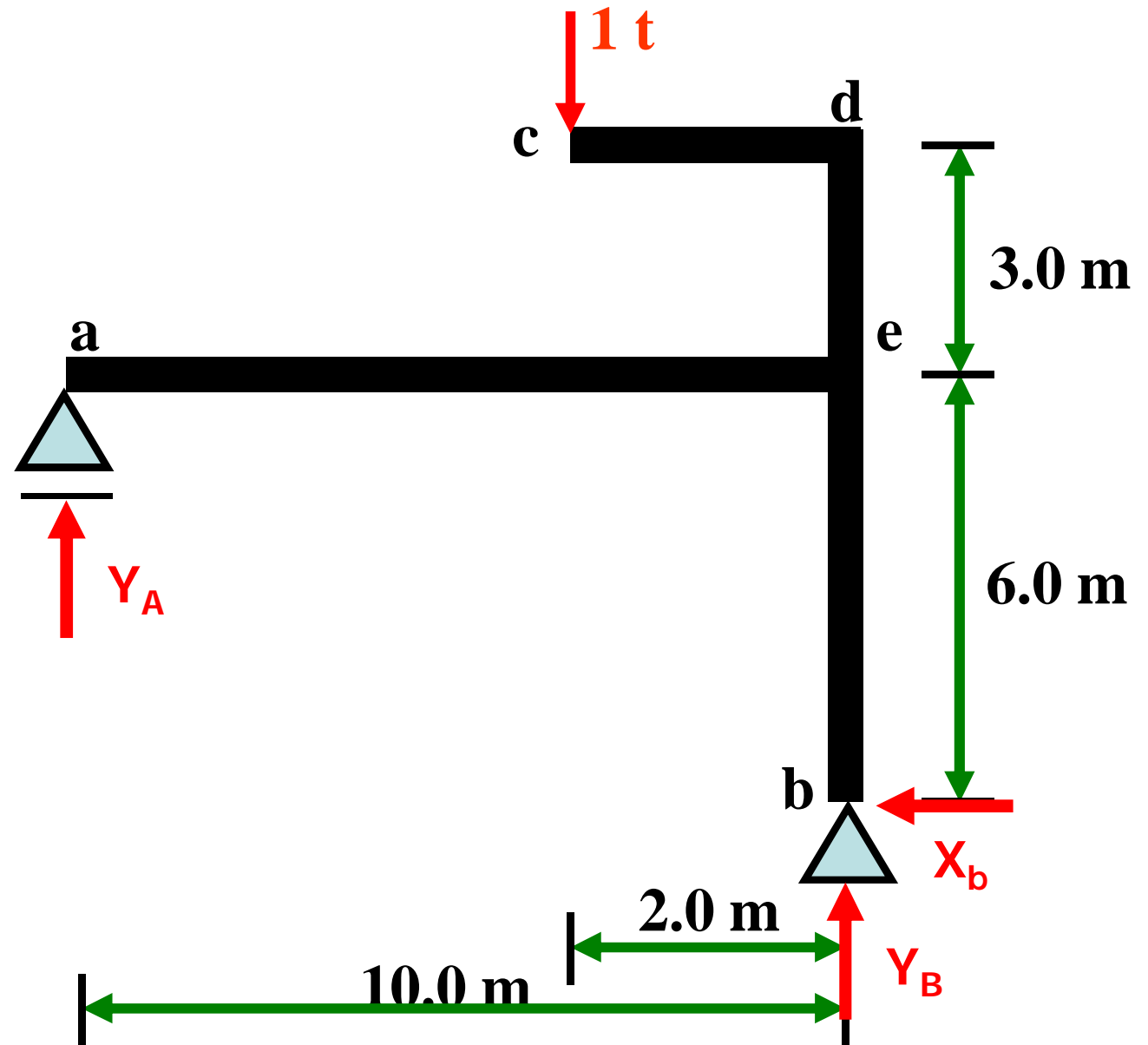
Example two



Required : Δ_{vc} and Δ_{hd} and θ_a due to settlement at **b** = 2cm.

For Δ_{vc}

المطلوب هو الازاحة
الرأسيه عند النقطة c



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 0.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



$$Y_B * 10$$

=

$$1 * 8$$

$$Y_B$$

=

$$0.8$$

$$\Sigma \mathbf{F}_y = 0.0$$



$$Y_A + 0.8$$

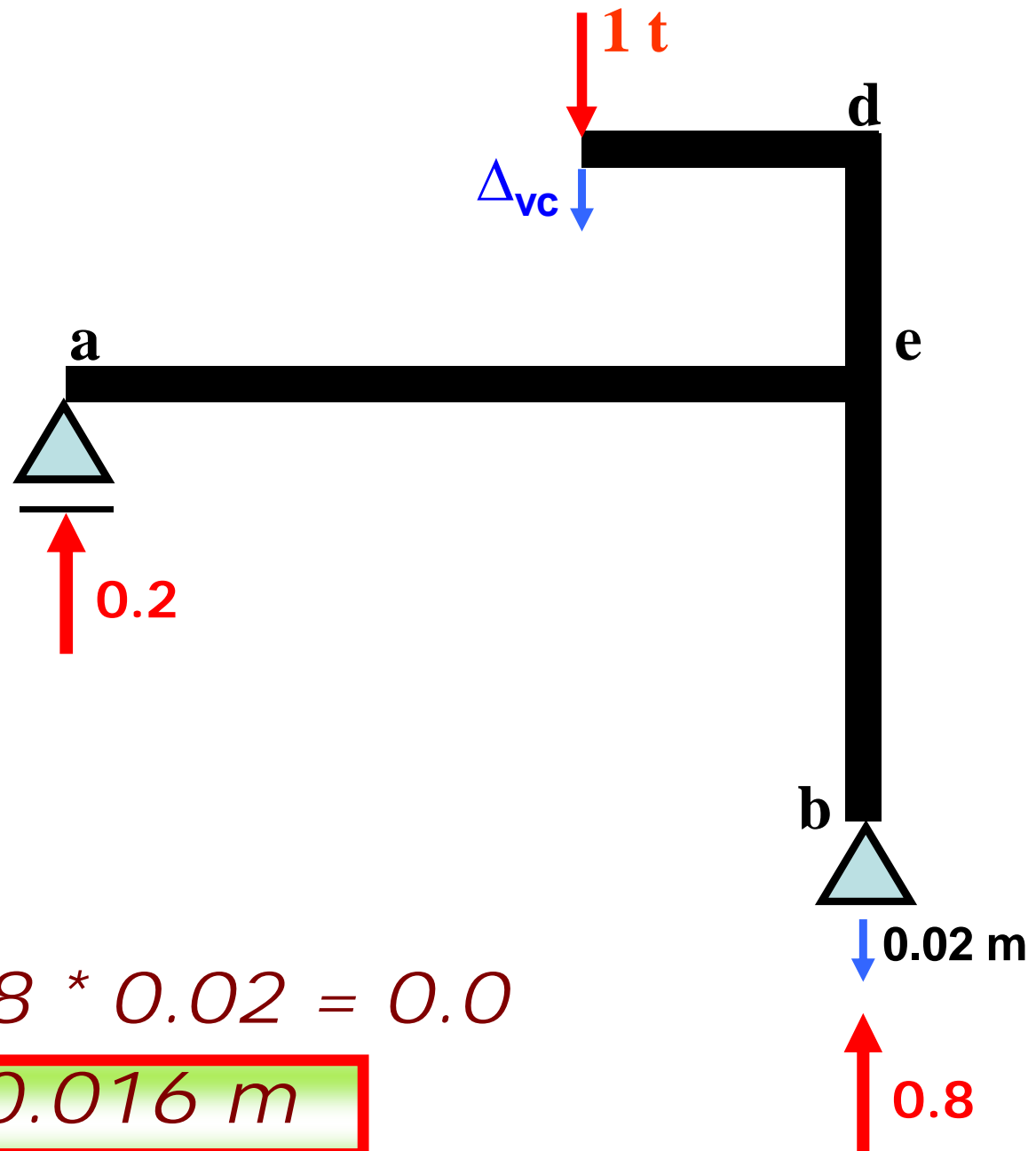
=

$$1$$

$$Y_A$$

=

$$0.2$$



القوة عكس الازاحة

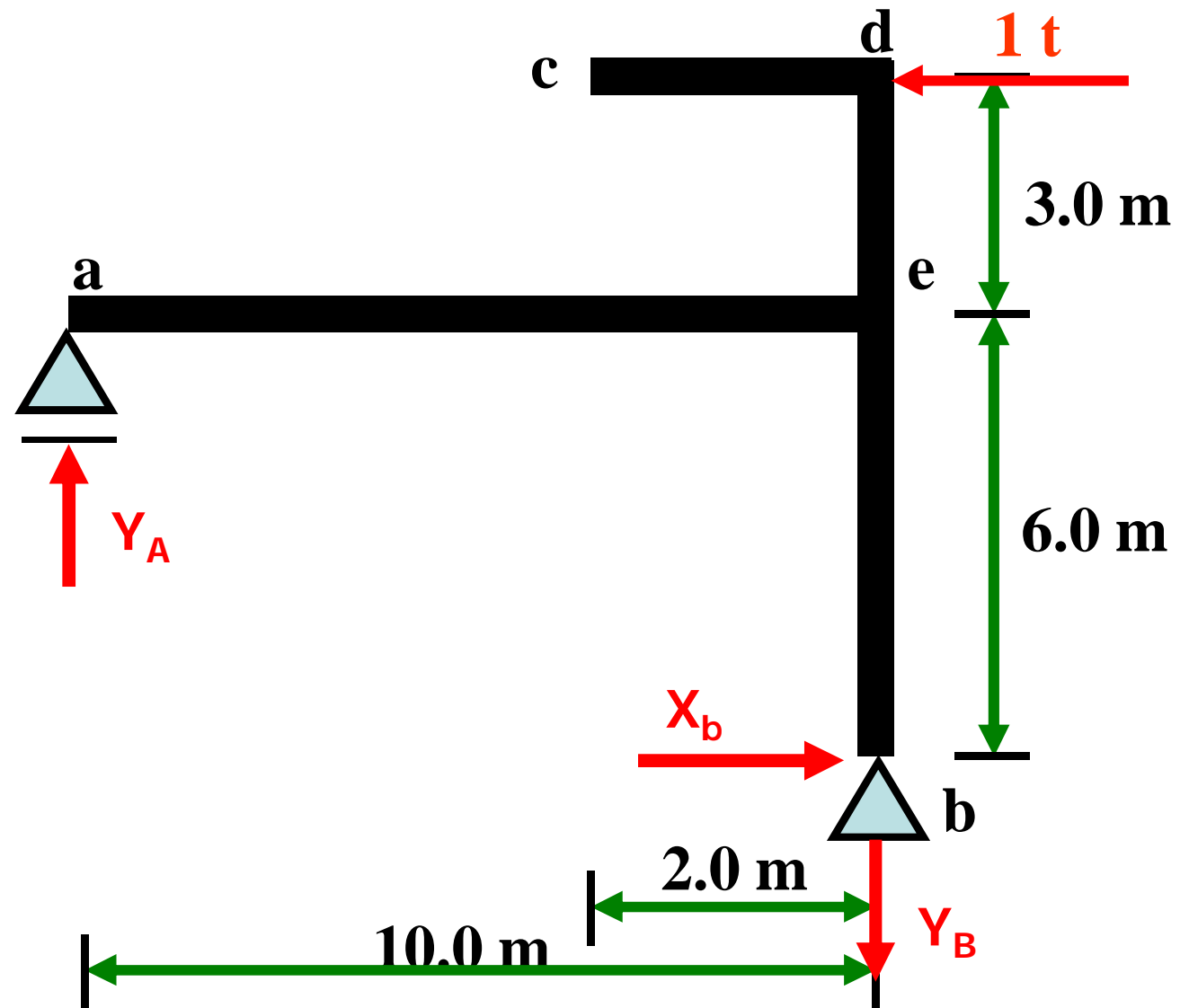
$$1 * \Delta_{vc} - 0.8 * 0.02 = 0.0$$

$$>> \Delta_{vc} = 0.016 \text{ m}$$

given

For Δ_{hd}

المطلوب هو الازاحة
الافقيه عند النقطة d



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 1.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



$$Y_B * 10$$

=

$$1 * 3 + 1 * 6$$

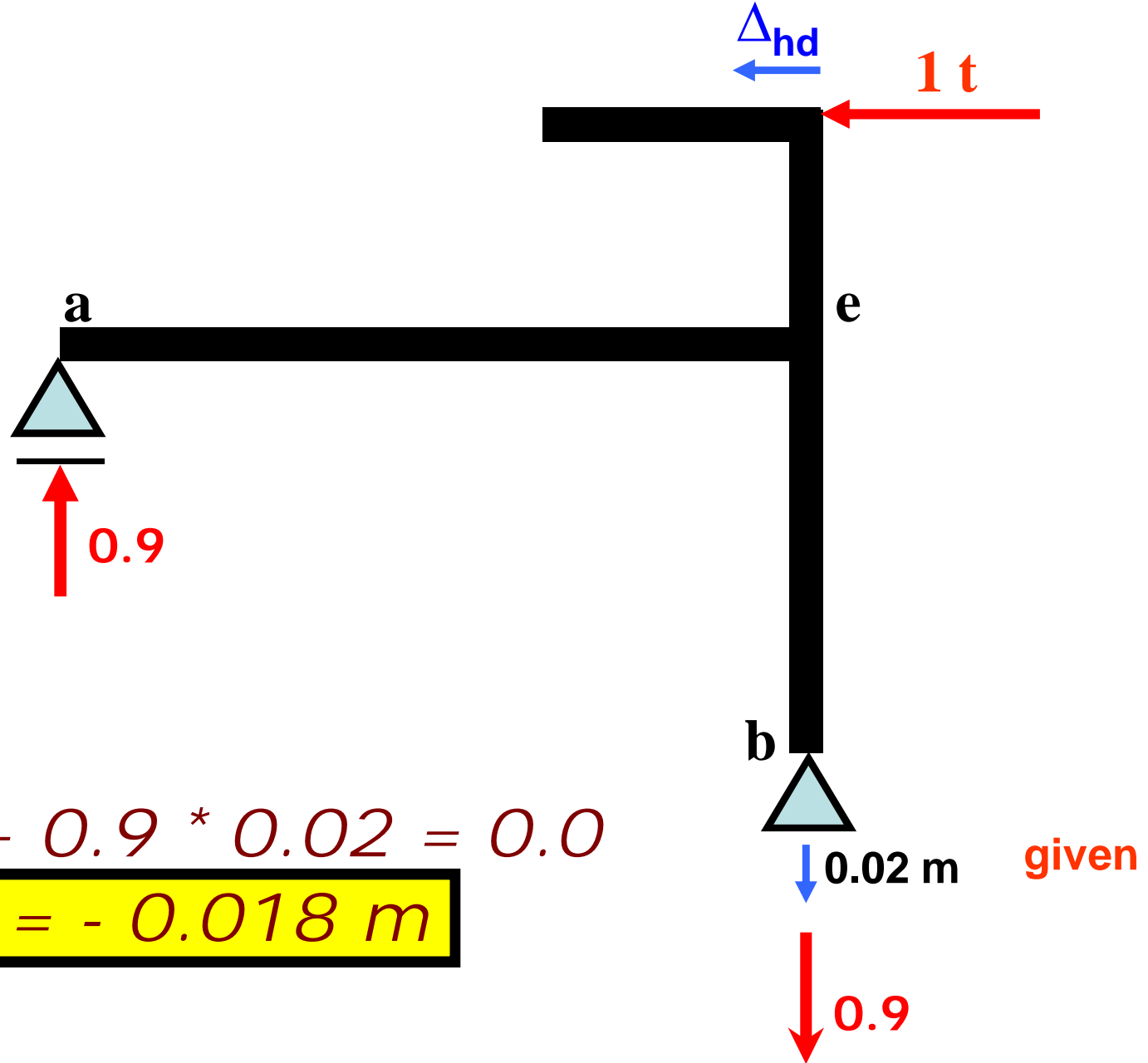
=

$$0.9$$

$$\Sigma \mathbf{F}_y = 0.0$$



$$Y_A = 0.9$$

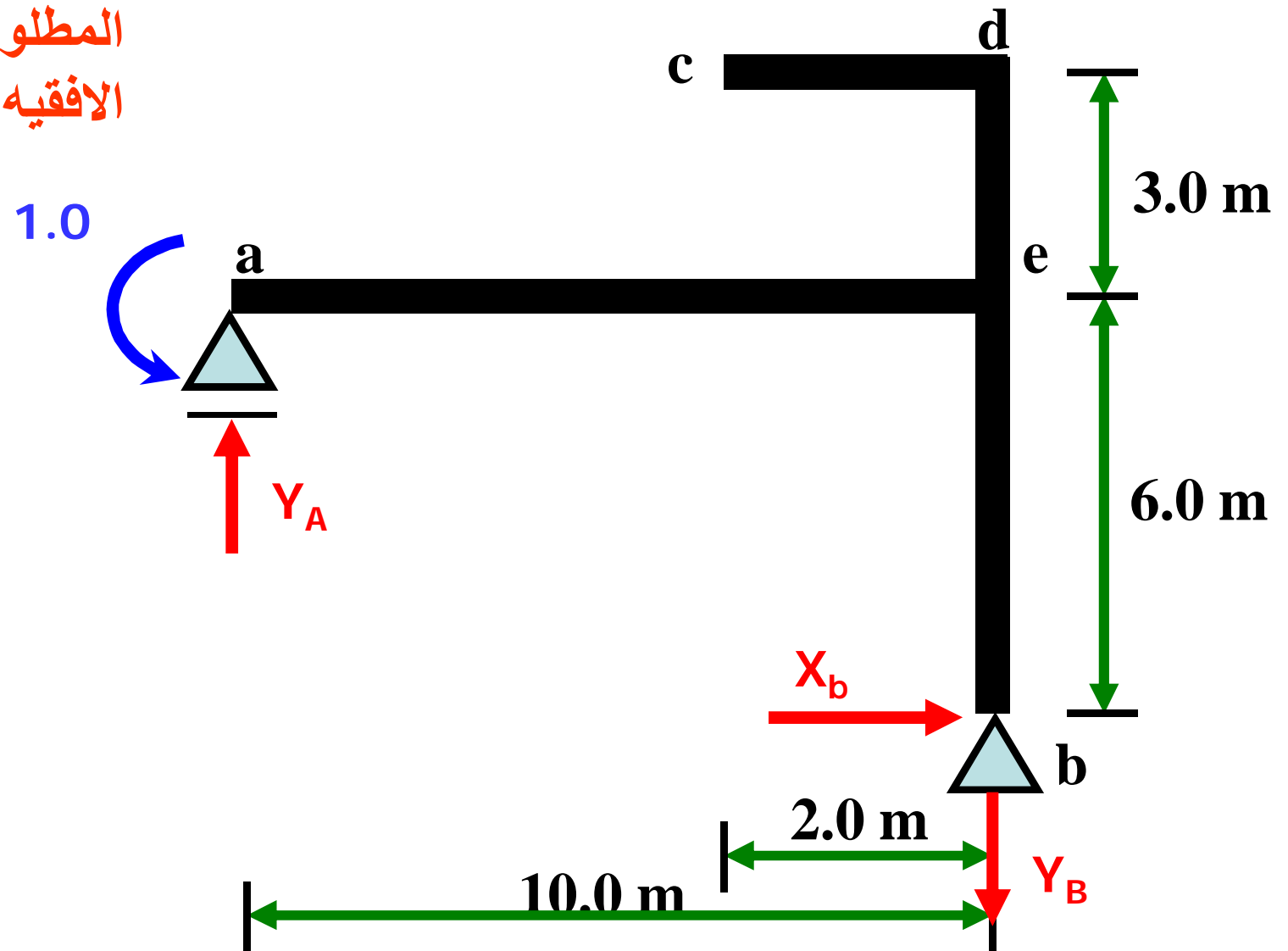


$$1 * \Delta_{hd} + 0.9 * 0.02 = 0.0$$

$$\gg \Delta_{hd} = -0.018 \text{ m}$$

For θ_a

المطلوب هو الإزاحة
الافقية عند النقطة d



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 0.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



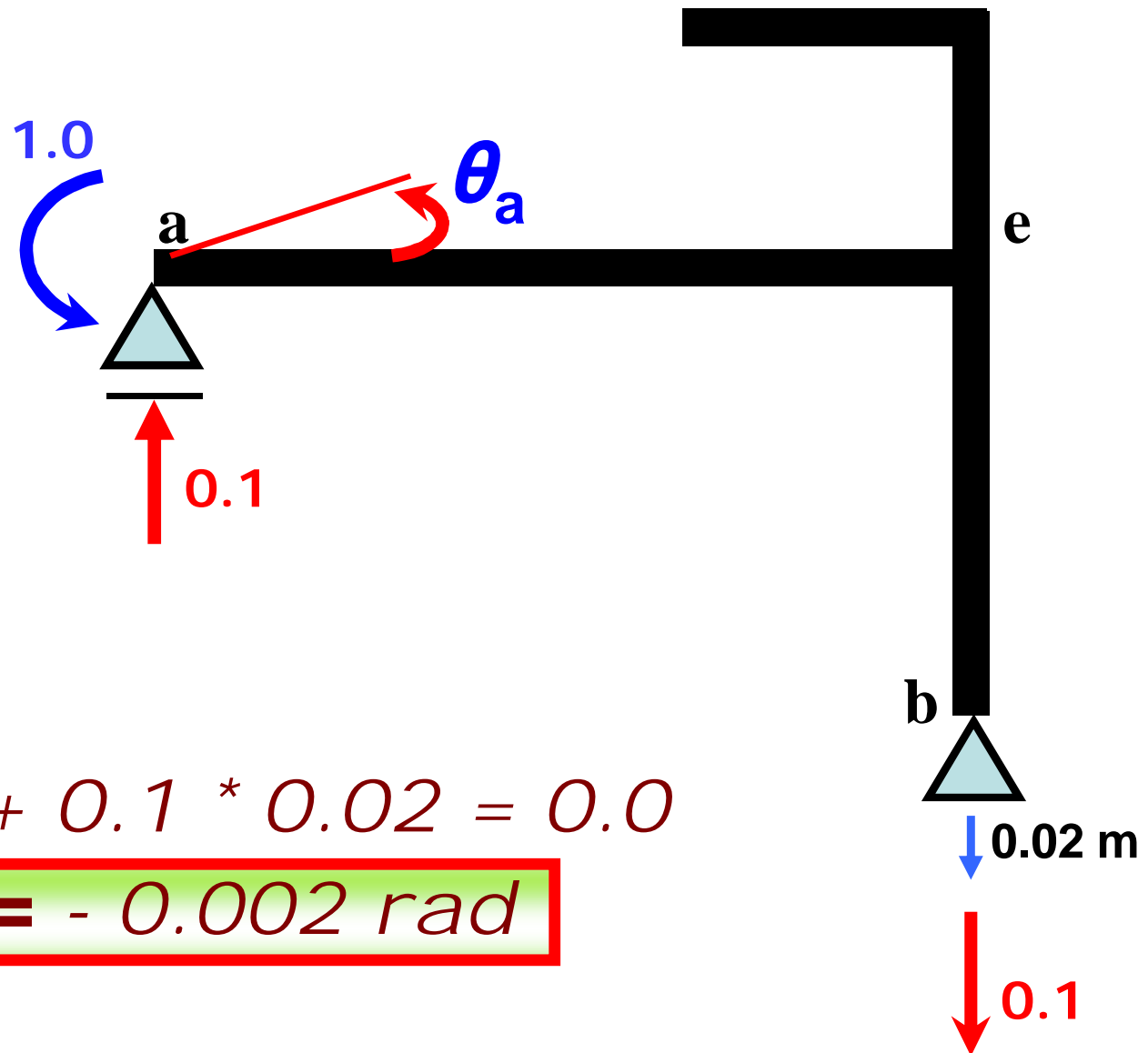
$$Y_B * 10 = 1$$

$$Y_B = 0.1$$

$$\Sigma \mathbf{F}_y = 0.0$$



$$Y_A = 0.1$$



$$1 * \theta_a + 0.1 * 0.02 = 0.0$$

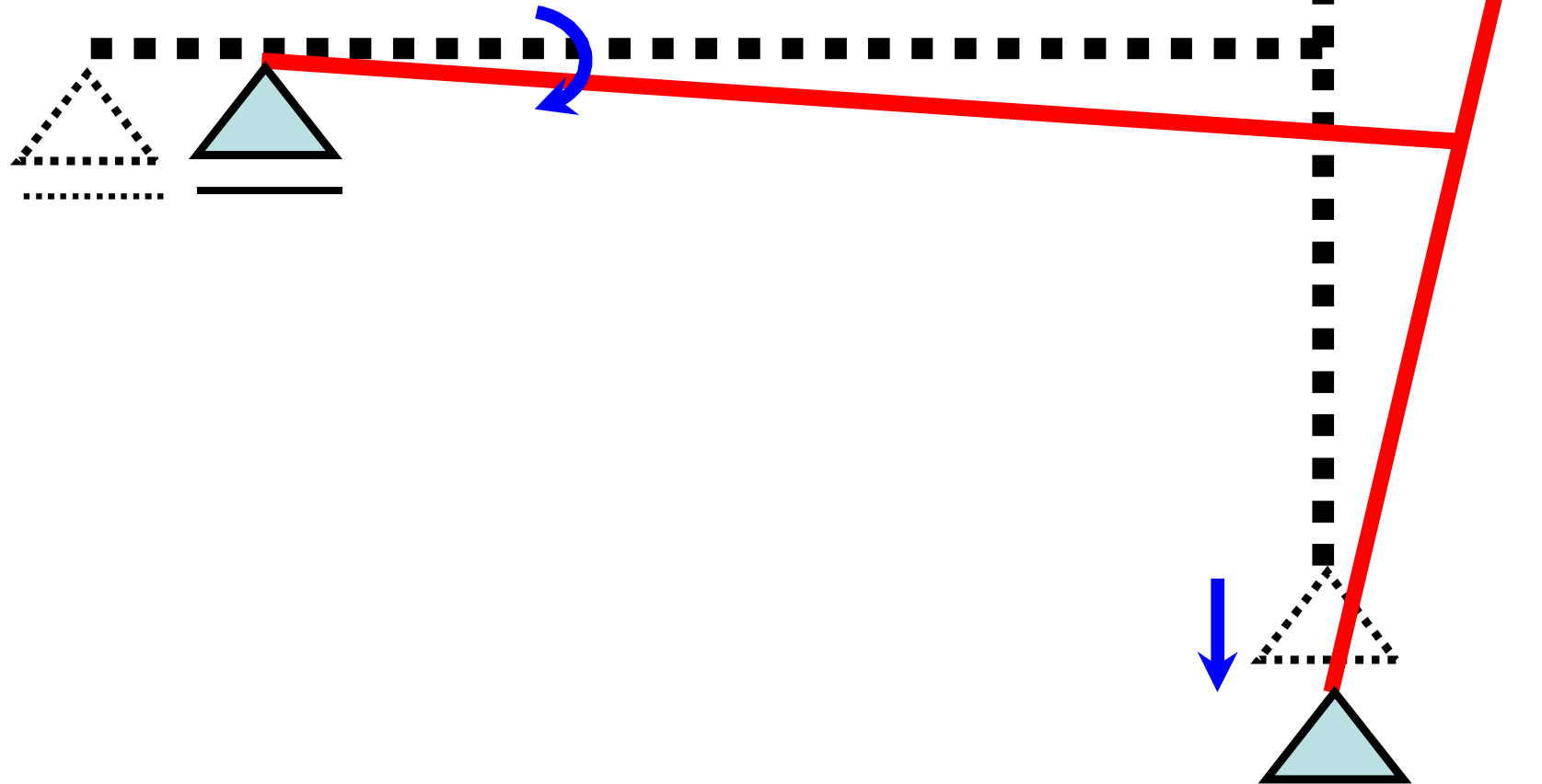
$$>> \theta_a = -0.002 \text{ rad}$$

given

$$>> \Delta v_c = 0.016 \text{ m}$$

$$>> \Delta \mathbf{hd} = -0.018 \text{ m}$$

$$>> \theta \mathbf{a} = -0.002 \text{ rad}$$



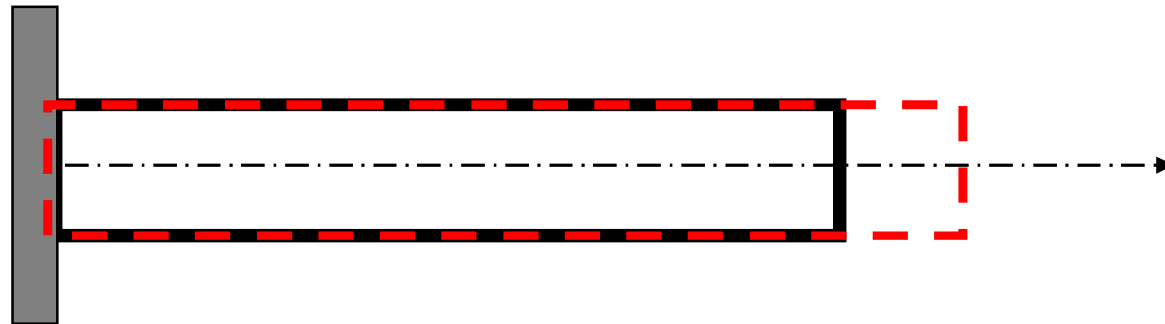
CASE OF TEMPRATURE

تأثير الحرارة

Case of uniform temprature.

طريقة الحل

- 1- يتم ازالة كل الاحمال من الكمرة.
- 2- يتم وضع 1ton عند النقطة المطلوب حساب الهبوط عندها.
- 3- يتم رسم الـ normal الناتج عن الـ 1ton.

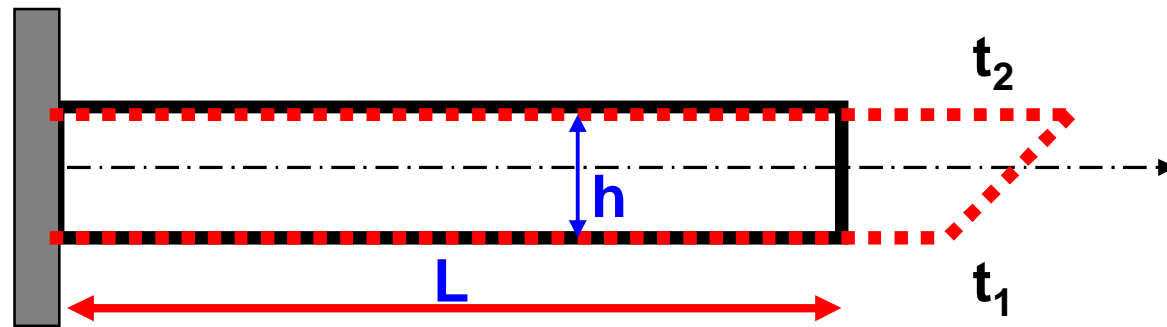


$$y \text{ or } \theta = \alpha * dt * (\text{Area of normal})$$

Case of non uniform temprature.

طريقة الحل

- 1- يتم ازالة كل الاحمال من الكمره.
- 2- يتم وضع 1ton عند النقطة المطلوب حساب الهبوط عندها.
- 3- يتم رسم الـ normal الناتج عن الـ 1ton.



$$y \text{ or } \theta = \alpha \cdot \left(\frac{t_2 + t_1}{2} \right) \cdot \Sigma \text{Area Normal} + \alpha \cdot \left(\frac{t_2 - t_1}{h} \right) \cdot \Sigma \text{Area of Moment}$$

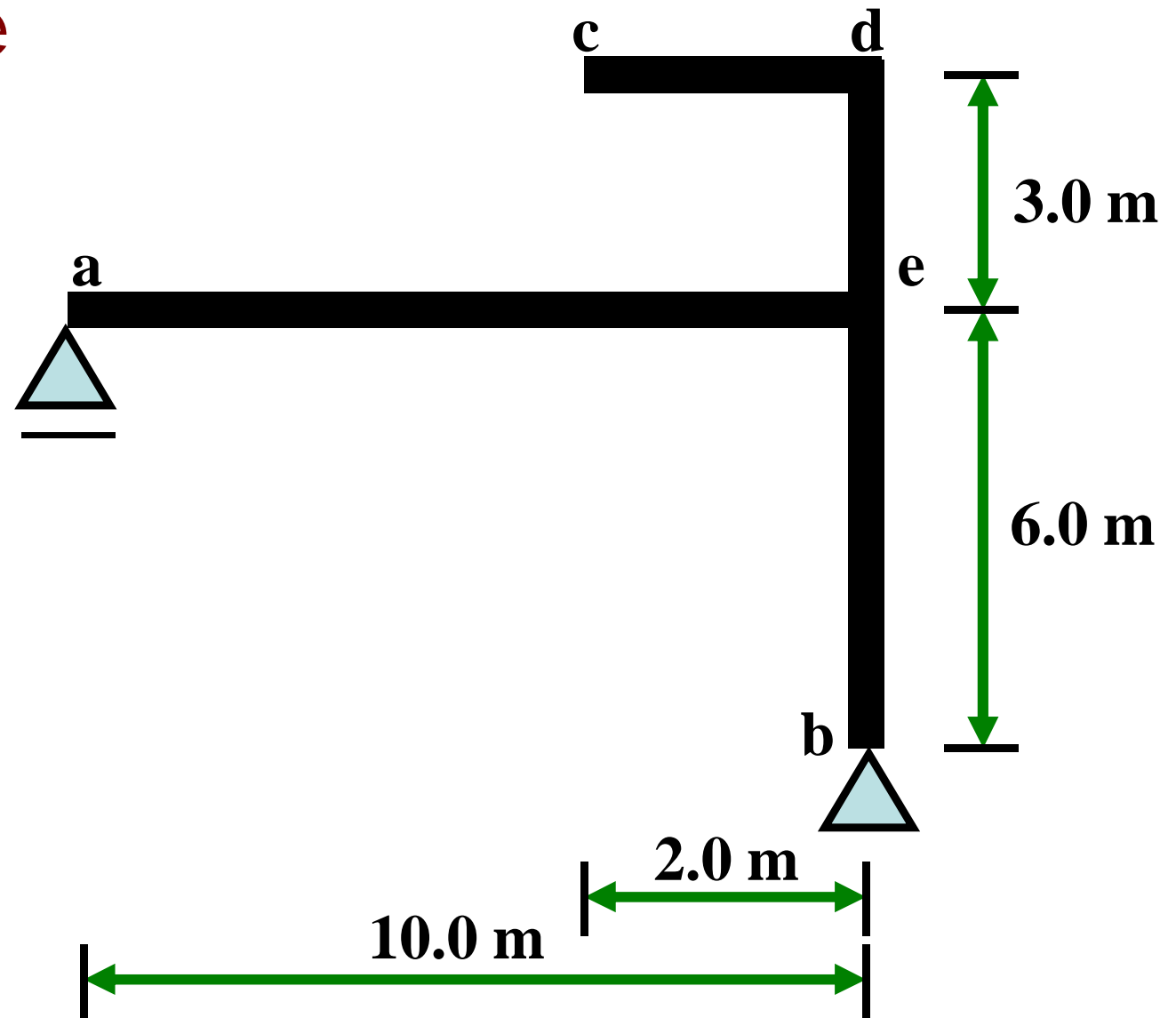
t_2 >>>> external temprature.

t_1 >>>> internal temprature.

h >>>> beam depth.

α >>>> thermal coeffiecnt.

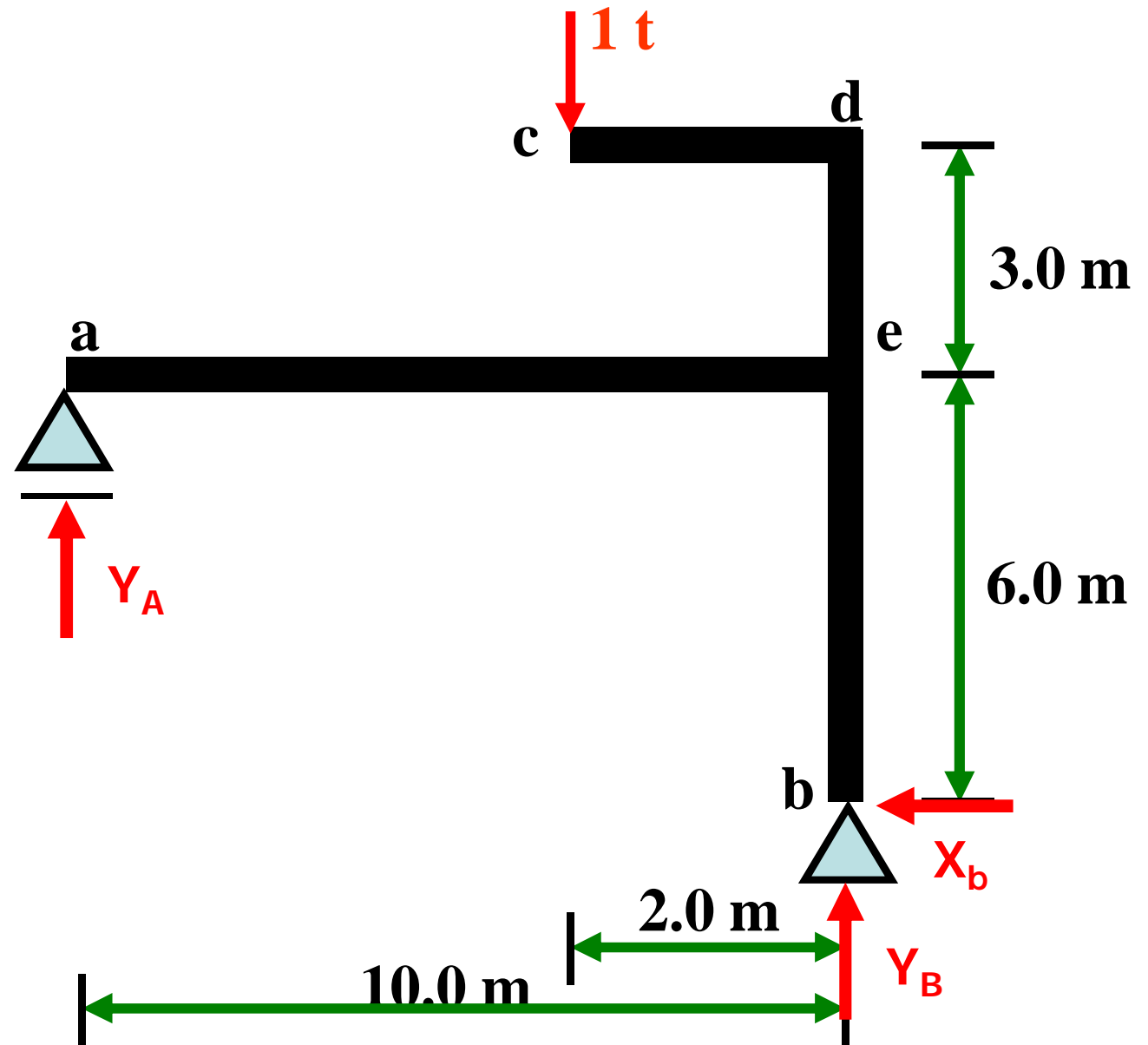
Example three



Required : Δ_{vc} and Δ_{hd} and θ_a due to uniform increase in temperature equal 20° . $\alpha = 10^{-5}$

For Δ_{vc}

المطلوب هو الازاحة
الرأسيه عند النقطة c



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 0.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



$$Y_B * 10$$

=

$$1 * 8$$

=

$$0.8$$

$$\Sigma \mathbf{F}_y = 0.0$$



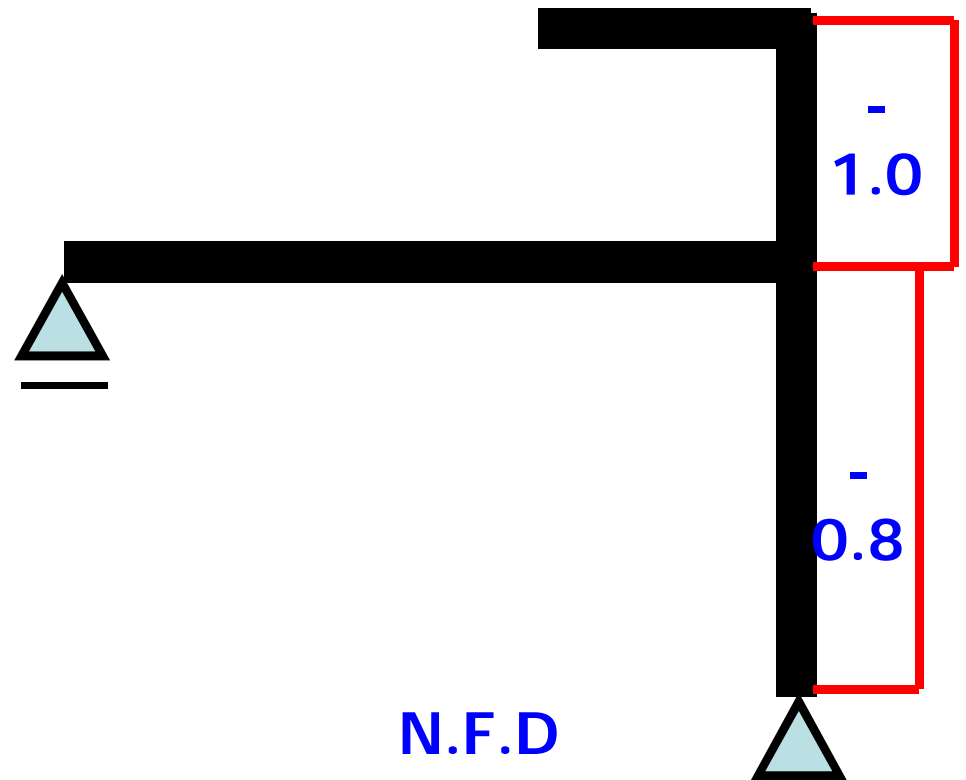
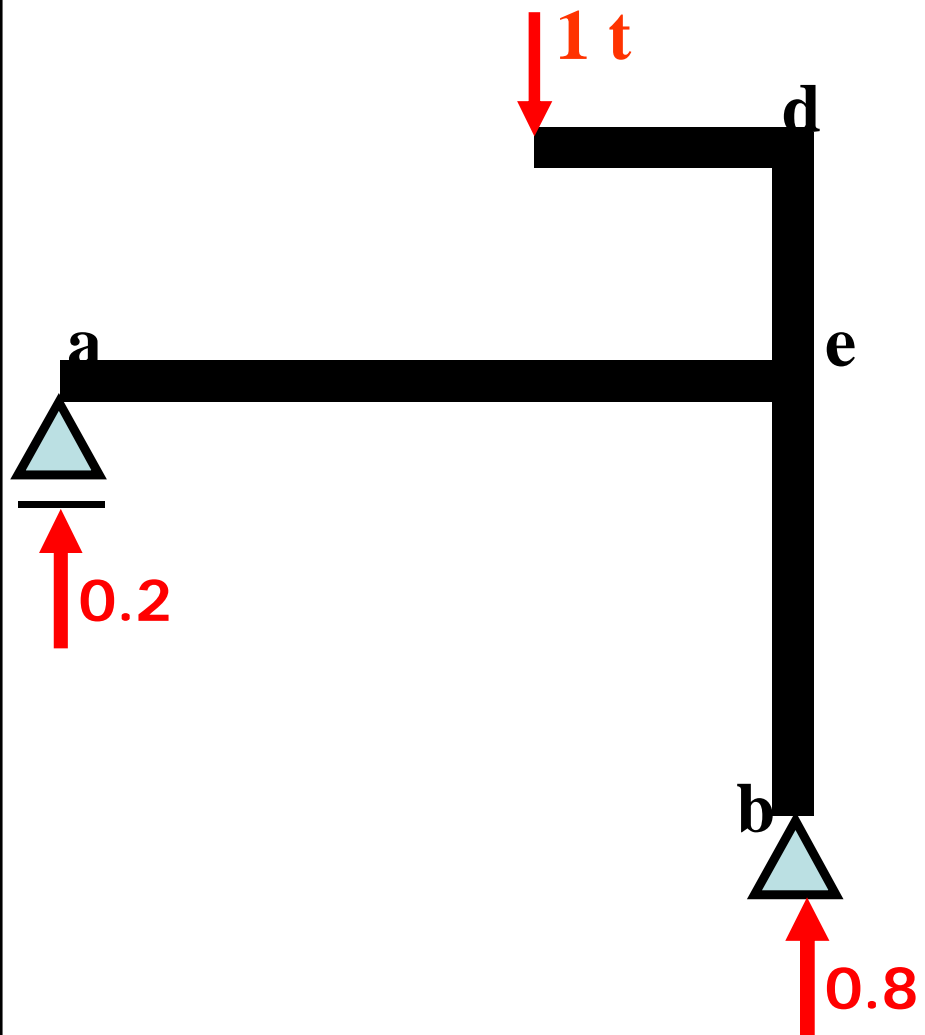
$$Y_A + 0.8$$

=

$$1$$

=

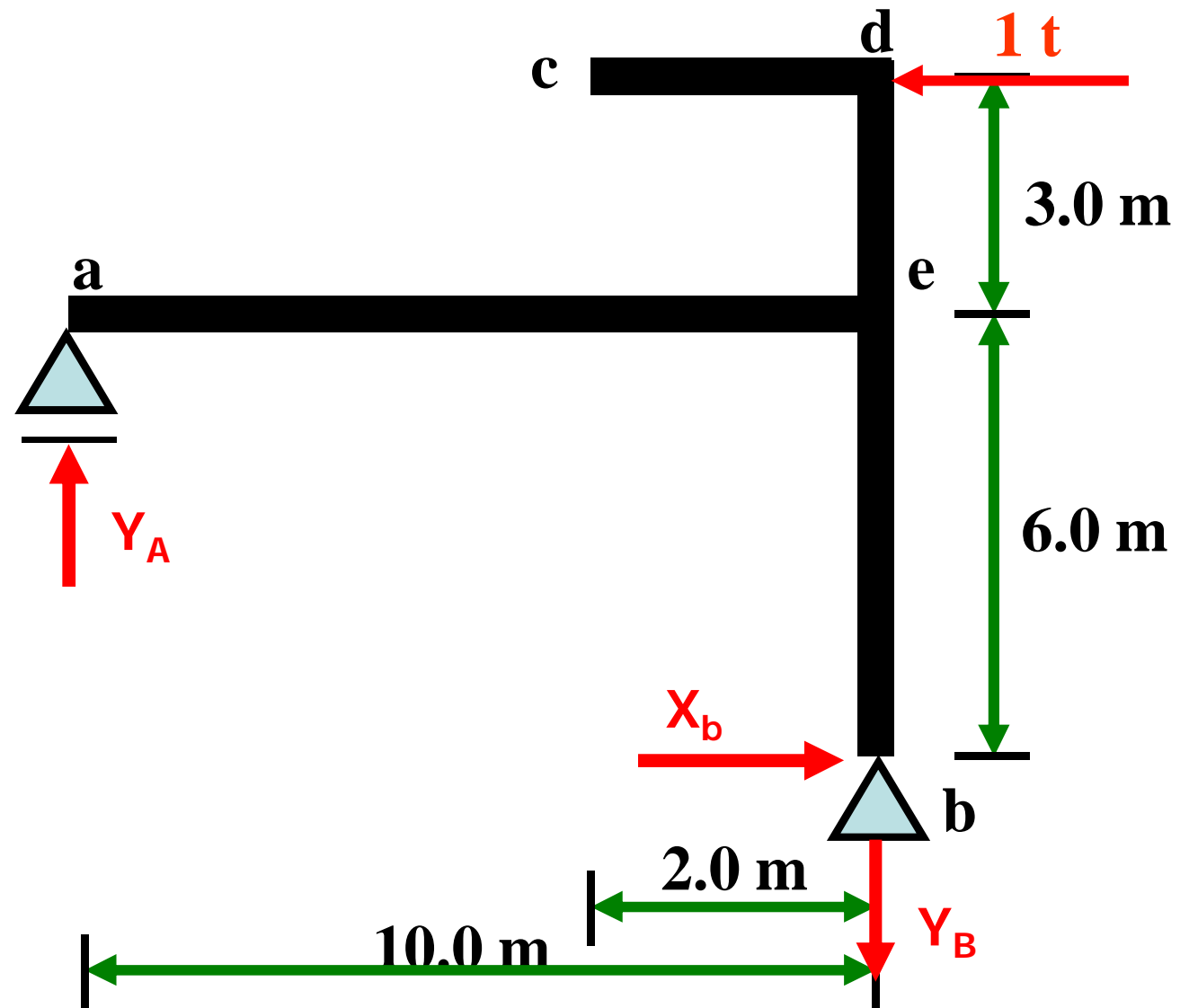
$$0.2$$



$$\Delta_{vc} = 10^{-5} * 20 * (-1*3 - 0.8*6) = -0.00156 \text{ m}$$

For Δ_{hd}

المطلوب هو الازاحة
الافقيه عند النقطة d



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 1.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



$$Y_B * 10$$

=

$$1 * 3 + 1 * 6$$

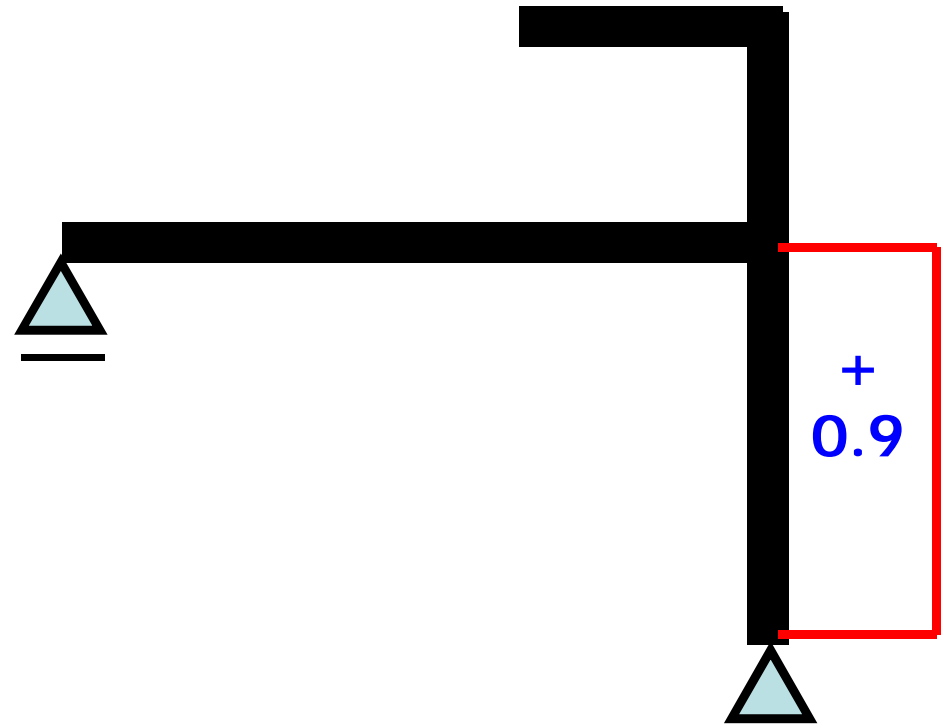
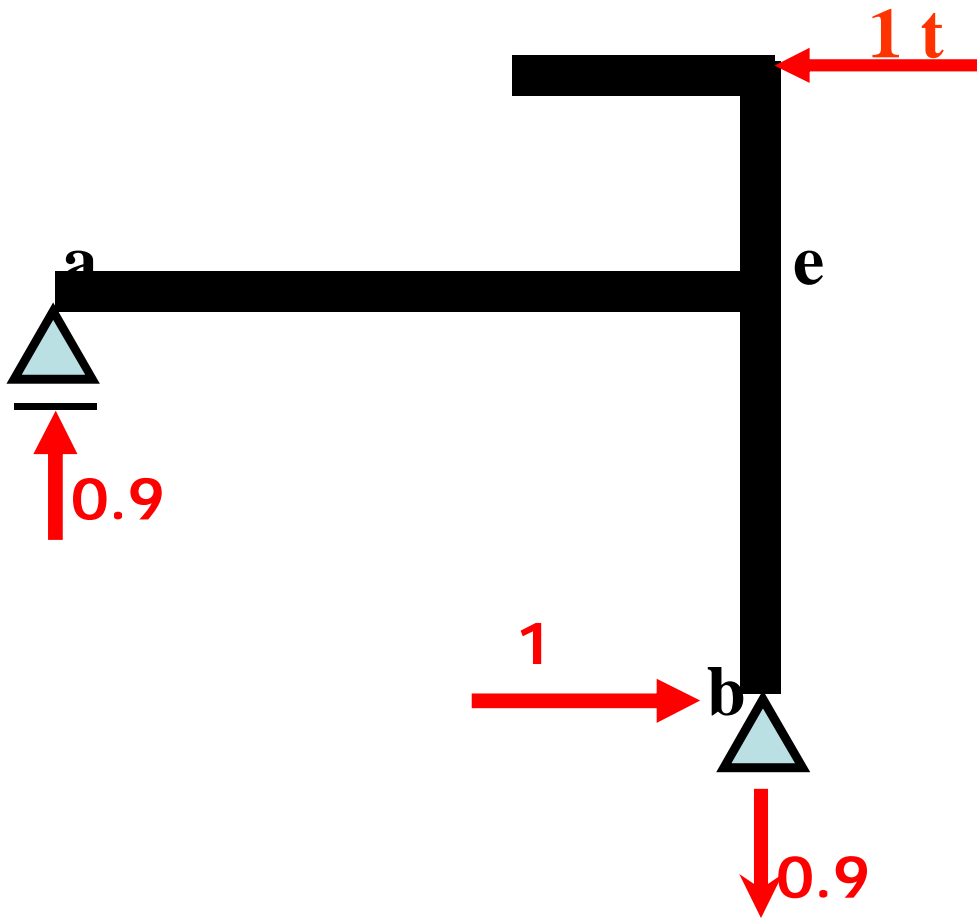
=

$$0.9$$

$$\Sigma \mathbf{F}_y = 0.0$$



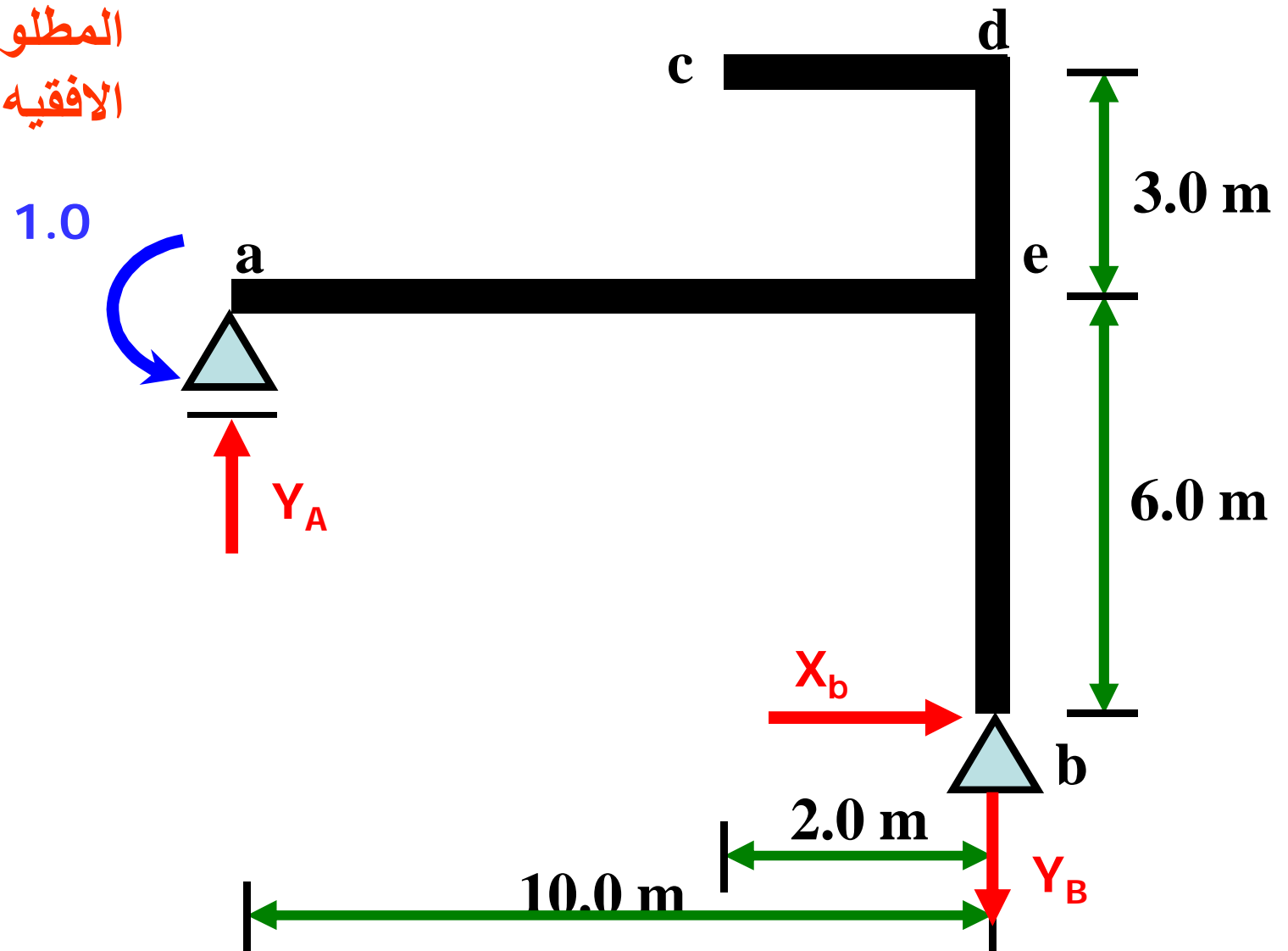
$$Y_A = 0.9$$



$$\Delta_{hd} = 10^{-5} * 20 * (+0.9 * 6) = 0.00108 \text{ m}$$

For θ_a

المطلوب هو الازاحة
الافقيه عند النقطة d



$$\Sigma \mathbf{F}_x = 0.0$$



$$X_b = 0.0$$

$$\Sigma \mathbf{M}_A = 0.0$$



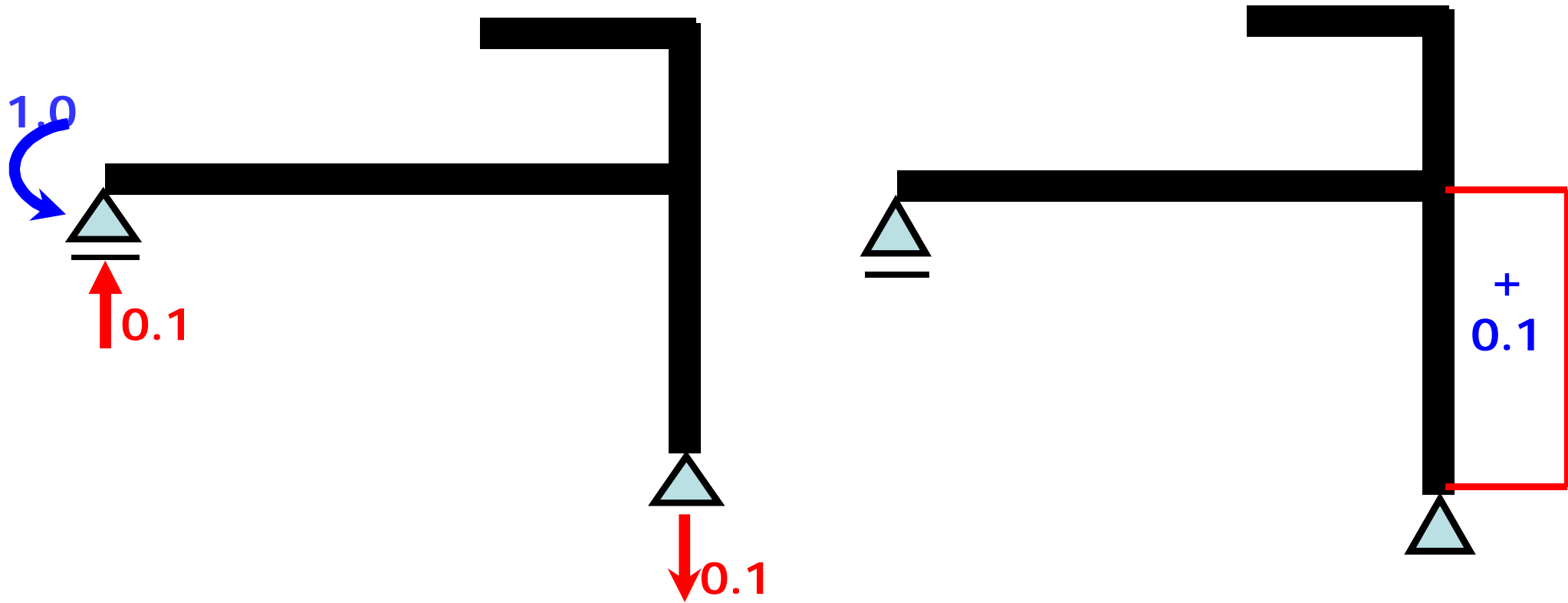
$$Y_B * 10 = 1$$

$$Y_B = 0.1$$

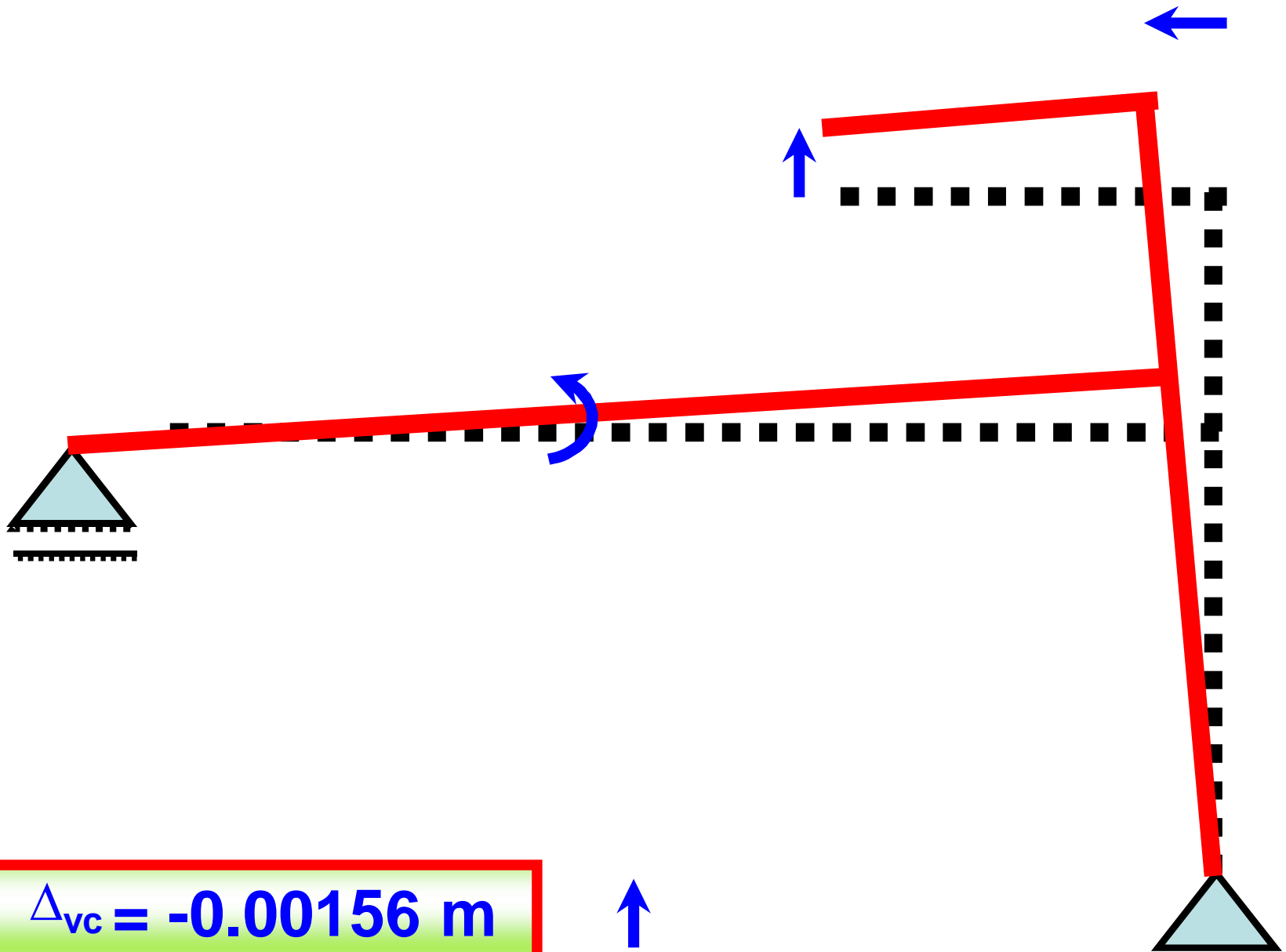
$$\Sigma \mathbf{F}_y = 0.0$$



$$Y_A = 0.1$$



$$\theta_a = 10^{-5} * 20 * (+0.1 * 6) = 0.00012 \text{ rad}$$



$$\Delta_{vc} = -0.00156 \text{ m}$$



$$\Delta_{hd} = 0.00108 \text{ m}$$

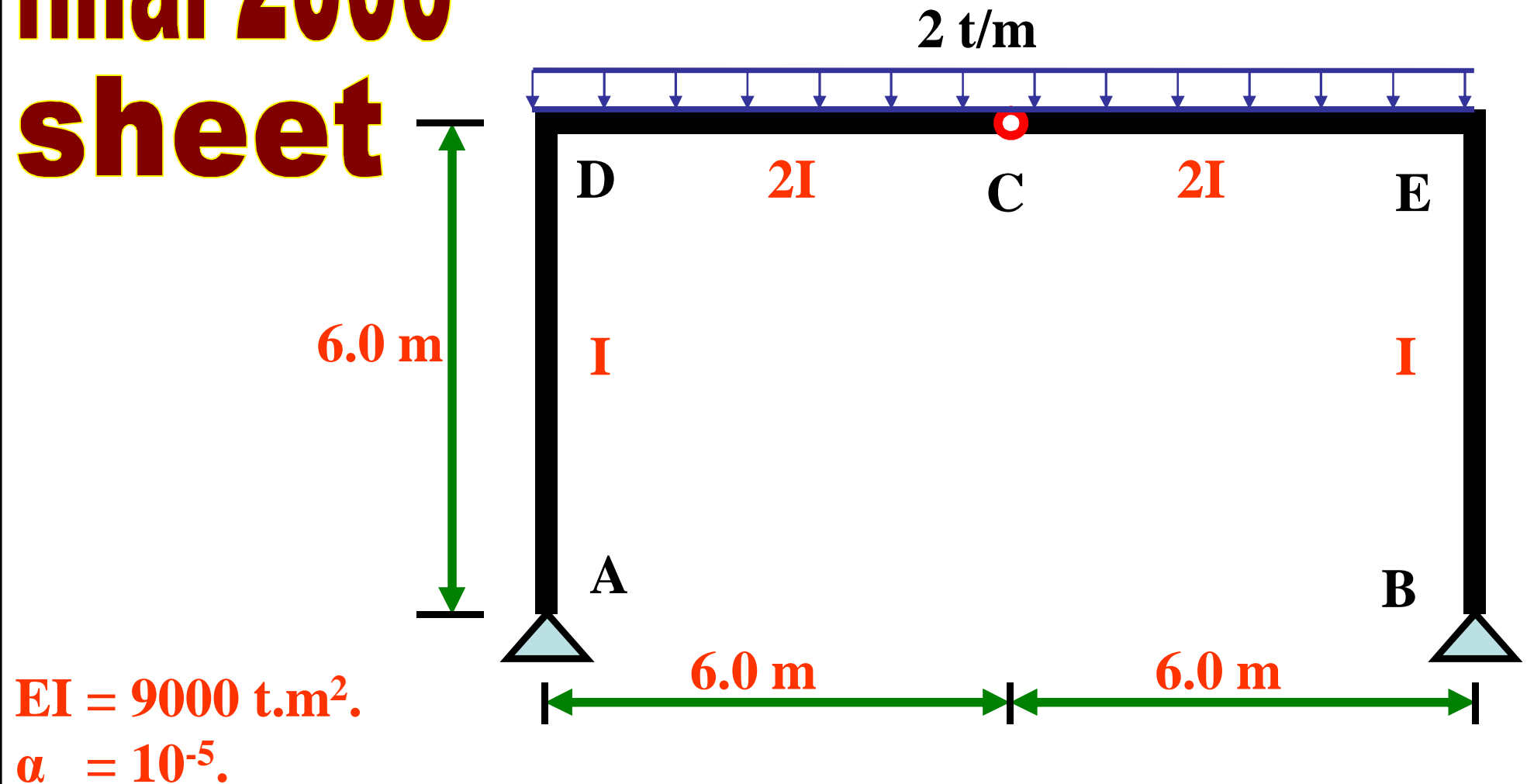


$$\theta_a = 0.00012 \text{ rad}$$



Exams problems

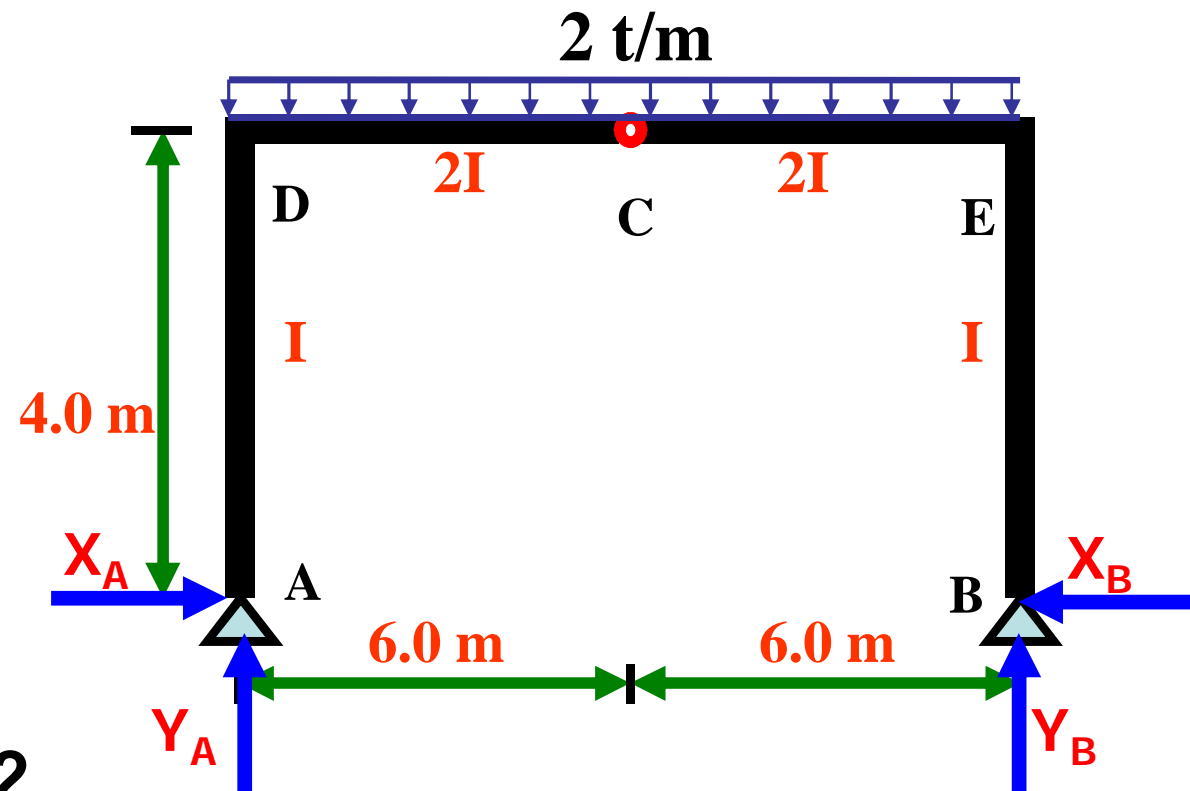
final 2006 sheet



For the Shown Frame, Calculate the Vertical Deflection at C, Horizontal Deflection at D and Draw the Elastic Curve due to:

- 1- Given Loads.
- 2- Horizontal Movement at B = 2cm to the Right.
- 3- Uniform Rise in Temperature 40° .

Reactions



$$\sum \mathbf{F}_Y = 0.0$$

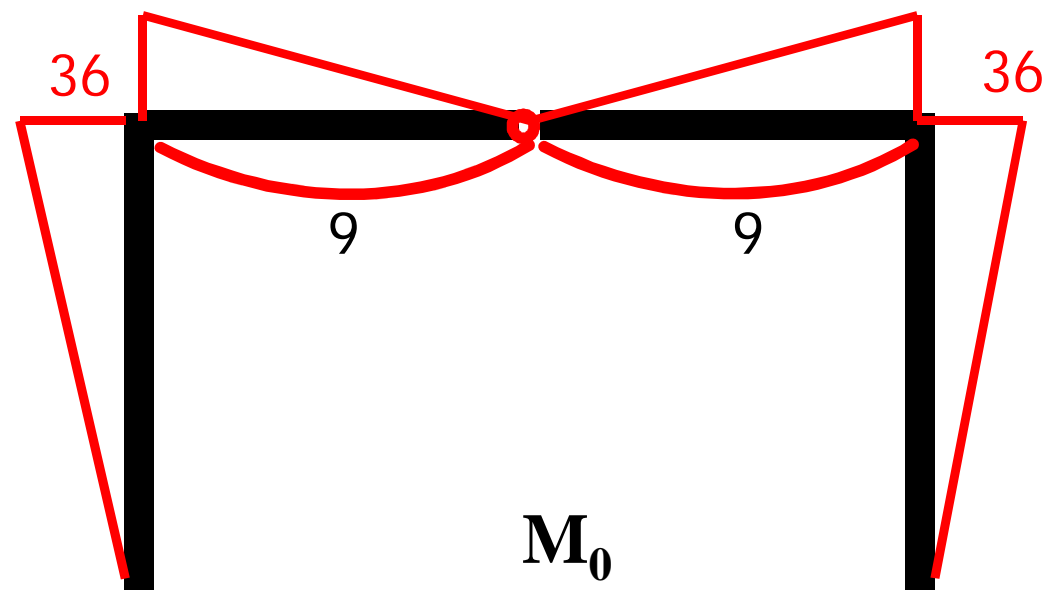
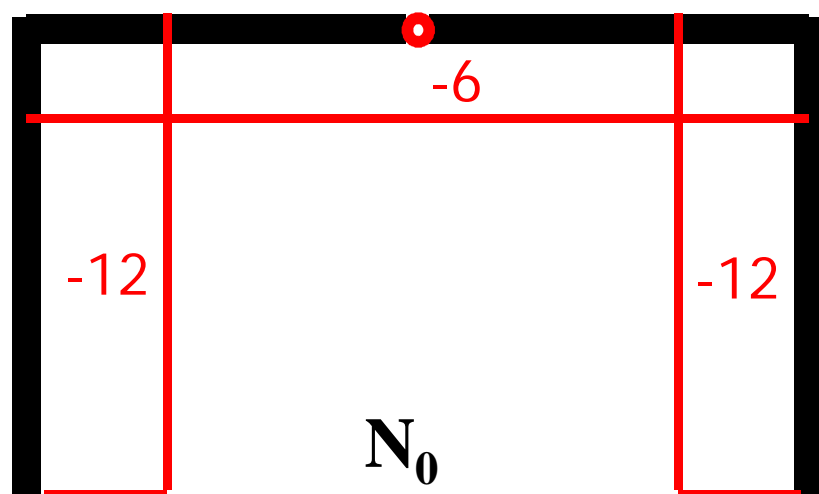
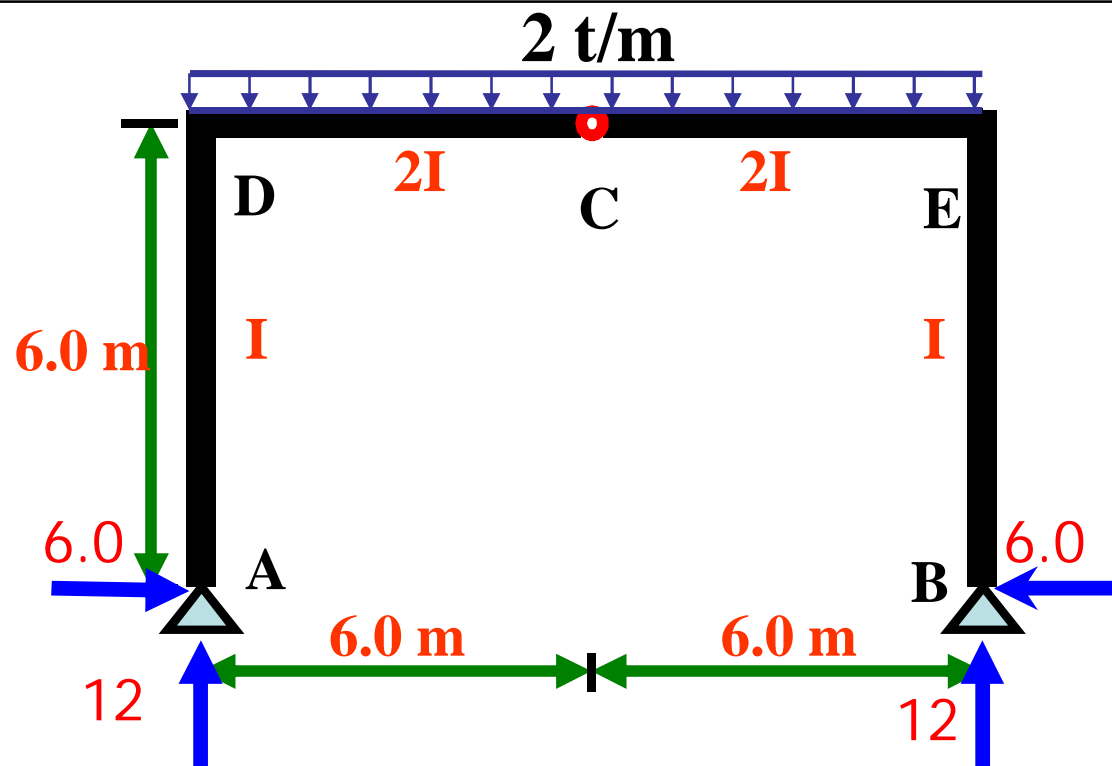
$$Y_A = Y_B = 24/2 = 12$$

$$\sum \mathbf{M}_{cr} = 0.0$$

$$X_B * 6.0 + 12 * 3 = Y_B * 6 \quad \gggg \quad X_B = 6 \text{ t}$$

$$\sum \mathbf{F}_X = 0.0$$

$$X_A = 6.0$$



For yc

$$\sum \mathbf{F}_Y = 0.0$$

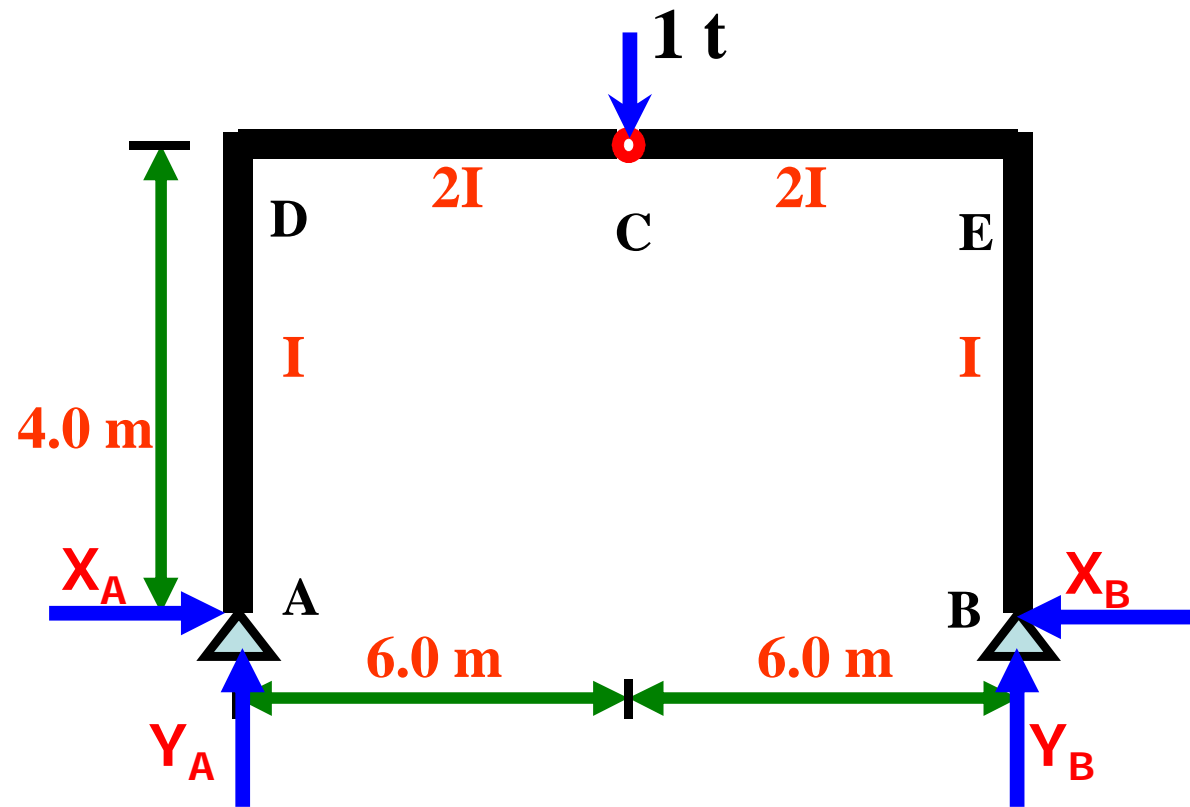
$$Y_A = Y_B = 1/2$$

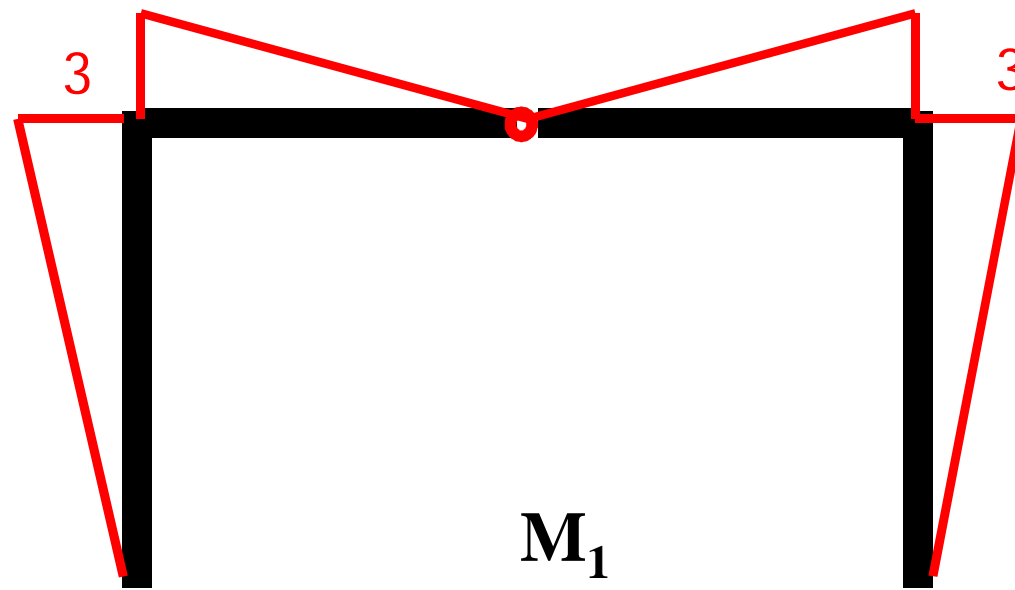
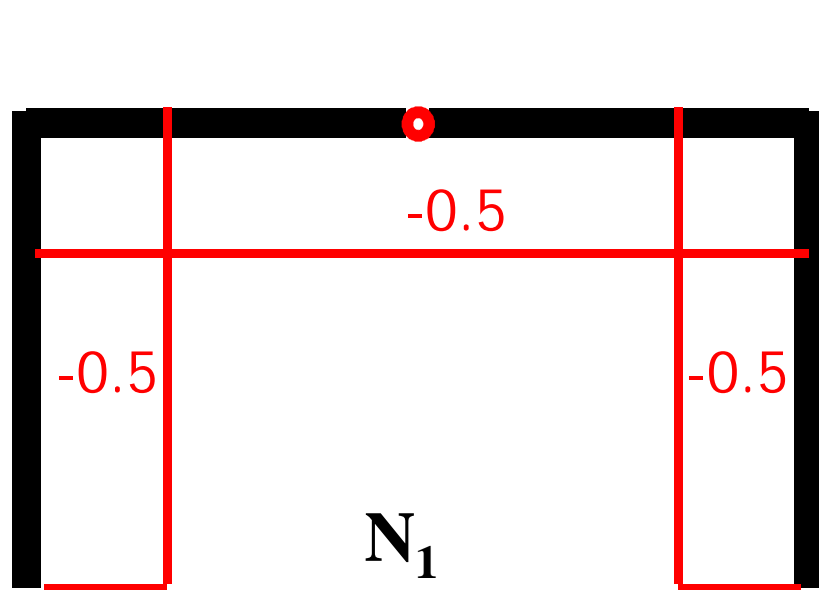
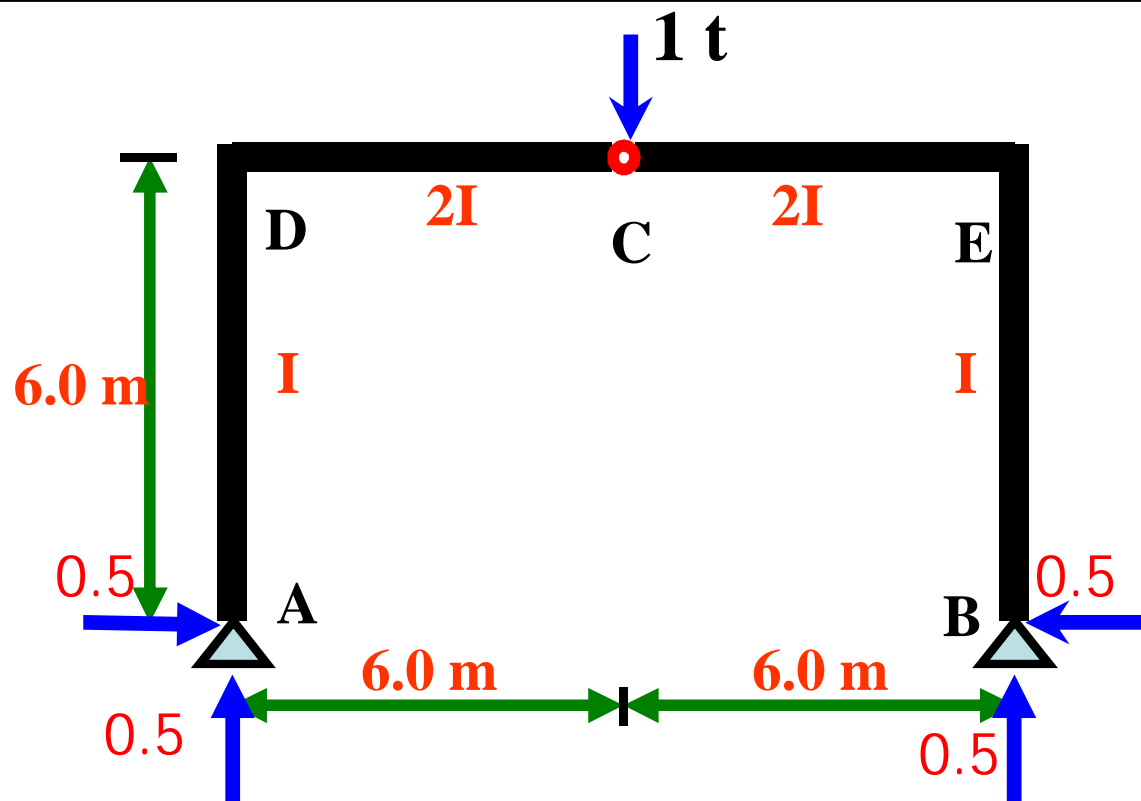
$$\sum \mathbf{M}_{cr} = 0.0$$

$$X_B * 6.0 = Y_B * 6 \quad \gggg \quad X_B = 0.5 \text{ t}$$

$$\sum \mathbf{F}_X = 0.0$$

$$X_A = 0.5$$





For hd

$$\Sigma \mathbf{M}_a = 0.0$$

$$1 * 6.0 = Y_B * 12$$

$$>>>> Y_B = 0.5 \text{ t}$$

$$\Sigma \mathbf{M}_{cr} = 0.0$$

$$X_B * 6.0 = Y_B * 6$$

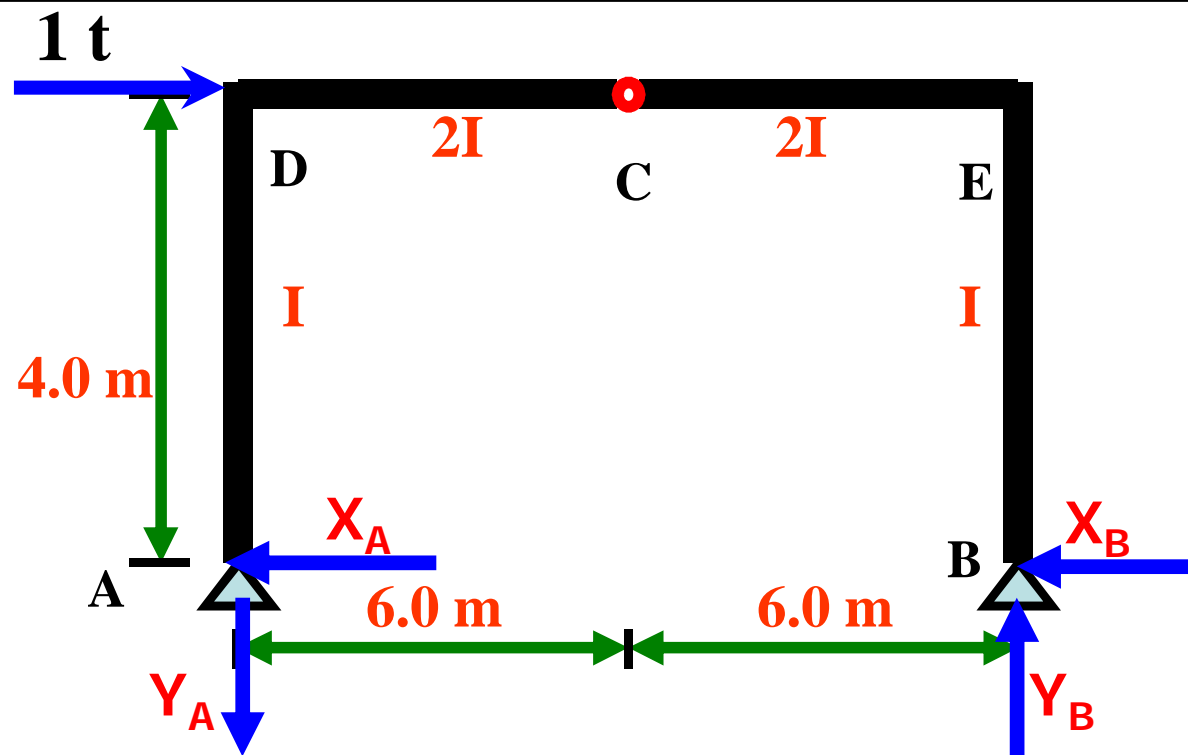
$$>>>> X_B = 0.5 \text{ t}$$

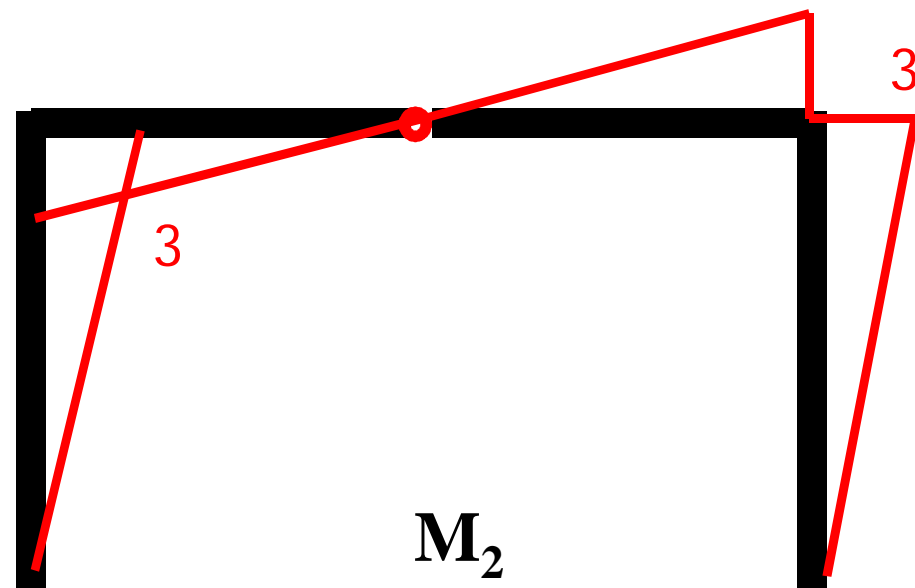
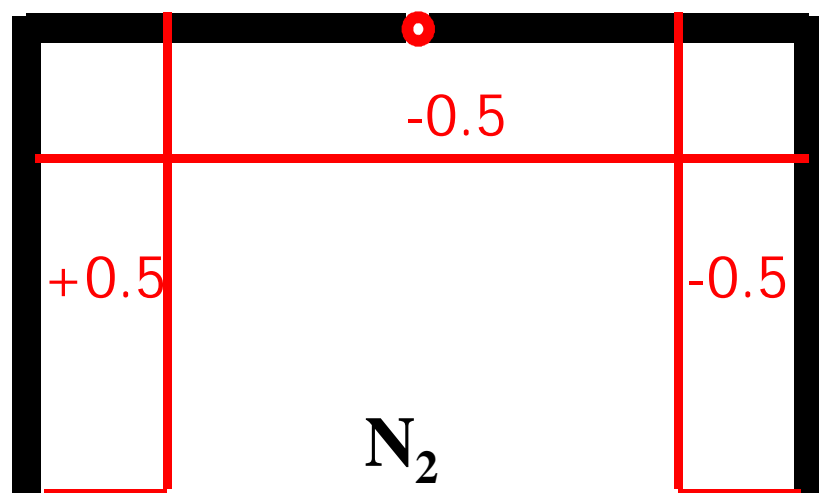
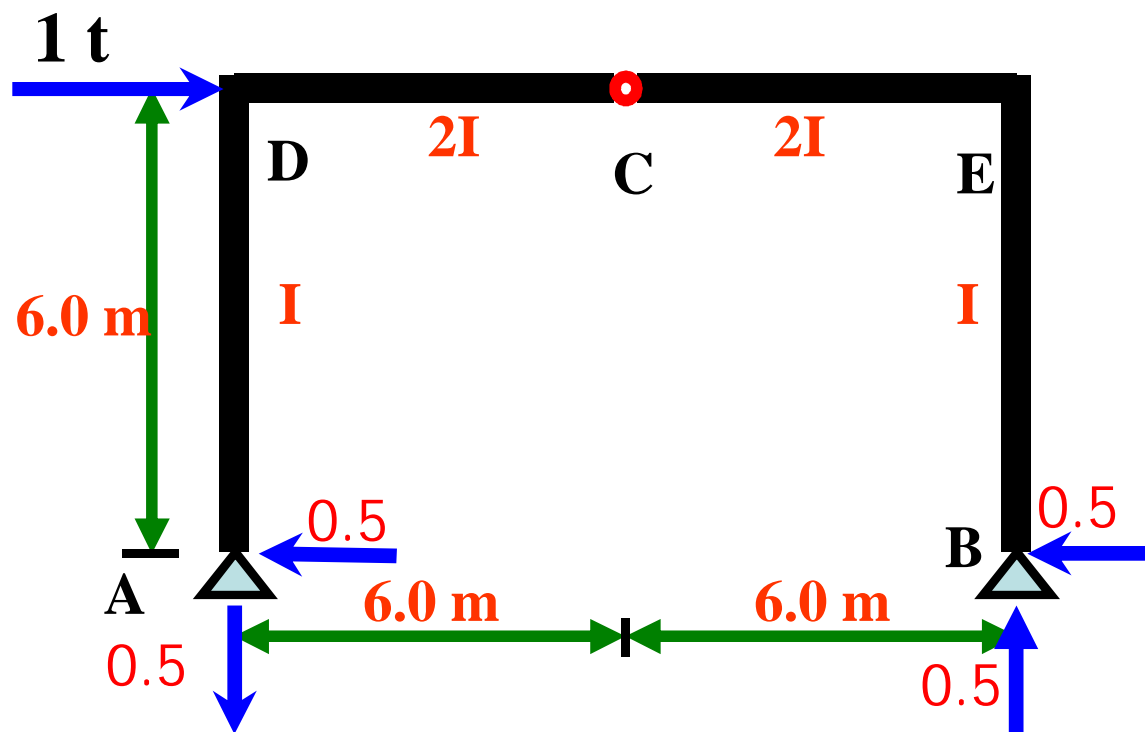
$$\Sigma \mathbf{F}_x = 0.0$$

$$X_A = 1 - 0.5 = 0.5$$

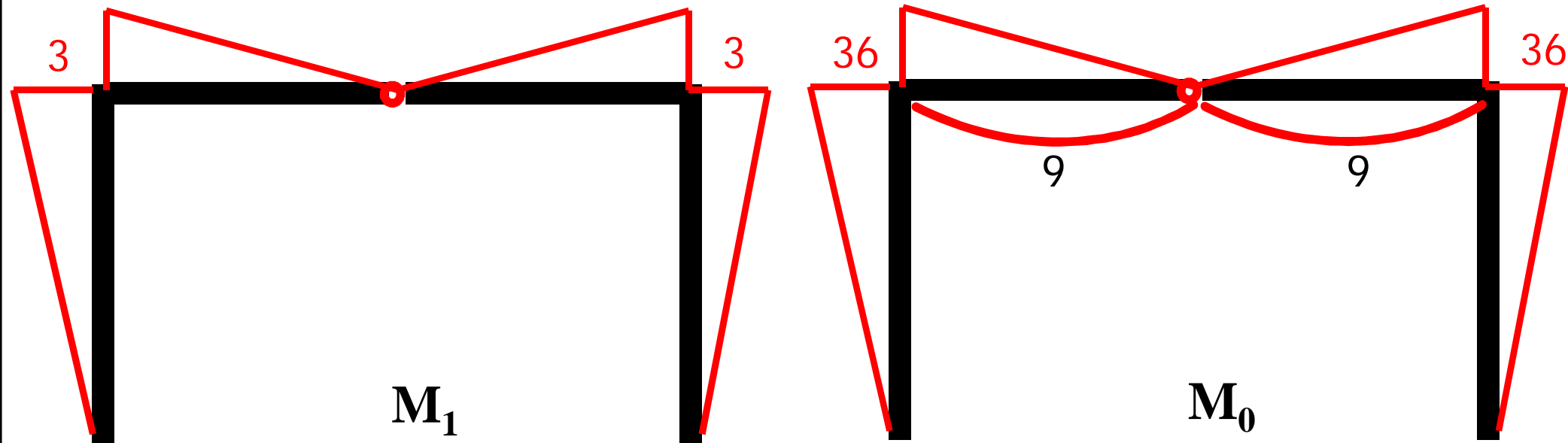
$$\Sigma \mathbf{F}_y = 0.0$$

$$Y_A = Y_B = 0.5$$





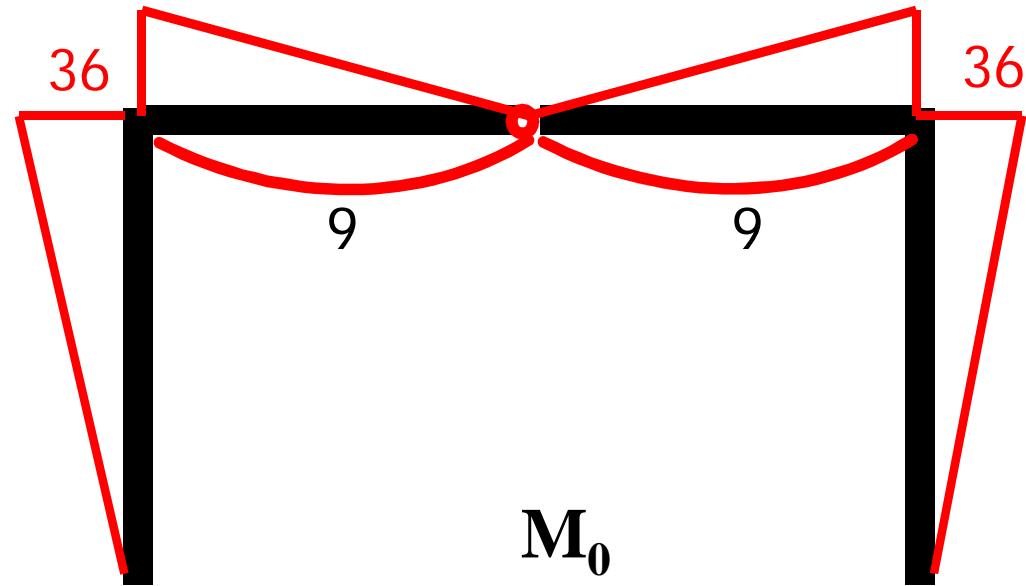
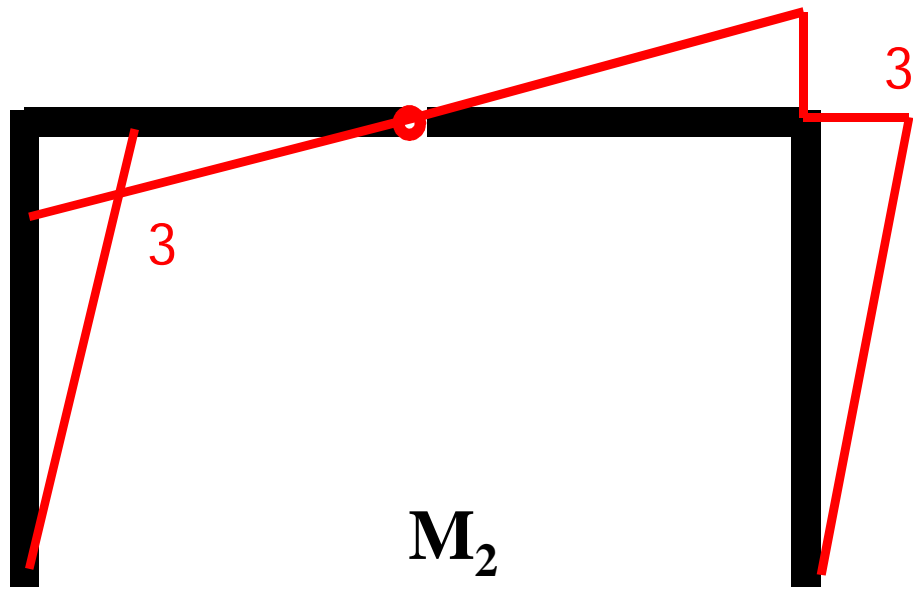
given loads



$$y_c = 2 * \left[\frac{1}{EI} \left[\frac{36 * 3 * 6}{3} \right] + \frac{1}{2EI} \left[\frac{36 * 3 * 6}{3} - \frac{2}{3} * 9 * 6 * 1.5 \right] \right]$$

$$= \frac{594}{EI} = 0.066m$$

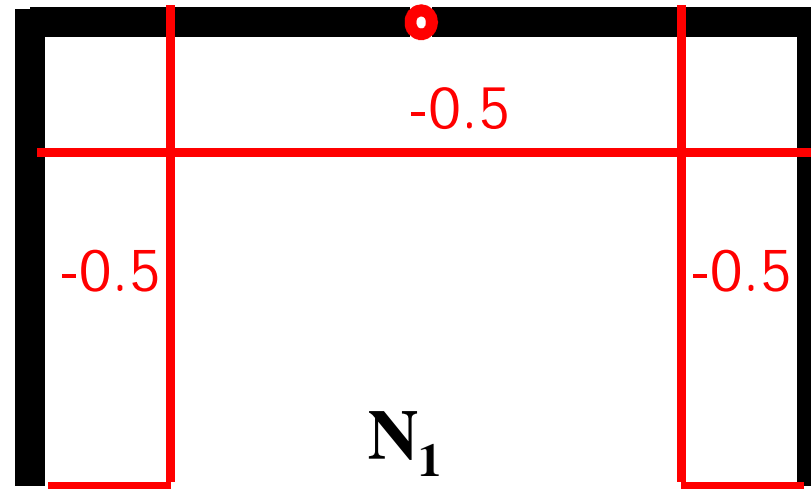
given loads



$$hd = 0.0$$

تکامل شکل متماثل مع شکل
متماثل عکسی یساوی صفر

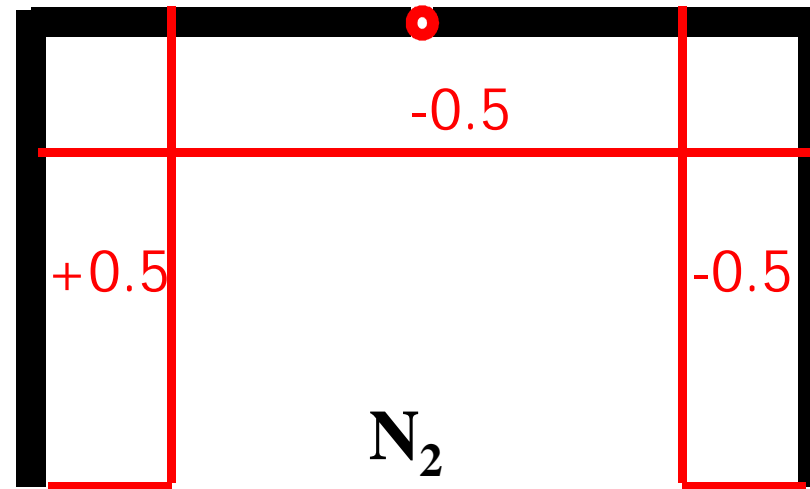
Uniform temperature=40



$$\text{Area of Normal} = -0.5 * 6 - 0.5 * 12 - 0.5 * 6 = -12$$

$$yc = \alpha * dt * \text{Area of Normal} = 10^{-5} * 40 * -12 = -0.0048m$$

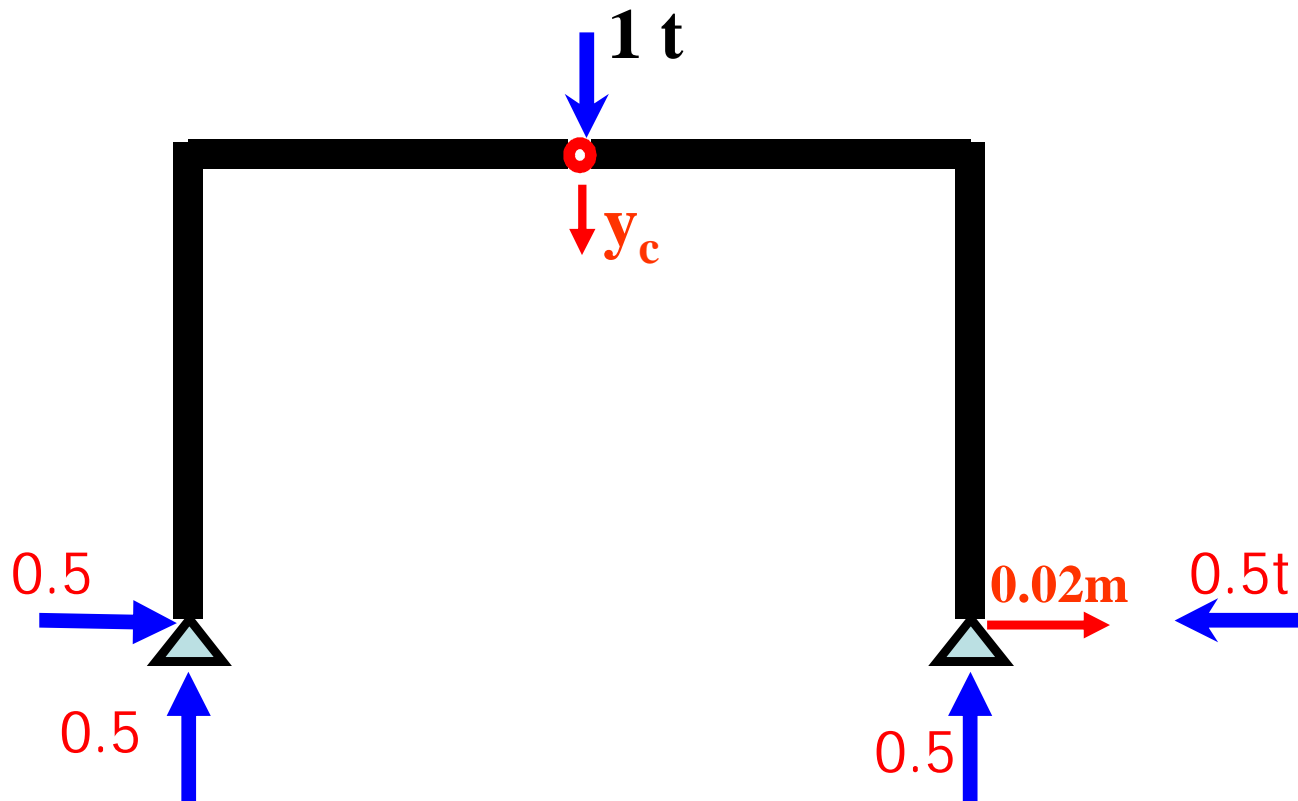
Uniform temprature=40



$$\text{Area of Normal} = +0.5 * 6 - 0.5 * 12 - 0.5 * 6 = -6$$

$$hd = \alpha * dt * \text{Area of Normal} = 10^{-5} * 40 * -6 = -0.0024m$$

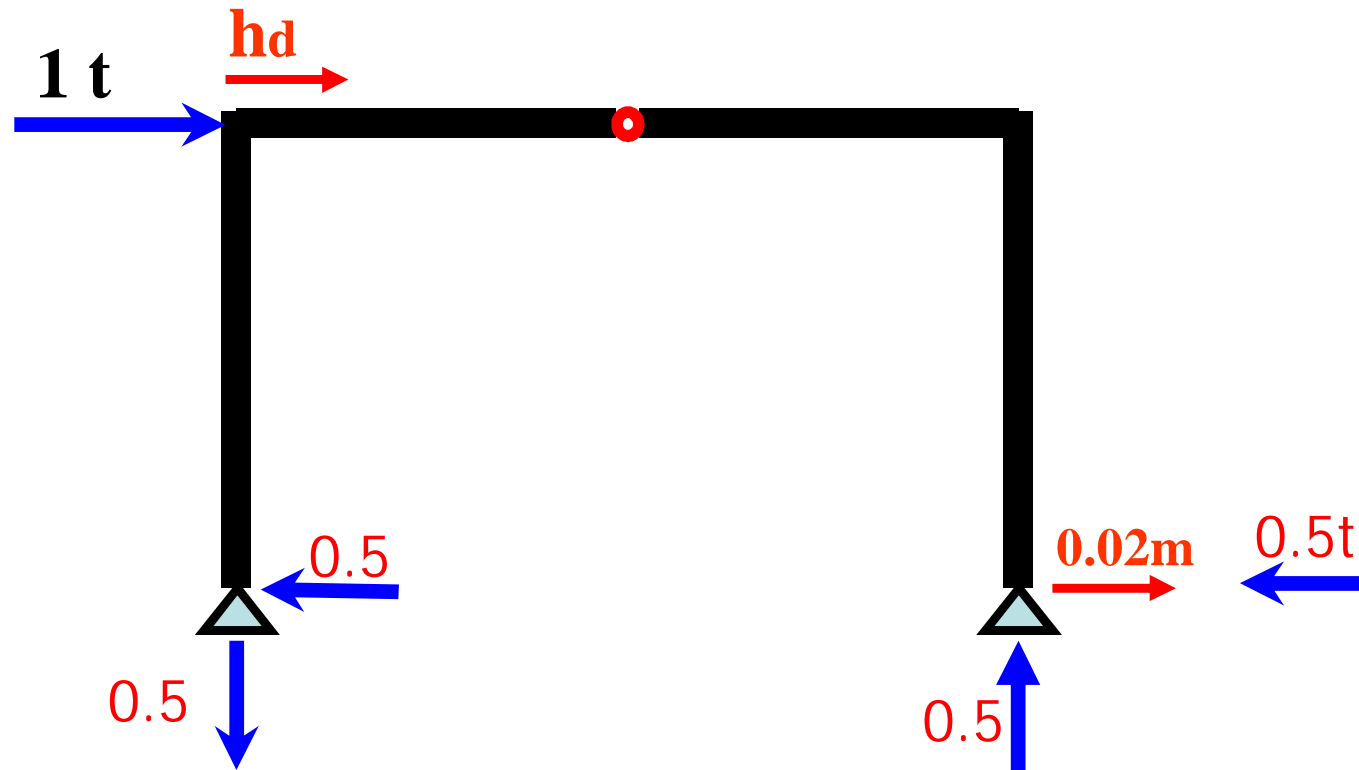
movement at b to the right = 0.02 m



$$1 * y_c - 0.5 * 0.02 = 0.0$$

$$y_c = 0.01 m$$

movement at b to the right = 0.02 m



$$1 * hd - 0.5 * 0.02 = 0.0$$

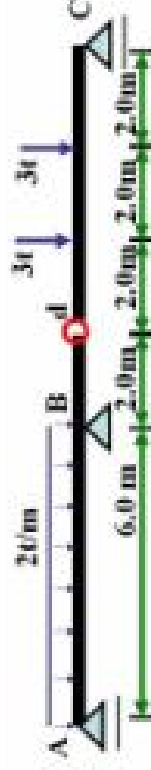
$$hd = 0.01\text{ m}$$



For the shown Structures, determine the Slope angle and deflection values as mentioned using the Virtual Work Method due to given load

4.1.

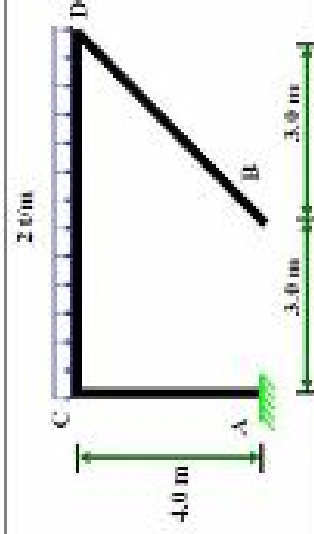
- Rotation at Point B
- Vertical Deflection at d
- Relative rotation at d
- Draw Elastic Curve



4.2.

- Rotation at Point C
- Horizontal Deflection at B
- Vertical Deflection at D
- Relative Movement Between B and C

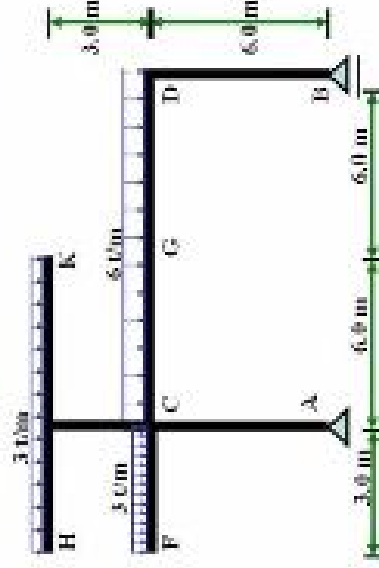
$$EI = 10000 \text{ t.m}^2$$



4.3.

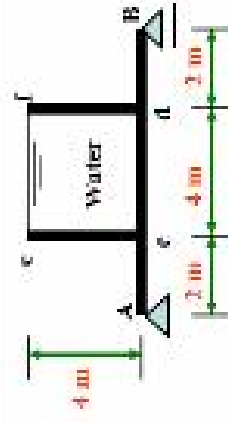
- Rotation at D
- Horizontal Displacement at B
- Vertical Deflection at G

$$EI = 12000 \text{ t.m}^2$$



4.4.

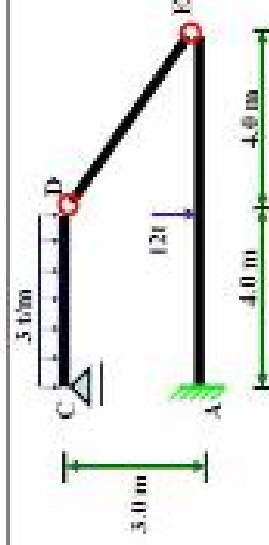
- Rotation at Point D
- Vertical Deflection at E



4.5.

- Horizontal Displacement at D
- Vertical Displacement at E
- Rotation at point C

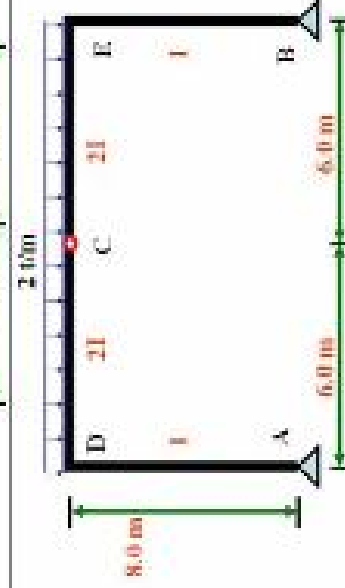
$$EI = 10000 \text{ t.m}^2$$



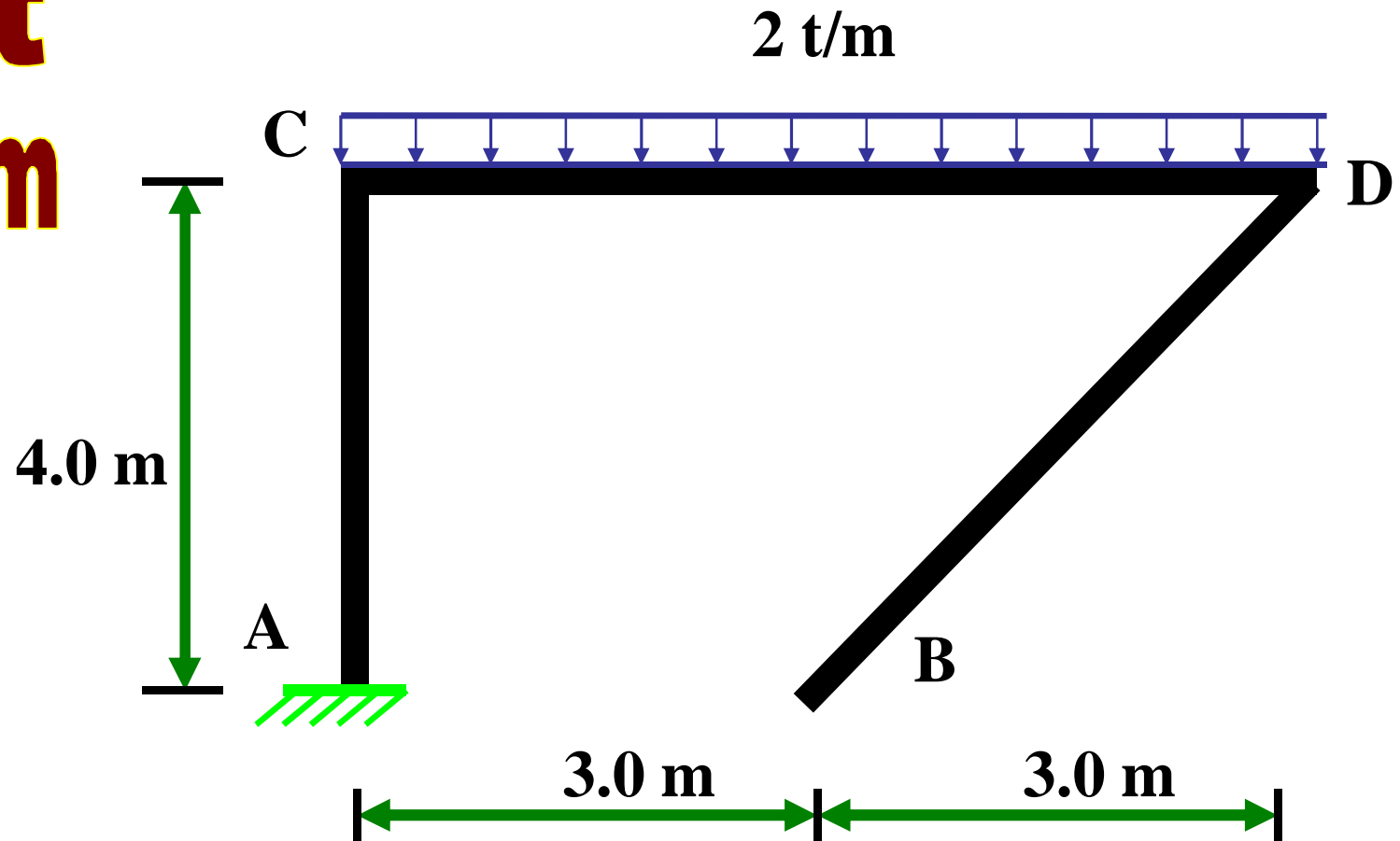
4.5.

- Vertical Deflection at C
- Horizontal Deflection at D
- Draw the Elastic Curve

$$EI = 9000 \text{ t.m}^2$$



sheet mid term

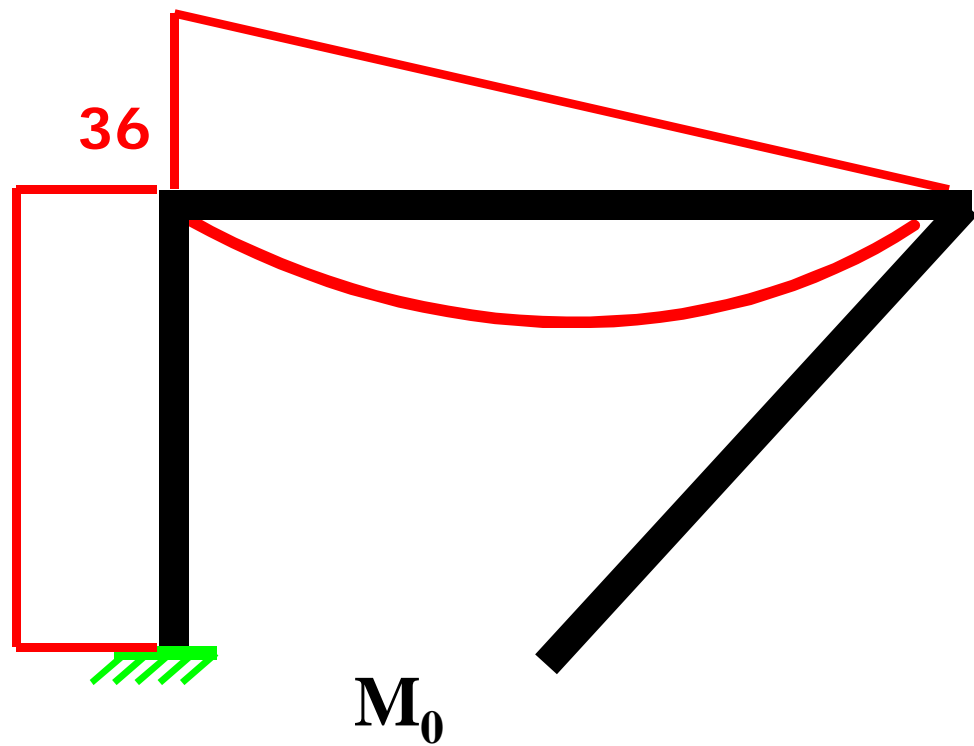
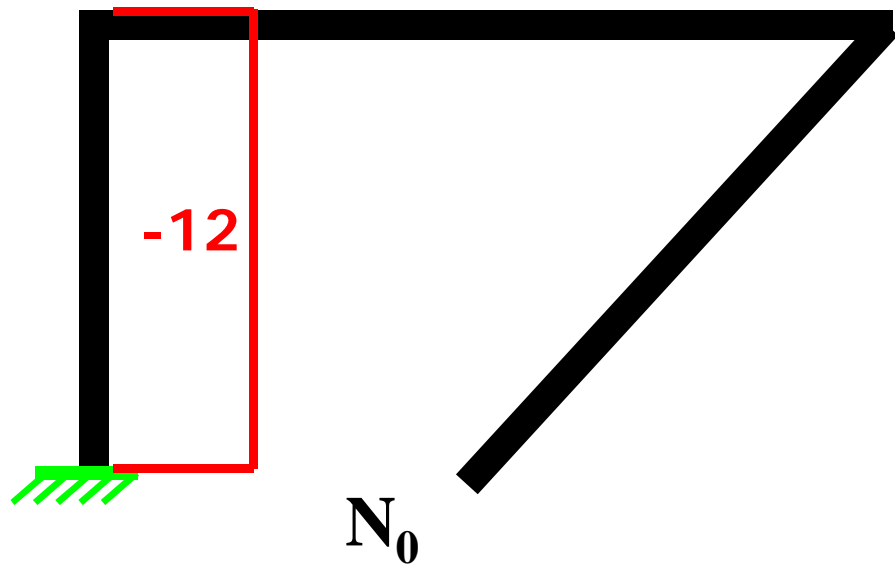
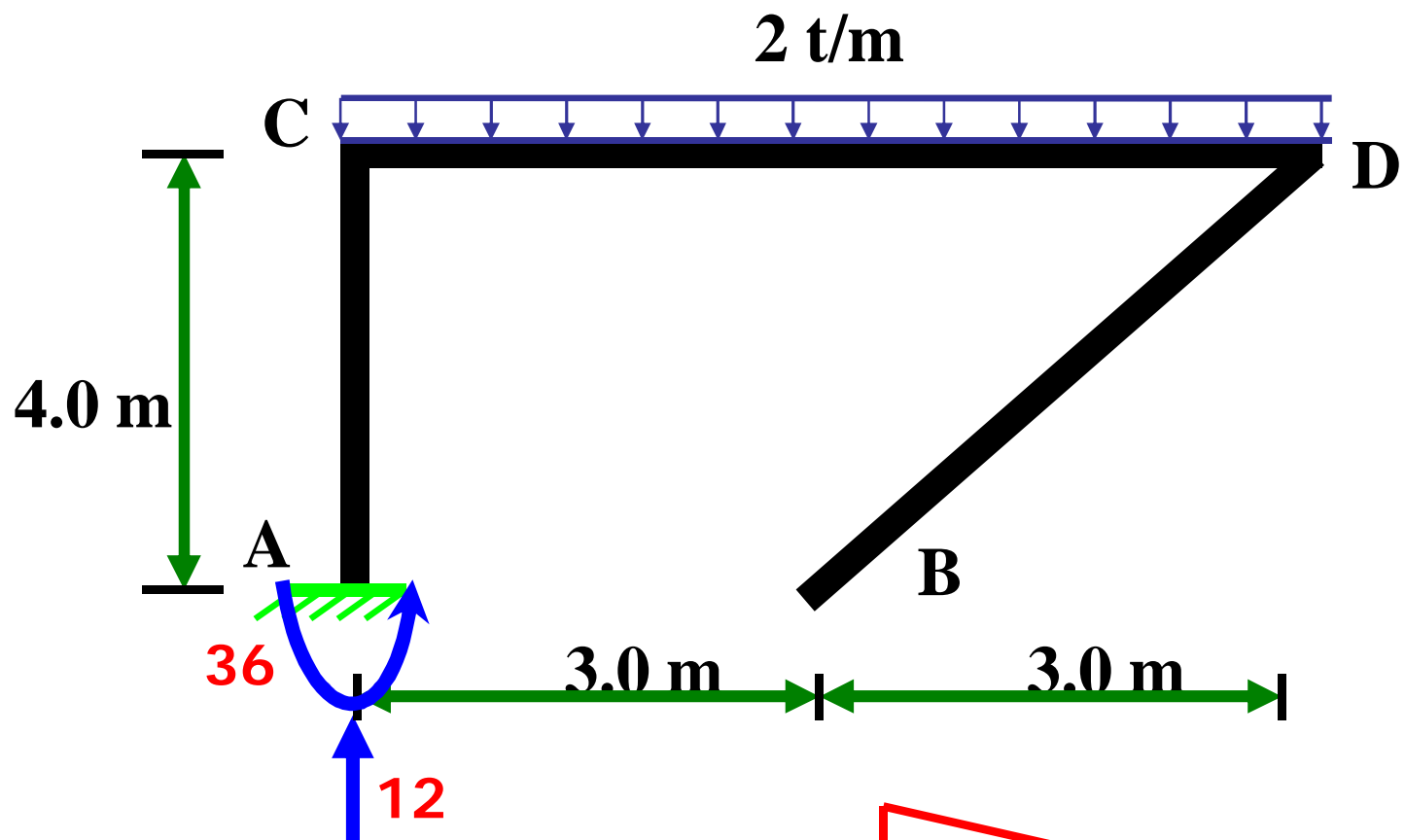


For the Shown Frame, Calculate the Rotation at Point D due to:

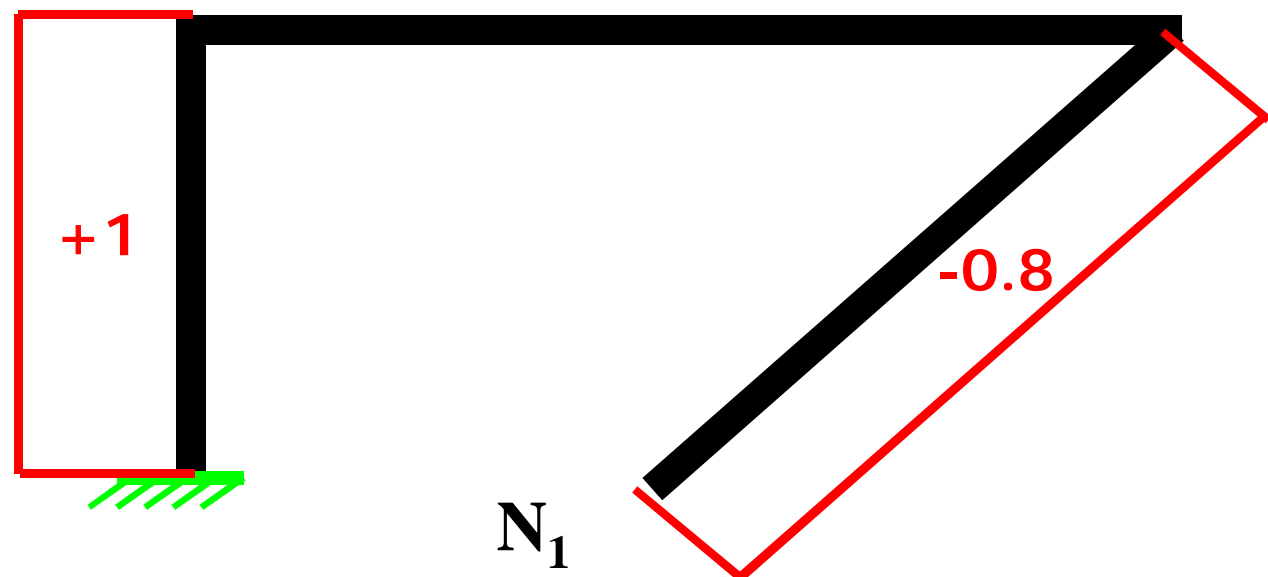
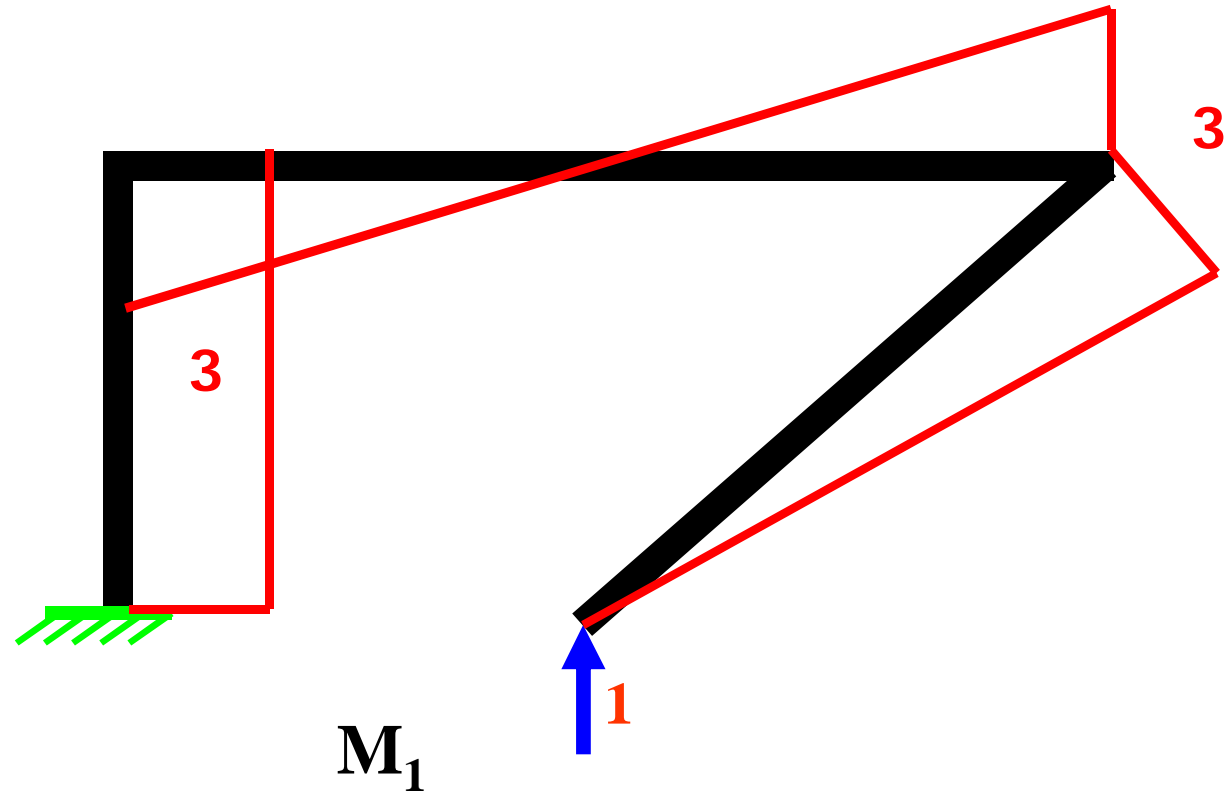
- 1- Given Loads.
- 2- Rotational Settlement at A = 1 rad.
- 3- Rise in Temperature 30° in All Members.

$$EI = 10000 \text{ t.m}^2.$$

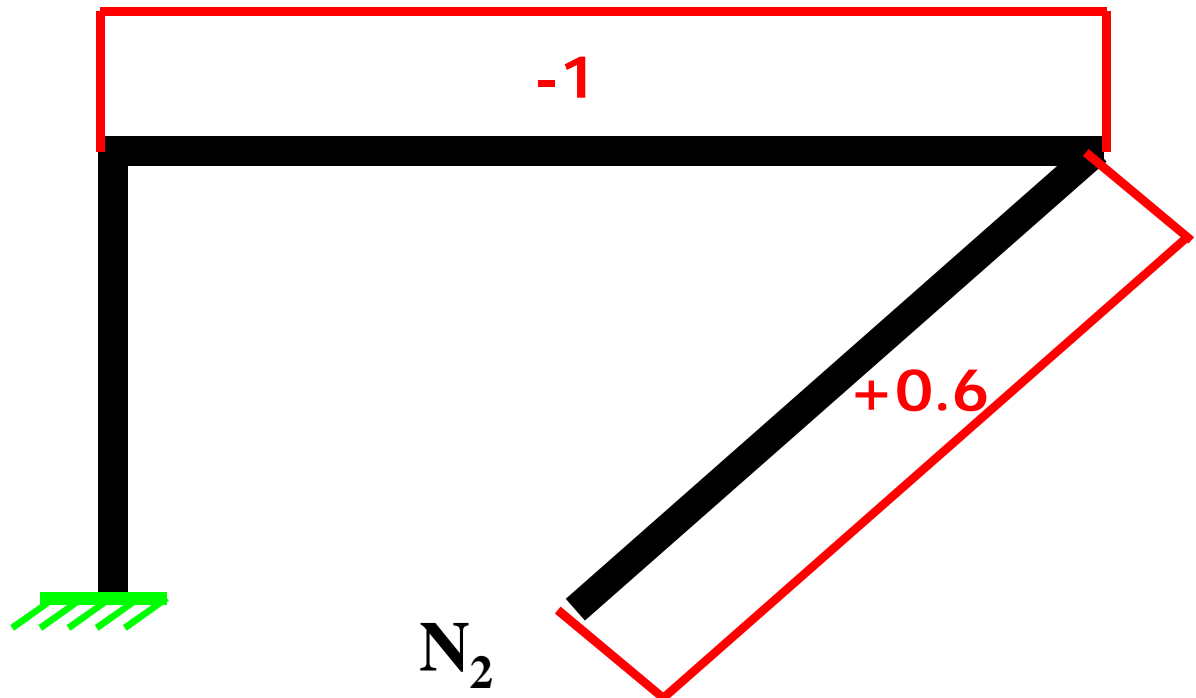
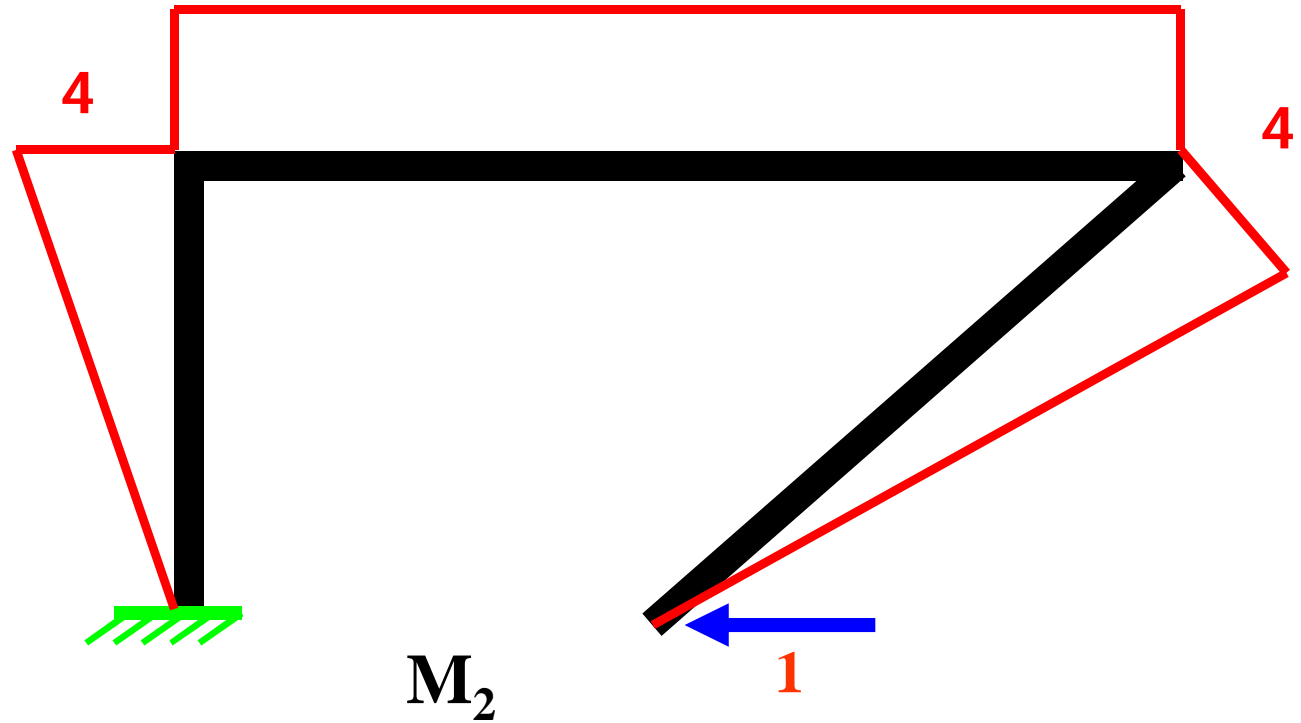
$$\alpha = 10^{-5}.$$



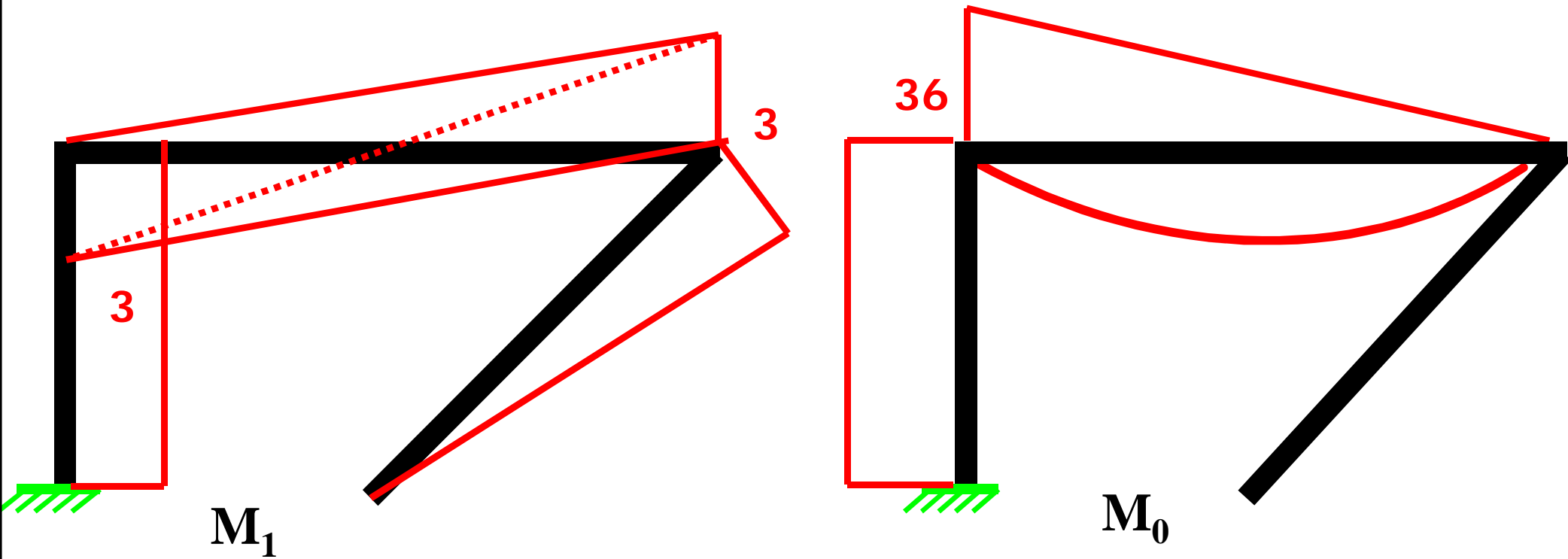
For yb



For xb

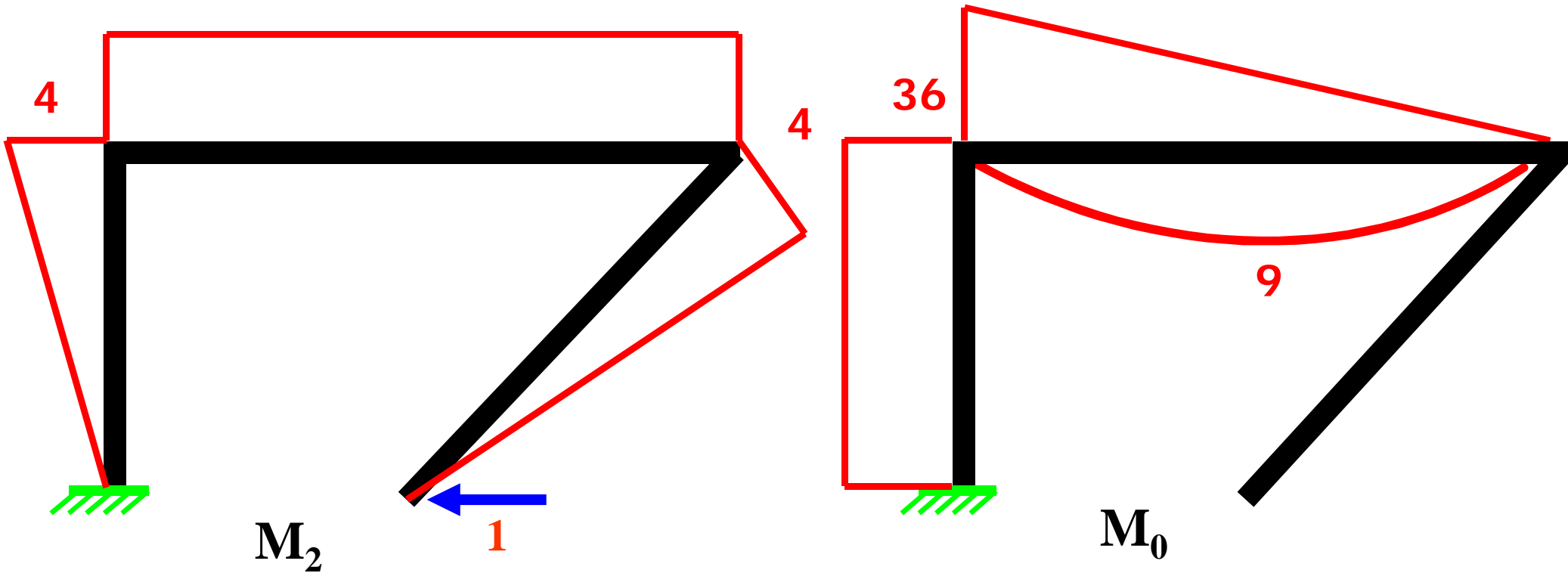


given loads



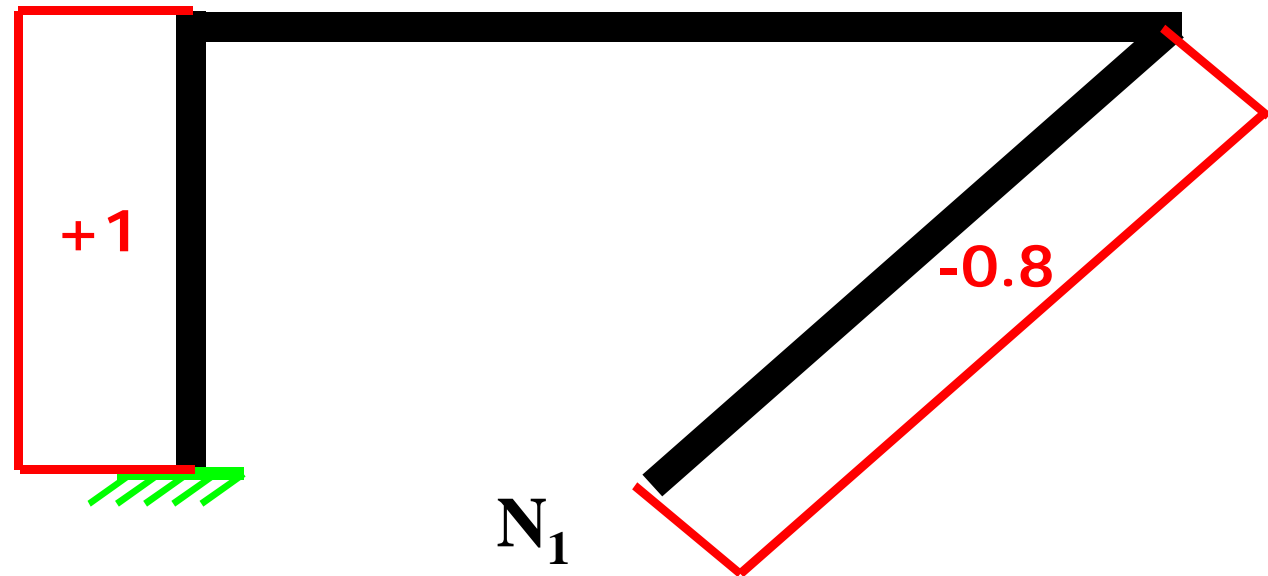
$$y_c = \frac{1}{EI} \left[\frac{36 * 3 * 6}{6} - \frac{36 * 3 * 6}{3} - 36 * 3 * 4 \right]$$

given loads



$$y_c = \frac{1}{EI} \left[\frac{36 * 4 * 6}{2} - \frac{2}{3} * 9 * 6 * 4 + \frac{36 * 4 * 4}{2} \right]$$

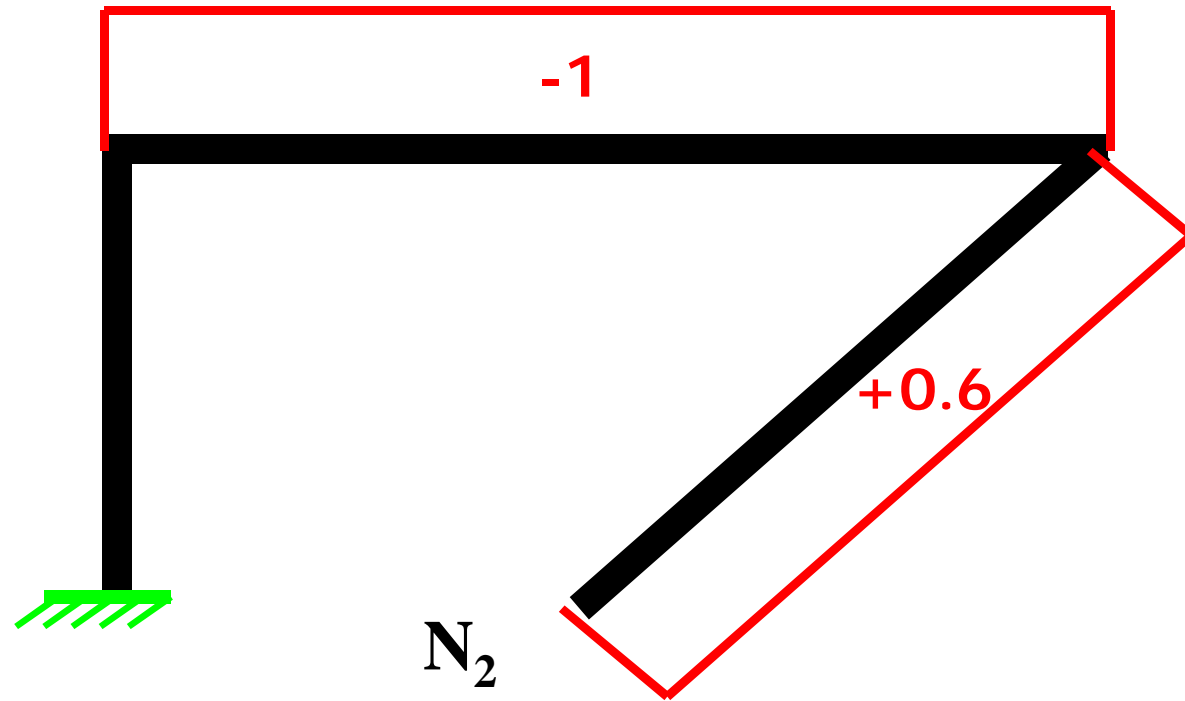
Uniform temperature=30



$$\text{Area of Normal} = -0.8 * 5 + 1 * 4 = 0.0$$

$$yc = \alpha * dt * \text{Area of Normal} = 10^{-5} * 30 * 0 = 0$$

Uniform temperature=30

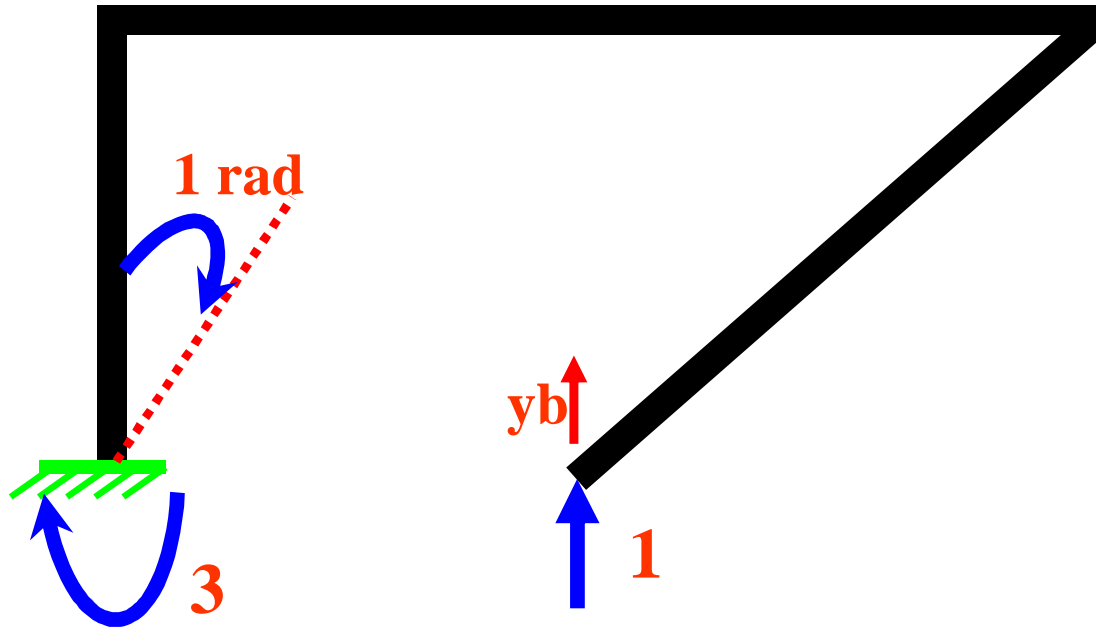


$$\text{Area of Normal} = -0.6 * 5 - 1 * 6 = -3$$

$$hd = \alpha * dt * \text{Area of Normal} = 10^{-5} * 30 * -3 =$$

rotation at b = 1 rad

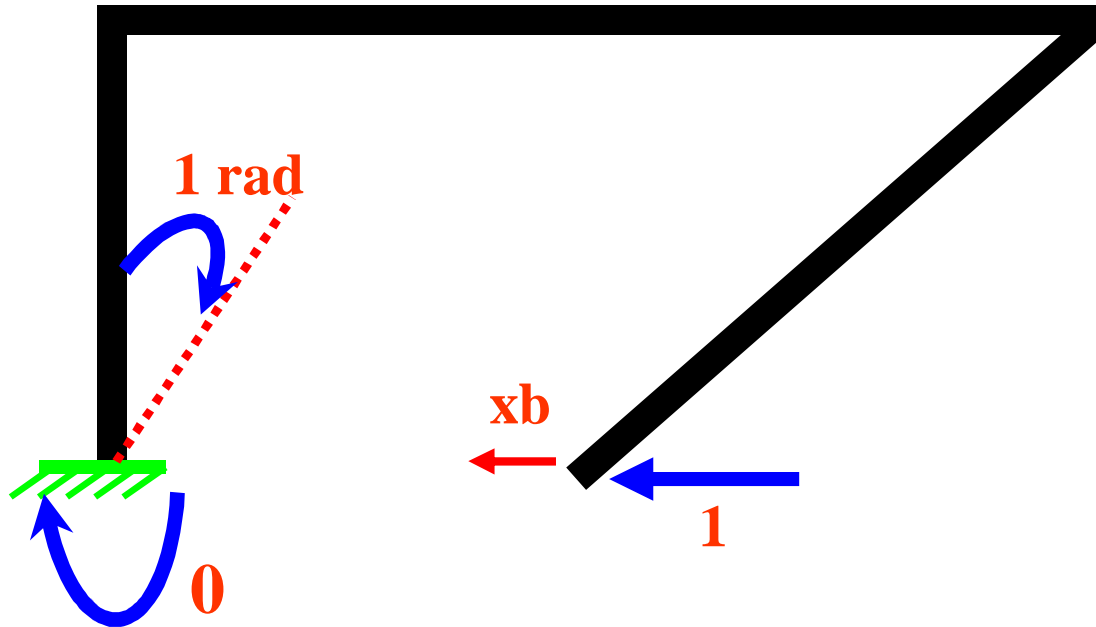
موجب یعنی مع عقارب الساعه



$$1 * y_b + 3 * 1 = 0.0$$

$$y_b = -3cm$$

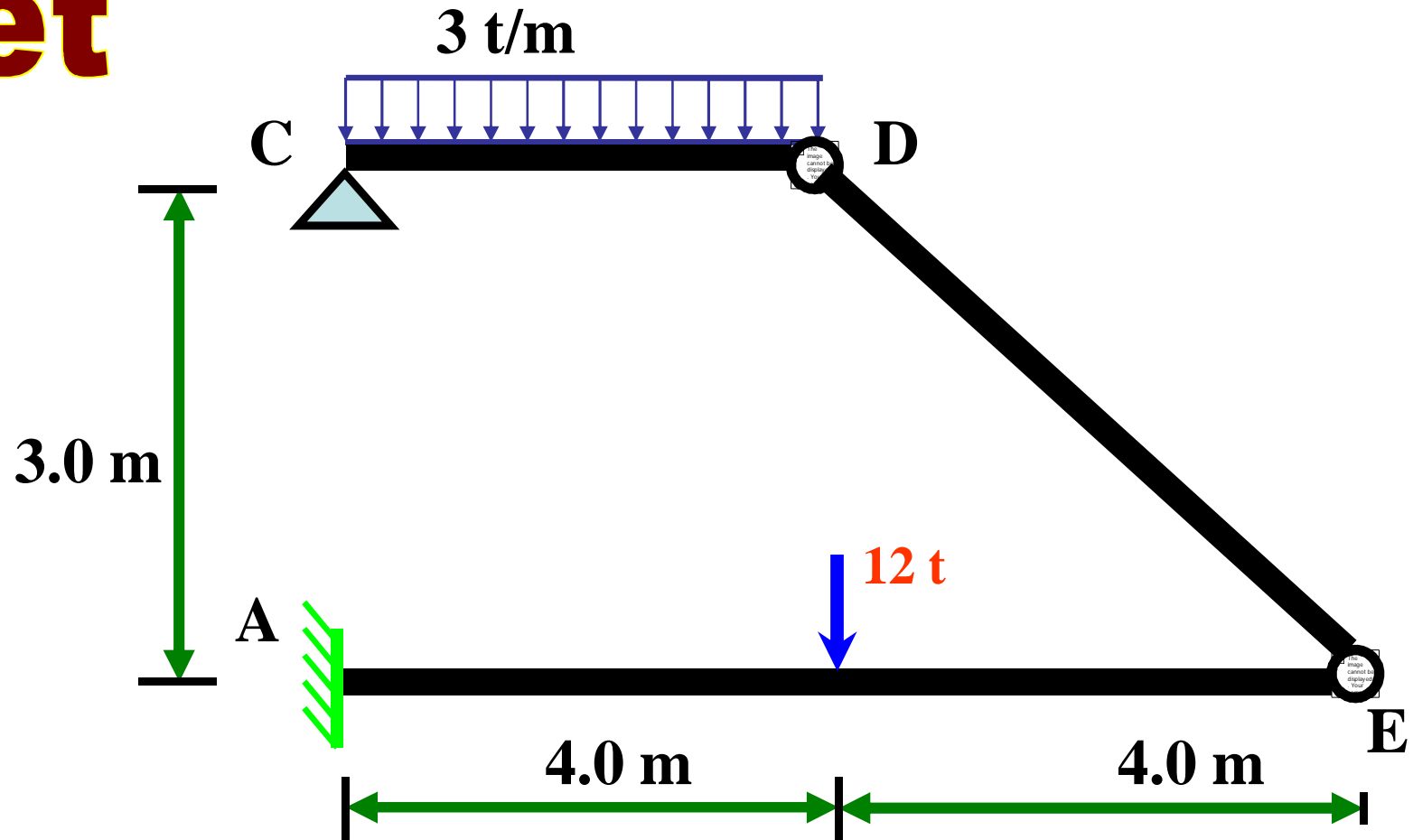
rotation at b = 1 rad



$$1 * xb - 0 * 1 = 0.0$$

$$xb = 0$$

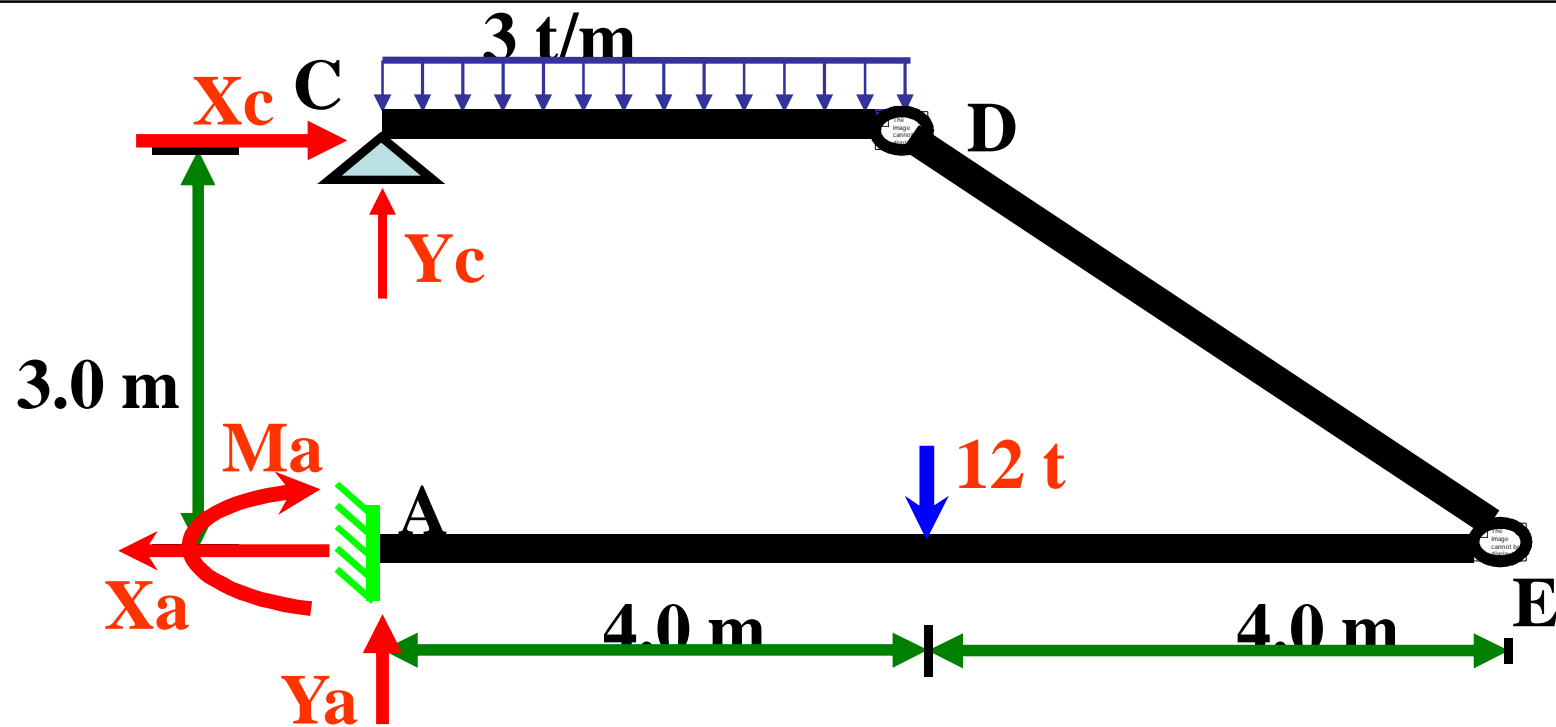
sheet



For the Shown Frame, Calculate:

- 1- horizontal displacement at D.
- 2- vertical displacement at E.
- 3- rotation at C.

$$EI = 10000 \text{ t.m}^2.$$



$$\sum \mathbf{M}_{DR} = 0.0$$

$$Y_c * 4 = 12 * 2 \Rightarrow Y_c = 6.0$$

$$\sum \mathbf{M}_{E \text{ up}} = 0.0$$

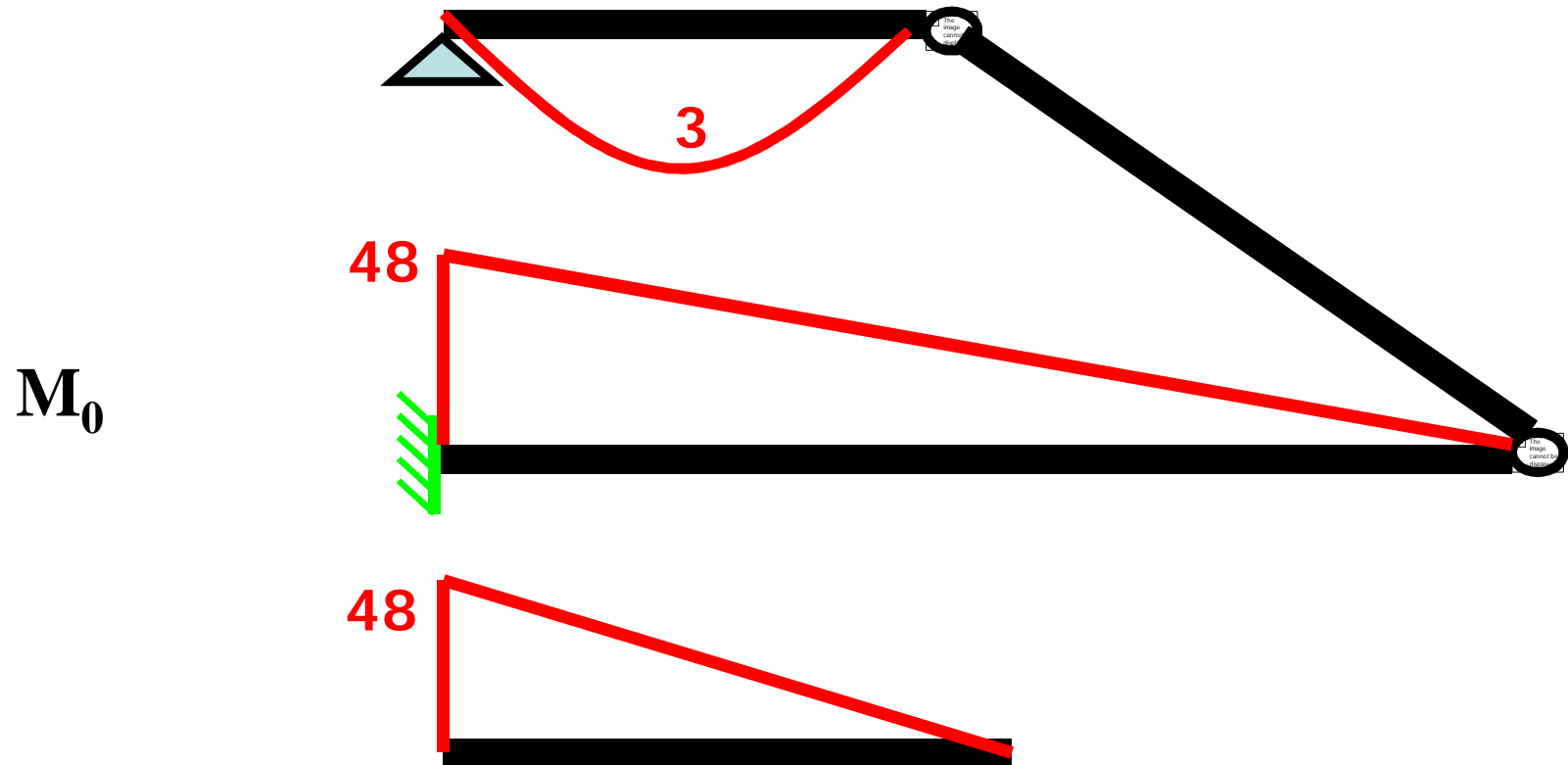
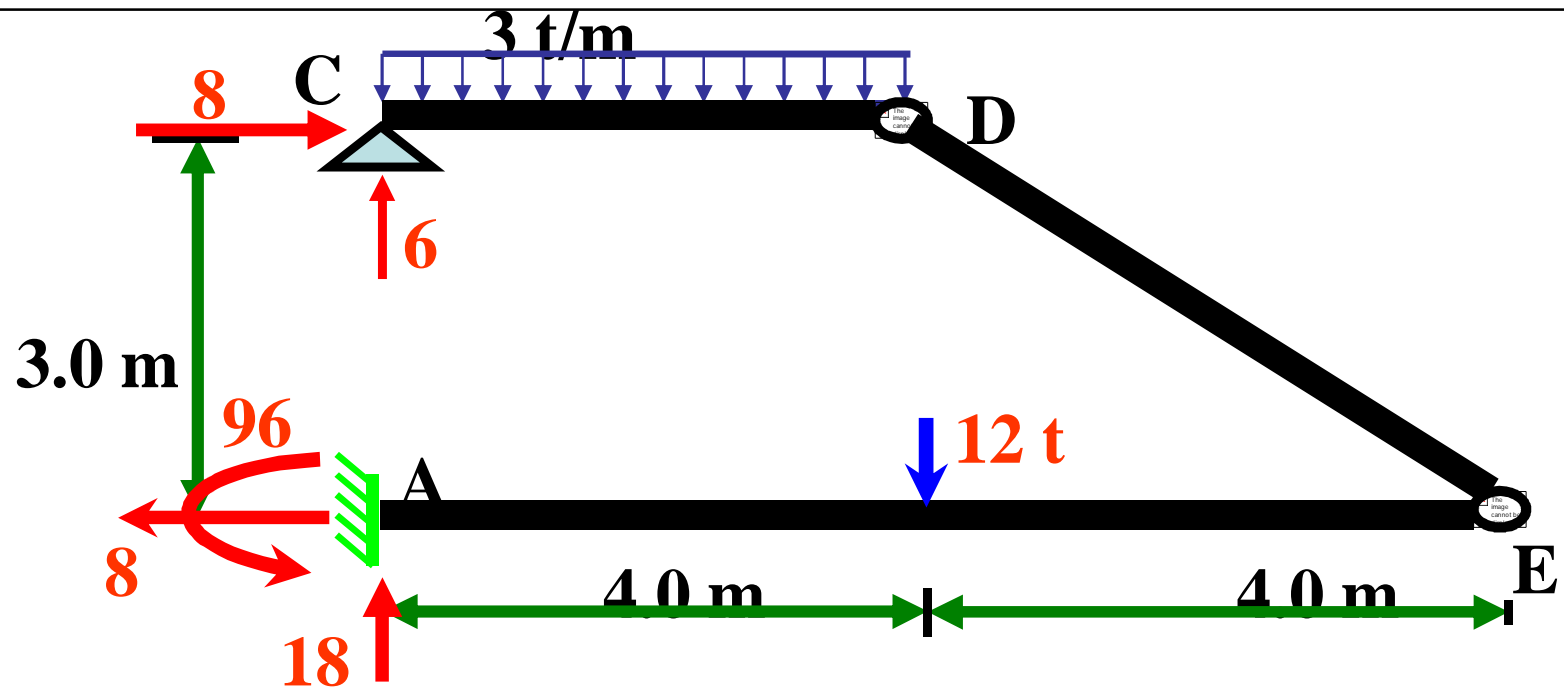
$$6 * 8 + X_c * 3 - 12 * 6 = 0 \Rightarrow X_c = 8.0$$

$$\sum \mathbf{F}_x = 0.0$$

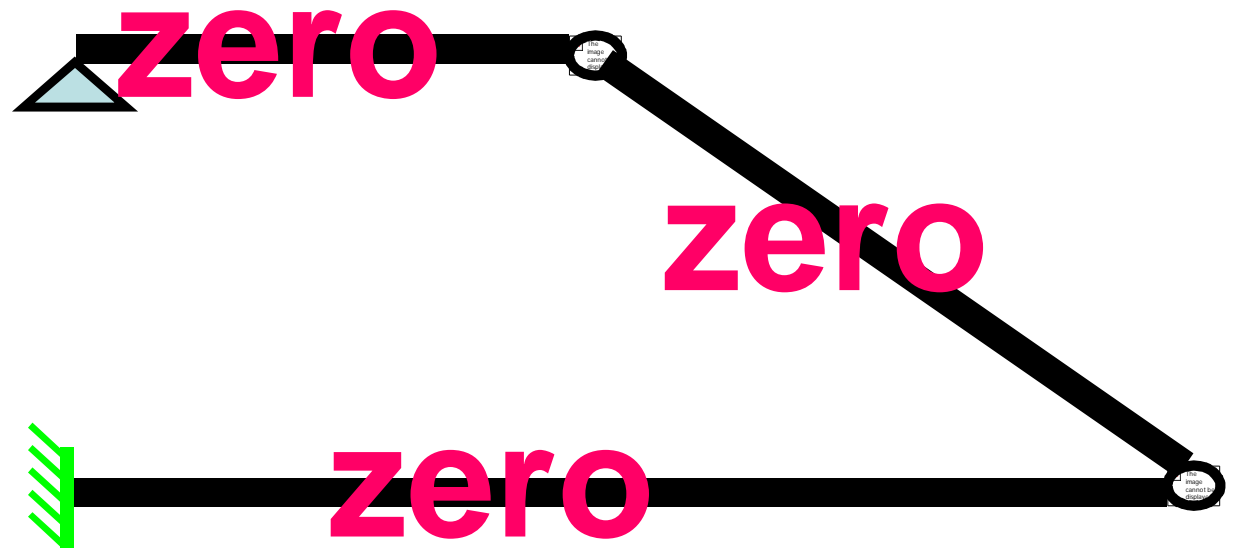
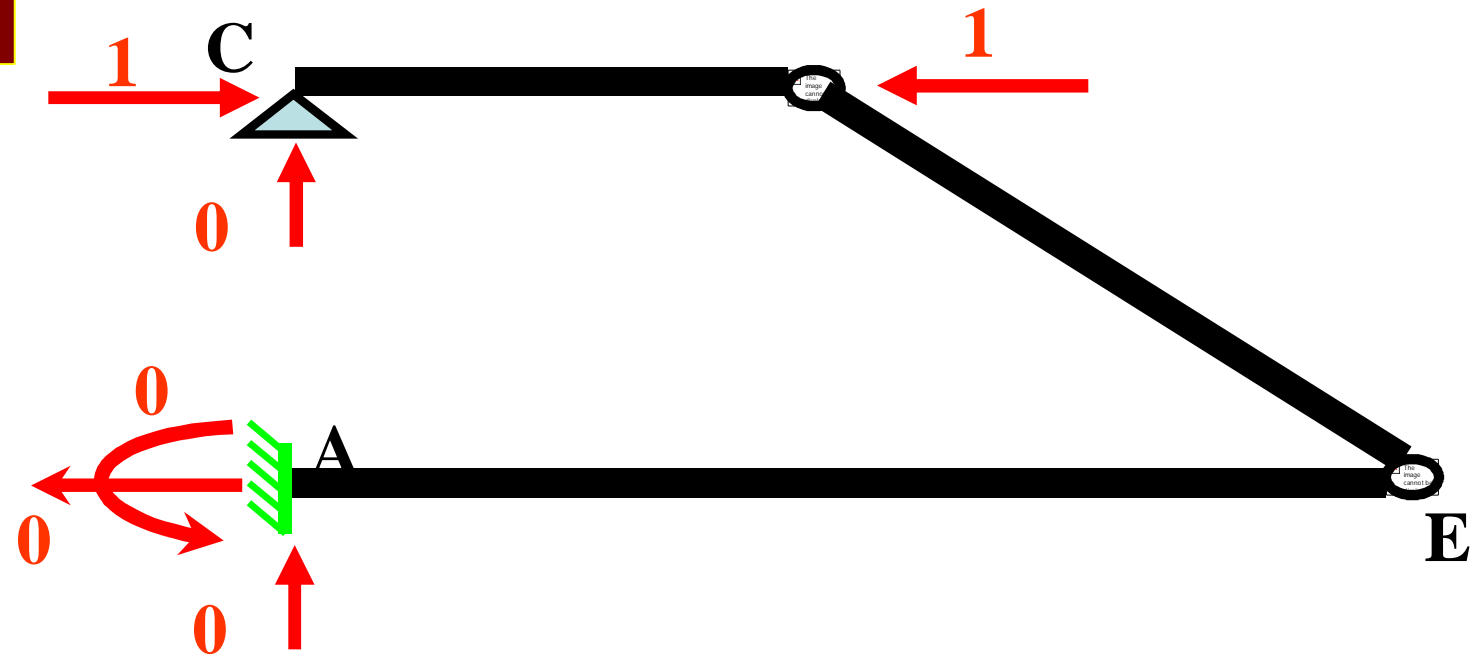
$$X_a = 8.0$$

$$\sum \mathbf{M}_A = 0.0$$

$$Ma - 8 * 3 - 12 * 2 - 12 * 4 = 0.0 \Rightarrow Ma = 96$$

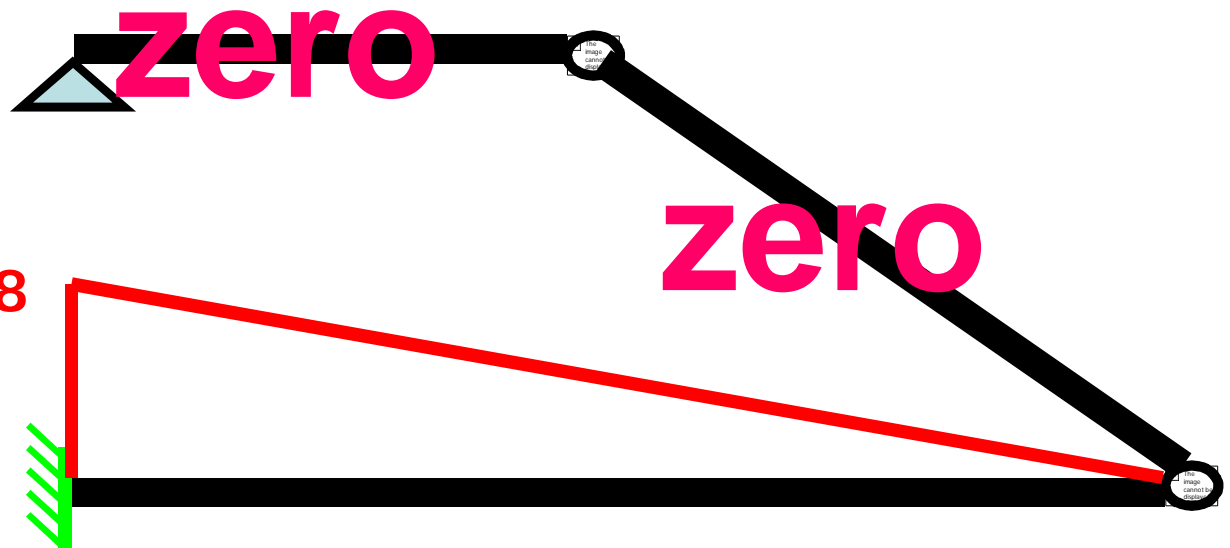
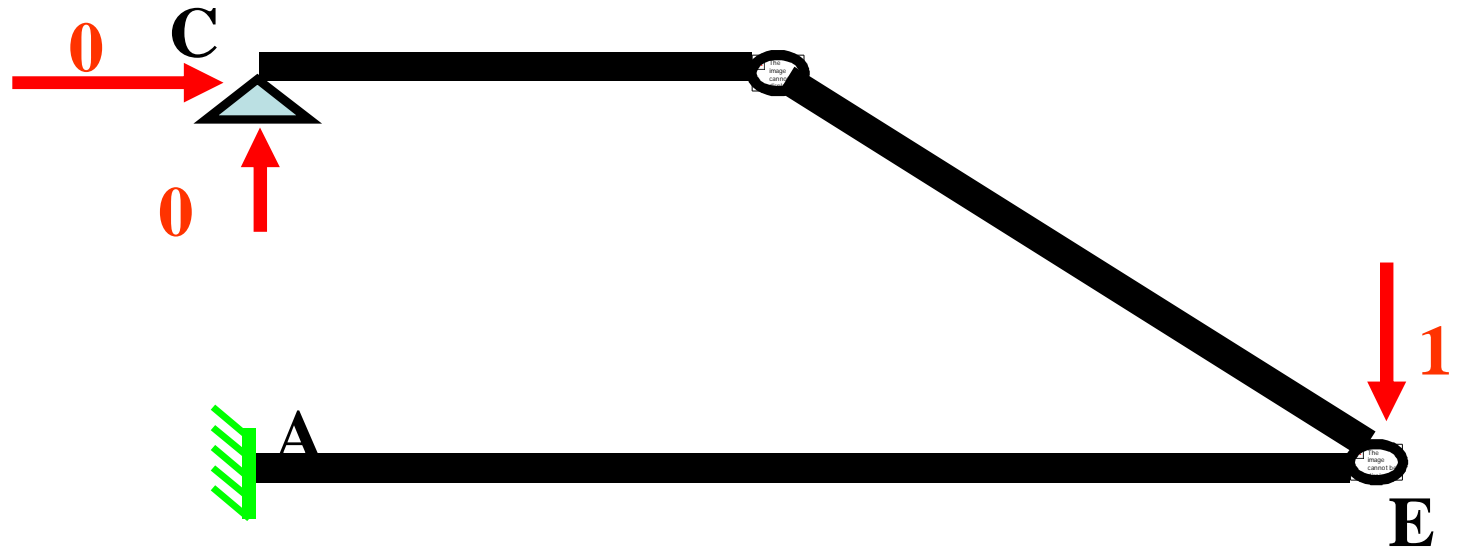


For hd

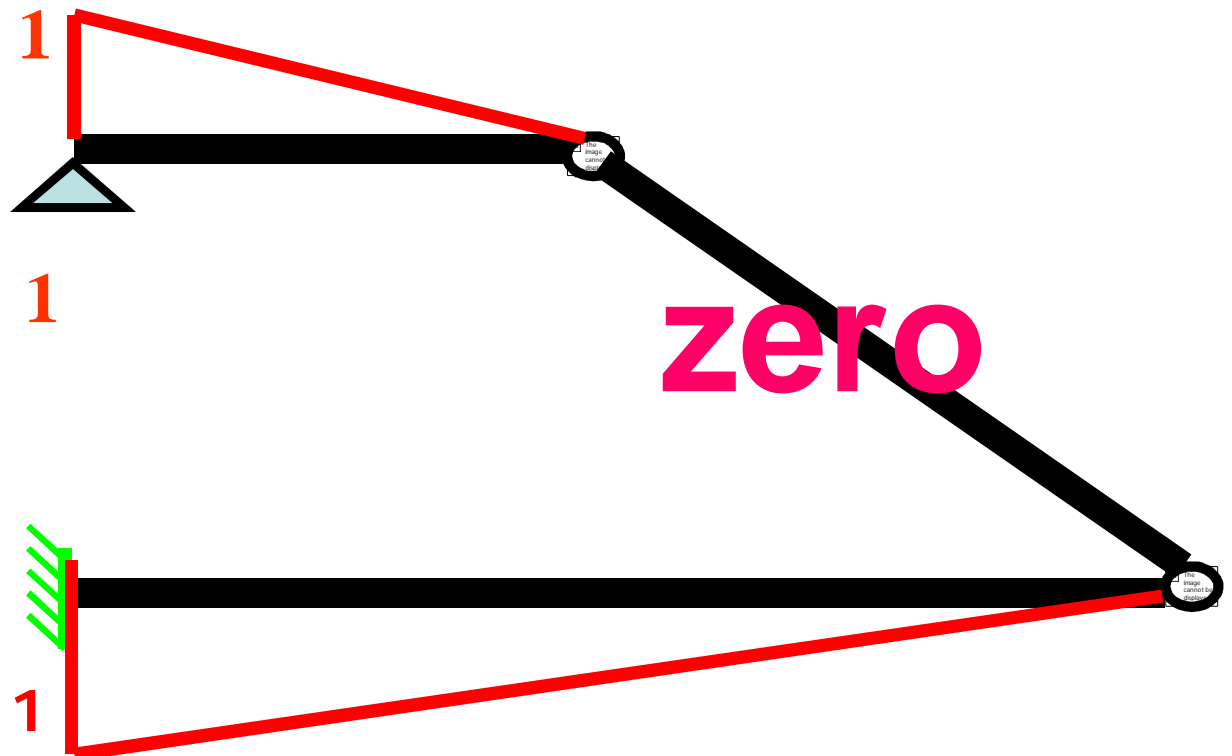
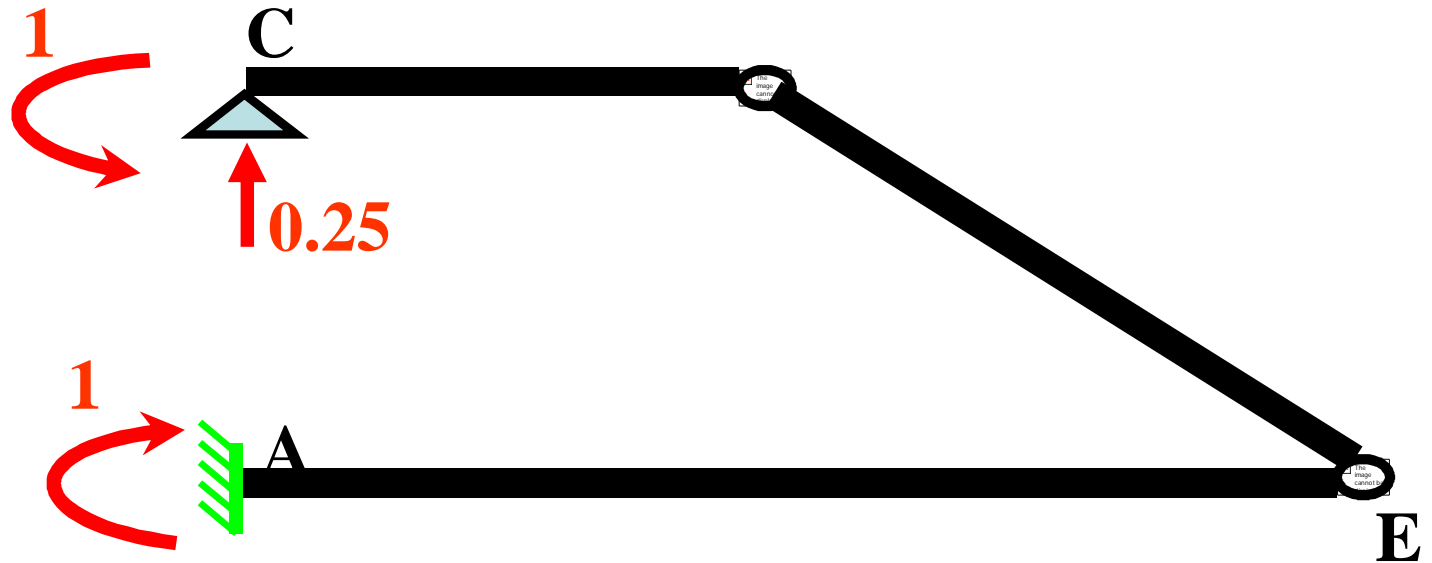


M_1

For ye



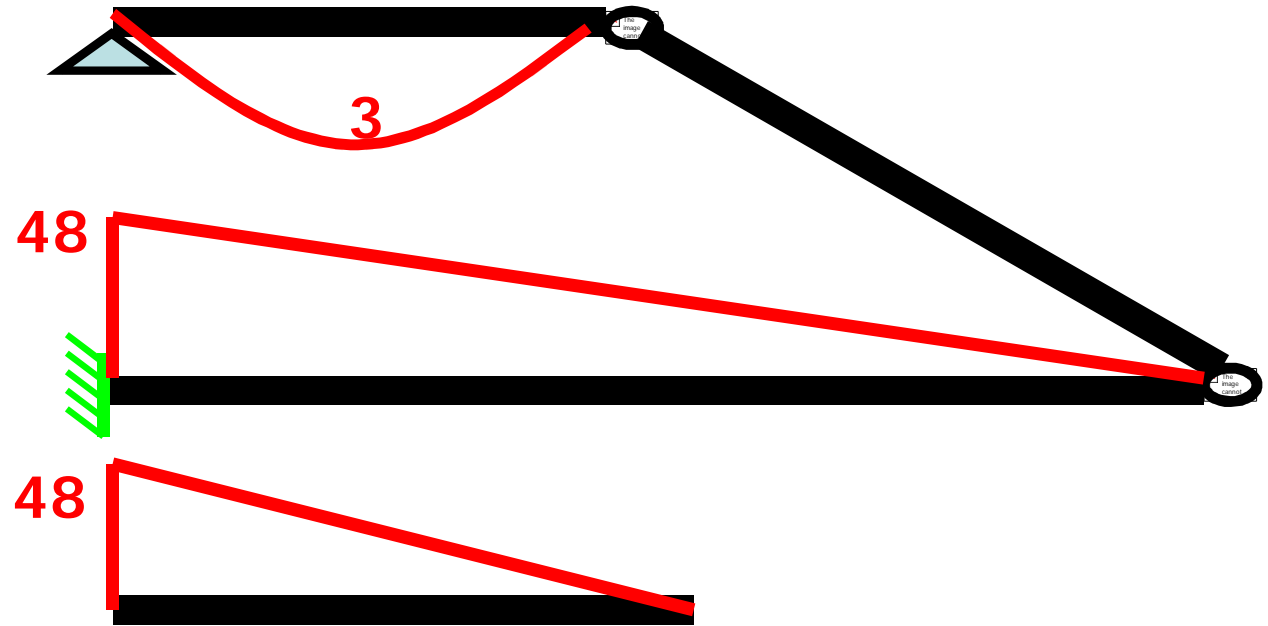
For θ_c



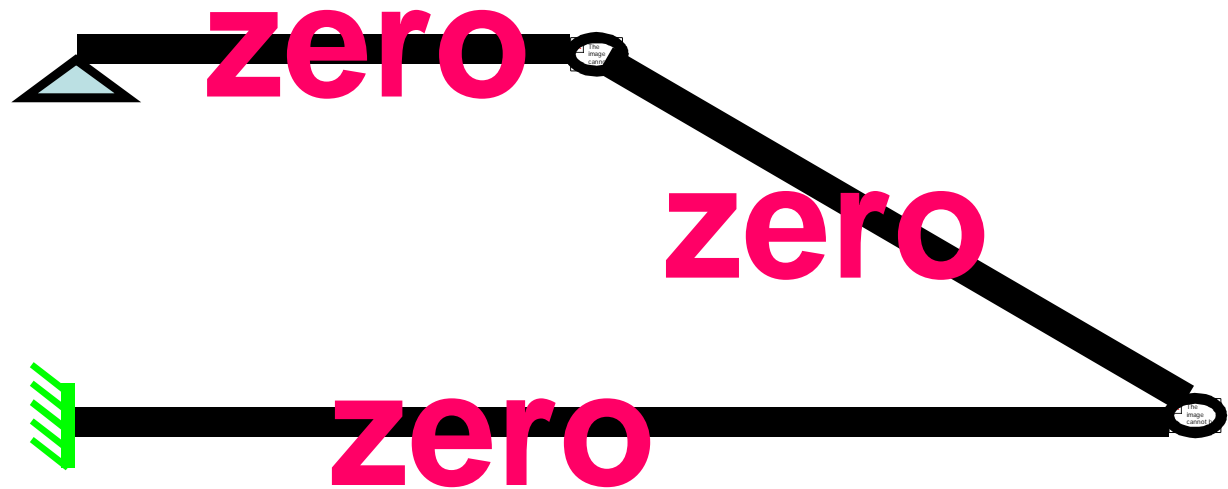
M_3

zero

M_0

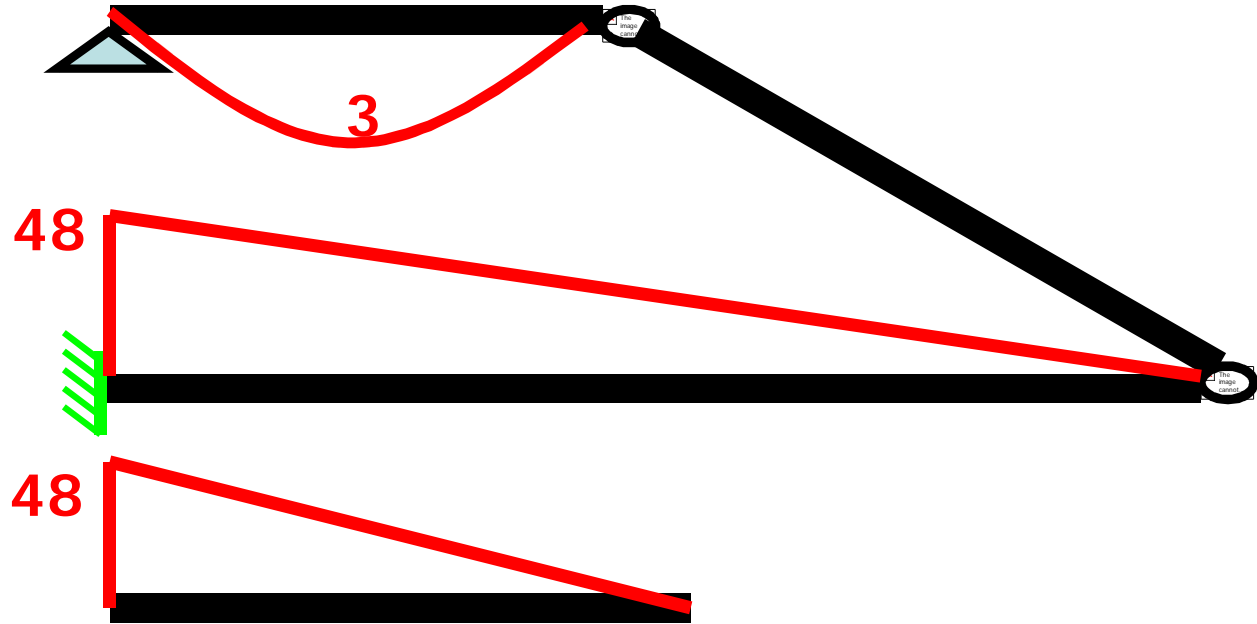


M_1

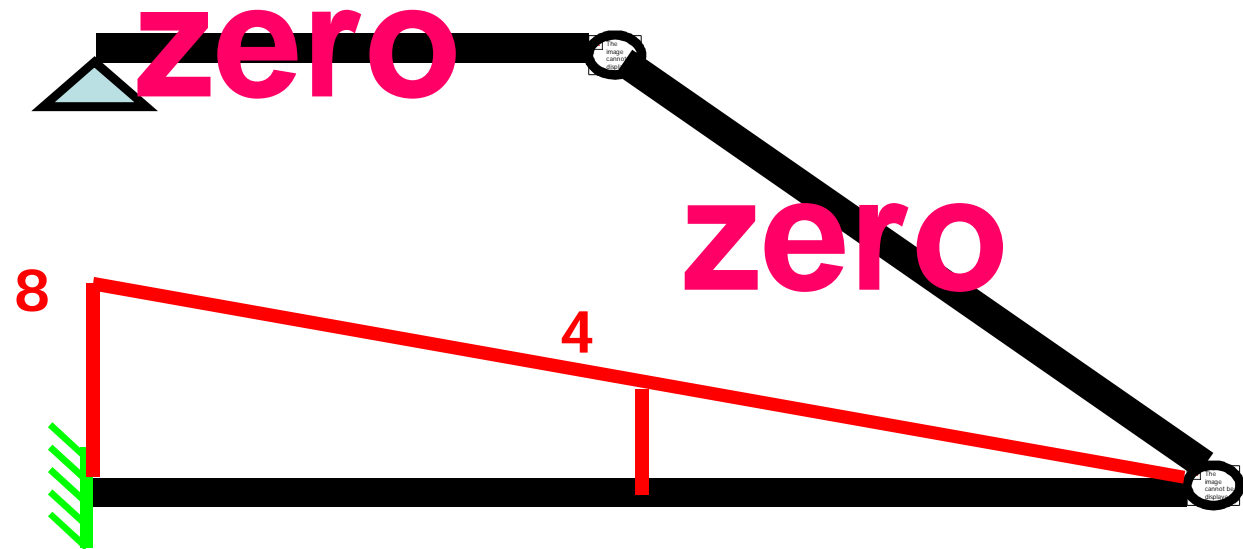


$$hd = 0.0$$

M_0

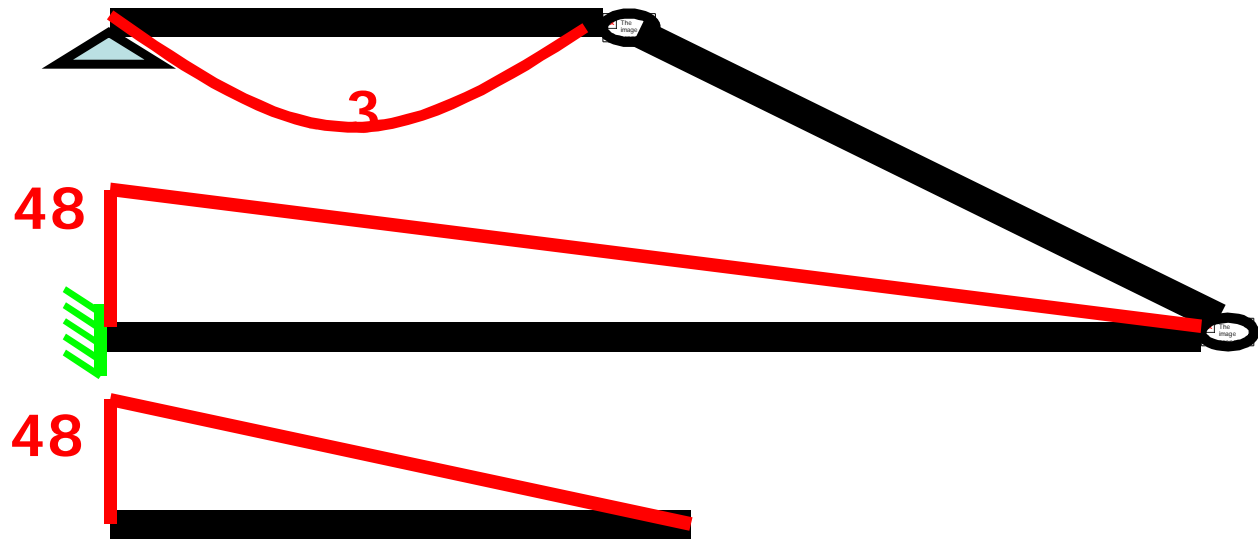


M_2

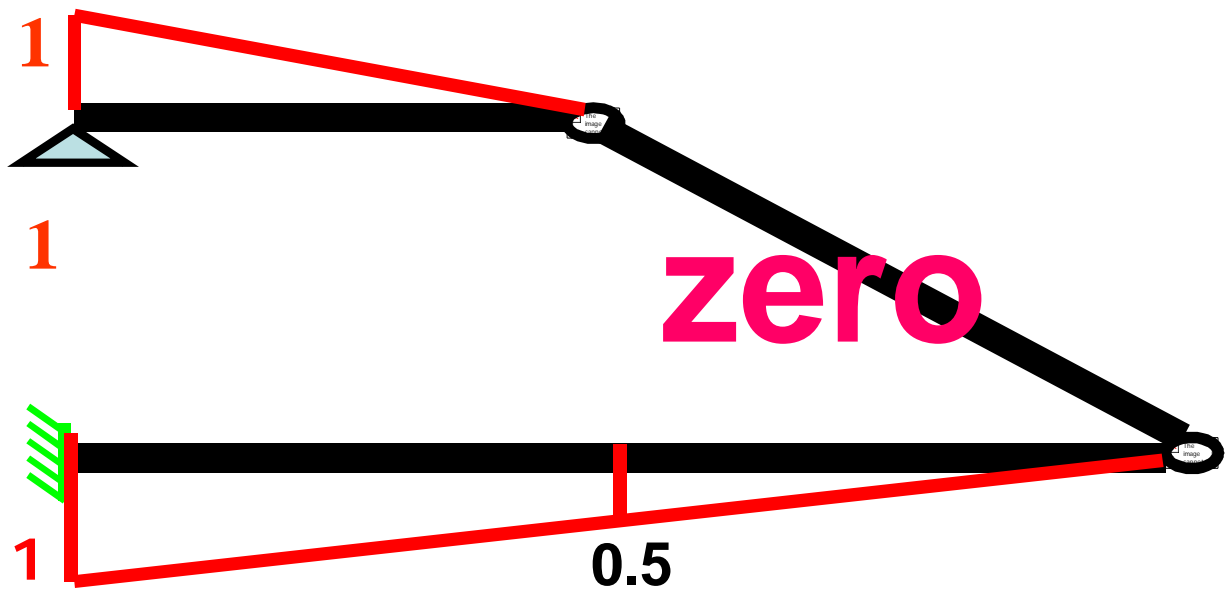


$$y_e = \frac{1}{EI} \left[\frac{48 * 8 * 8}{3} + \frac{4}{3} (48 * 8 + 0.5(48 * 4)) \right] =$$

M_0



M_3



$$\theta_c = \frac{1}{EI} \left[\frac{-48 \cdot 1 \cdot 8}{3} - \frac{4}{3} (48 \cdot 1 + 0.5(48 \cdot 0.5)) - 2/3 \cdot 3 \cdot 4 \cdot 0.5 \right] =$$

VERTUAL WORK

TRUSSES

خطوات الحل:

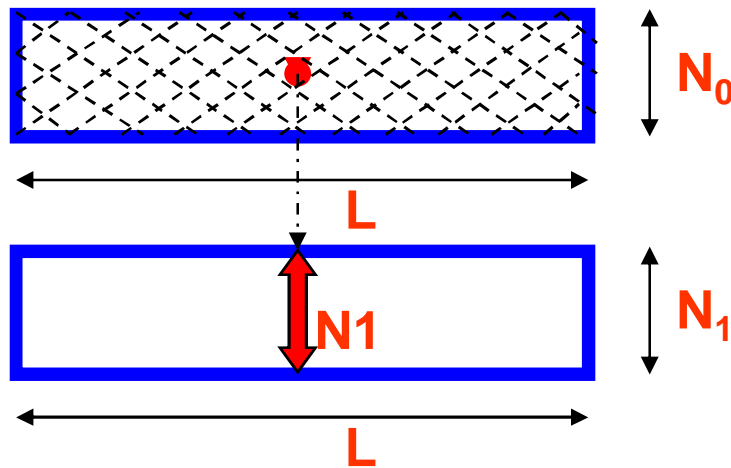
1- يتم رسم الـ normal على الـ truss المعطى.

Find N_0

2- يتم إزالة الاحمال وضع حمل 1ton ورسم الـ normal على الـ truss.

Find N_d

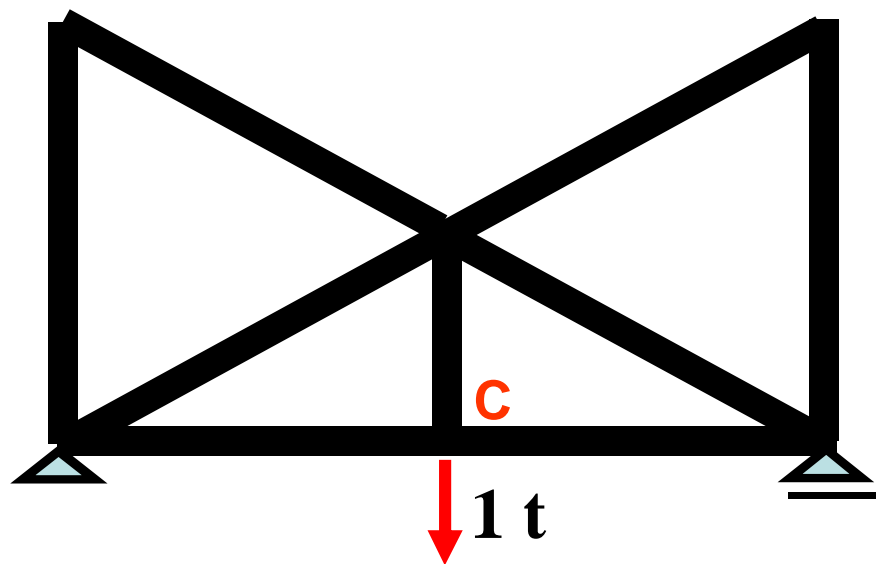
3- يتم تكامل الشكل الاساسى بالخطوه 1 والشكل المرسوم للمطلوب الاول بالخطوه 2 والتكامل هنا سيكون تكامل مساحات:



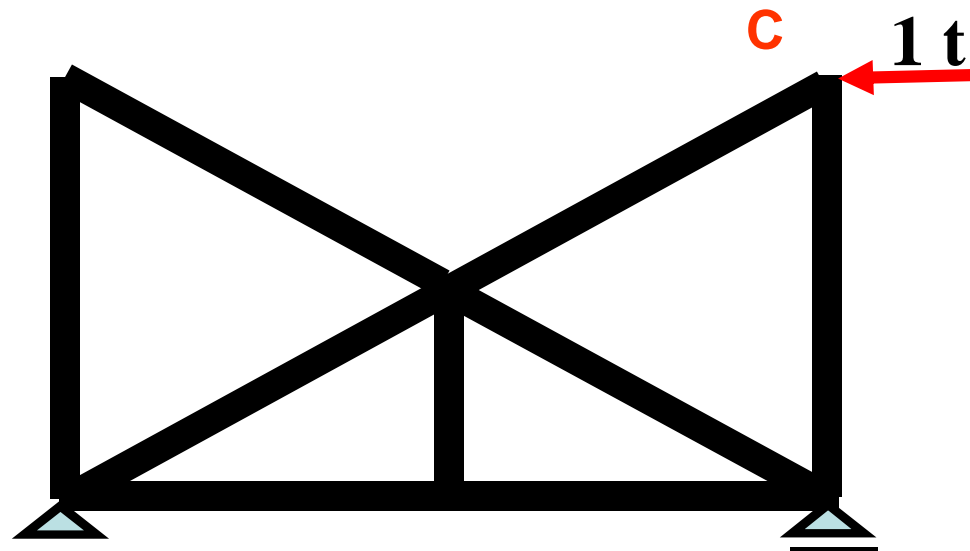
كل رسومات الـ normal فى الـ truss
تكون عبارة عن مستطيلات

$$\omega_{\text{internal}} = \int \frac{N_0 \cdot N_1}{EA} \cdot dl = \frac{\sum (N_0 \cdot N_1 \cdot L)}{EA}$$

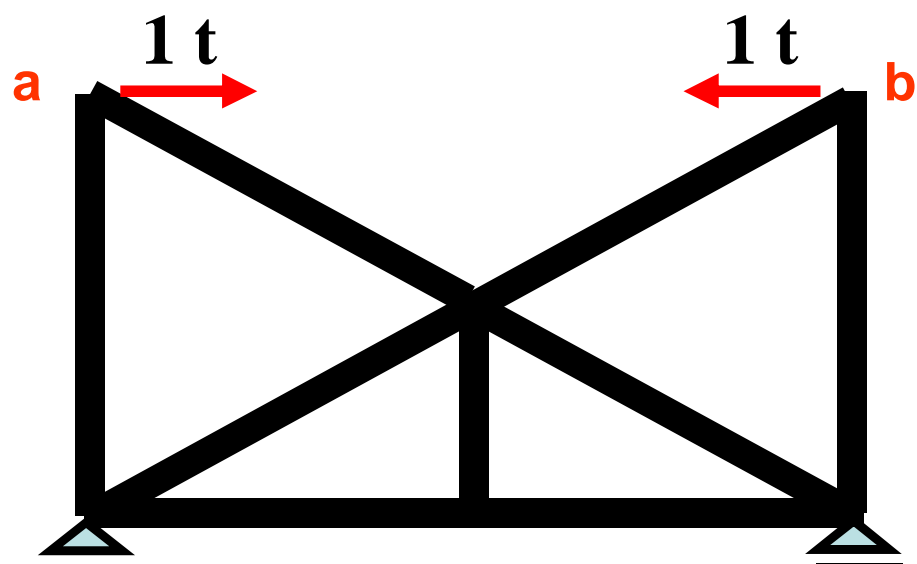
for δvc



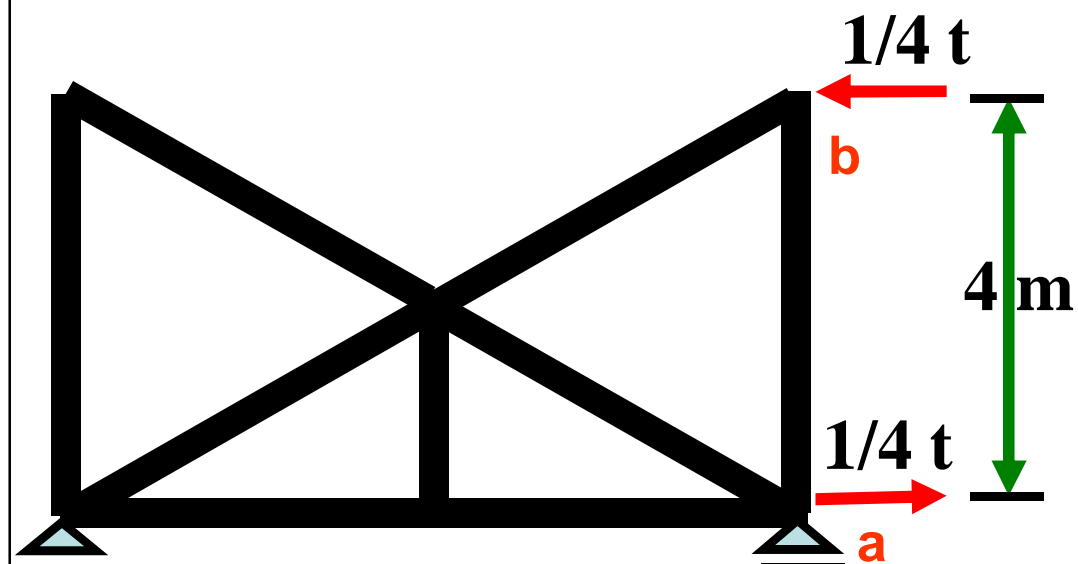
for δhc



for $rel(ab)$



for θab



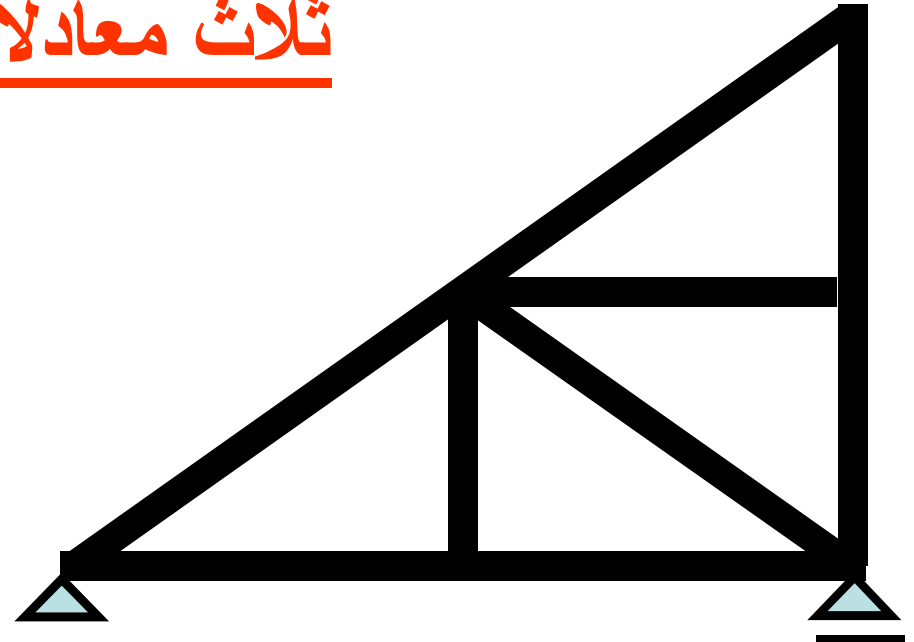
مراجعة سريعة على بعض
المصطلحات من المعلومات السابقة

ثلاث معادلات وثلاث مجاهيل

$$\sum \mathbf{F}_x = 0.0$$

$$\sum \mathbf{F}_y = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$



خمس معادلات وخمس مجاهيل

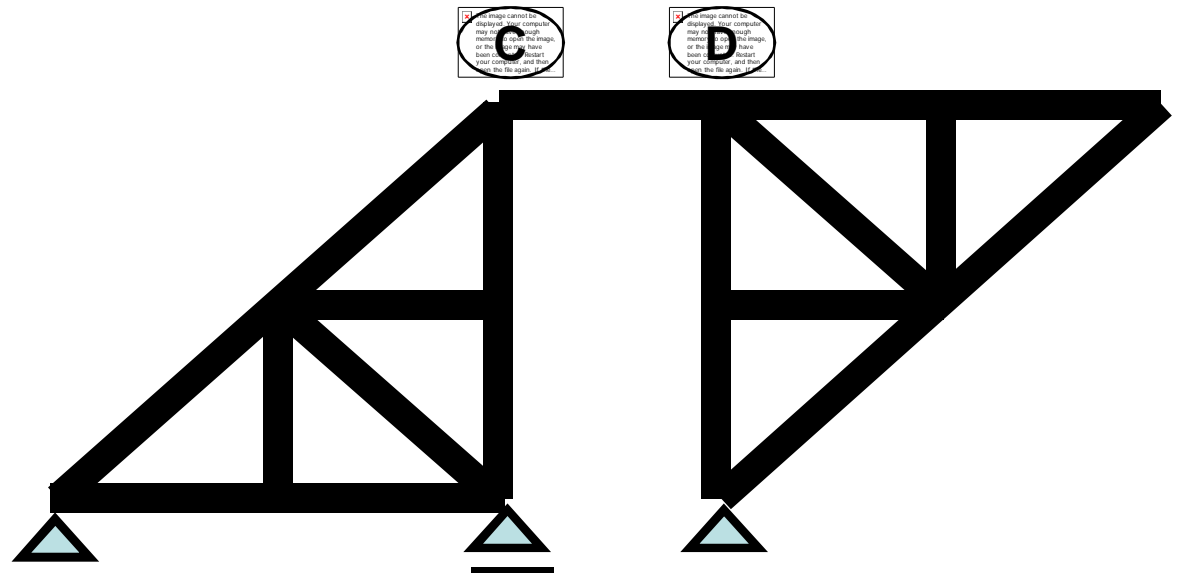
$$\sum \mathbf{F}_x = 0.0$$

$$\sum \mathbf{F}_y = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$

$$\sum \mathbf{M}_{CR} = 0.0$$

$$\sum \mathbf{M}_{DR} = 0.0$$



خمس معادلات وخمس مجاهيل

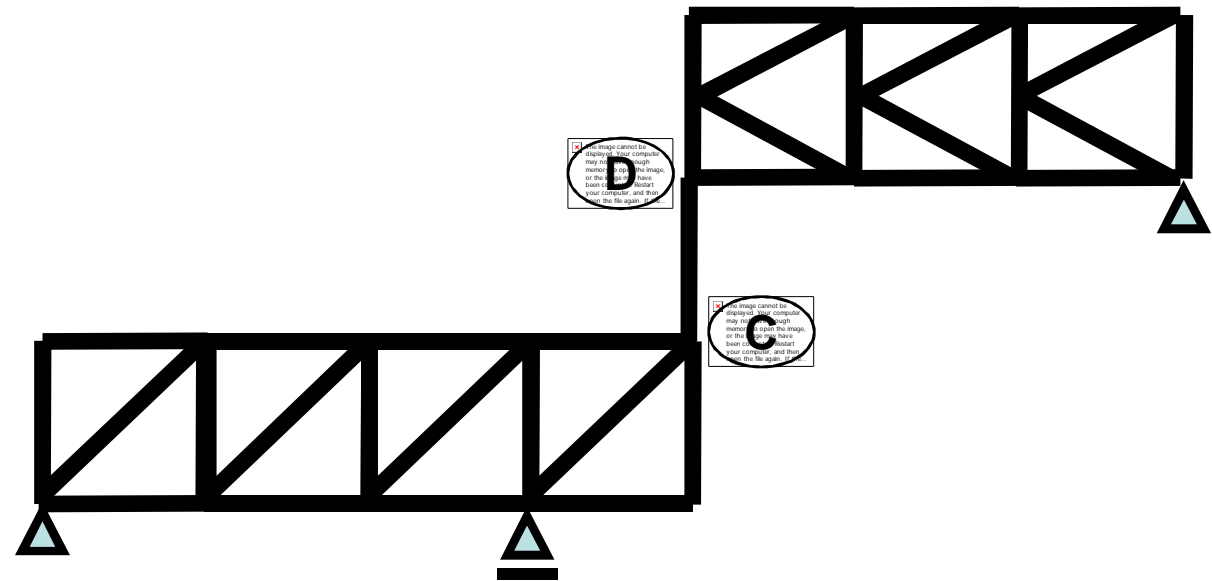
$$\sum \mathbf{F}_x = 0.0$$

$$\sum \mathbf{F}_y = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$

$$\sum \mathbf{M}_{CR} = 0.0$$

$$\sum \mathbf{M}_{DR} = 0.0$$



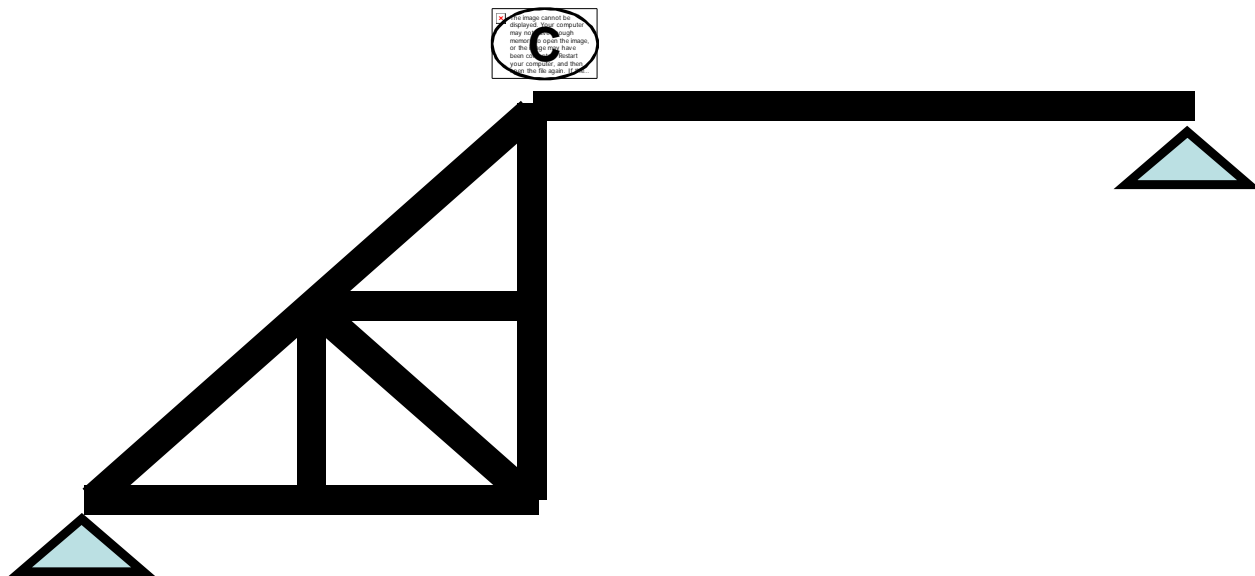
اربع معادلات واربع مجاهيل

$$\sum \mathbf{F}_x = 0.0$$

$$\sum \mathbf{F}_y = 0.0$$

$$\sum \mathbf{M}_A = 0.0$$

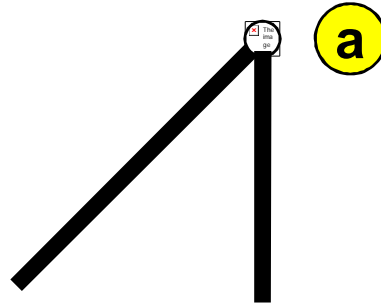
$$\sum \mathbf{M}_{CR} = 0.0$$



Zero members

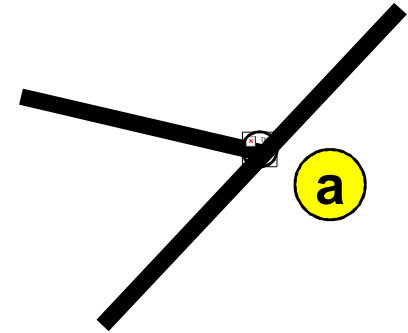
• Joint

- No loads
- Only two members
- The two members are zeros



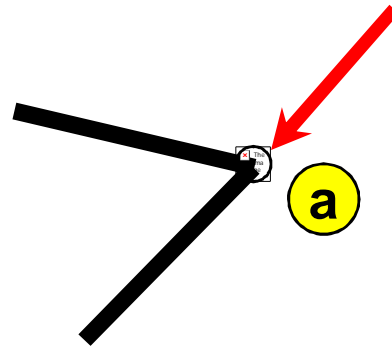
• Joint

- No loads
- عنصرين في اتجاه واحد والثالث مخالف
- $0.0 =$ العنصر الثالث

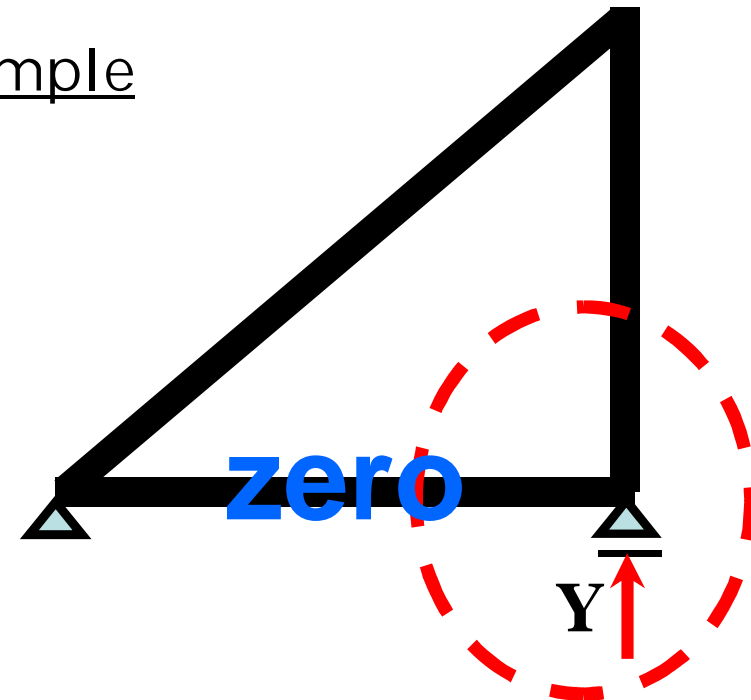


• Joint

- قوة باستقامة عنصر والثالث مخالف
- $0.0 =$ العنصر الثالث



• Example

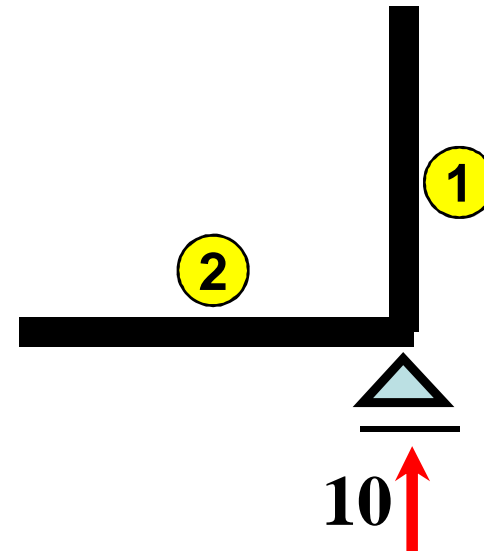


Some famous joints

Joint one

$$F_1 = 10 \quad \downarrow$$

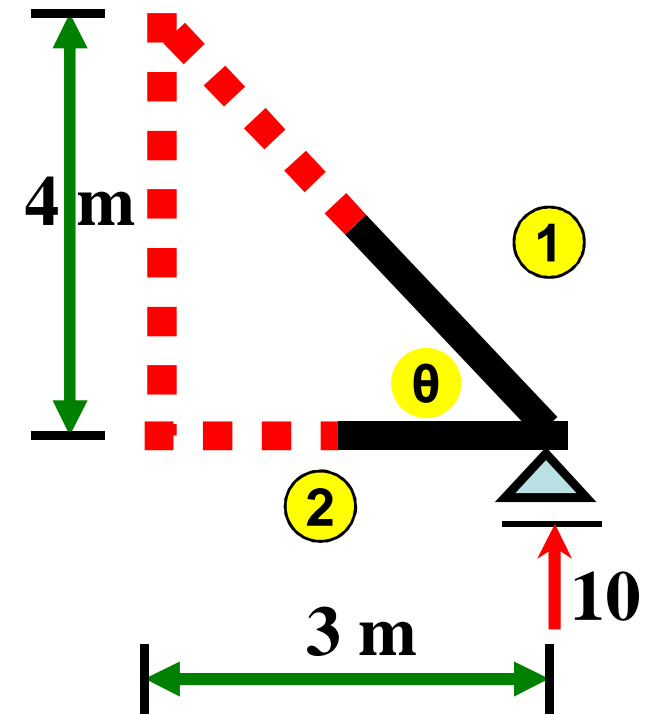
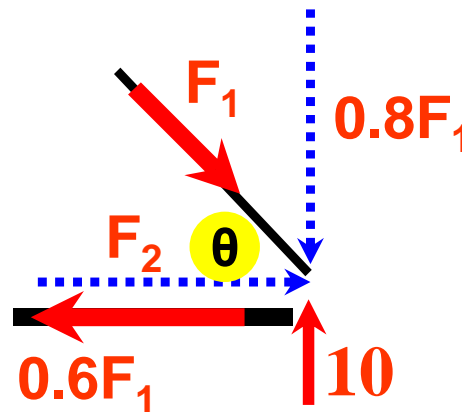
$$F_2 = 0.0$$




Joint two

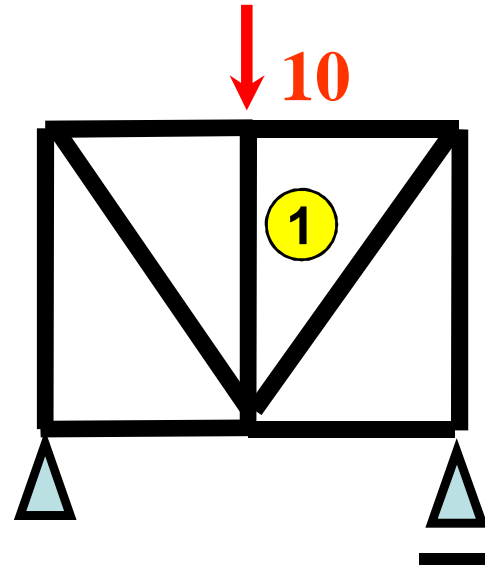
$$\begin{aligned} \sum F_y &= 0 \\ 0.8F_1 - 10 &= 0 \\ F_1 &= 12.5 \end{aligned}$$

$$\begin{aligned} \sum F_x &= 0 \\ 12.5 * 0.6 - F_2 &= 0 \\ F_1 &= 7.5 \end{aligned}$$





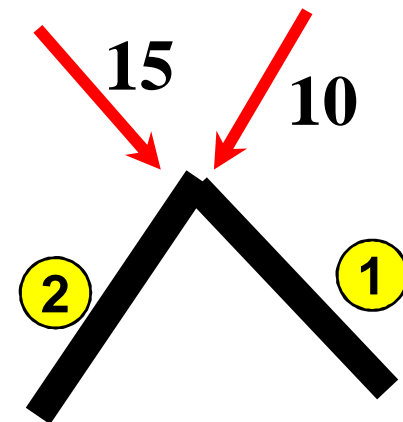
Joint three

$F1 = 15$ 

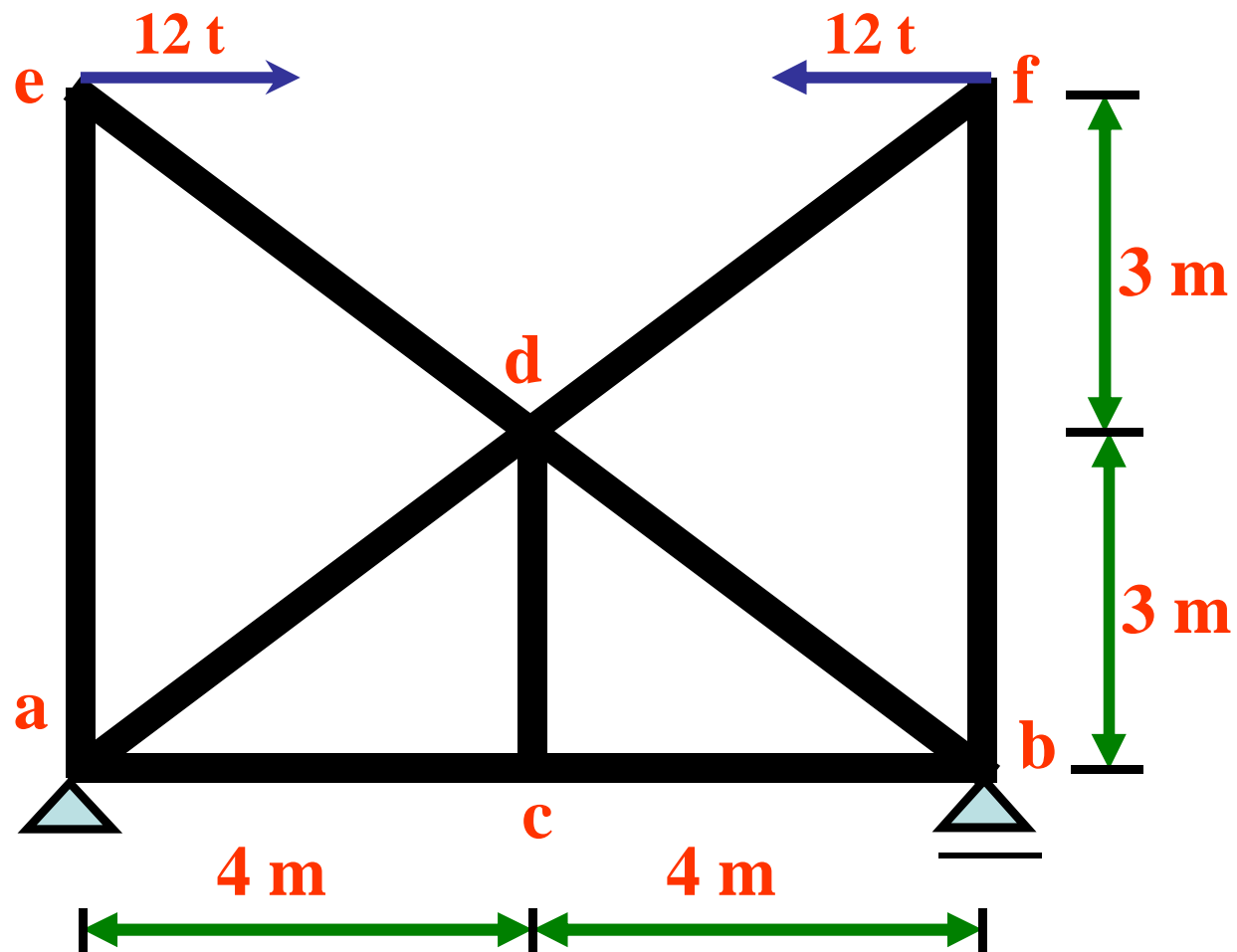


Joint four

$F1 = 15$ 
 $F2 = 10$ 



Example One

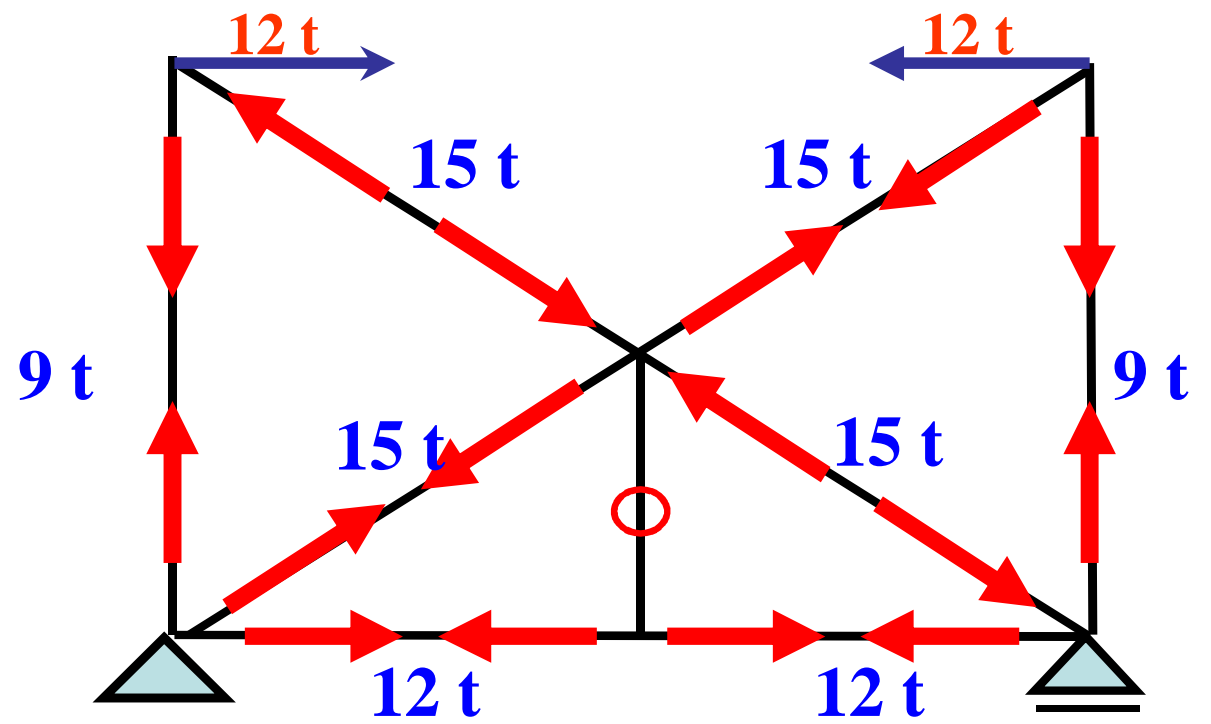


Required

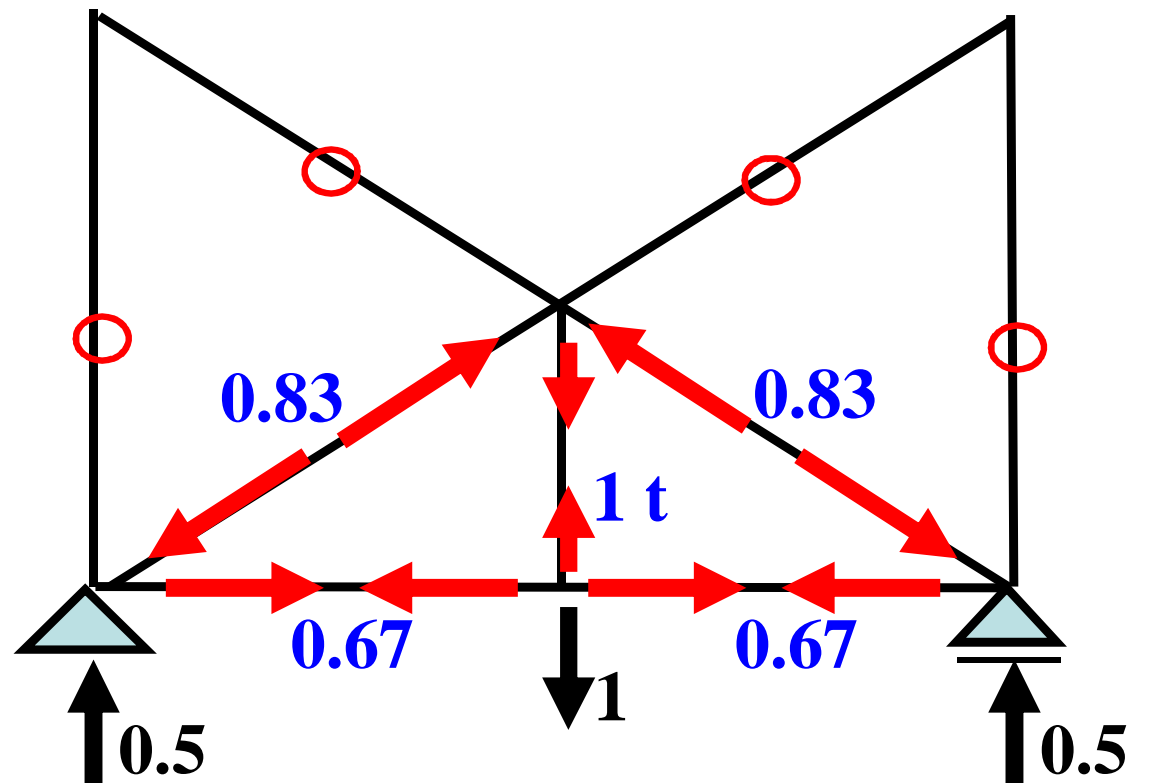
$EA = 10000 \text{ t}$

δ_{vc} , $\delta_{rel}(fe)$, and $\theta(fb)$

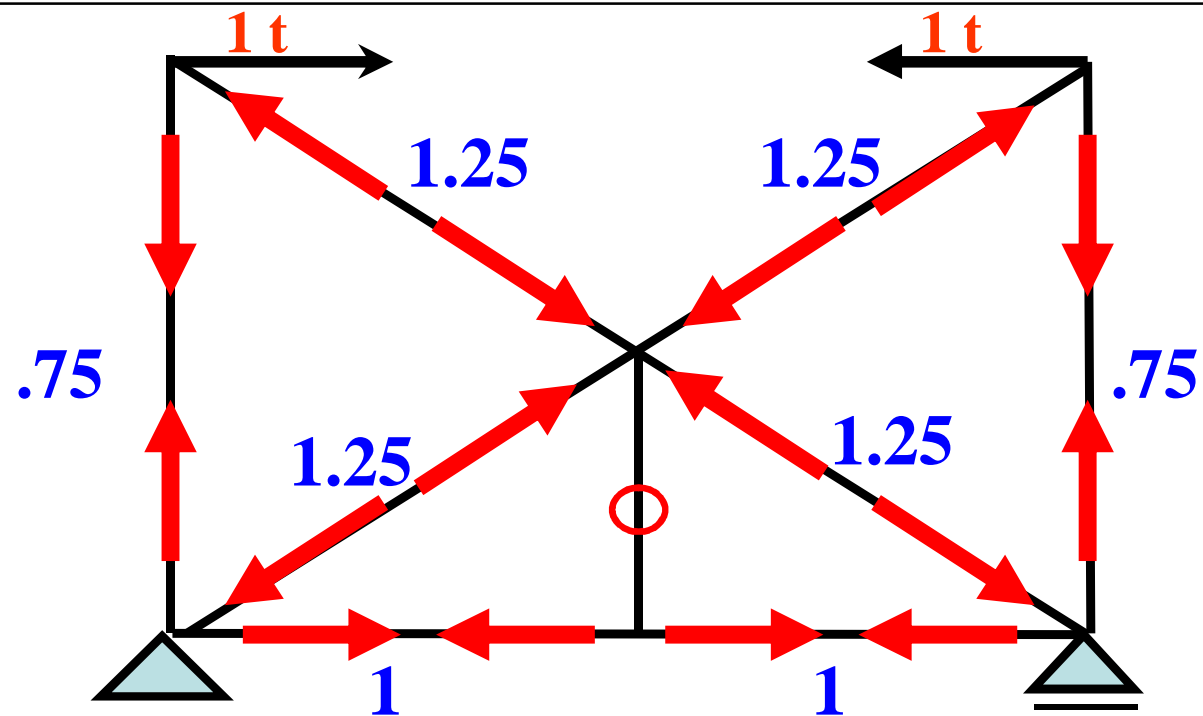
N_0



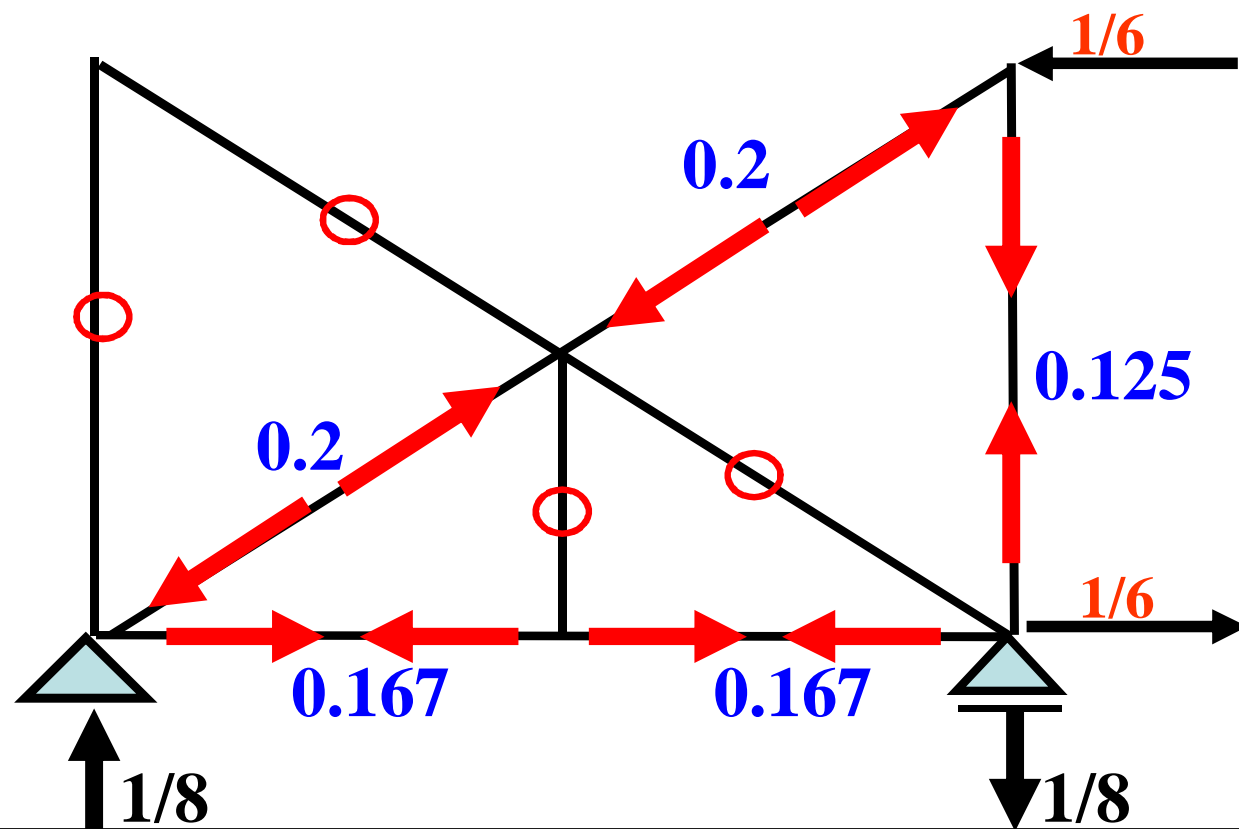
N_1



N_2



N_3



member	L	N ₀	N ₁	N ₂	N ₃	N ₀ N ₁ L	N ₀ N ₂ L	N ₀ N ₃ L
Ac	4	12	0.67	1	0.167	32	48	8
Cb	4	12	0.67	1	0.167	32	48	8
Ae	6	9	0	0.75	0	0	40.5	0
Cd	3	0	1	0	0	0	0	0
Bf	6	9	0	0.75	0.125	0	40.5	6.75
Ad	5	-15	-0.83	-1.25	-0.2	62.5	93.75	15.625
Df	5	-15	0	-1.25	-0.2	0	93.75	15.625
Bd	5	-15	-0.83	-1.25	0	62.5	93.75	0
De	5	-15	0	-1.25	0	0	93.75	0
Σ						189	552	54

$$\delta_{vc} = \frac{\Sigma N_0 N_1 L}{EA} = \frac{189}{EA}$$

$$\delta_{rel(fe)} = \frac{\Sigma N_0 N_2 L}{EA} = \frac{552}{EA}$$

$$\theta(fb) = \frac{\Sigma N_0 N_3 L}{EA} = \frac{54}{EA}$$

- 1- يتم ازالة كل الاحمال من الكمرة.
- 2- يتم وضع واحد طن عند النقطة المطلوب عندها الهبوط.
- 3- يتم حساب الـ **reaction** الناتج عن الواحد طن.

$$\sum (F * y + M * \theta) = 0.0$$

للمثال السابق

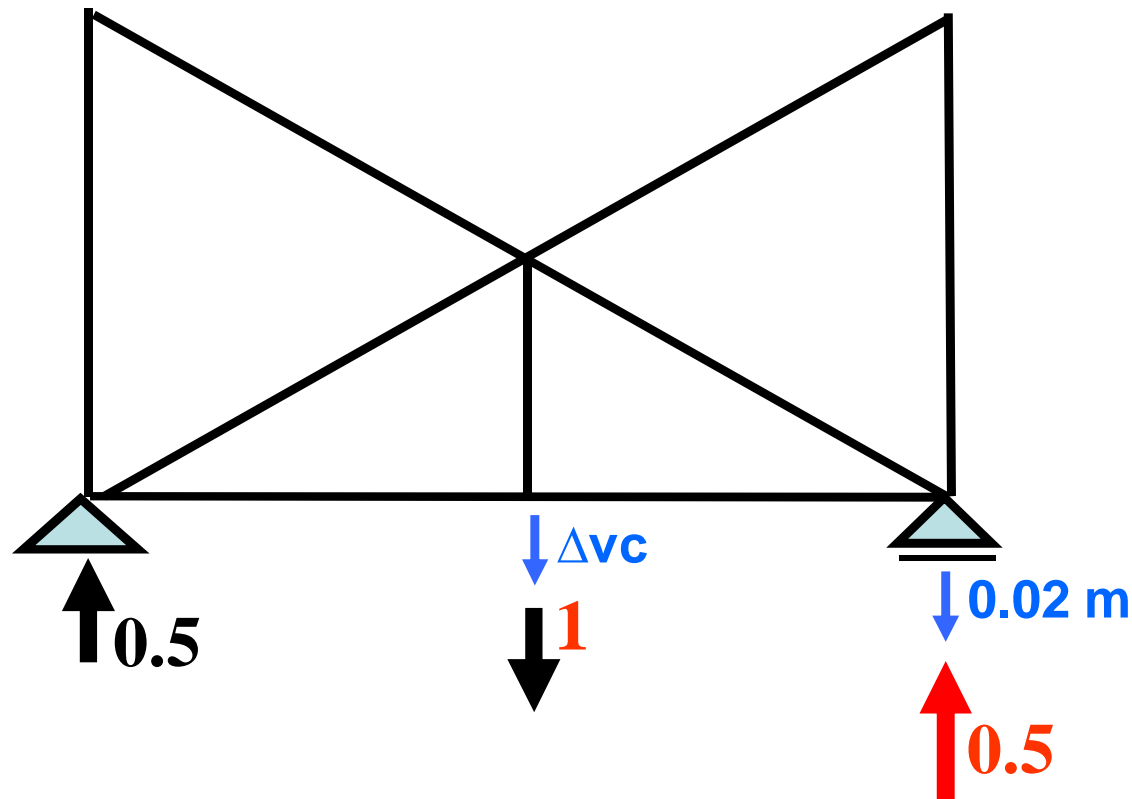
إذا كان المطلوب الثاني هو

Required

δvc , $\delta rel(fe)$, and $\theta(fb)$ due to settlement at $B = 2cm$

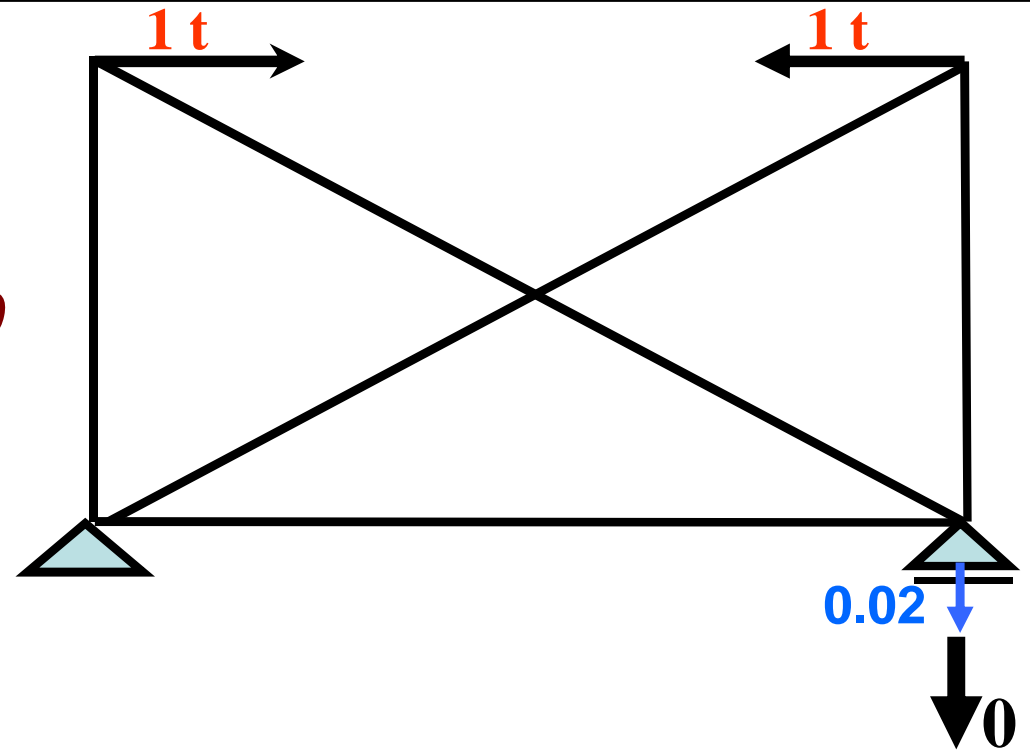
$$1 * \Delta vc - 0.5 * 0.02 = 0.0$$

$$>> \Delta vc = 0.01 m$$



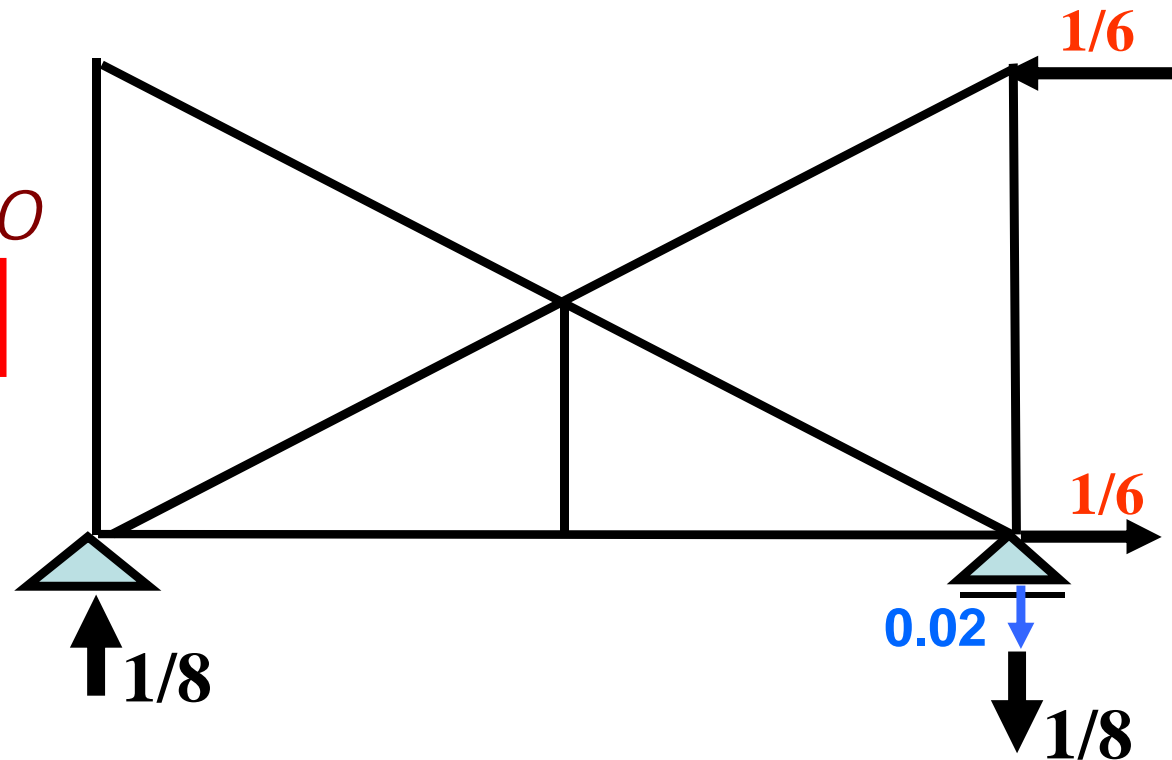
$$1 * \Delta \mathbf{rel}(ef) - 0 * 0.02 = 0.0$$

$$>> \Delta \mathbf{rel}(ef) = 0.0$$



$$1 * \theta_{fb} + 1/8 * 0.02 = 0.0$$

$$>> \theta_{fb} = 0.0025 \text{ rad}$$



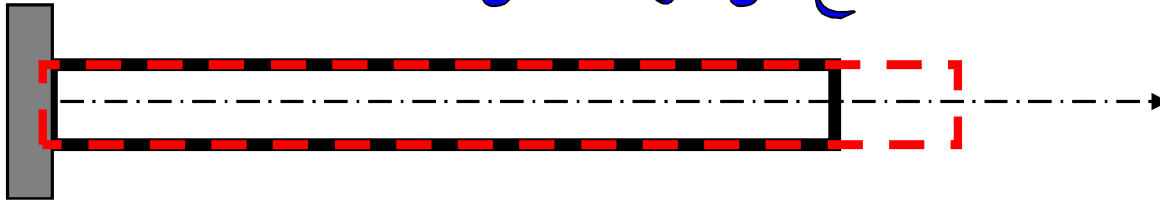
CASE OF TEMPRATURE

تأثير الحرارة

Case of uniform temprature.

طريقة الحل

- 1- يتم ازالة كل الاحمال من الكمرة.
- 2- يتم وضع واحد طن عند النقطة المطلوب عندها الهبوط.
- 3- يتم رسم normal الناتج عن الواحد طن.



$$y \text{ or } \theta = \alpha * dt * (\text{Area of normal}) = \alpha * dt * \sum (N*L)$$

للمثال السابق

اذا كان المطلوب الثالث هو

Required

δvc , $\delta rel(fe)$, and $\theta(fb)$ due to uniform rise in temp in all members

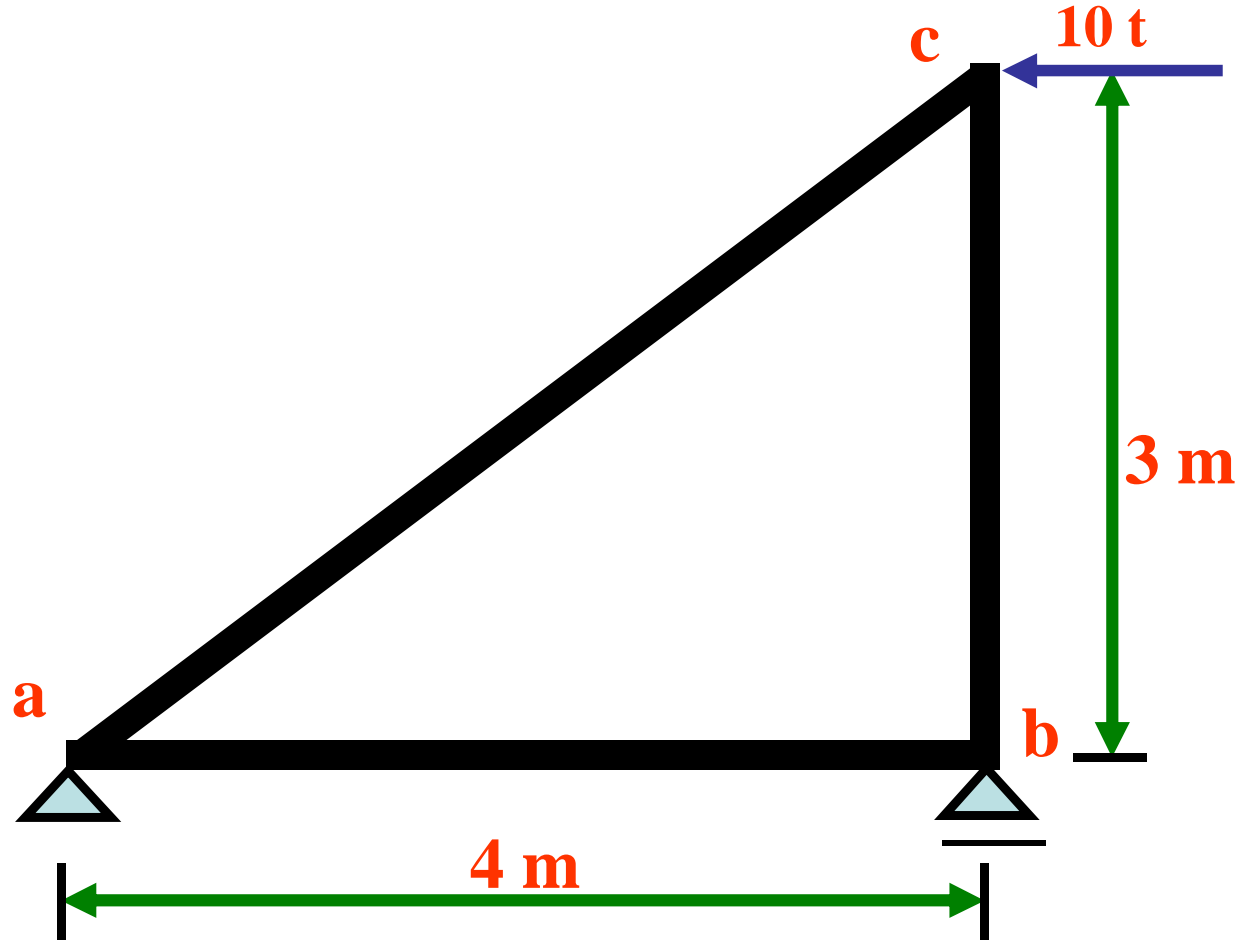
member	L	N ₀	N ₁	N ₂	N ₃	N ₁ L	N ₂ L	N ₃ L
Ac	4	12	0.67	1	0.167	2.68	4	0.67
Cb	4	12	0.67	1	0.167	2.68	4	0.67
Ae	6	9	0	0.75	0	0	4.5	0
Cd	3	0	1	0	0	3	0	0
Bf	6	9	0	0.75	0.125	0	4.5	0.75
Ad	5	-15	-0.83	-1.25	-0.2	-4.15	-6.25	-1
Df	5	-15	0	-1.25	-0.2	0	-6.25	-1
Bd	5	-15	-0.83	-1.25	0	-4.15	-6.25	0
De	5	-15	0	-1.25	0	0	-6.25	0
Σ						0.0	-12.5	0.083

$$\delta vc = \alpha . \Delta t . \Sigma (N_1 L) = 10^{-5} * 40 * 0 = 0$$

$$\delta rel(fe) = \alpha . \Delta t . \Sigma (N_2 L) = 10^{-5} * 40 * -12.5$$

$$\theta(fb) = \alpha . \Delta t . \Sigma (N_3 L) = 10^{-5} * 40 * 0.83$$

Example Two



Required δh_c

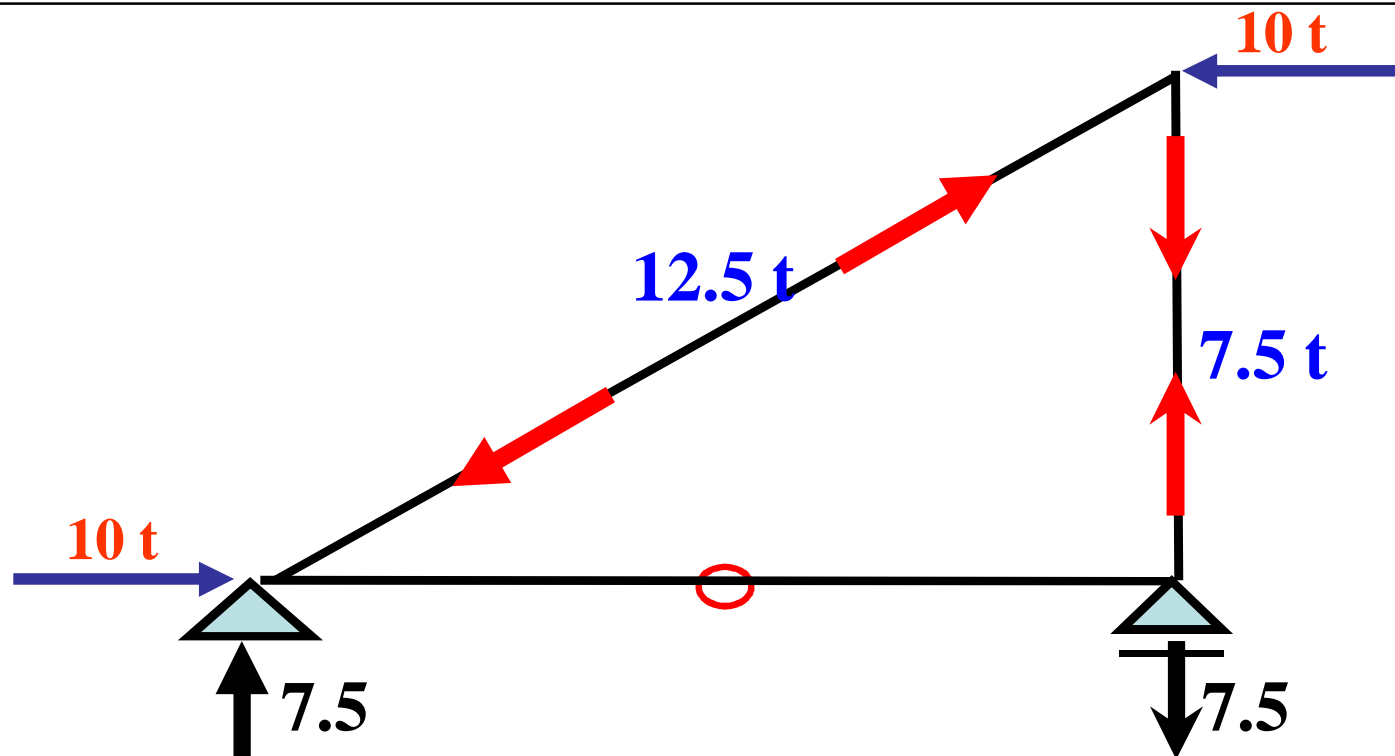
1 – given loads

$EA=10000 \text{ t}$

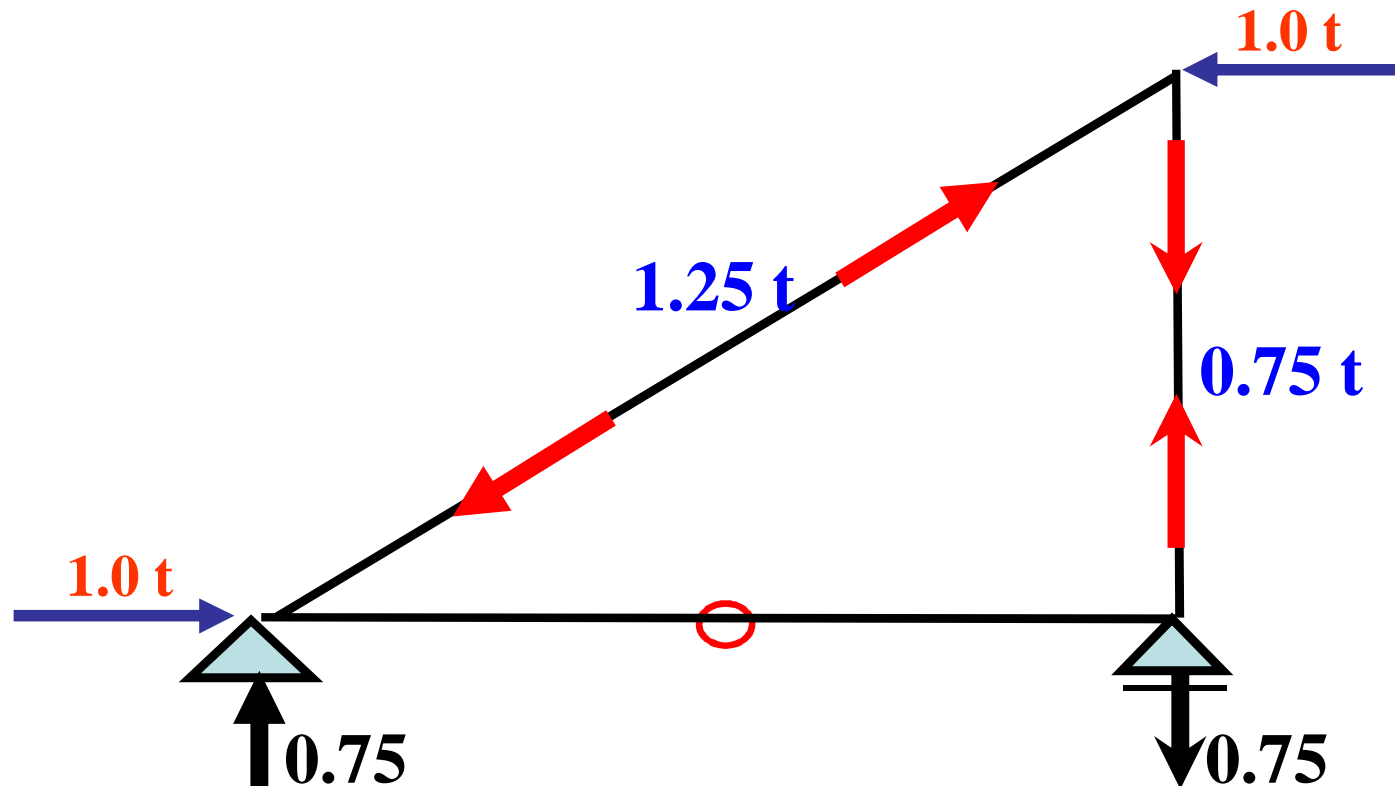
2 – settlement at $b=2\text{cm}$

3 – uniform rise in temperature 20° in member ac

N_0



N_1



member	L	N_0	N_1	$N_0 N_1 L$	$N_1 L$
AB	4	0	0	0	
BC	4	7.5	0.75	22.5	
CA	6	-12.5	-1.25	93.75	-7.5
Σ				116.25	-7.5

$$\delta_{hc} = \frac{\Sigma N_0 N_1 L}{EA} = \frac{116.25}{EA}$$

Given loads

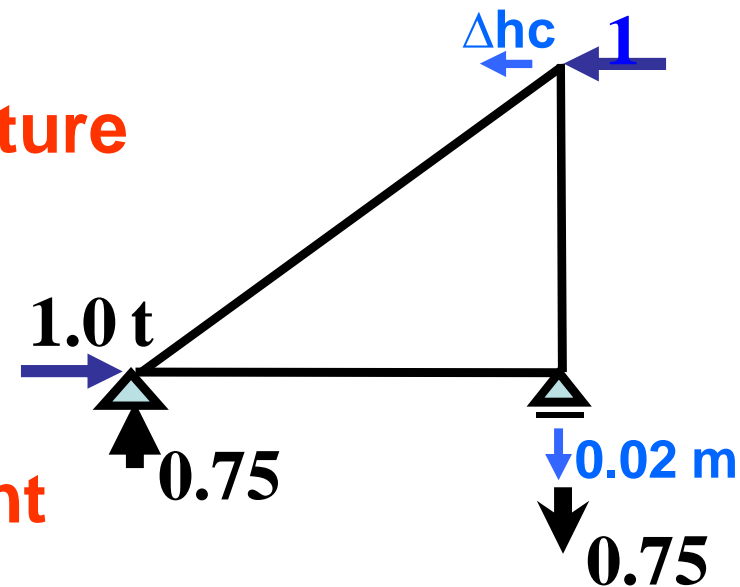
لم يتم حساب القيم هنا لان هذه العناصر لم يحدث فيها تاثر بالحراره

$$\delta_{hc} = \alpha \cdot \Delta t \cdot (\Sigma N_1 L) = 10^{-5} * 20 * -7.5 \quad \text{temperature}$$

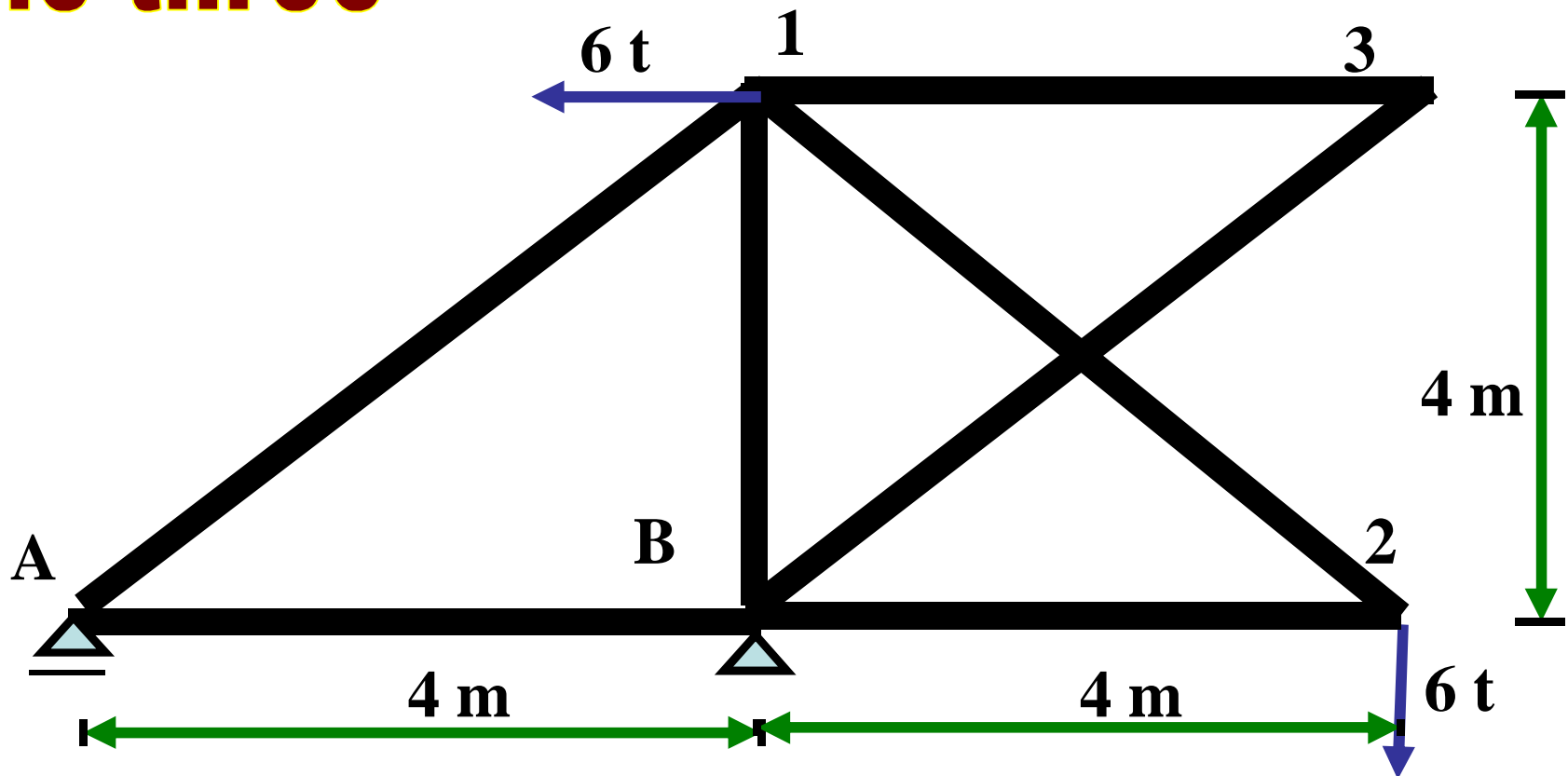
$$1 * \Delta_{hc} + 0.75 * 0.02 = 0.0$$

$$>> \Delta_{hc} = -0.015 \text{ m}$$

settlement



Example three

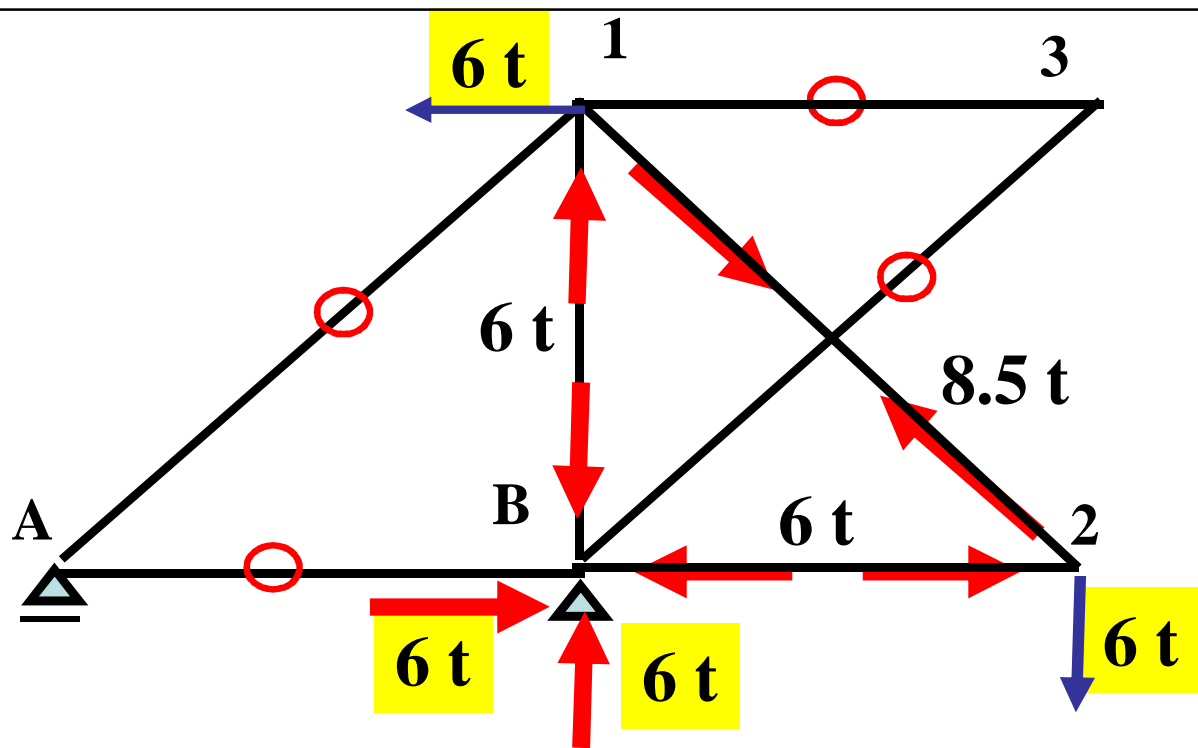


For the shown truss determine the horizontal displacement at point 1 and relative displacement between 2 and 3 due to:

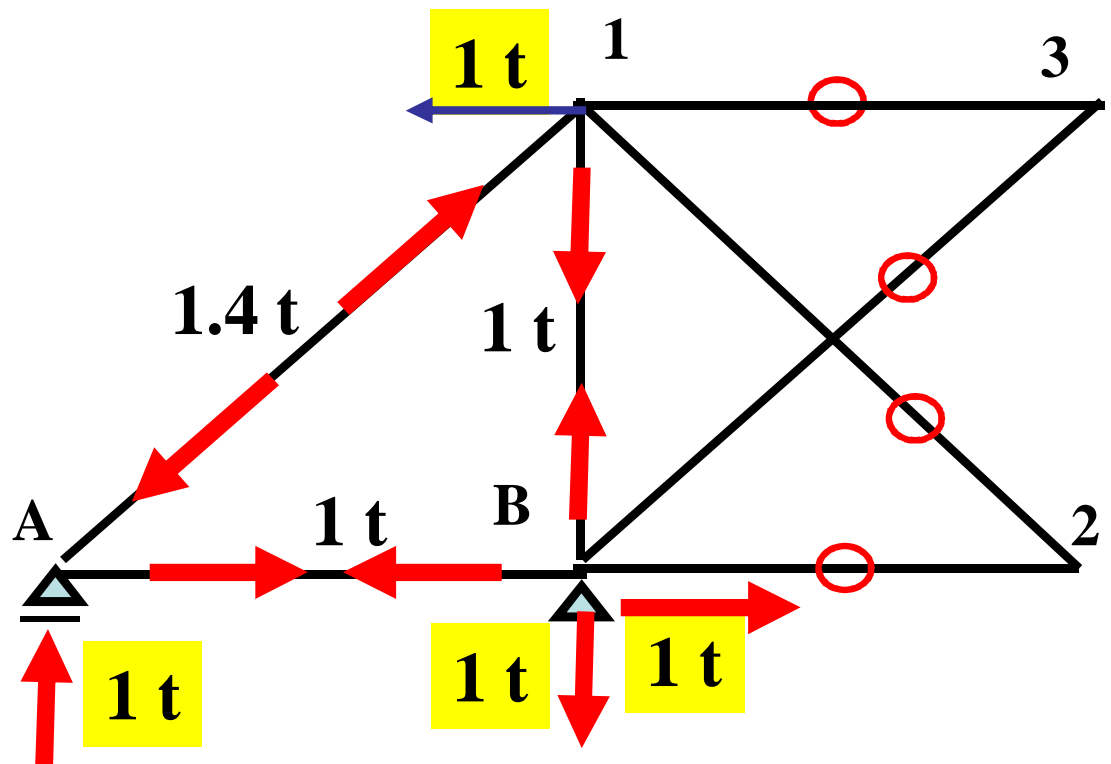
- 1- Given Load.
- 2- Settlement at B=1.5 cm.
- 3- Rise In Temperature 30° In Chord AB2.

$EA=10000 \text{ ton}$ $\alpha = 10^{-5}$

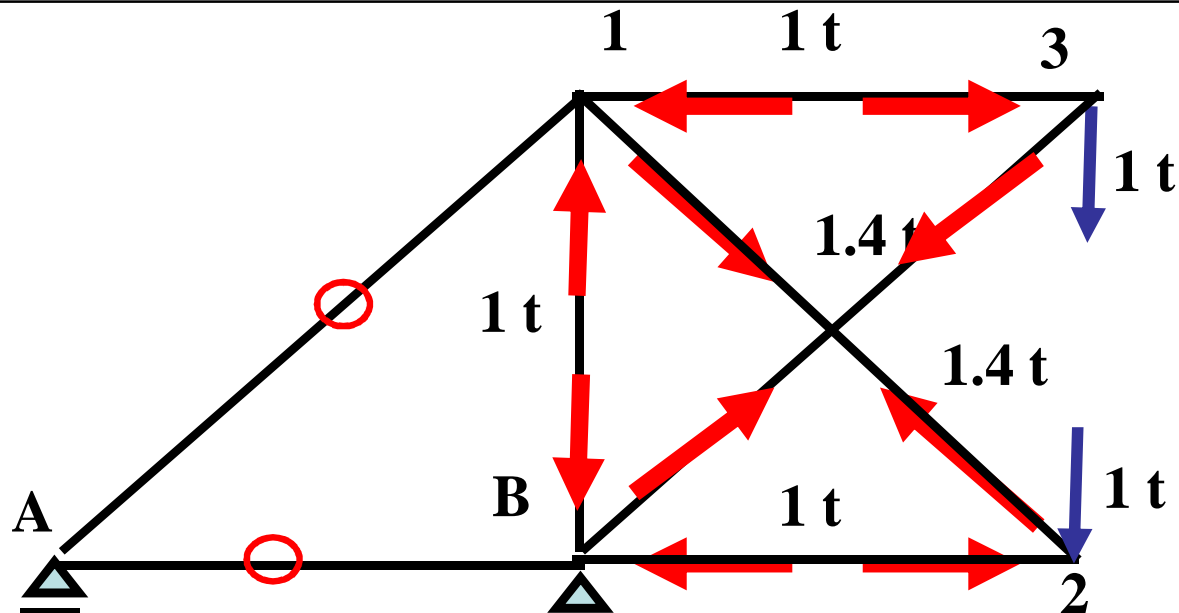
N_0



N_1



N_2



member	L	N_0	N_1	N_2	$N_0 N_1 L$	$N_0 N_2 L$	$N_1 L$	$N_2 L$
AB	4	0	1	0	0	0	4	0
B2	4	-6	0	-1	0	24	0	-4
13	4	0	0	-1	0	0		
B1	4	-6	1	-1	-24	24		
A1	7	0	-1.4	0	0	0		
B3	7	0	0	1.4	0	0		
12	7	8.5	0	1.4	0	84		
Σ					-24	132	4	-4

$$\delta h1 = \frac{\sum N_0 N_1 L}{EA} = \frac{-24}{10000}$$

$$\delta 23 = \frac{\sum N_0 N_2 L}{EA} = \frac{132}{10000}$$

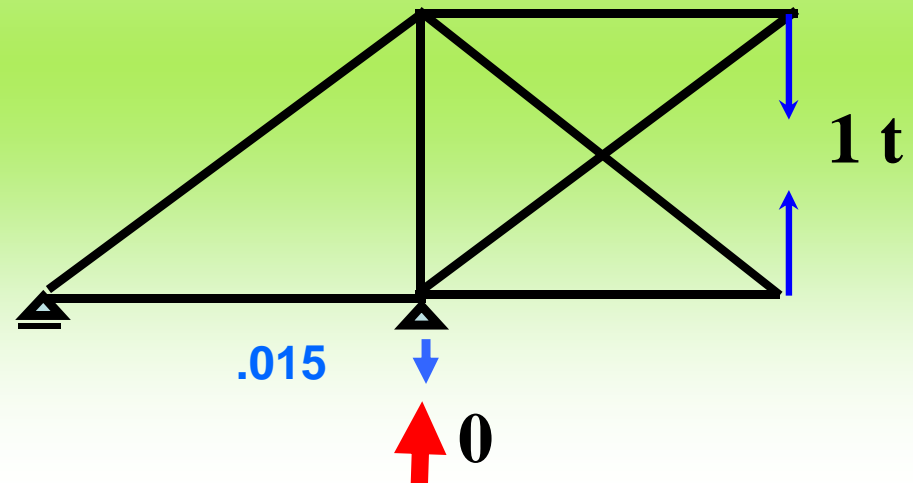
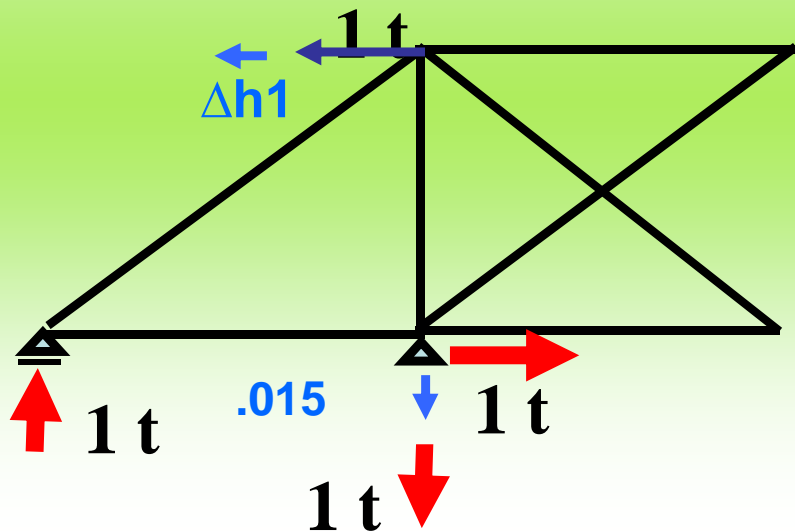
Given loads

$$\delta h1 = \alpha \cdot \Delta t \cdot (\sum N_1 L) = 10^{-5} * 30 * 4$$

temperature

$$\delta 23 = \alpha \cdot \Delta t \cdot (\sum N_1 L) = 10^{-5} * 30 * -4$$

settlement



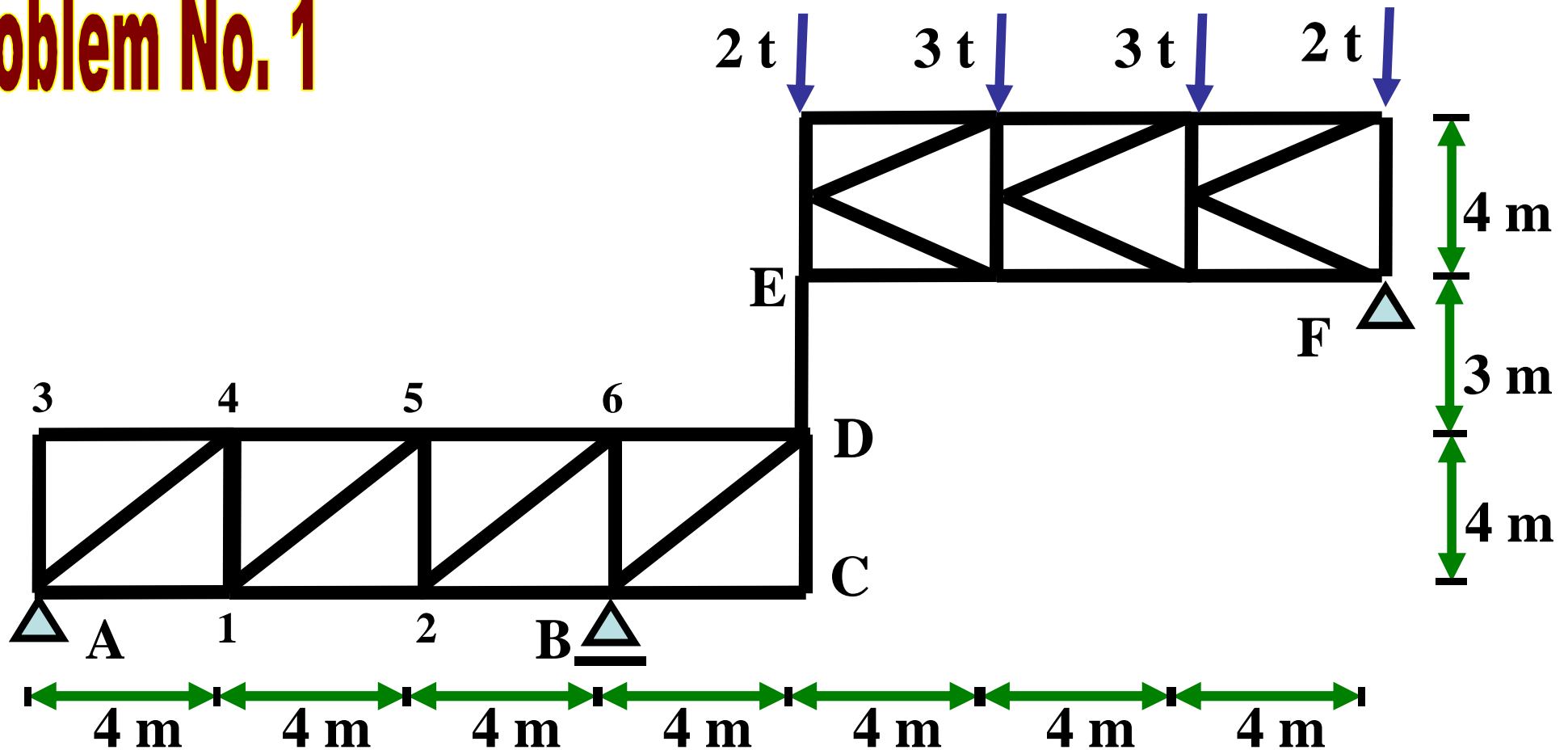
$$1 * \Delta h1 + 1 * 0.015 = 0.0$$

$$\Rightarrow \Delta h1 = -0.015 \text{ m}$$

$$1 * \Delta 23 + 0 * 0.015 = 0.0$$

$$\Rightarrow \Delta h23 = 0.0$$

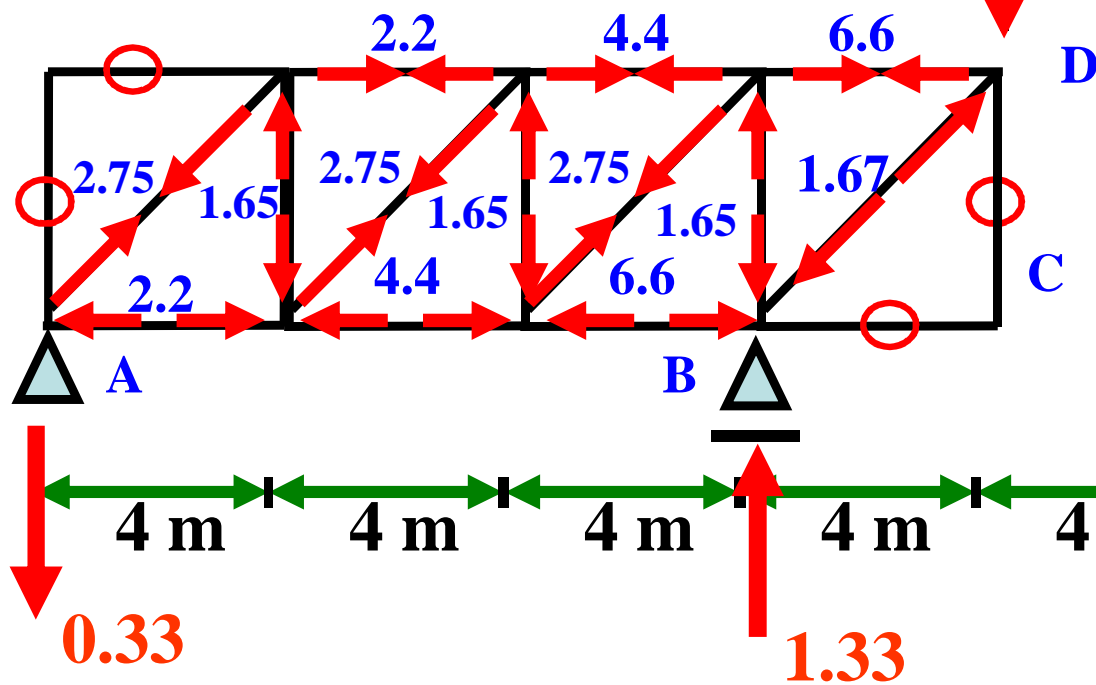
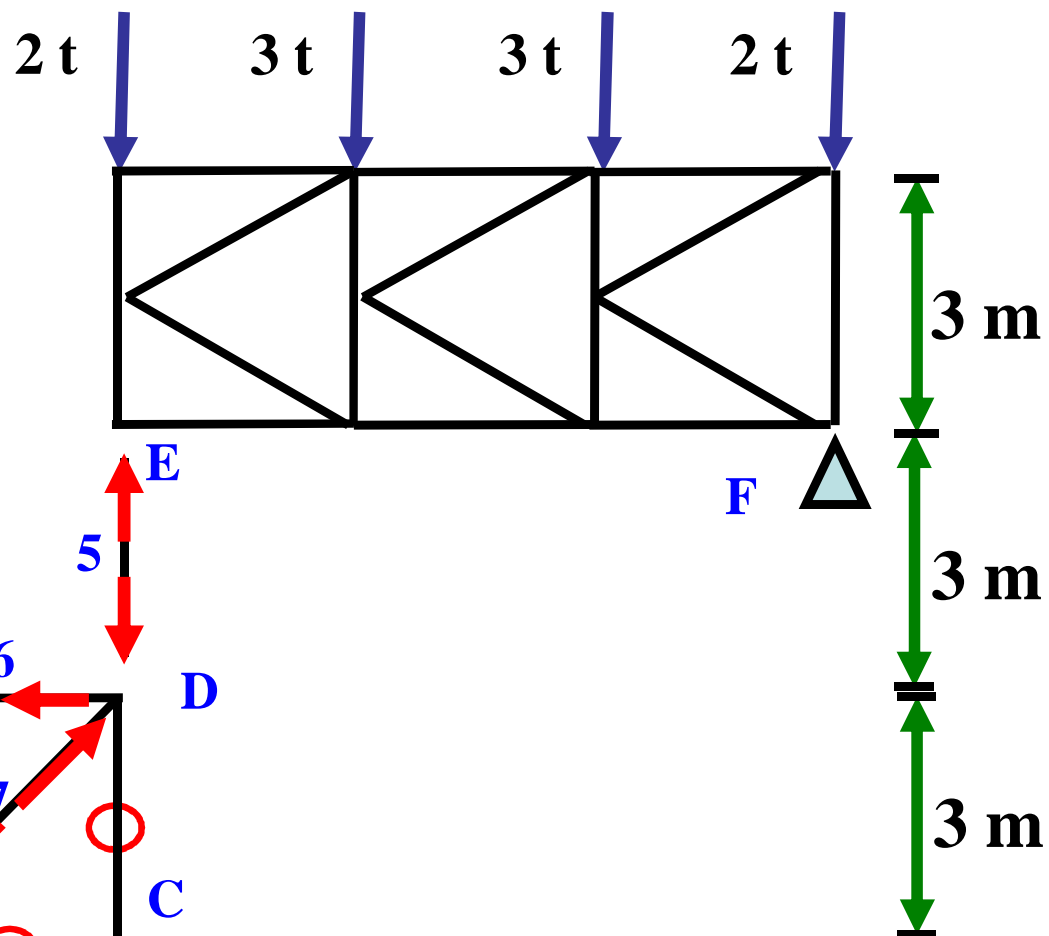
Problem No. 1



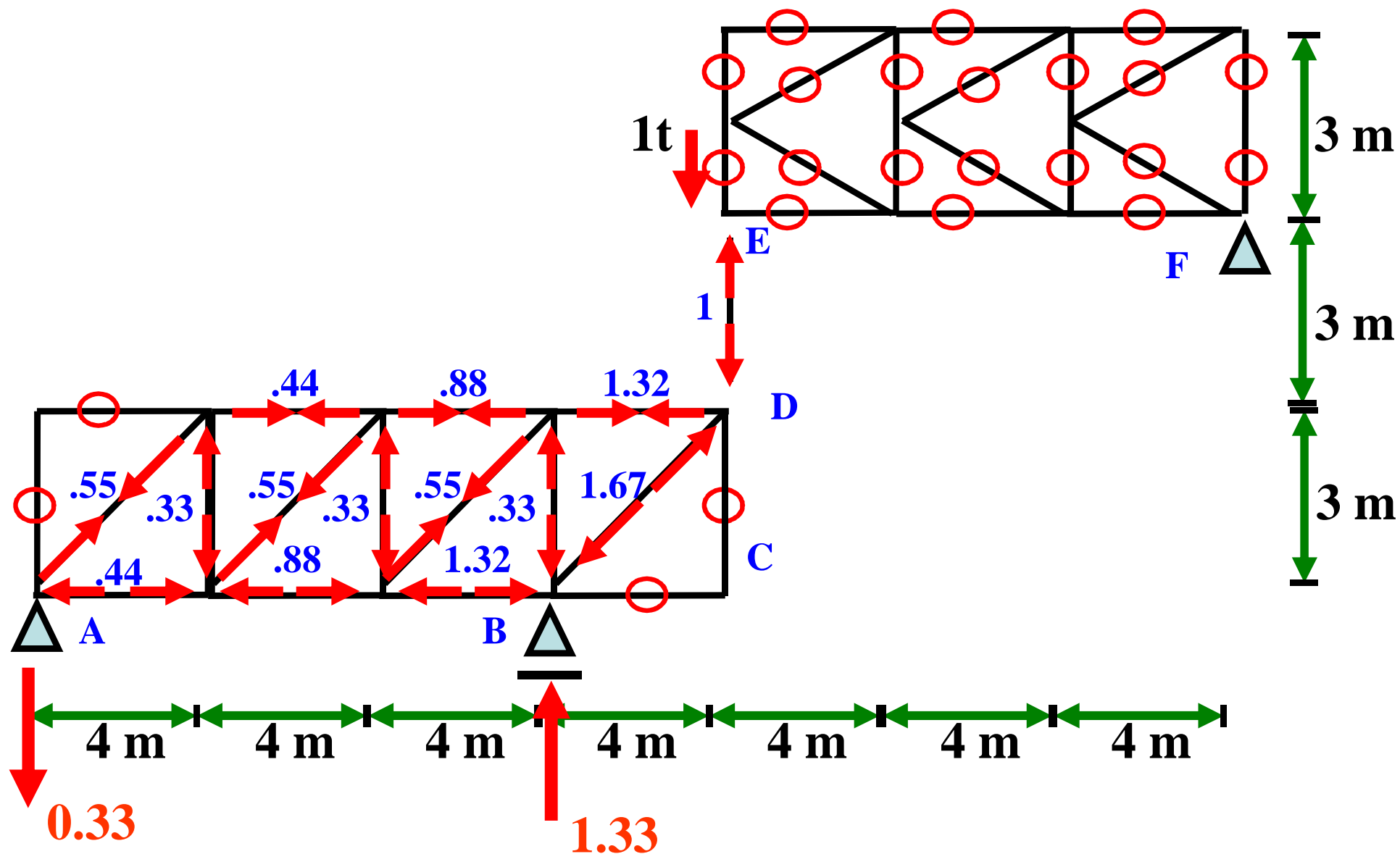
For the shown truss determine the vertical deflection E and the rotation of member CD due to:

- 1- Given Load.
- 2- Vertical and horizontal movement at A = 2cm downward and 3cm to the left. $L/EA = .003 \text{ cm/t}$.

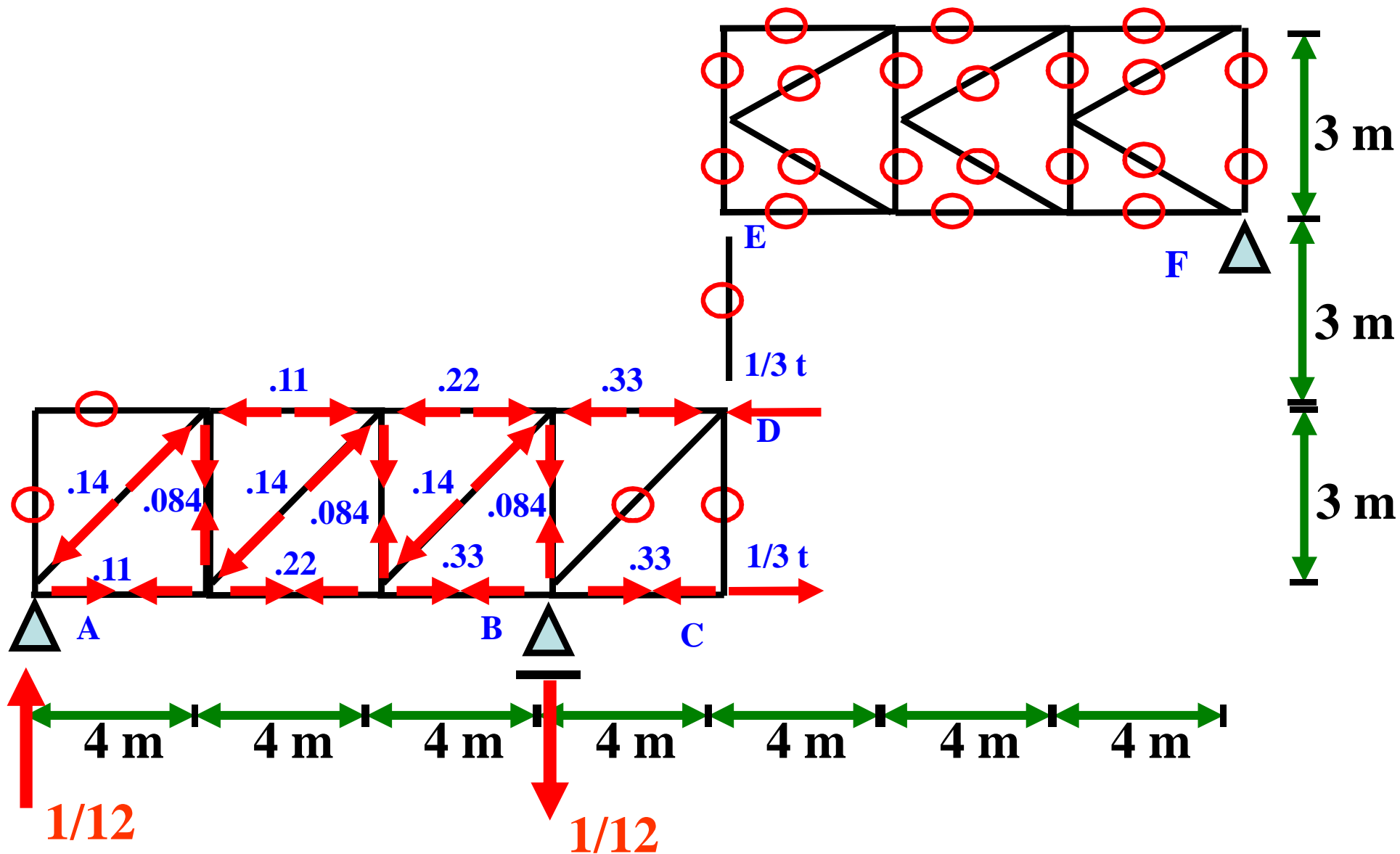
N_0



N_1



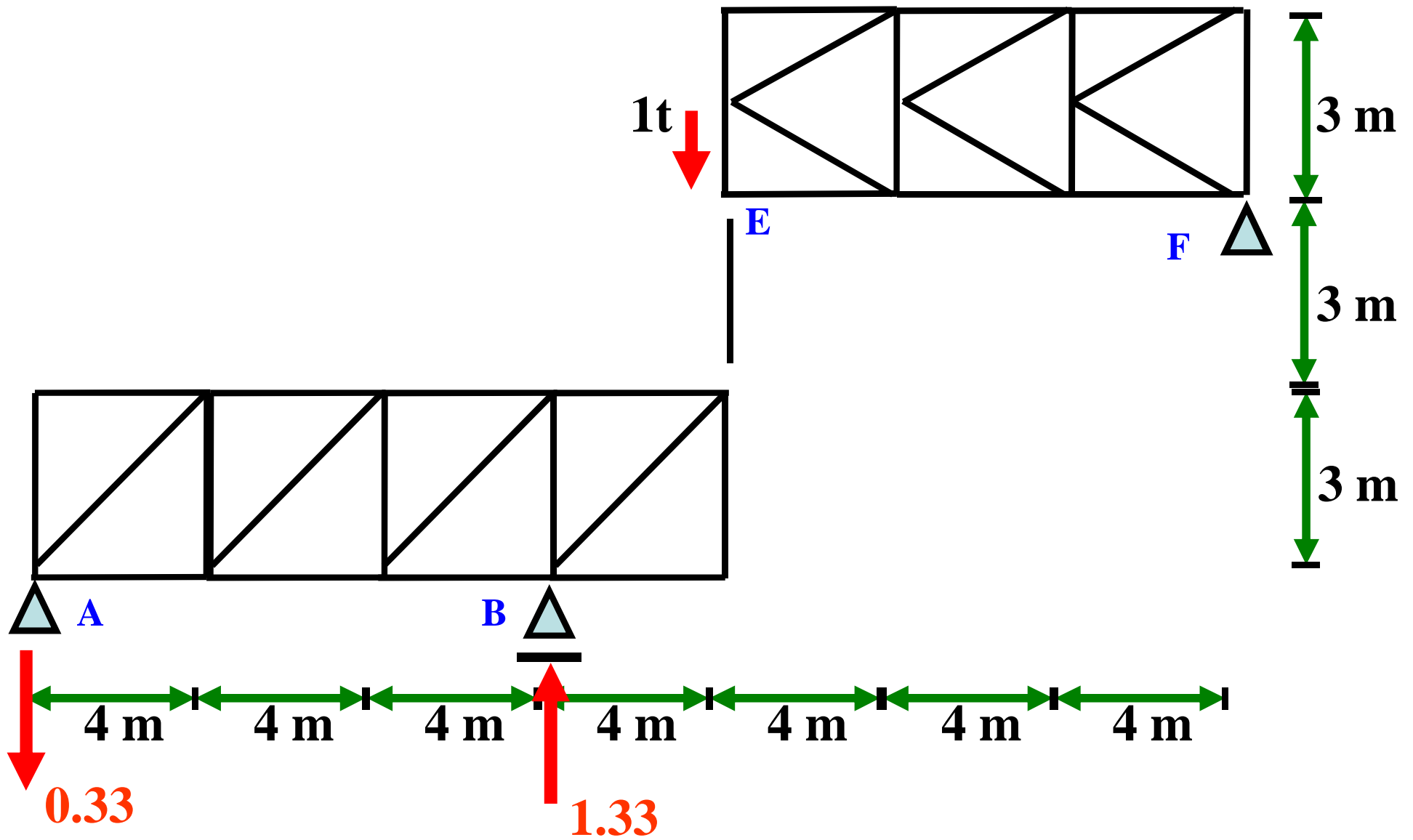
N_2



member	N ₀	N ₁	N ₂	N ₀ N ₁	N ₀ N ₂
A1	-2.2	-0.44	0.11	0.968	-0.242
12	-4.4	-0.88	0.22	3.872	-0.968
2b	-6.6	-1.32	0.33	8.712	-2.178
bc	0.0	0.0	0.33	0	0
34	0.0	0.0	0.0	0	0
45	2.2	0.44	0.11	0.968	0.242
56	4.4	0.88	0.22	3.872	0.968
6d	6.6	1.32	0.33	8.712	2.178
A3	0.0	0.0	0	0	0
14	-1.65	-0.33	0.084	0.55	-0.1386
25	-1.65	-0.33	0.084	0.55	-0.1386
B6	-1.65	-0.33	0.084	0.55	-0.1386
Cd	0.0	0.0	0	0	0
A4	2.75	0.55	0.14	1.5	1.51
15	2.75	0.55	0.14	1.5	1.51
26	2.75	0.55	0.14	1.5	1.51
bd	-1.67	-1.67	0	2.78	0
dE	-5.0	-1.0	0	5	0
Σ				36.2	4.11

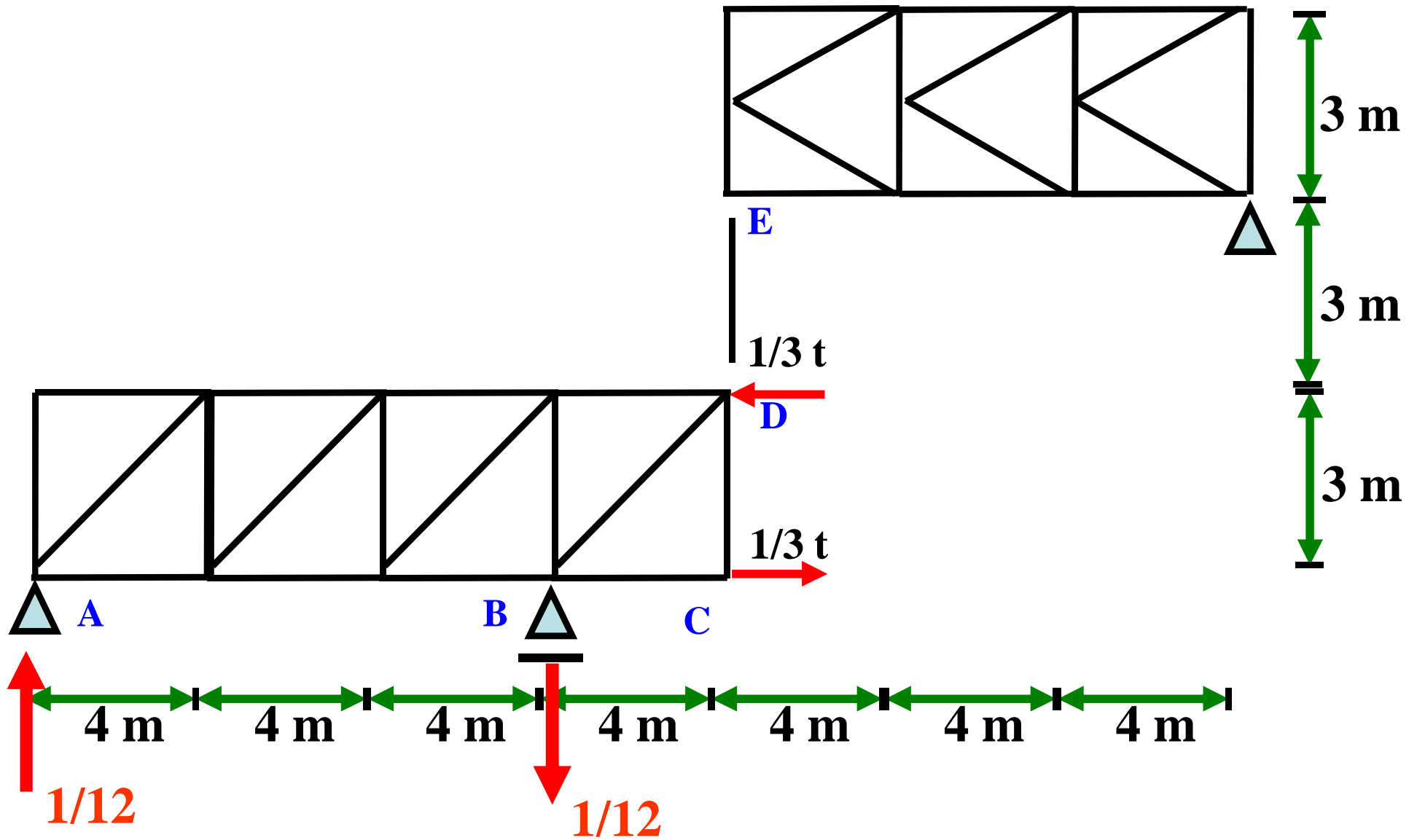
$$\delta ve = \frac{L}{EA} * \sum N_0 N_1 = 0.003 * 36.2$$

$$\theta(fb) = \frac{L}{EA} * \sum N_0 N_2 = 0.003 * 4.11$$



$$1 * \Delta v E + 0.33 * 0.02 + 0 * .03 = 0.0$$

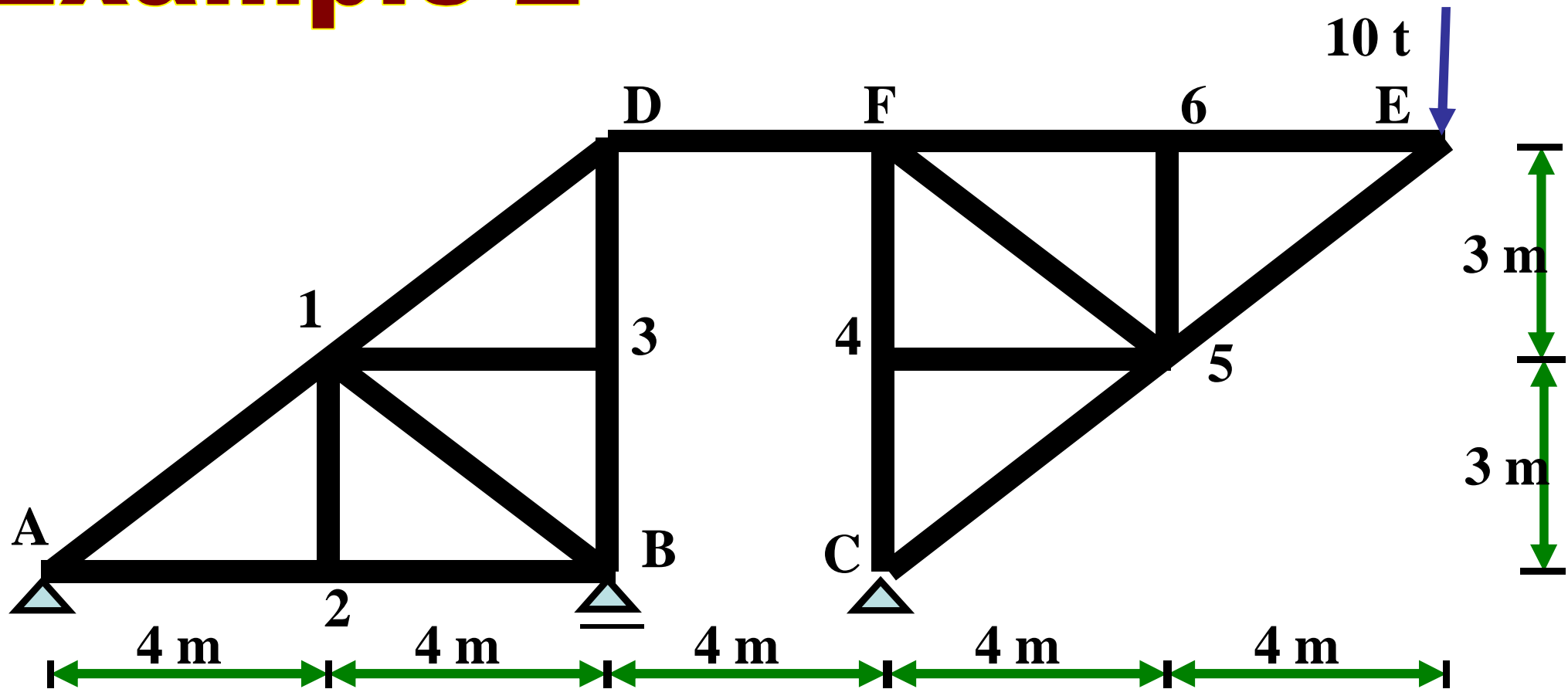
$$\gg \Delta v E = - 0.0066 \text{ m}$$



$$1 * \theta_{DC} - 1/12 * 0.02 + 0 * .03 = 0.0$$

$$>> \theta_{DC} = 0.0016 \text{ m}$$

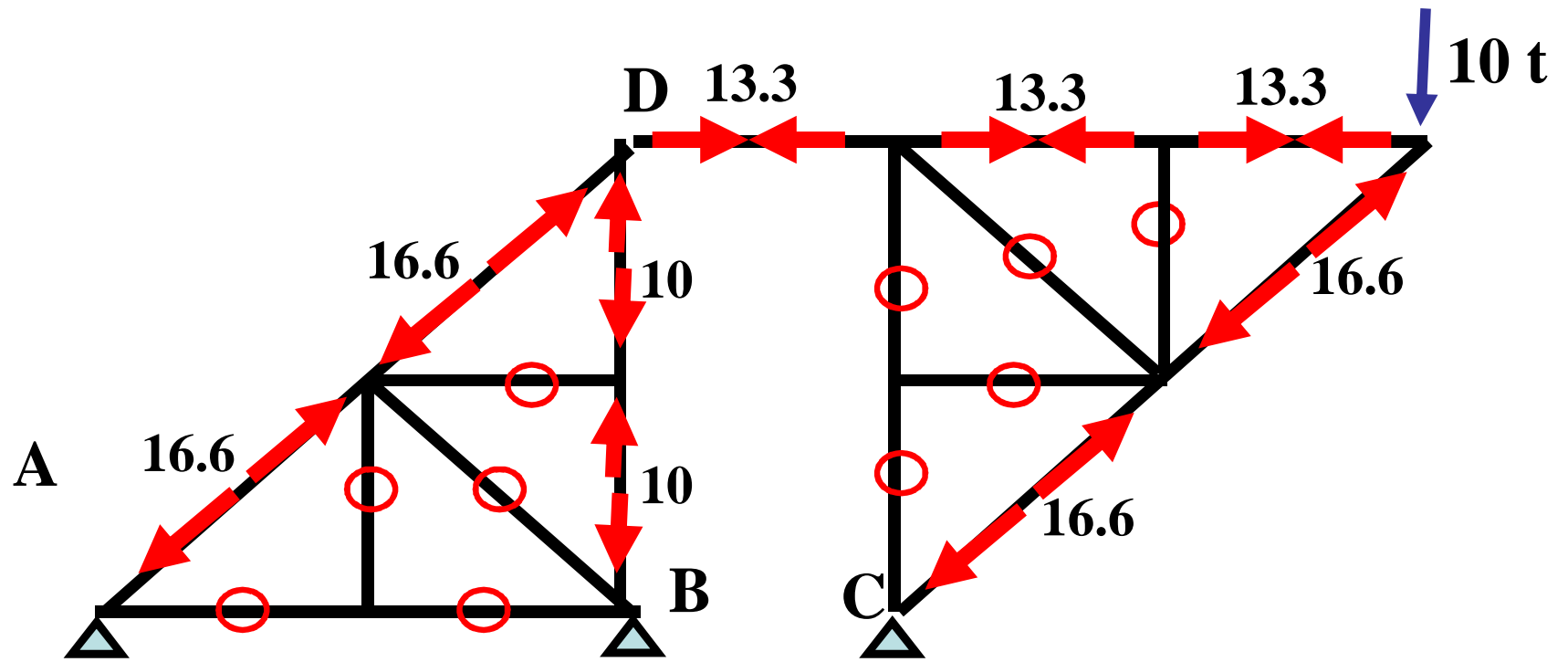
Example 2



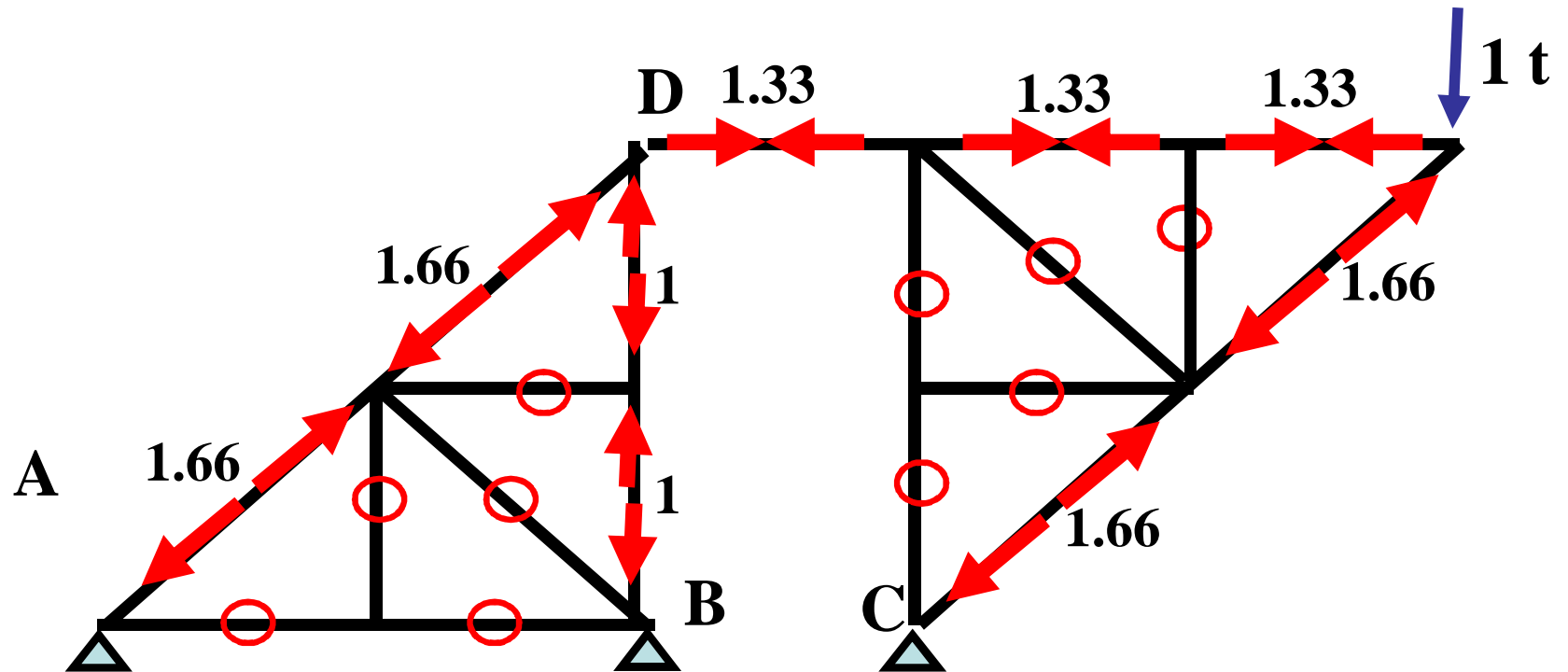
For the shown truss determine the vertical deflection E and horizontal displacement at D due to:

- 1- Given Load.
- 2- Settlement Of Support A=2cm And Horizontal Displacement At Support C=3cm To The Right.
- 3- Rise In Temperature 30° In All Members $EA=10000 \text{ ton}$ $\alpha = 10^{-5}$

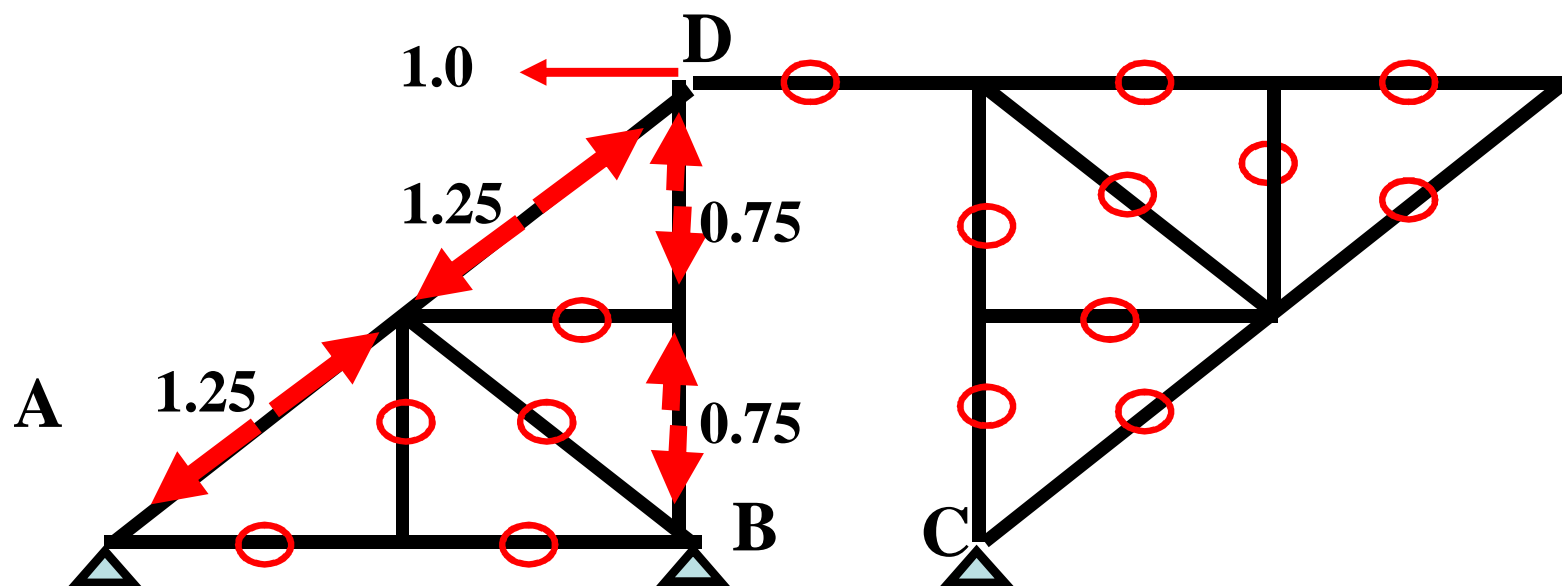
N_0



N_1



N_2



member	L	N ₀	N ₁	N ₂	N ₀ N ₁ L	N ₀ N ₂ L	N ₁ L	N ₂ L
A2	4	0	0	0	0	0	0	0
2b	4	0	0	0	0	0	0	0
13	4	0	0	0	0	0	0	0
45	4	0	0	0	0	0	0	0
Df	4	13.3	1.33	0	71.11	0	5.33	0
F6	4	13.3	1.33	0	71.11	0	5.33	0
6e	4	13.3	1.33	0	71.11	0	5.33	0
21	3	0	0	0	0	0	0	0
B3	3	-10	-1.0	-0.75	30	22.5	-3	-2.25
3d	3	-10	-1.0	-0.75	30	22.5	-3	-2.25
C4	3	0	0	0	0	0	0	0
4f	3	0	0	0	0	0	0	0
56	3	0	0	0	0	0	0	0
B1	5	0	0	0	0	0	0	0
5f	5	0	0	0	0	0	0	0
A1	5	16.67	1.667	1.25	138.9	104	8.33	6.25
1d	5	16.67	1.667	1.25	138.9	104	8.33	6.25
C5	5	-16.67	-1.667	0	138.9	0	-8.33	0
5e	5	-16.67	-1.667	0	138.9	0	-8.33	0
298 Σ					768.8	253	10	8

$$\delta_{ve} = \frac{\sum N_0 N_1 L}{EA} = \frac{768.8}{10000}$$

$$\delta_{hd} = \frac{\sum N_0 N_2 L}{EA} = \frac{253}{10000}$$

Given loads

$$\delta_{ve} = \alpha \cdot \Delta t \cdot (\sum N_1 L) = 10^{-5} * 30 * 10$$

temperature

$$\delta_{hd} = \alpha \cdot \Delta t \cdot (\sum N_1 L) = 10^{-5} * 30 * 8$$

settlement

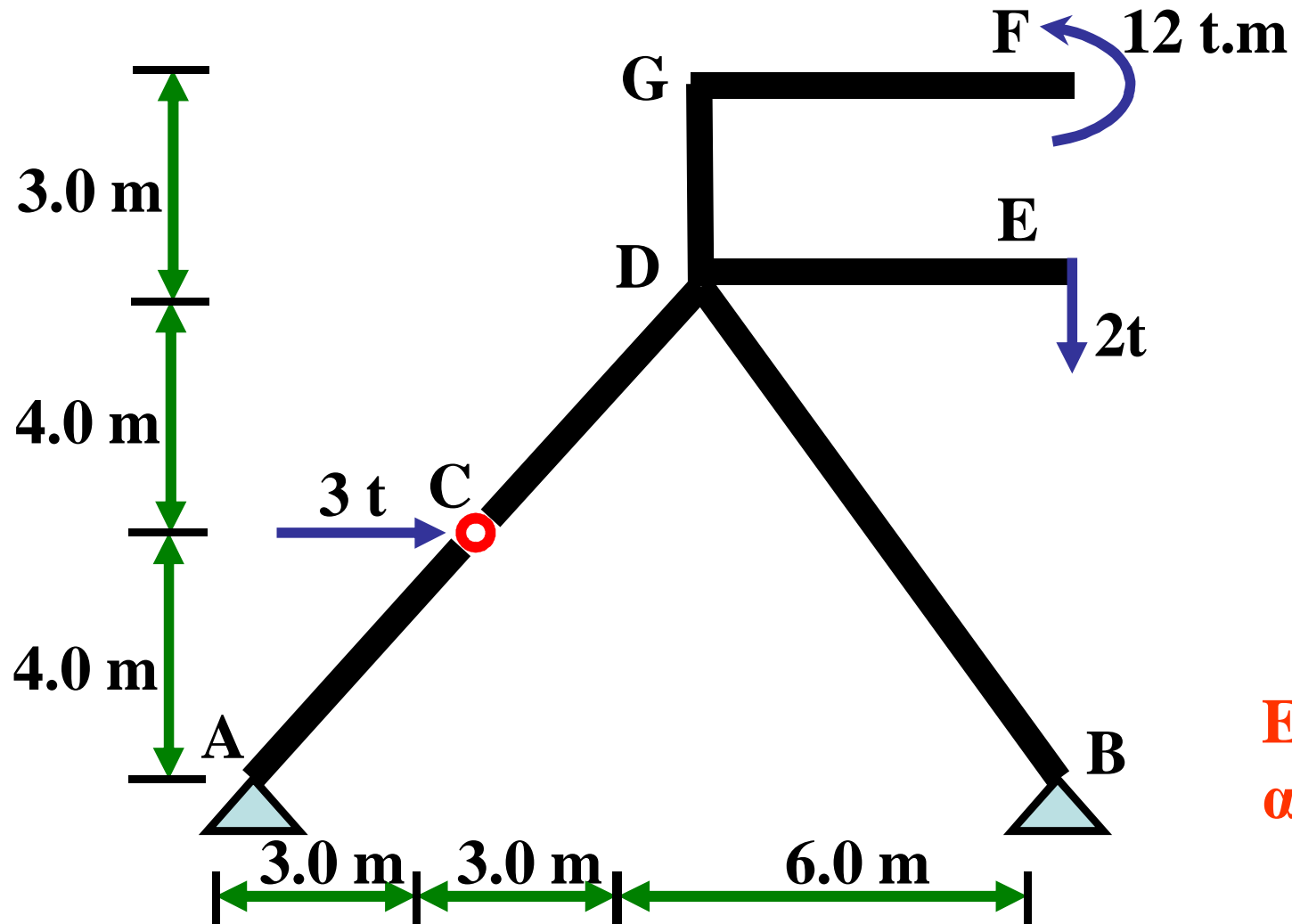
$$1 * \Delta \mathbf{ve} + 3 * 1.33 - 1 * 2 = 0.0$$

$$>> \Delta \mathbf{ve} = -2 \text{ cm}$$

$$1 * \Delta \mathbf{hd} + 0.75 * 2 + 3 * 0 = 0.0$$

$$>> \Delta \mathbf{hd} = -1.5 \text{ cm}$$

Problem No. 3



$$EI = 10000 \text{ t.m}^2.$$

$$\alpha = 10^{-5}.$$

For the Shown Frame, Calculate the Rotation at Point D due to:

- 1- Given Loads.
- 2- Settlement at A = 2cm.
- 3- Rise in Temperature 30° in All Members.

$$\Sigma \mathbf{M}_A = 0.0$$

$$3 \cdot 4 + 2 \cdot 12 - 12 - Y_B \cdot 12 = 0 \Rightarrow Y_B = 2.0 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

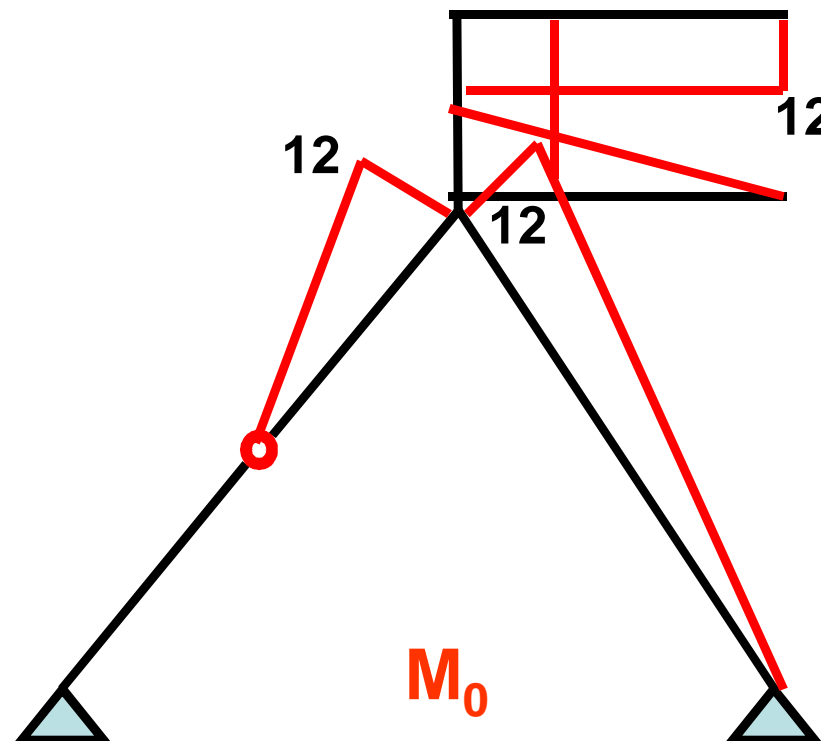
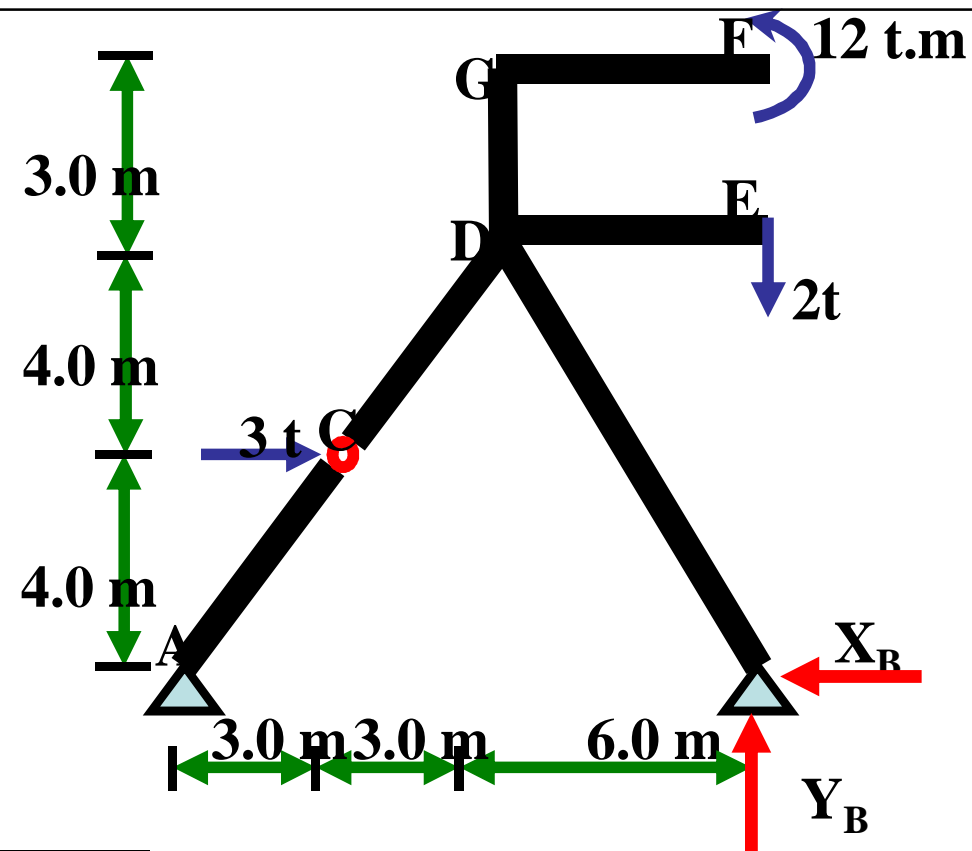
$$Y_A = 2 - 2 = 0.0$$

$$\Sigma \mathbf{M}_{CL} = 0.0$$

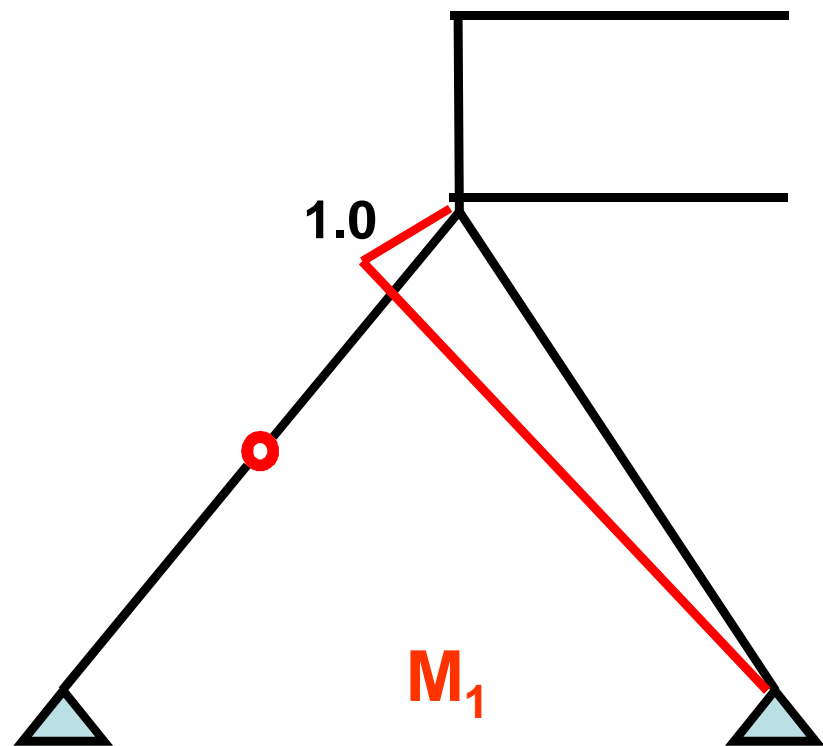
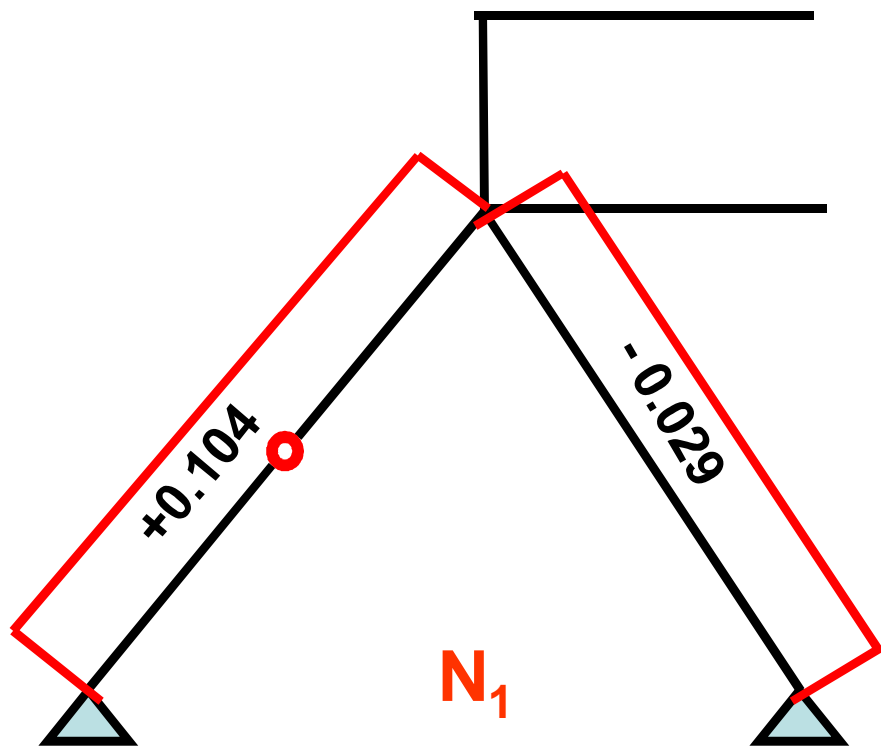
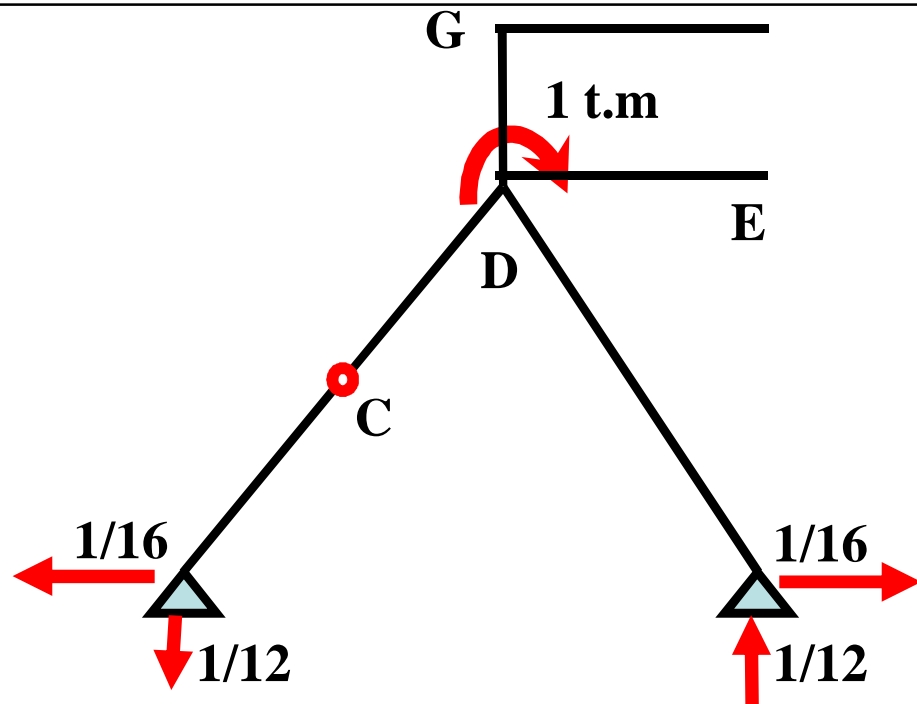
$$0 \cdot 3 - X_A \cdot 4 = 0.0 \Rightarrow X_A = 0.0$$

$$\Sigma \mathbf{F}_X = 0.0$$

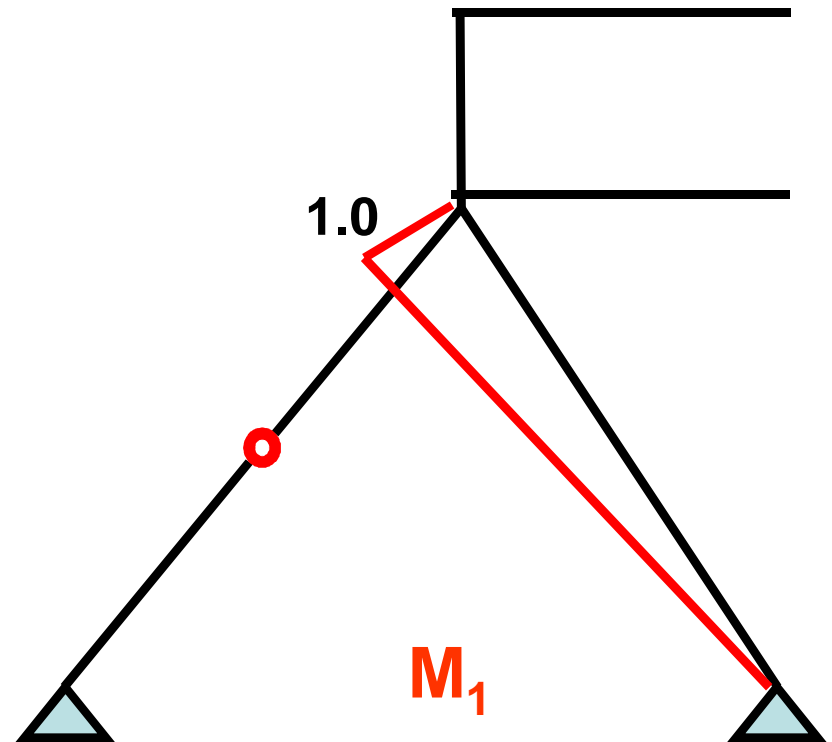
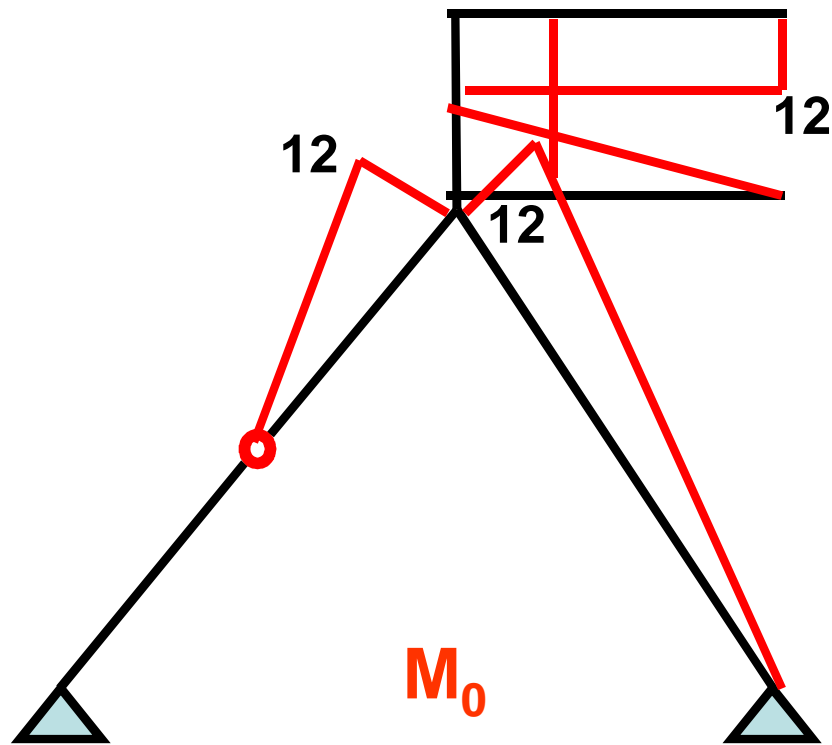
$$X_B = 3.0$$



For θ_d

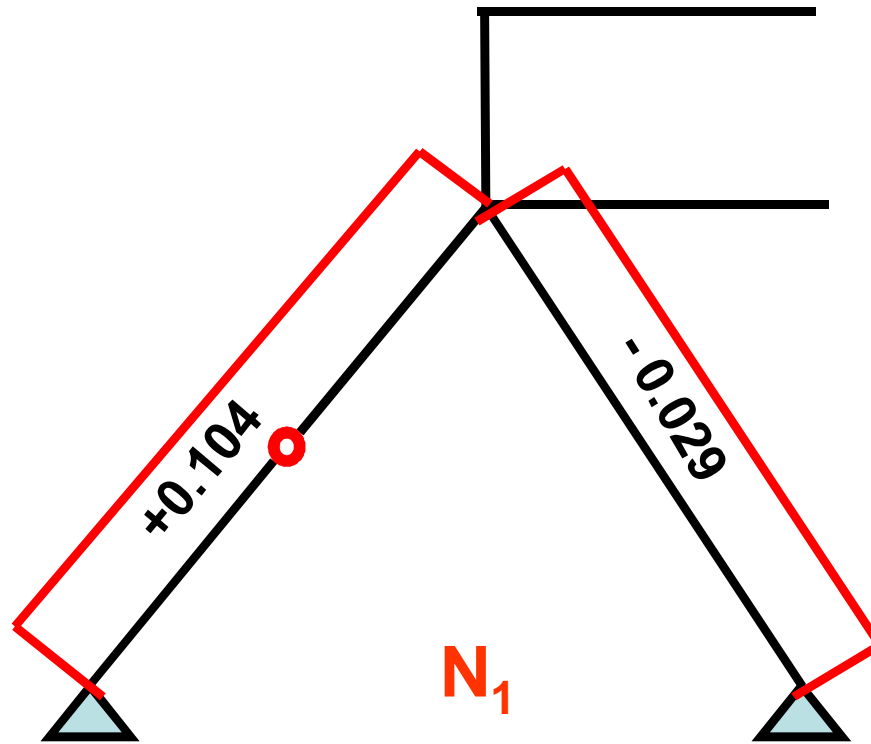


given loads



$$\theta_d = \frac{1}{EI} \left[\frac{-1 * 12 * 10}{3} \right] = -4 * 10^{-4} \text{ rad}$$

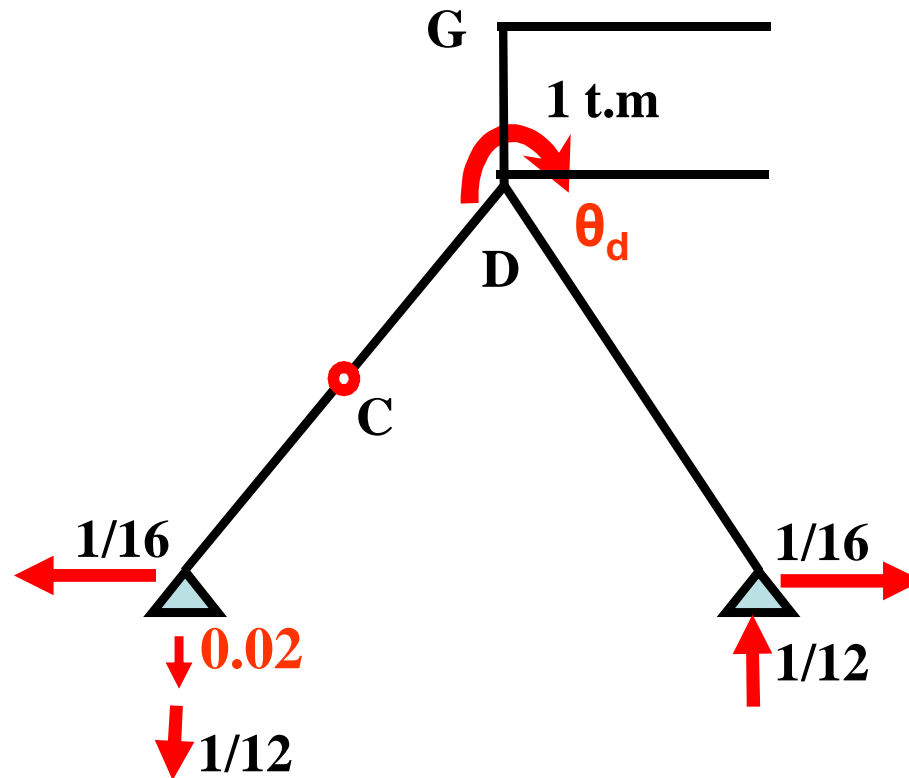
Uniform temperature=30



$$\text{Area of Normal} = -0.209 * 10 + 0.104 * 10 = -1.05$$

$$\theta_d = \alpha * dt * \text{Area of Normal} = 10^{-5} * 30 * -1.05 = 0$$

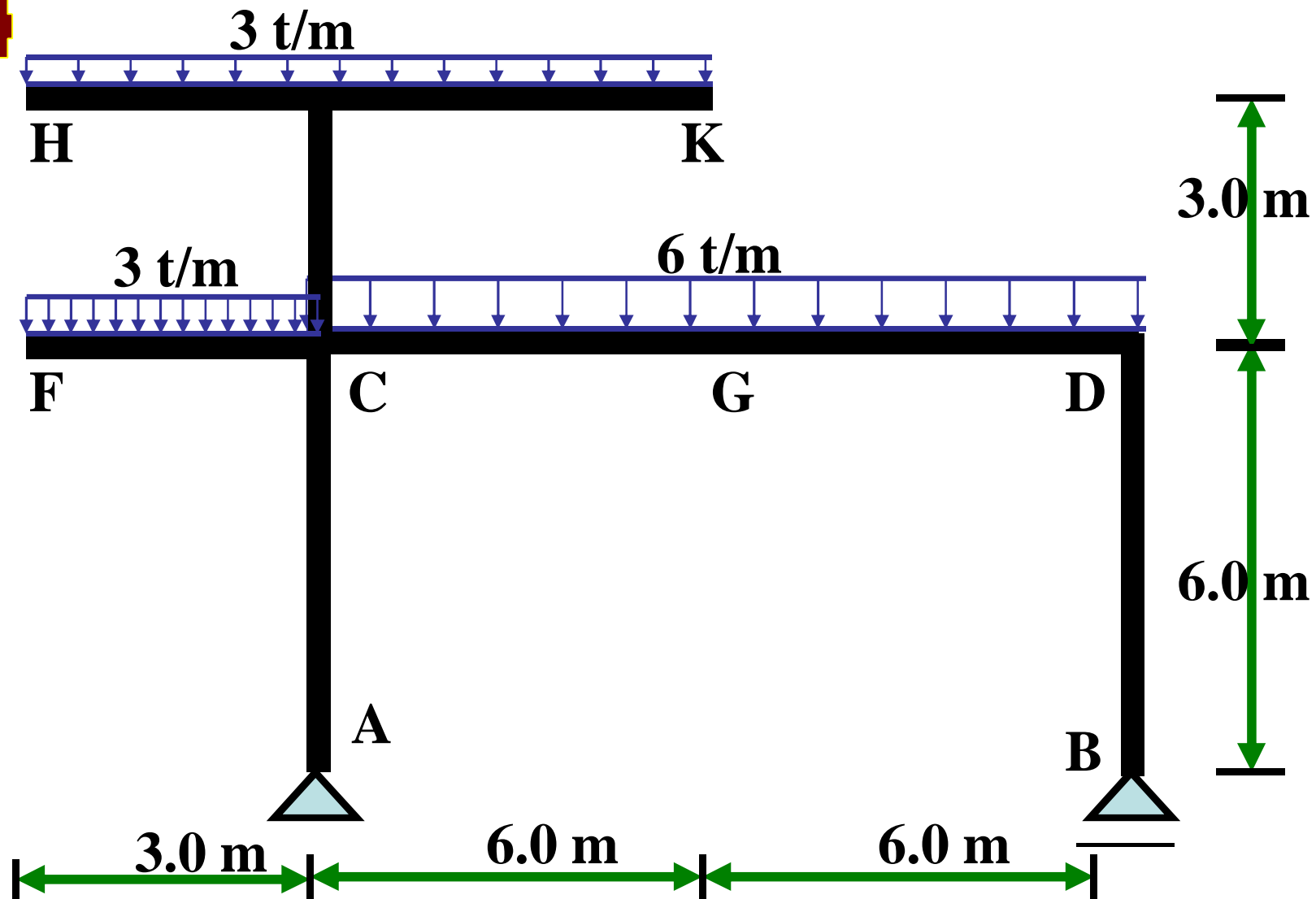
due settlement at A = 2 cm



$$1 * \theta_d + 1/12 * 0.02 = 0.0$$

$$\theta_d = -0.0167 \text{ rad}$$

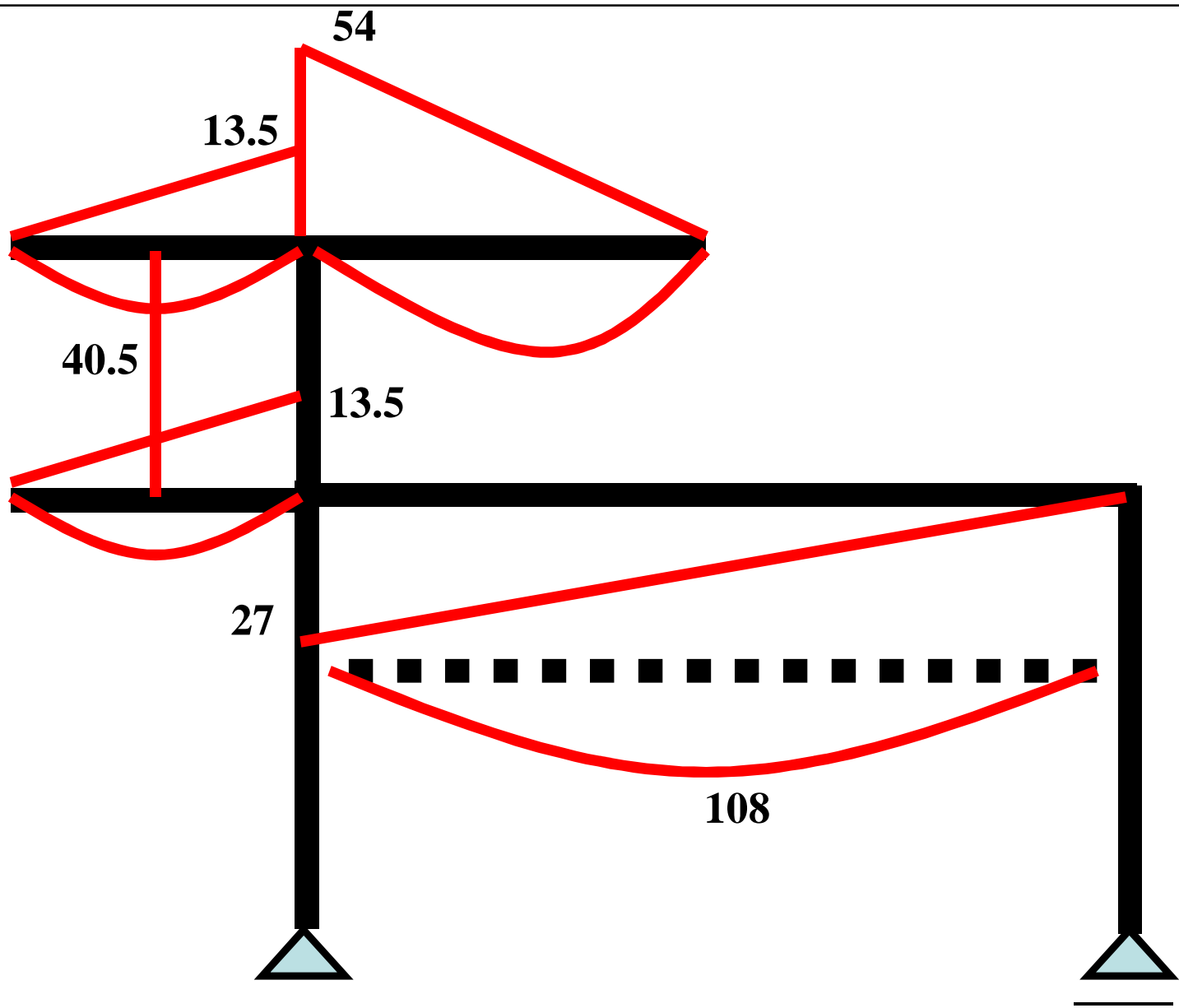
Problem No. 4



For the Shown Frame, Calculate the Rotation at D, Horizontal Displacement at B and Vertical Deflection at G due to:

1- Given Loads.

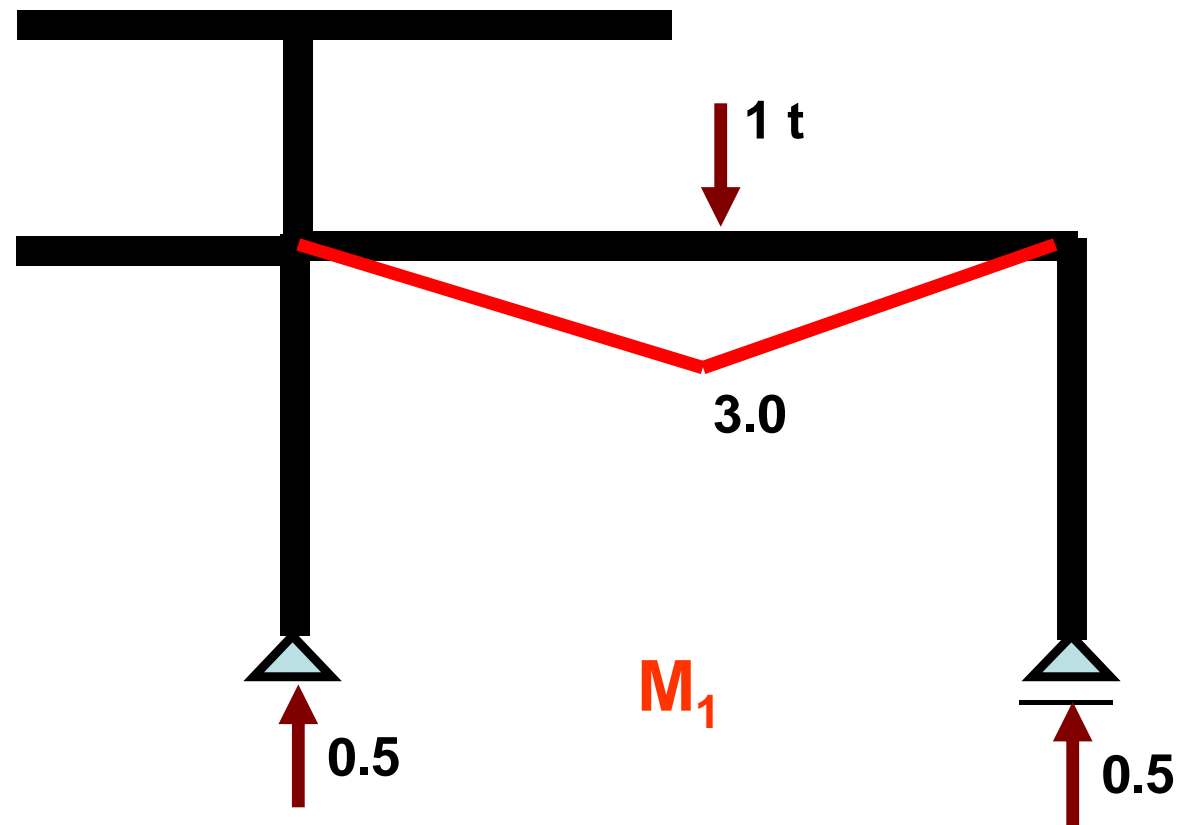
2- Downward Settlement at A = 2cm.



M_0

For Δ_{vg}

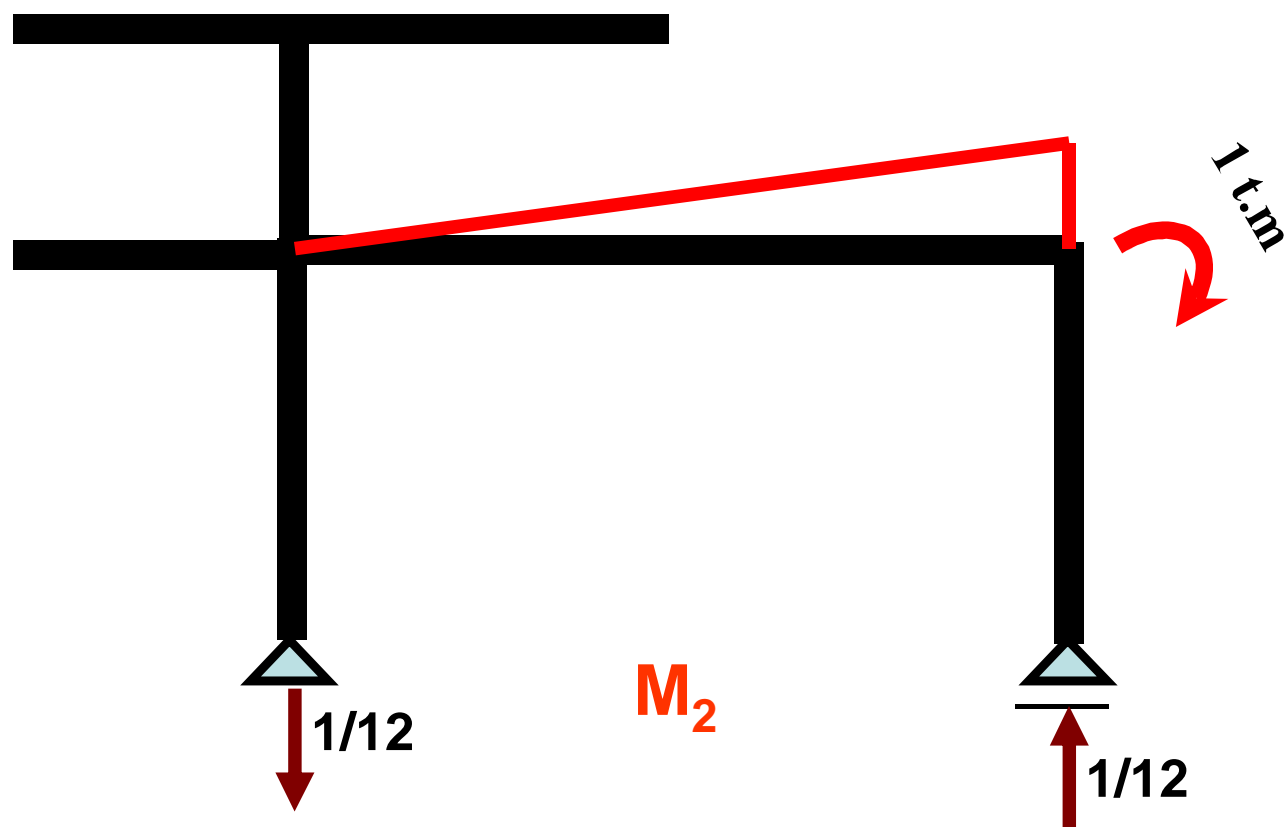
given loads



$$\Delta_{vg} = \frac{1}{EI} \left[-\frac{1}{2} * 3 * 12 * 13.5 + 2 \left(\frac{2}{3} * 108 * 6 * 5/8 * 3 \right) \right] = 0.155$$

For θ_d

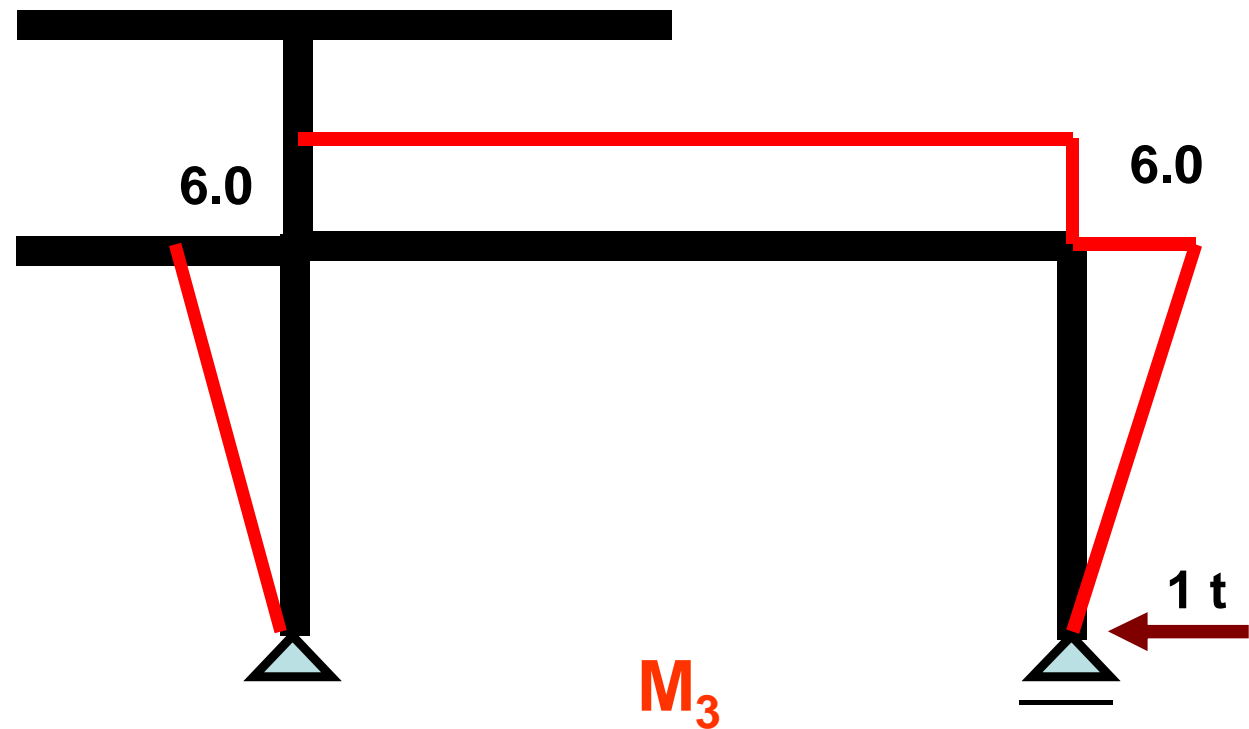
given loads



$$\theta_d = \frac{1}{EI} \left[-\frac{1 * 27 * 12}{6} - \left(\frac{2}{3} * 108 * 12 * 0.5 \right) \right] = -0.004 \text{ rad}$$

For Δ_{hb}

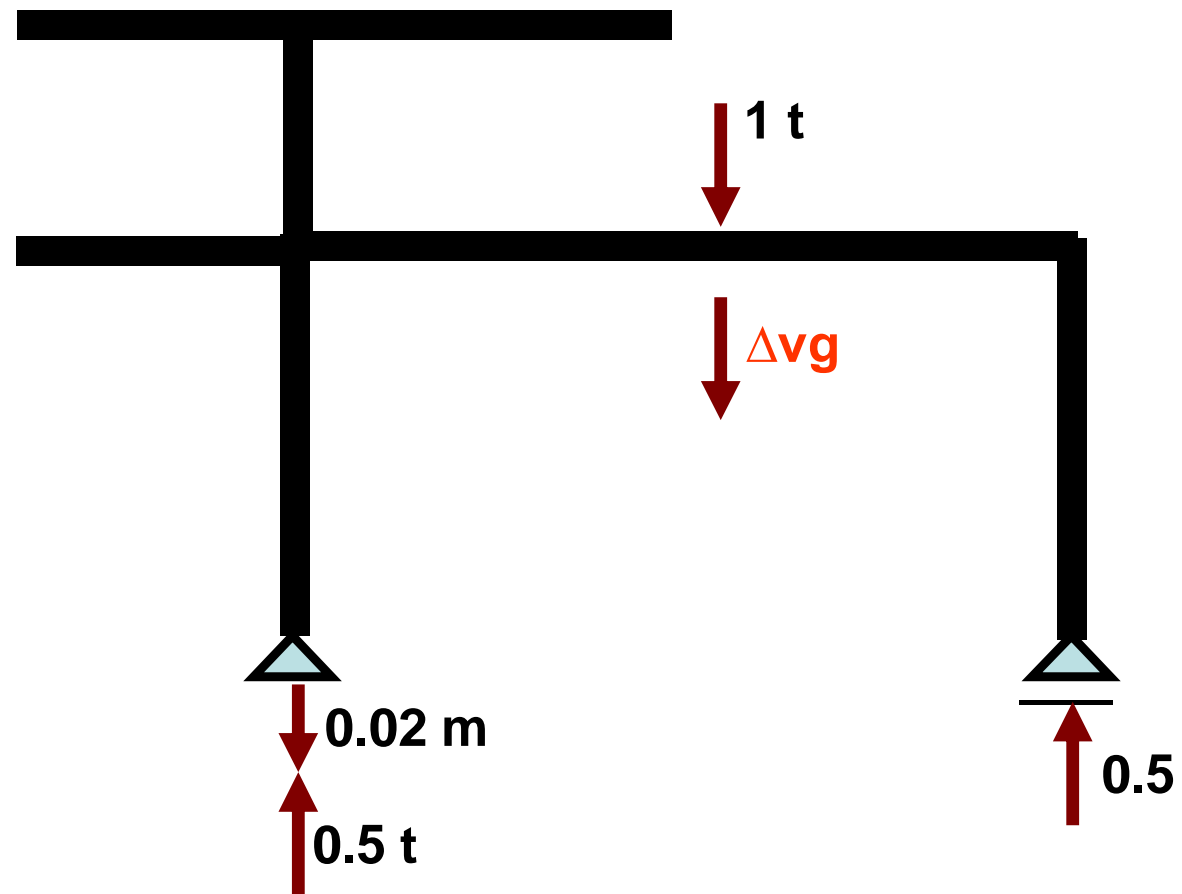
given loads



$$\Delta_{hb} = \frac{1}{EI} \left[-\frac{27 \cdot 6 \cdot 12}{2} - \frac{2}{3} \cdot 108 \cdot 12 \cdot 6 \right] = -0.513m$$

Downward Settlement at A = 2cm

For Δ_{vg}

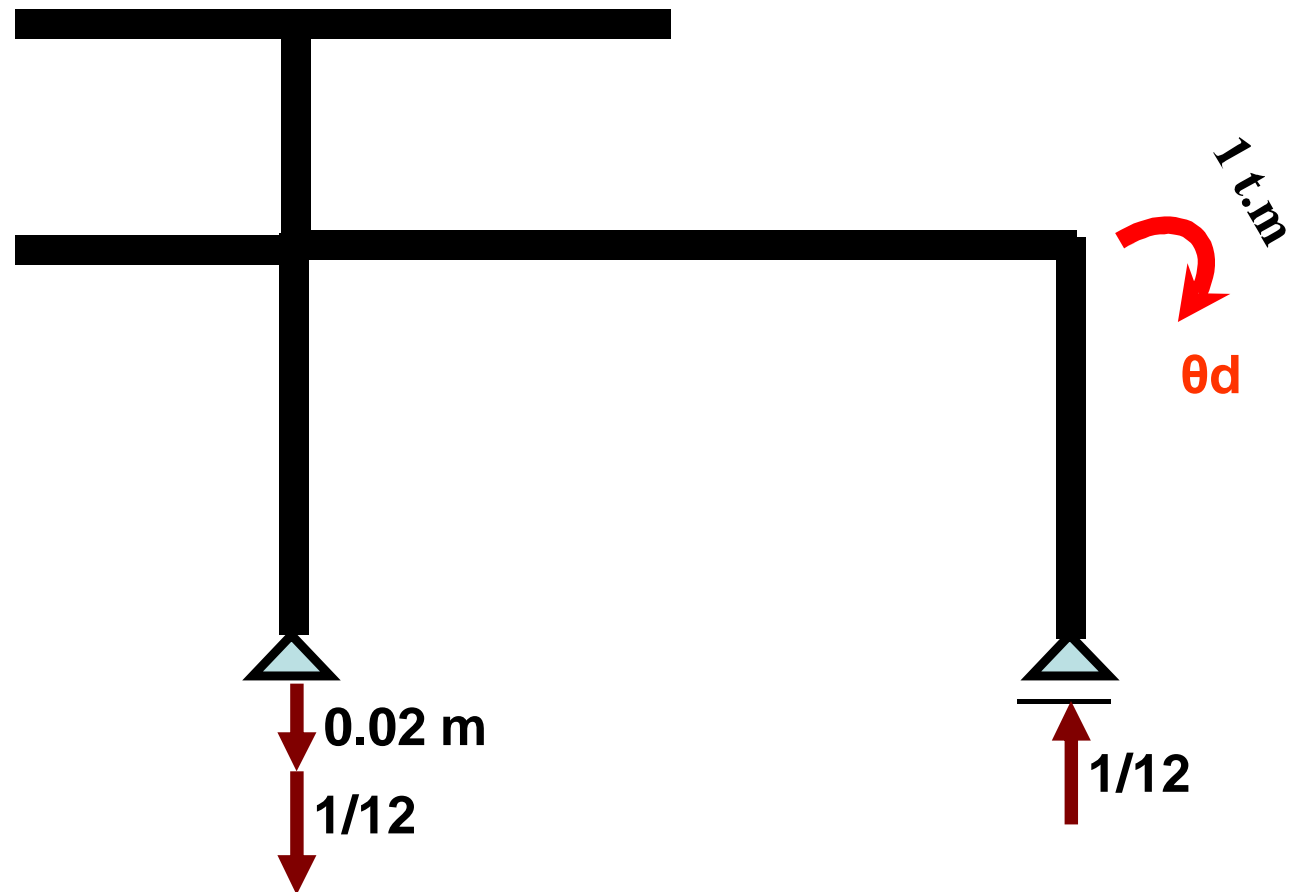


$$1 * \Delta_{vg} - 0.5 * 0.02 = 0.0$$

$$\Delta_{vg} = 0.01 \text{ m}$$

Downward Settlement at A = 2cm

For θ_d

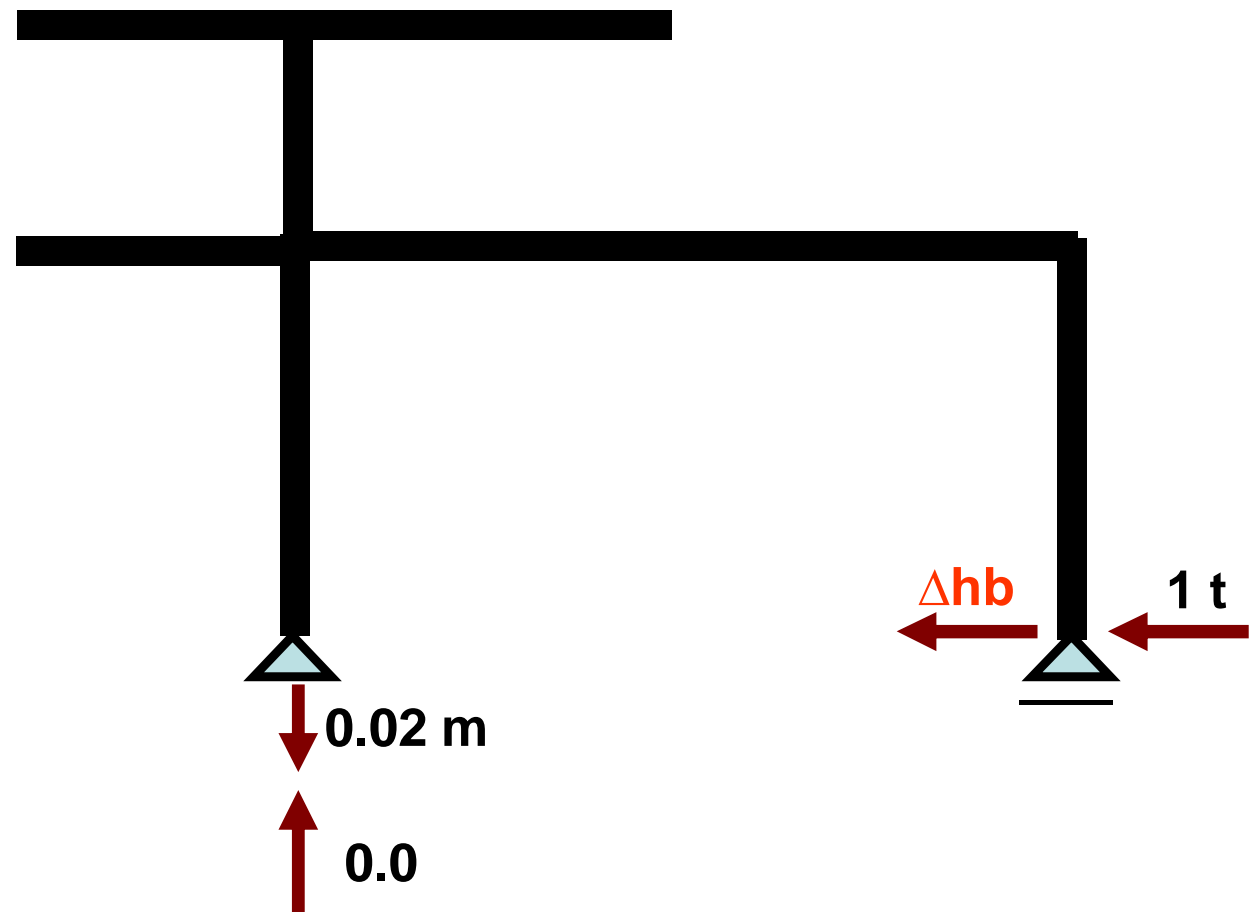


$$1 * \theta_d + 1/12 * 0.02 = 0.0$$

$$\theta_d = -0.0167 \text{ rad}$$

Downward Settlement at A = 2cm

For Δ_{hb}



$$1 * \Delta_{hb} + 0.0 * 0.02 = 0.0$$

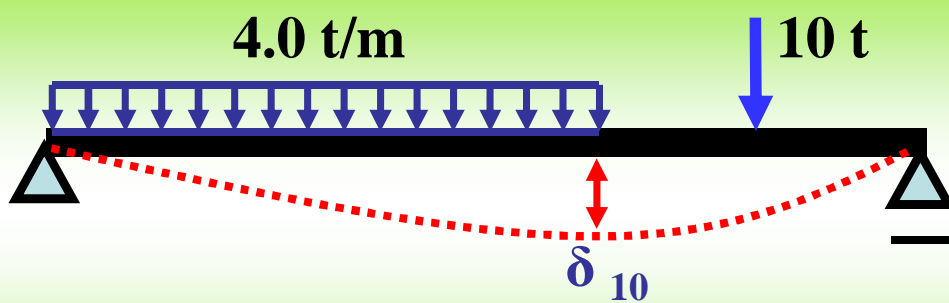
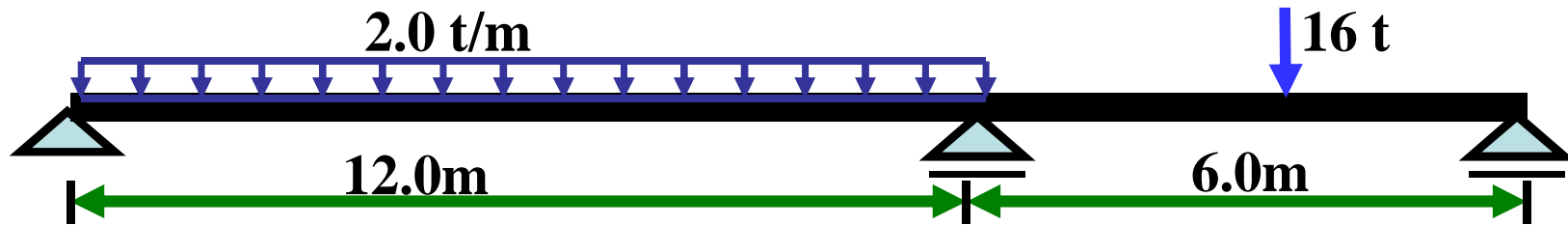
$$\Delta_{hb} = 0.0$$

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

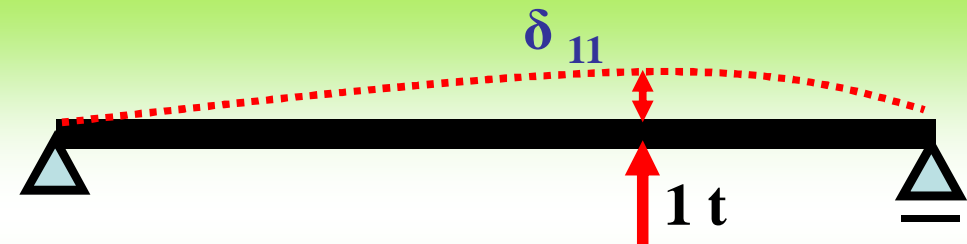
Consistant Deformation

Eng : Aymman abdo

الهدف من دراسة المنشآت (indeterminate structures) بعد دراسة الهبوط هي ان اى support يضاف للكمرة سيتسبب فى زيادة معادلة للهبوط عند هذه النقطة وبالتالى يمكن استخدامها لحساب المجاهيل الزيادة

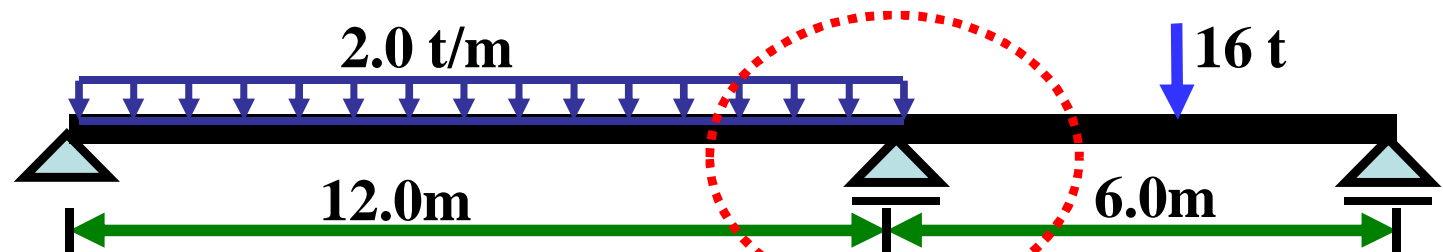


δ_{10} = support حالة عدم وجود



δ_{11} = deformation حالة وجود 1t لاعلى

$R * \delta_{11}$ = reaction حالة وجود كامل لاعلى

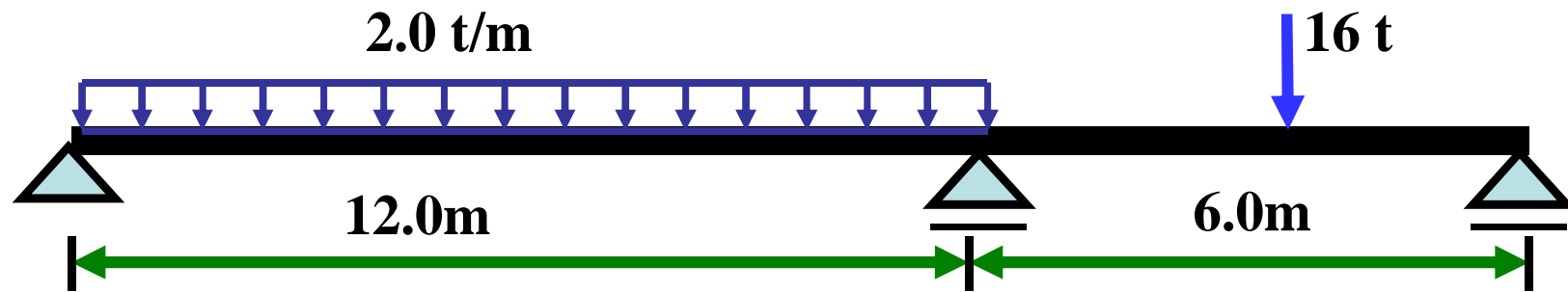


الهبوط النهائى عند هذا الـ 0.0 = support

>>>

$$\delta_{10} + R * \delta_{11} = 0.0$$

Example 1



Draw B.M.D , S.F.D, and N.F.D $EI = 10000$

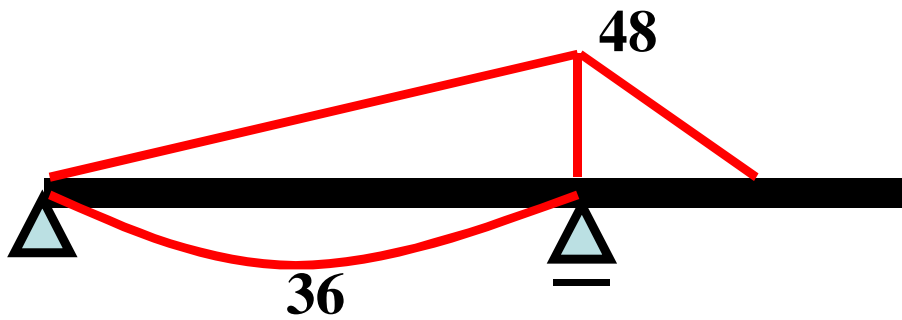
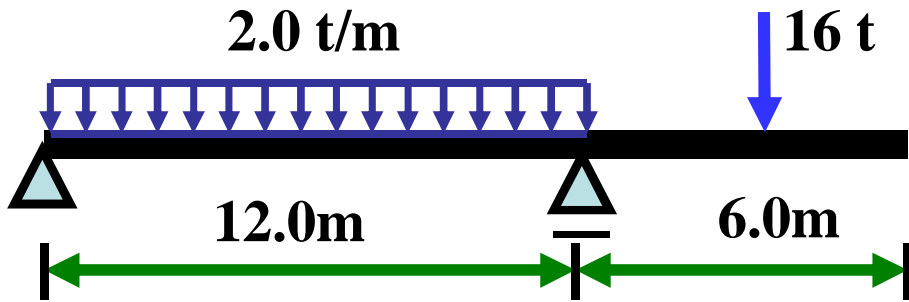
خطوات الحل:

1- يتم عمل main system وذلك بإزالة ال reaction الزيادة على الكمرة المعطاه بشرط انها تبقى متزنه.

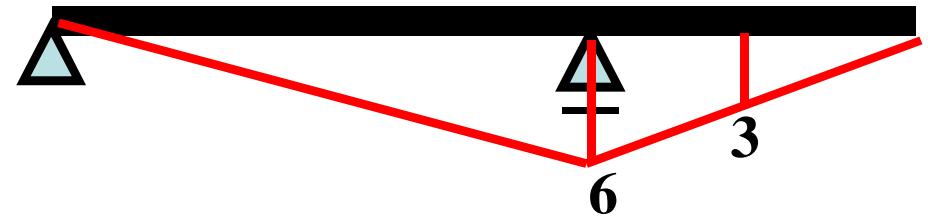
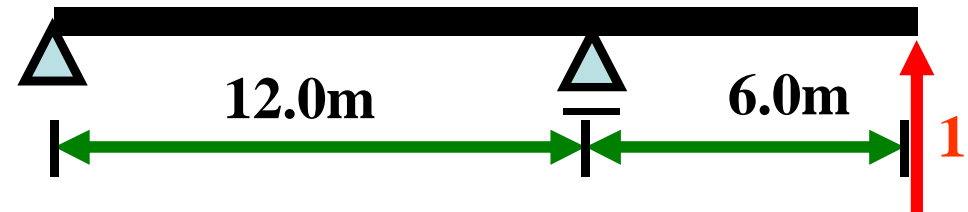
main system



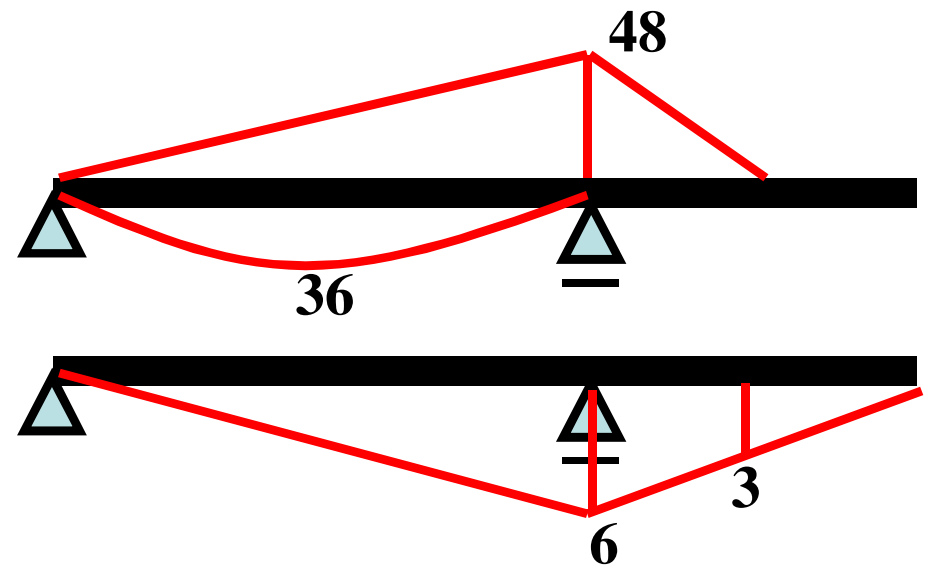
2- draw (M_0) [loads + $X=0$]



3- draw (M_1) [no loads + $X=1$]



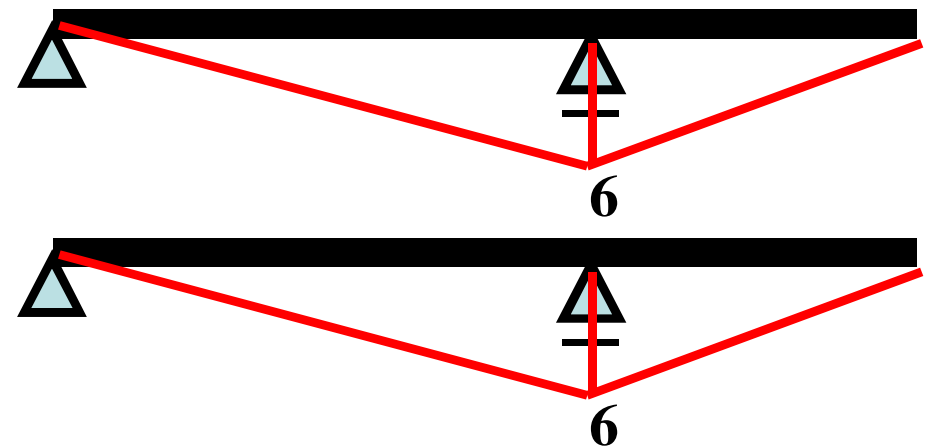
$\delta_{10} = M_1$ مع M_0 تكامل



$$\delta_{10} = \frac{1}{10000} \left[-\frac{48 \cdot 6 \cdot 12}{3} + \left(\frac{2}{3} \cdot 36 \cdot 12 \cdot \frac{1}{2} \cdot 6 \right) - \frac{3}{3} \cdot (6 \cdot 48 + 0.5 \cdot (3 \cdot 48)) \right]$$

$$= -\frac{648}{10000} m$$

$\delta_{11} = M_1$ مع M_1 تكامل

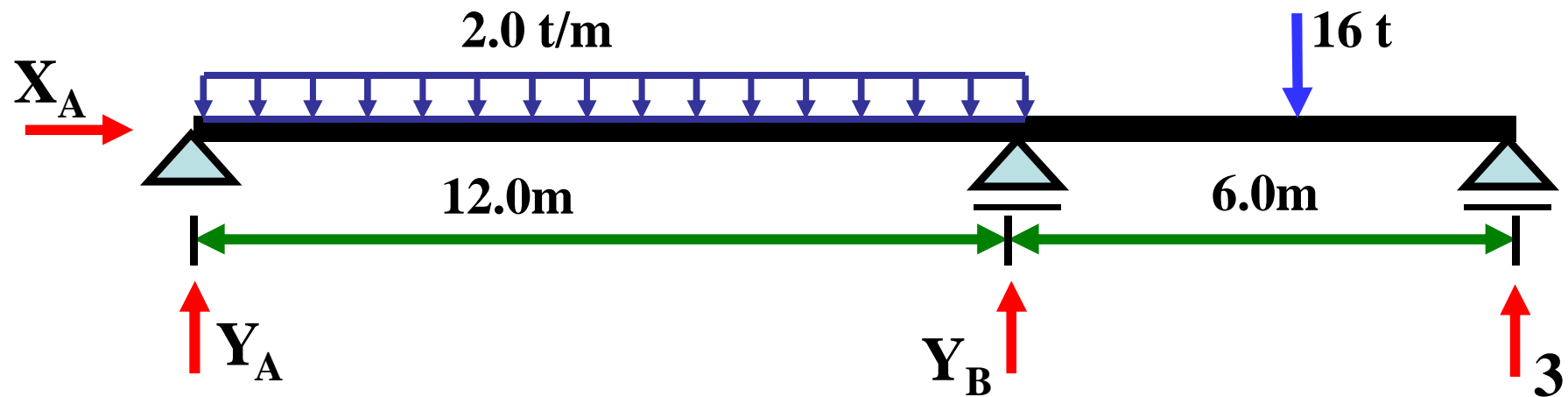


$$\delta_{11} = \frac{1}{10000} \left[\frac{6 \cdot 6 \cdot 12}{3} + \frac{6 \cdot 6 \cdot 6}{3} \right]$$

$$= \frac{216}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{648}{10000} + X * \frac{216}{10000} = 0.0 \gg X = 3 \text{ ton}$$



$$\Sigma \mathbf{M}_A = 0.0$$

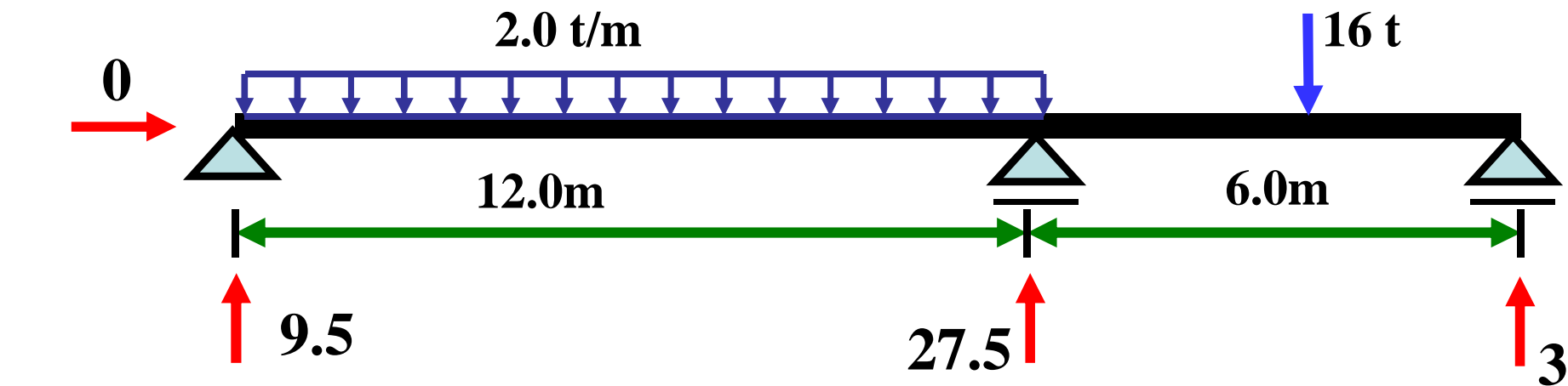
$$24 * 6 + 16 * 15 - Y_B * 12 - 3 * 18 = 0 \gg Y_B = 27.5 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

$$Y_A = 24 + 16 - 27.5 - 3 = 9.5$$

$$\Sigma \mathbf{F}_X = 0.0$$

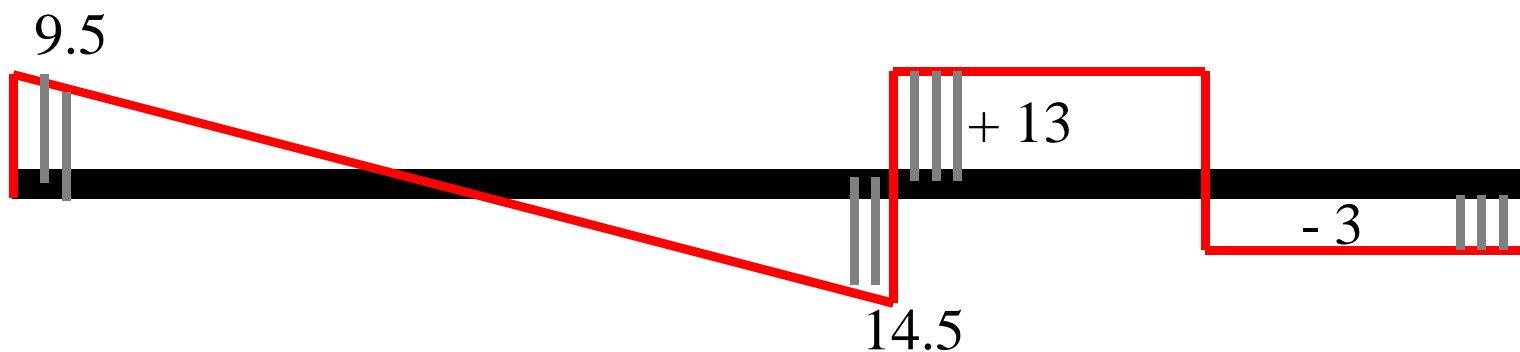
$$X_A = 0.0$$



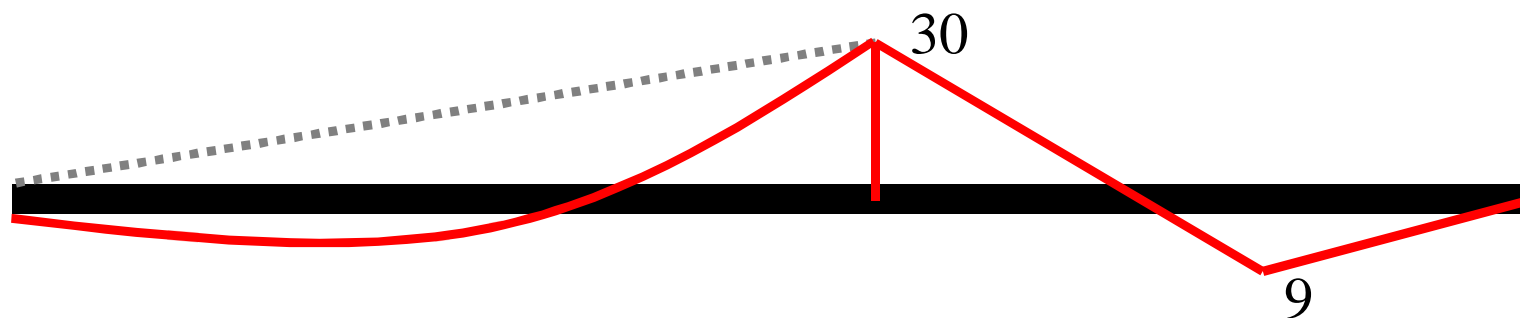
N.F.D

ZERO

S.F.D



B.M.D

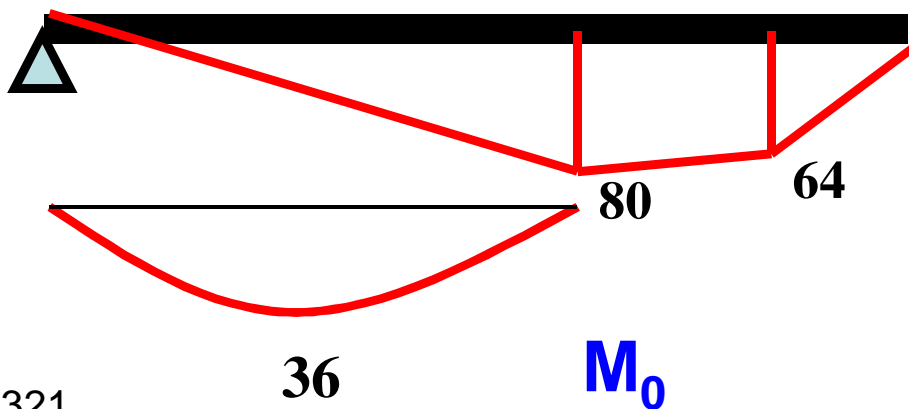
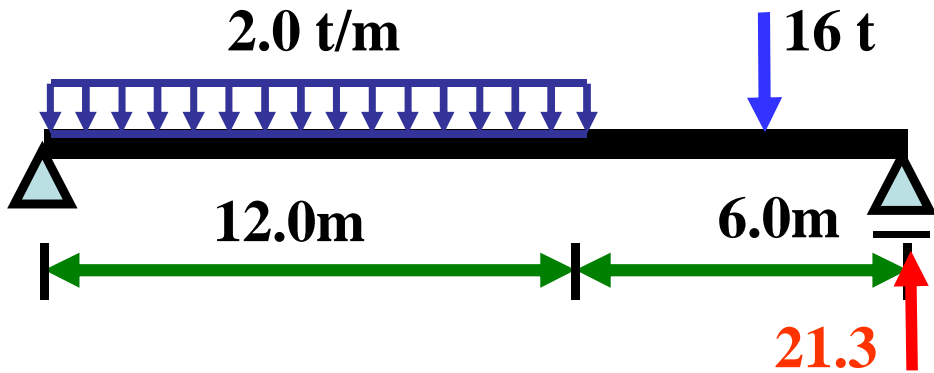


main system

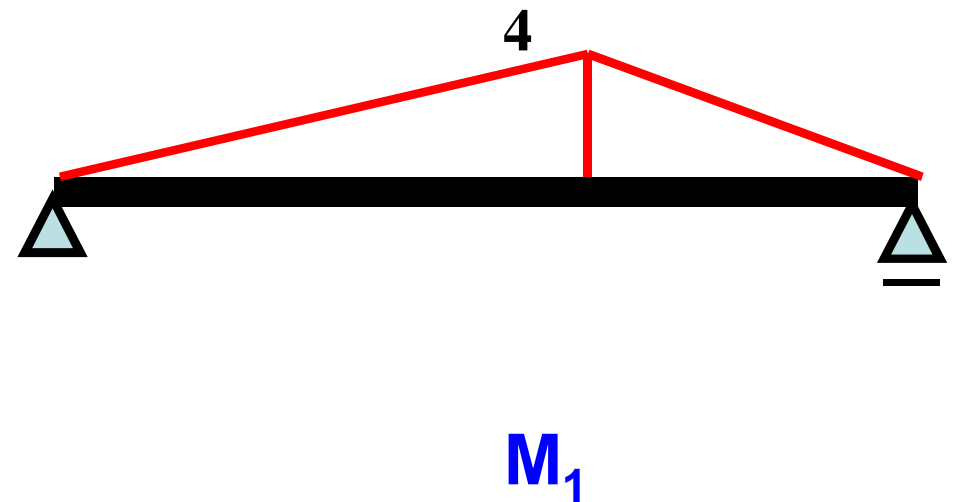
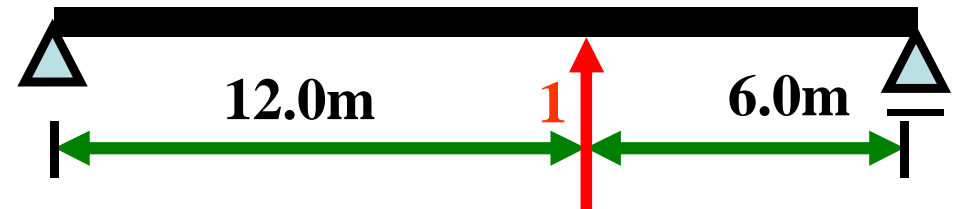
حل اخر:



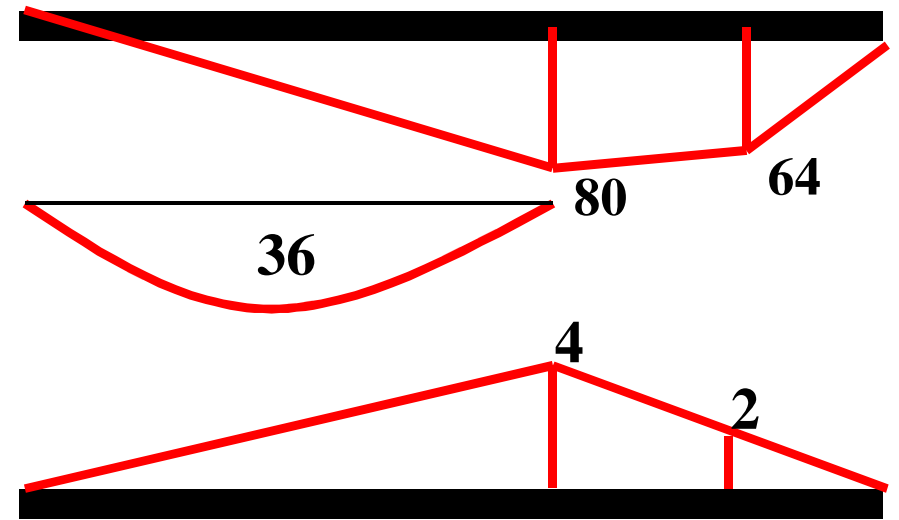
2- draw (M_0) [loads + $X=0$]



3- draw (M_1) [no loads + $X=1$]



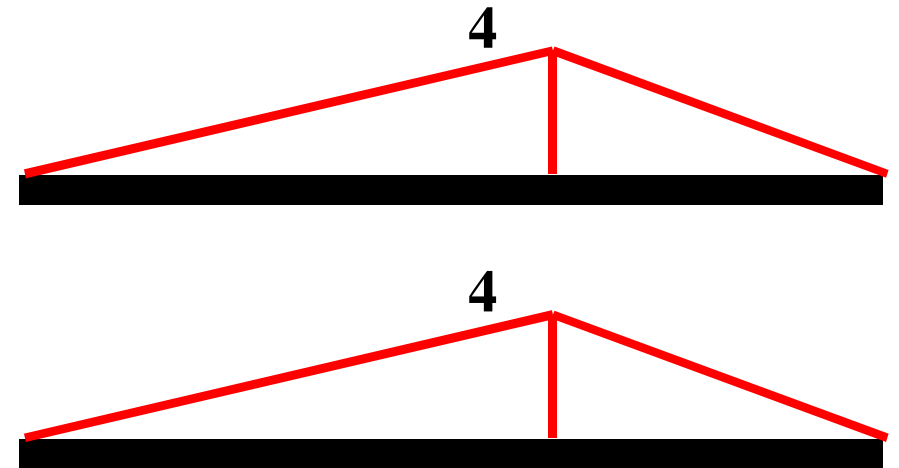
$\delta_{10} = M_1$ مع M_0 تكامل



$$\delta_{10} = \frac{1}{10000} \left[-\frac{80 \cdot 4 \cdot 12}{3} - \left(\frac{2}{3} \cdot 36 \cdot 12 \cdot 2 \right) - \frac{3}{3} \cdot (4 \cdot 80 + 64 \cdot 2 + 0.5 \cdot (4 \cdot 64 + 2 \cdot 80)) - 2 \cdot 64 \cdot 3/3 \right]$$

$$= -\frac{2640}{10000} m$$

$\delta_{11} = M_1$ مع M_1 تكامل

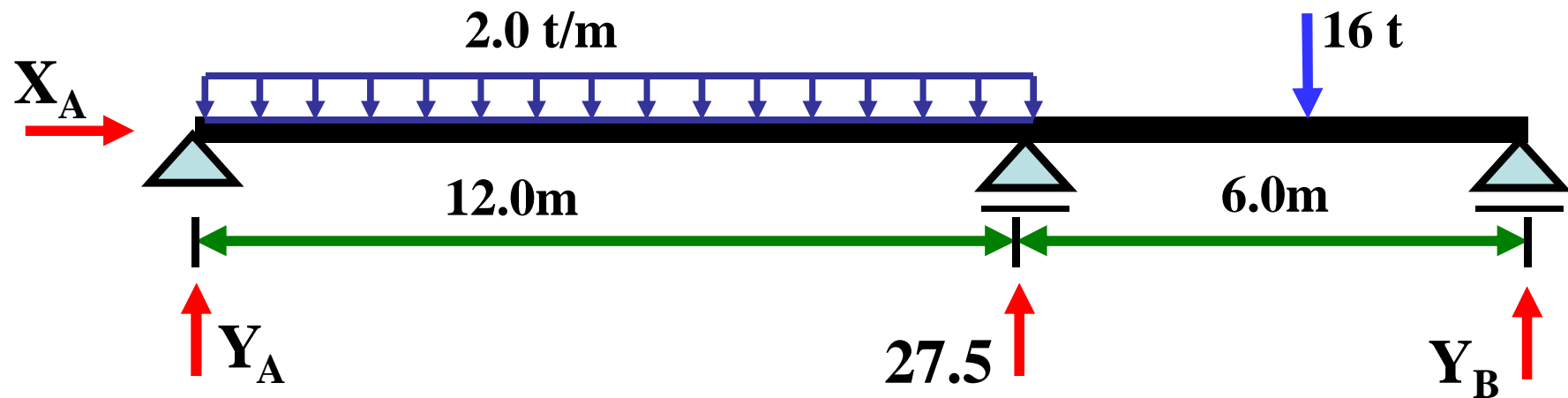


$$\delta_{11} = \frac{1}{10000} \left[\frac{4 \cdot 4 \cdot 12}{3} + \frac{4 \cdot 4 \cdot 6}{3} \right]$$

$$= \frac{96}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{2640}{10000} + X * \frac{96}{10000} = 0.0 \gg X = 27.5 \text{ ton}$$



$$\Sigma \mathbf{M}_A = 0.0$$

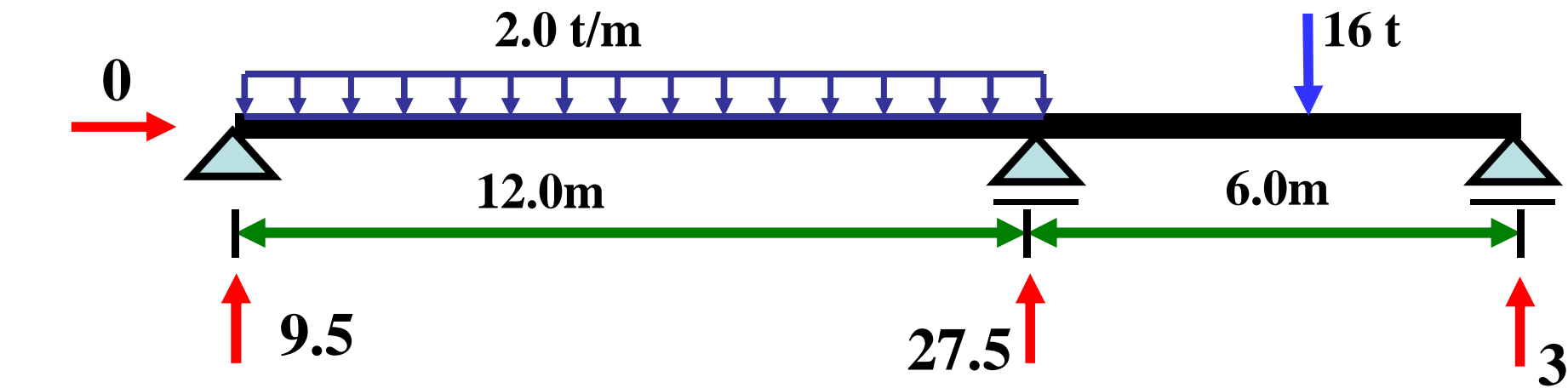
$$24 * 6 + 16 * 15 - Y_c * 18 - 27.5 * 12 = 0 \gg Y_c = 3.0 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

$$Y_A = 24 + 16 - 27.5 - 3 = 9.5$$

$$\Sigma \mathbf{F}_X = 0.0$$

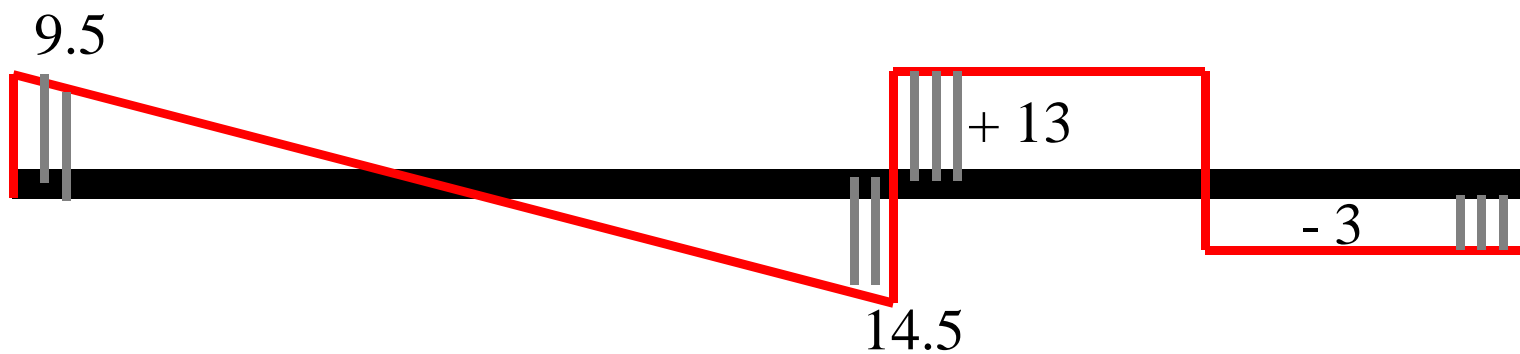
$$X_A = 0.0$$



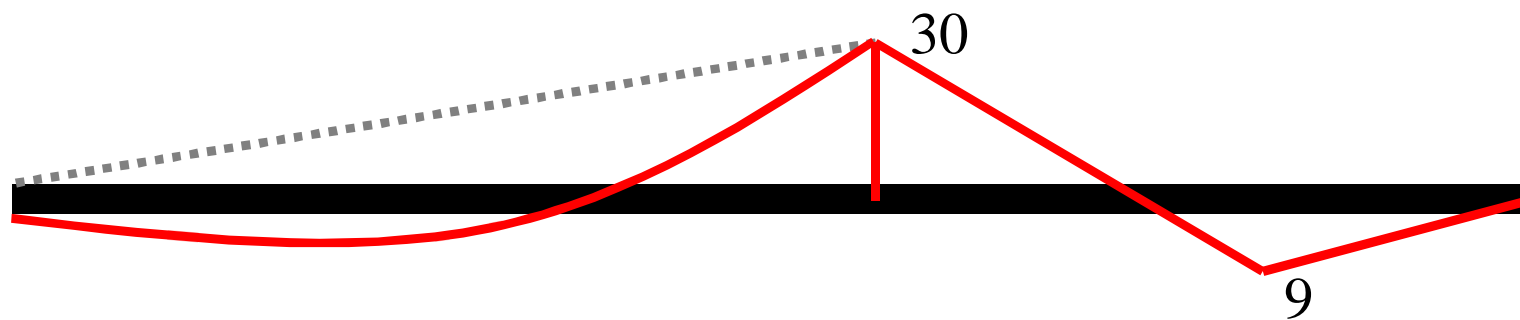
N.F.D

ZERO

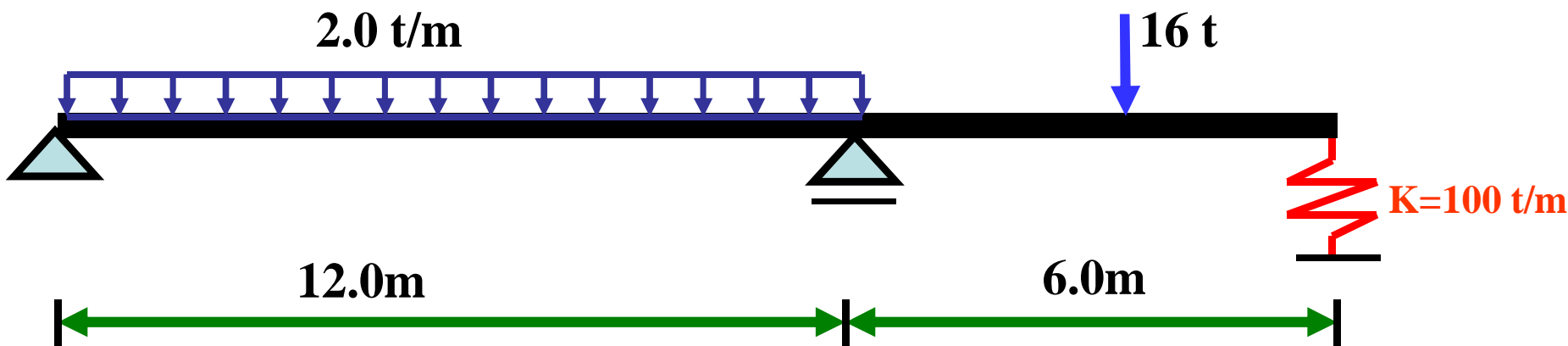
S.F.D



B.M.D



Example 2



Draw B.M.D , S.F.D, and N.F.D EI = 10000

الهبوط النهائي عند هذا الـ

$$R / k = \Delta = \text{spring}$$

>>>

$$\delta_{10} + R * \delta_{11} = - R / K$$

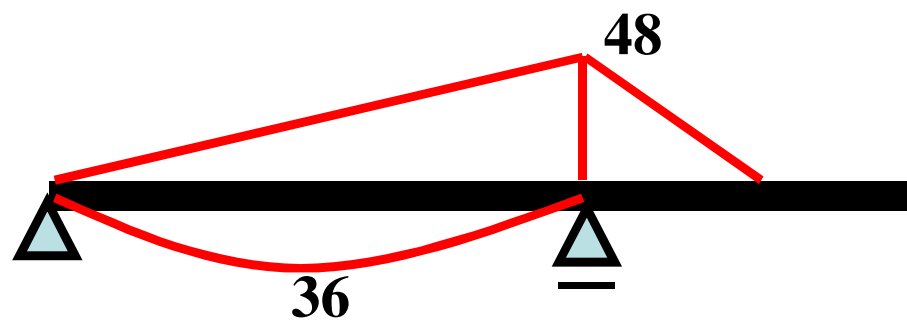
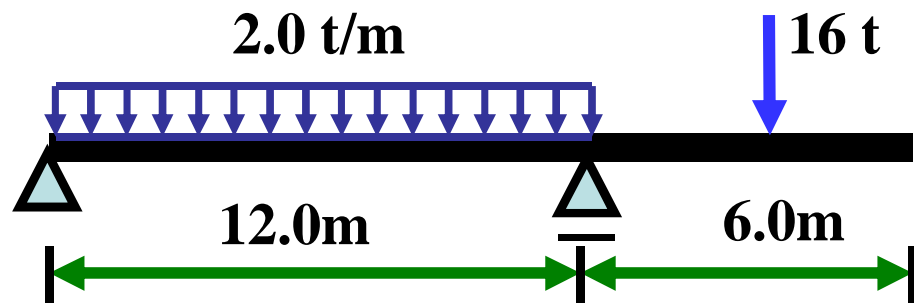
>>>

$$\delta_{10} + R * (\delta_{11} + 1/k) = 0$$

main system

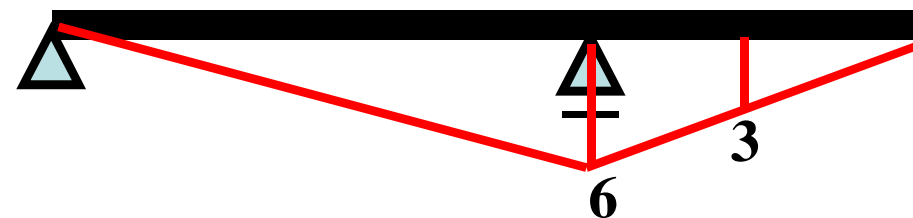
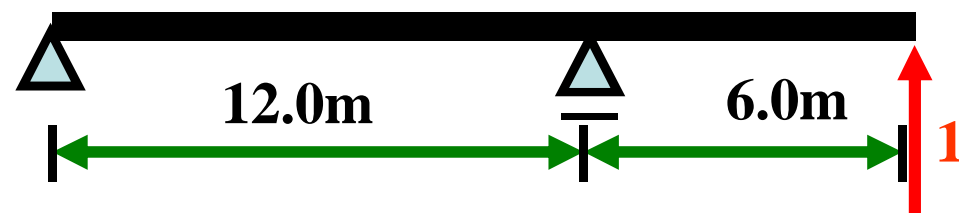


2- draw (M_0) [loads + $X=0$]



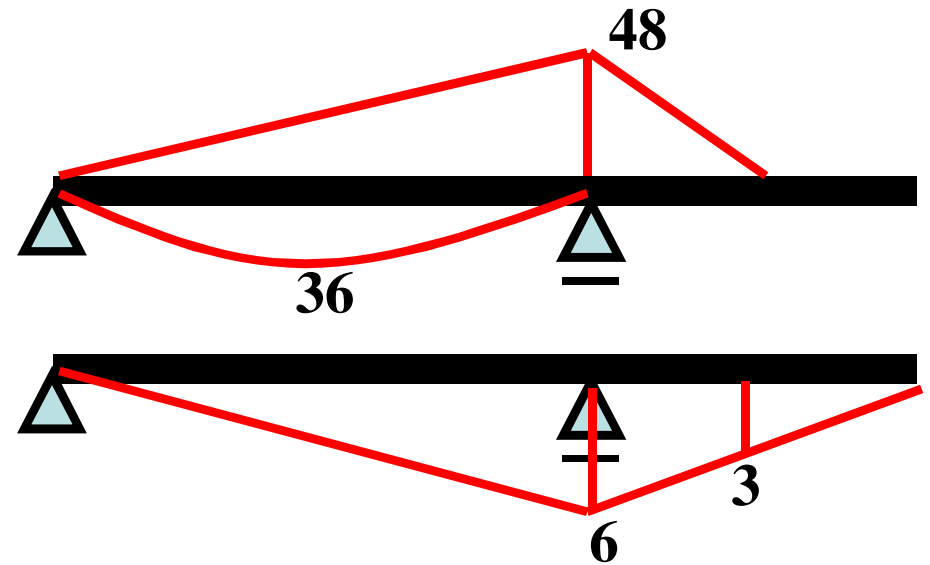
M_0

3- draw (M_1) [no loads + $X=1$]



M_1

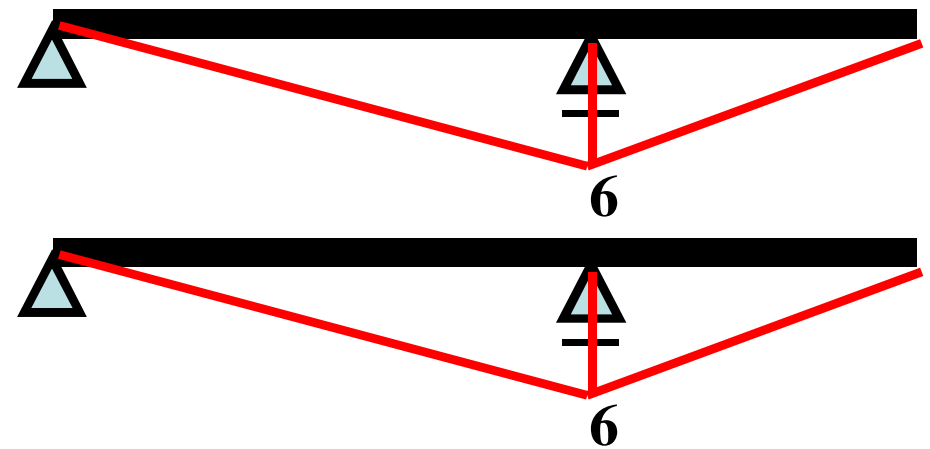
$\delta_{10} = M_1$ مع M_0 تكامل



$$\delta_{10} = \frac{1}{10000} \left[-\frac{48 * 6 * 12}{3} + \left(\frac{2}{3} * 36 * 12 * 1/2 * 6 \right) - \frac{3}{3} * (6 * 48 + 0.5 * (3 * 48)) \right]$$

$$= -\frac{648}{10000} m$$

$\delta_{11} = M_1$ مع M_1 تكامل

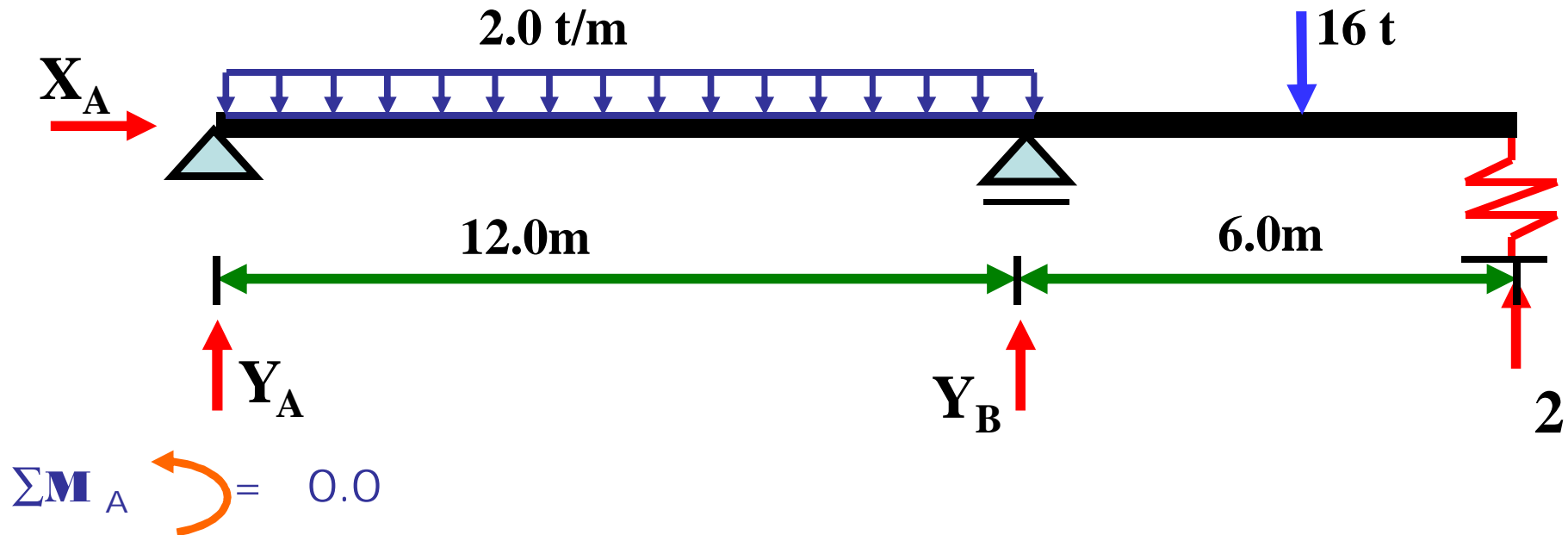


$$\delta_{11} = \frac{1}{10000} \left[\frac{6 * 6 * 12}{3} + \frac{6 * 6 * 6}{3} \right]$$

$$= \frac{216}{10000} m$$

$$\delta_{10} + X \cdot (\delta_{11} + 1/k) = 0.0$$

$$-\frac{648}{10000} + X \cdot \left(\frac{216}{10000} + \frac{1}{100} \right) = 0.0 \gg X = 2 \text{ ton}$$



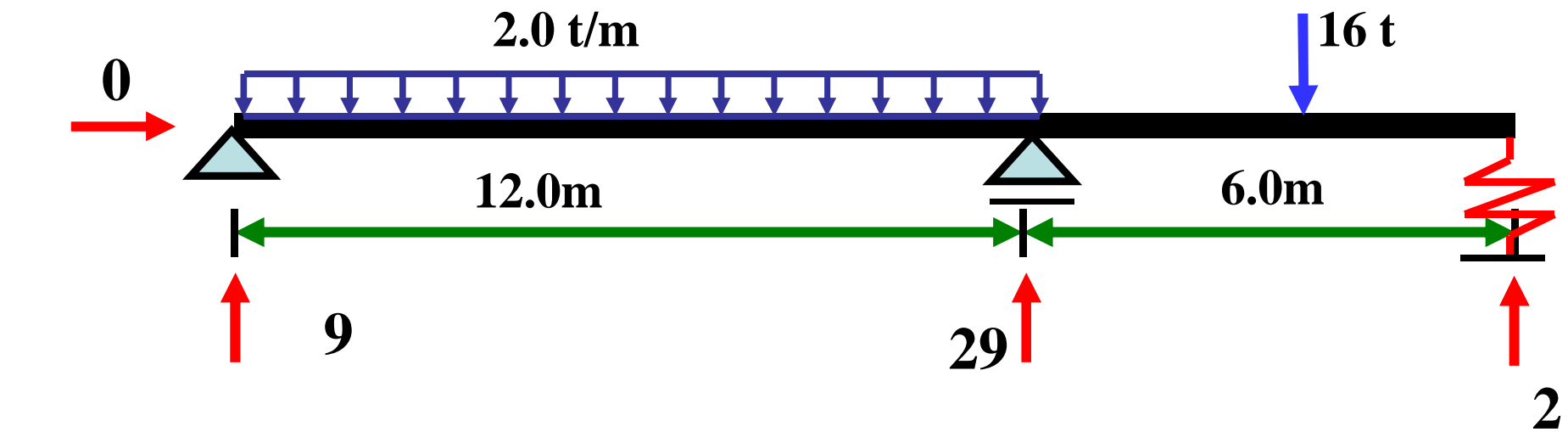
$$24 \cdot 6 + 16 \cdot 15 - Y_B \cdot 12 - 2 \cdot 18 = 0 \gg Y_B = 29 \text{ t}$$

$$\sum F_Y = 0.0$$

$$Y_A = 24 + 16 - 29 - 2 = 9$$

$$\sum F_X = 0.0$$

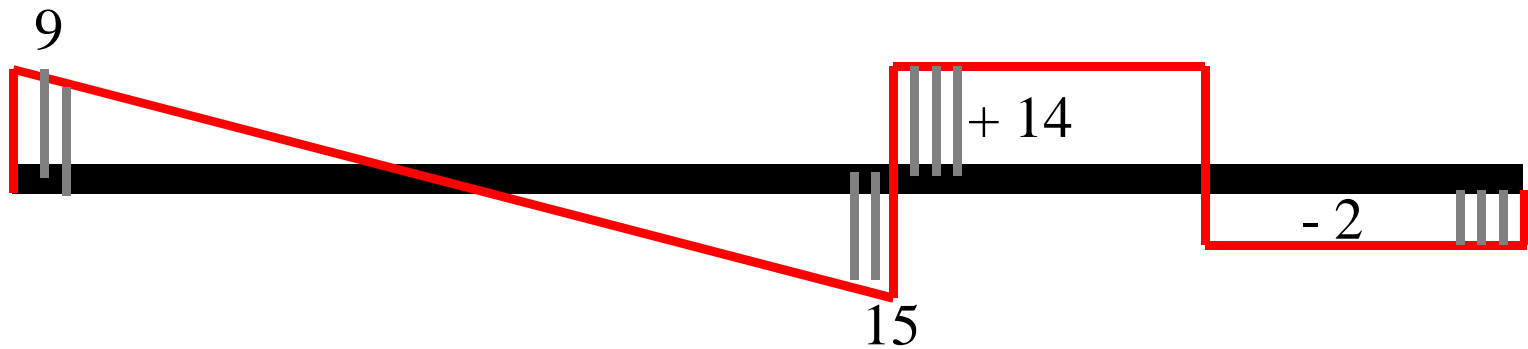
$$X_A = 0.0$$



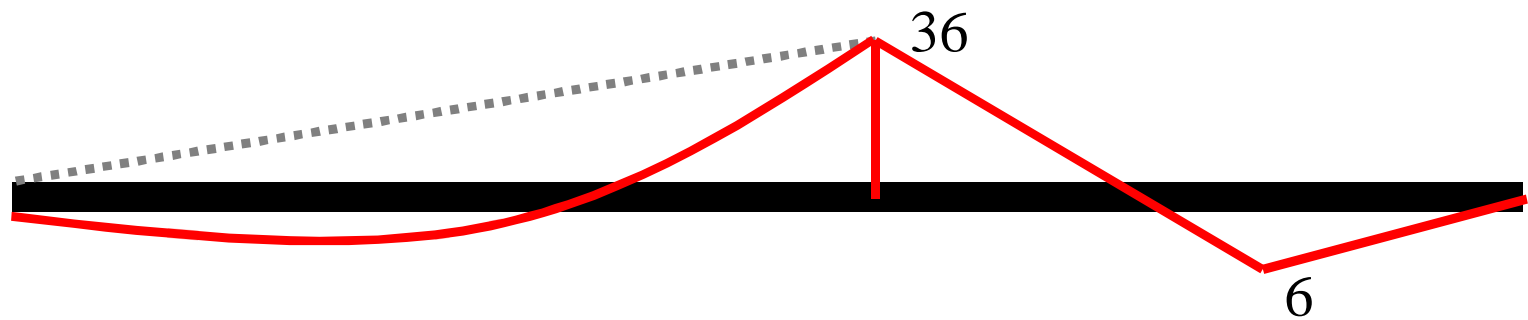
N.F.D

ZERO

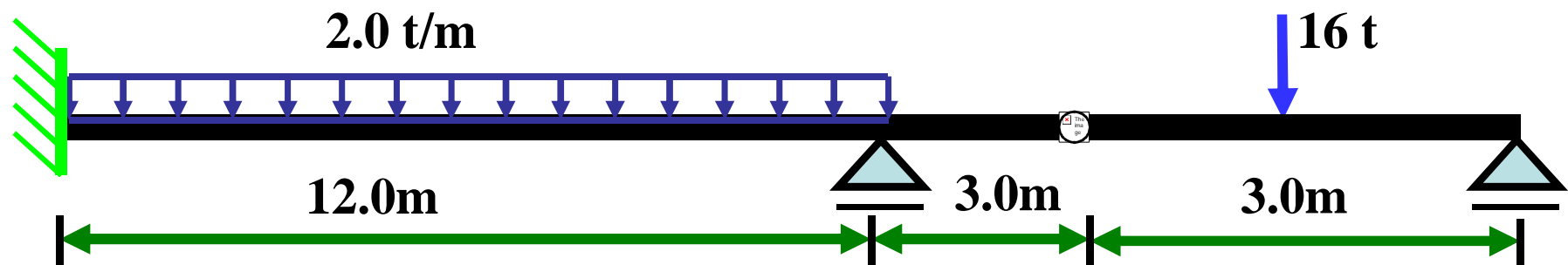
S.F.D



B.M.D

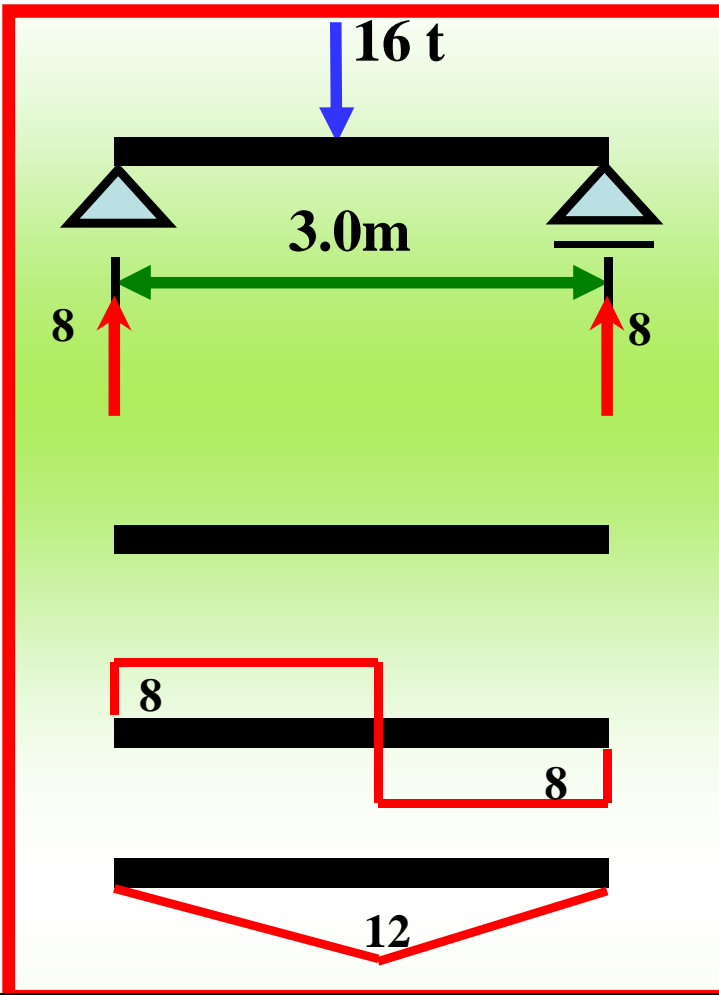
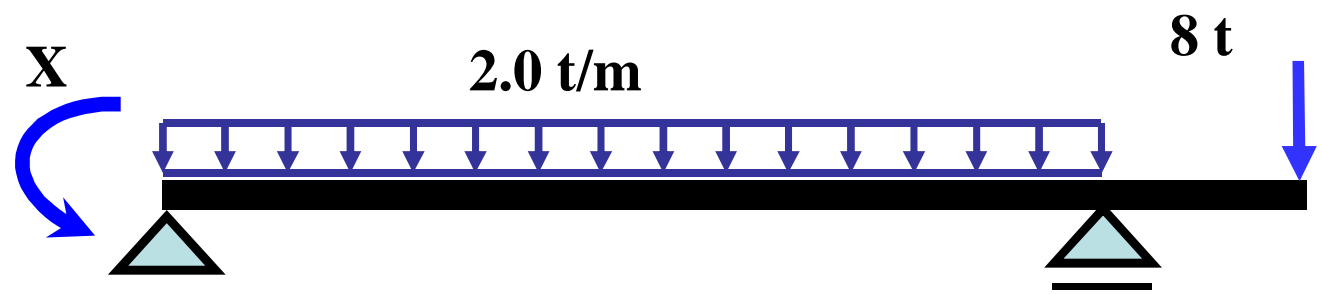


Example 3

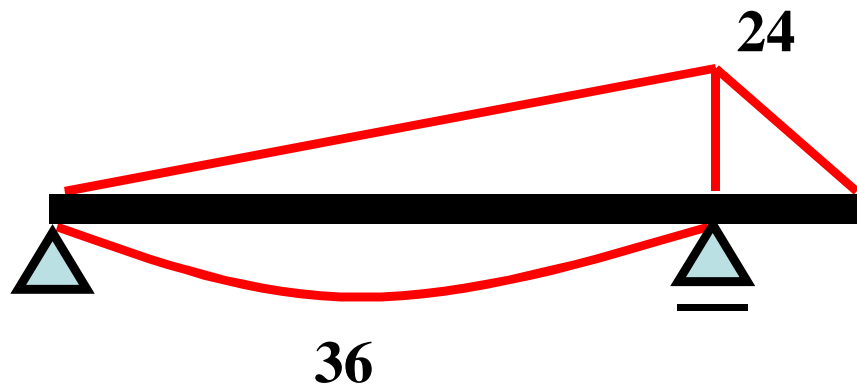
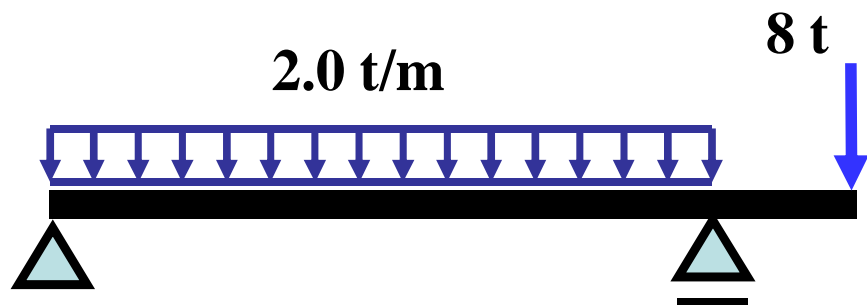


Draw B.M.D , S.F.D, and N.F.D $EI = 10000$

main system

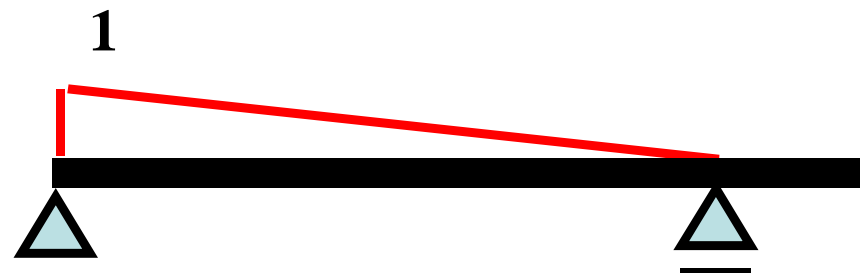


2- draw (M_0) [loads + $X=0$]



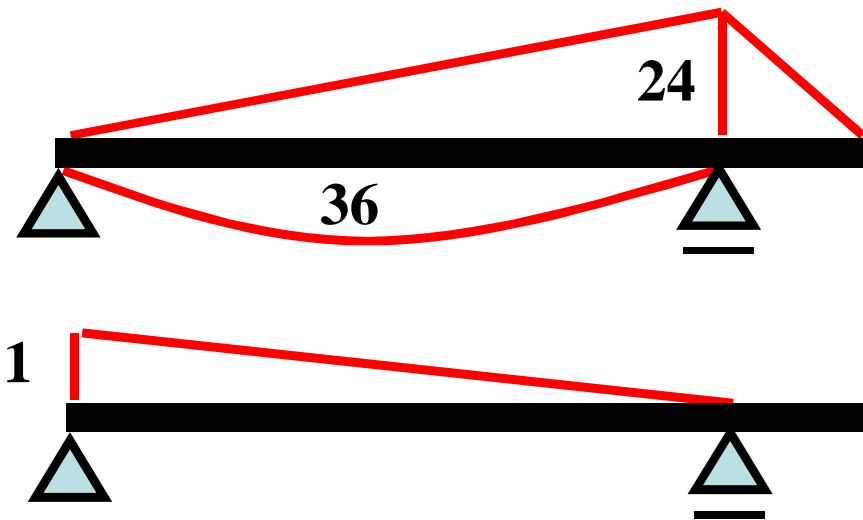
M_0

3- draw (M_1) [no loads + $X=1$]



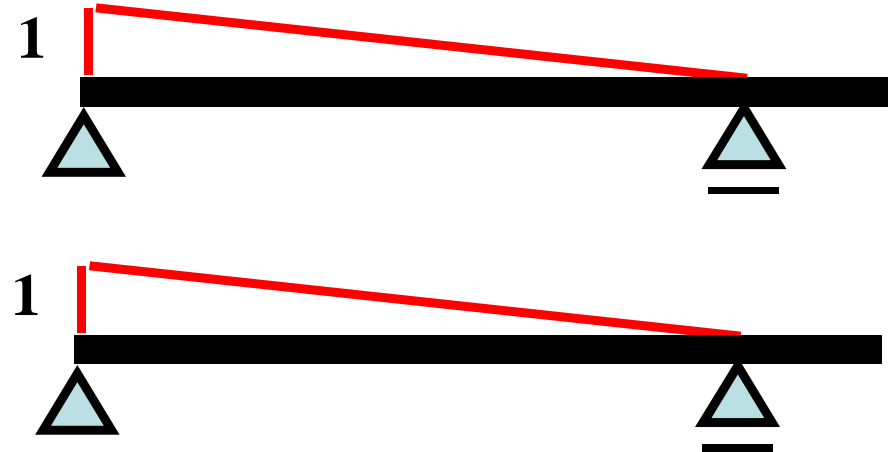
M_1

تکامل مع M_0 $\delta_{10} = M_1$



$$\delta_{10} = \frac{1}{10000} \left[\frac{24 * 1 * 12}{6} - \left(\frac{2}{3} * 36 * 12 * 0.5 \right) \right] = - \frac{96}{10000} m$$

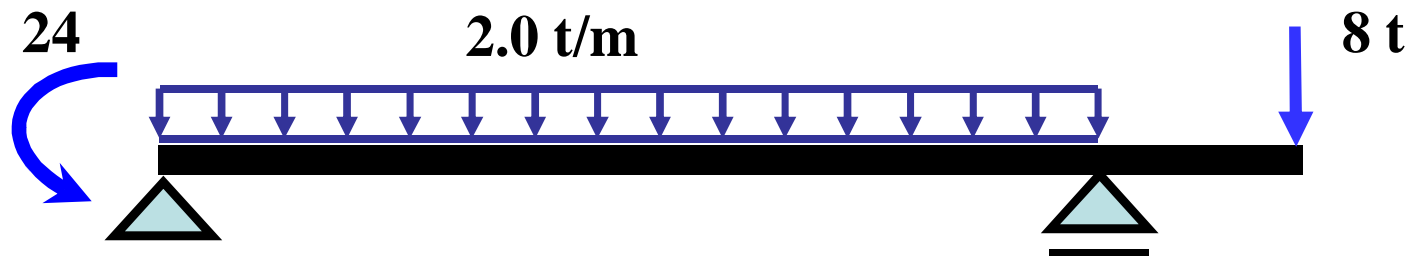
تکامل مع M_1 $\delta_{11} = M_1$



$$\delta_{11} = \frac{1}{10000} \left[\frac{1 * 1 * 12}{3} \right] = \frac{4}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{96}{10000} + X * \frac{4}{10000} = 0.0 \gg X = 24 \text{ t.m}$$



$$\Sigma \mathbf{M}_A = 0.0$$

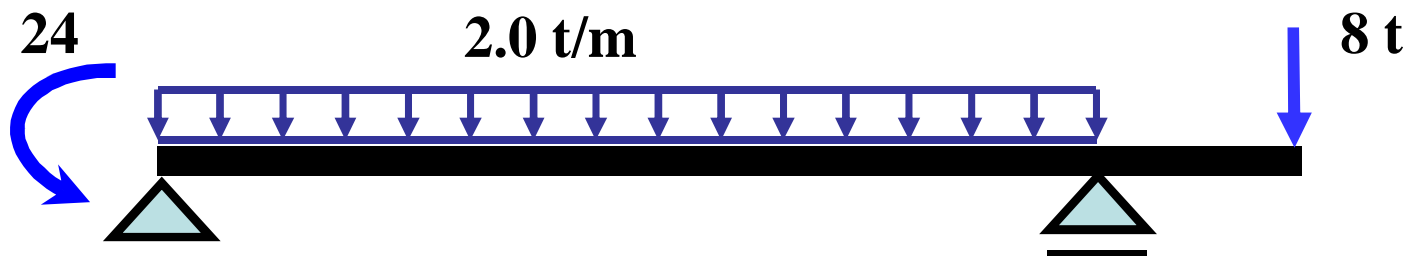
$$16 * 6 - Y_B * 12 + 8 * 15 - 24 = 0.0 \gg Y_B = 16 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

$$Y_A = 24 + 8 - 16 = 16$$

$$\Sigma \mathbf{F}_X = 0.0$$

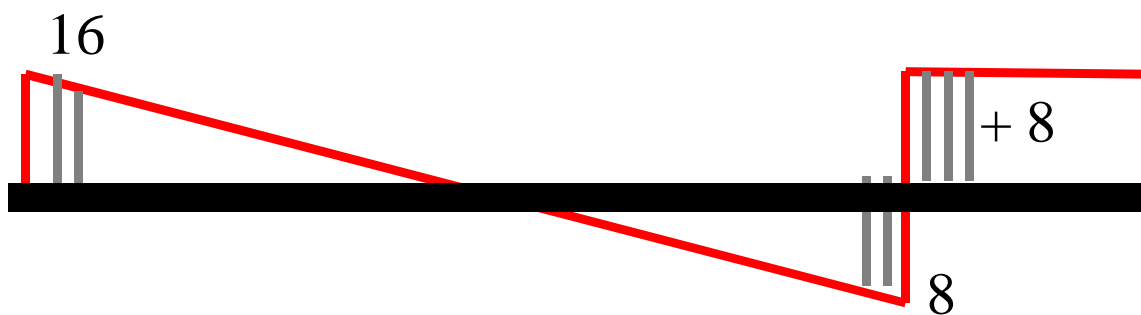
$$X_A = 0.0$$



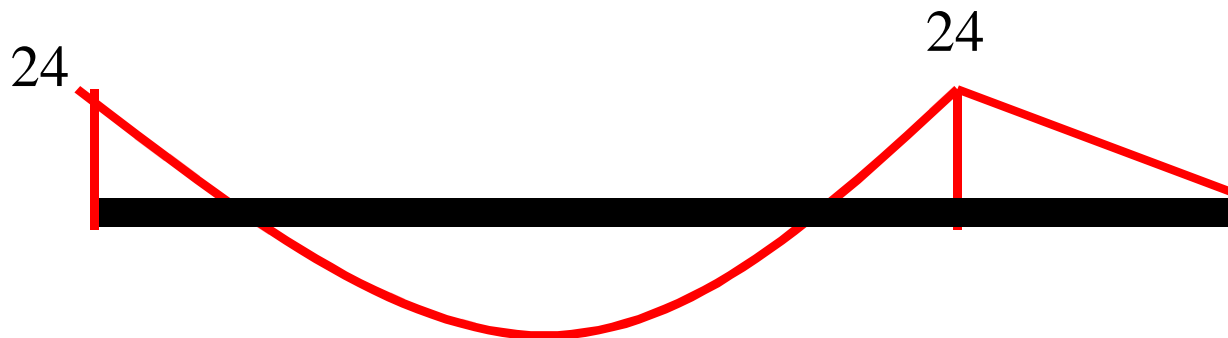
N.F.D

ZERO

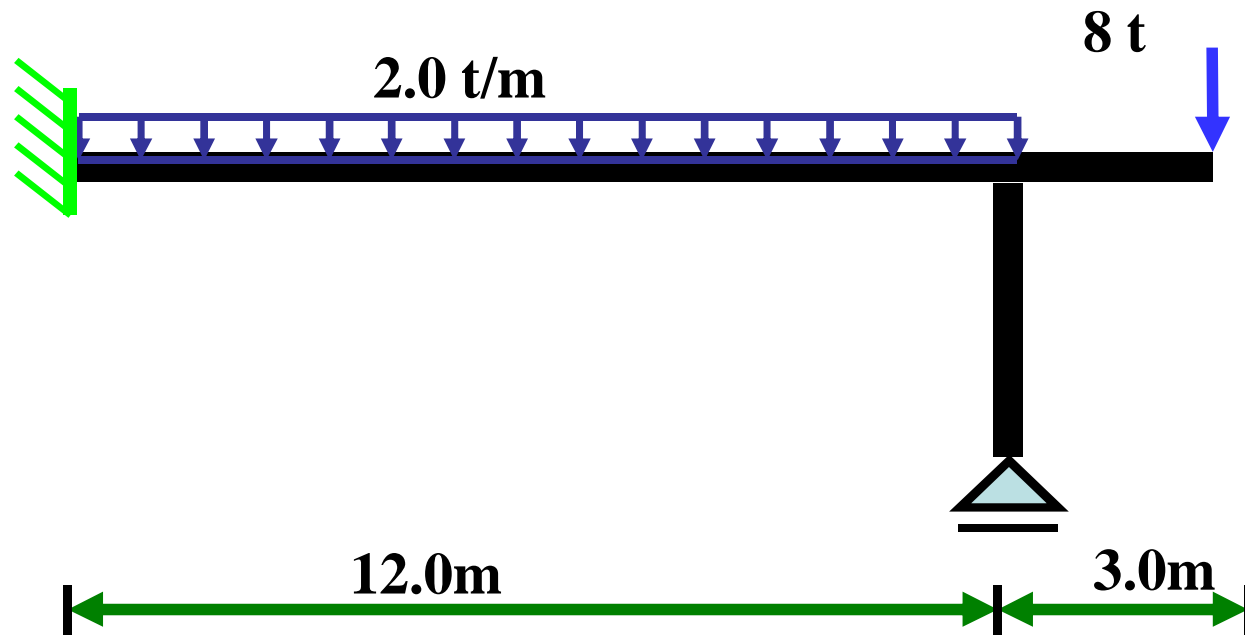
S.F.D



B.M.D

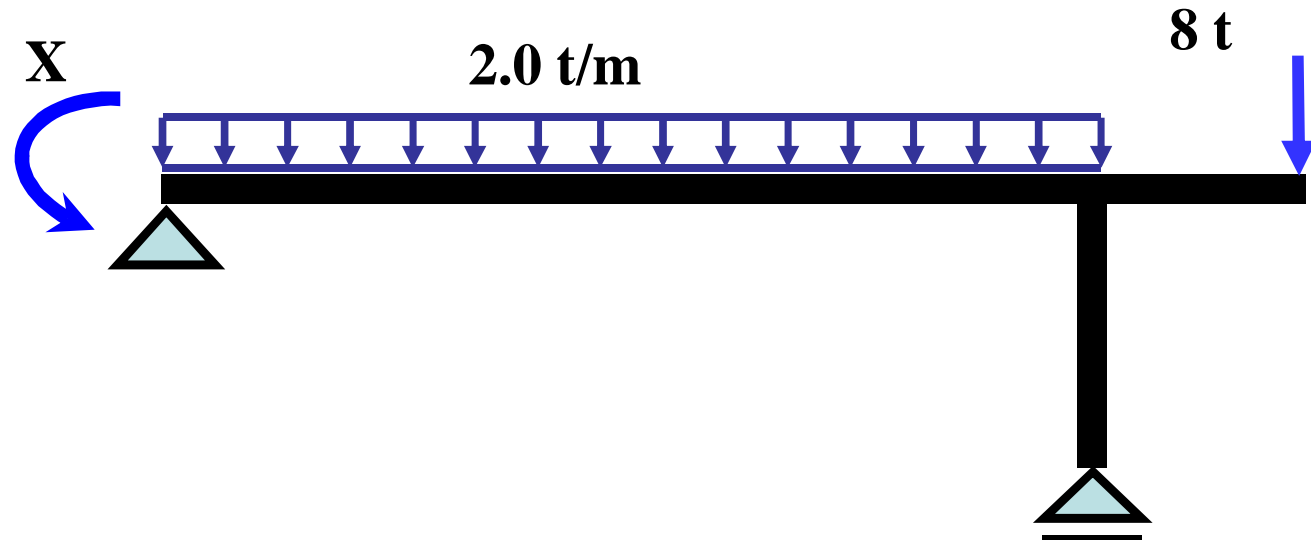


Example 4

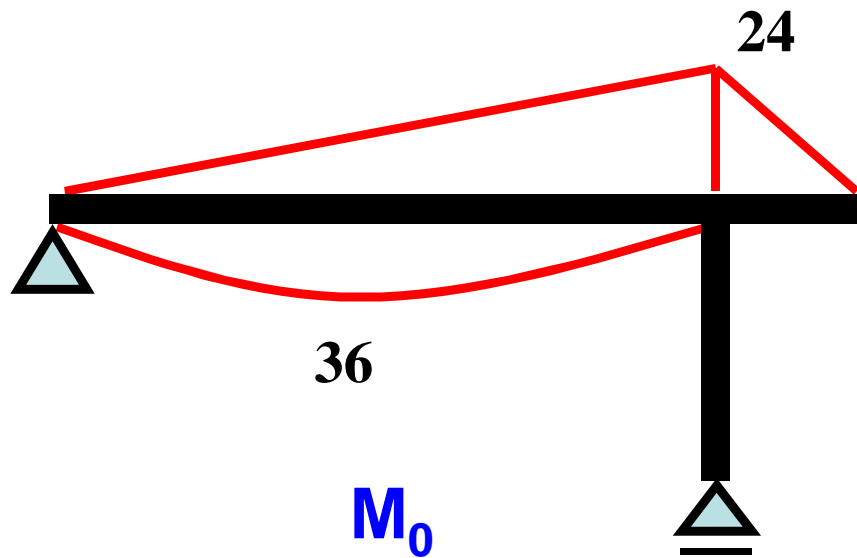
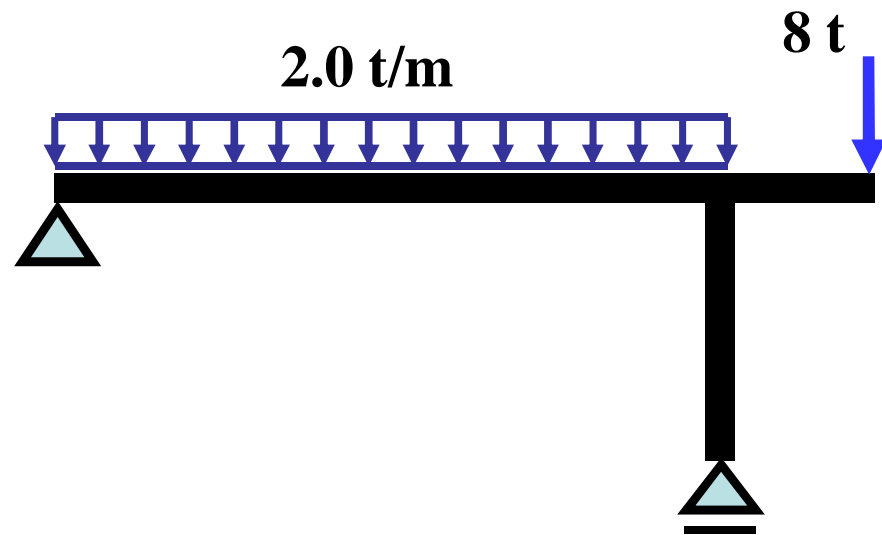


Draw B.M.D , S.F.D, and N.F.D $EI = 10000$

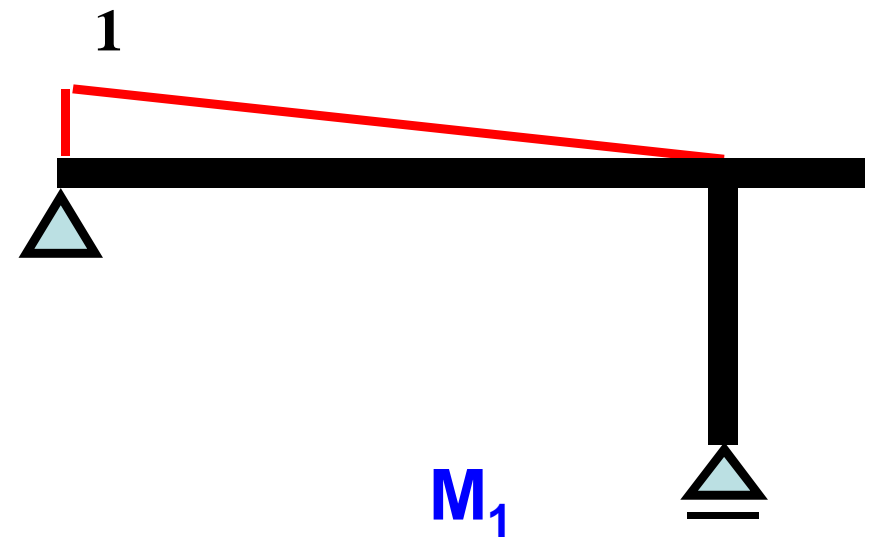
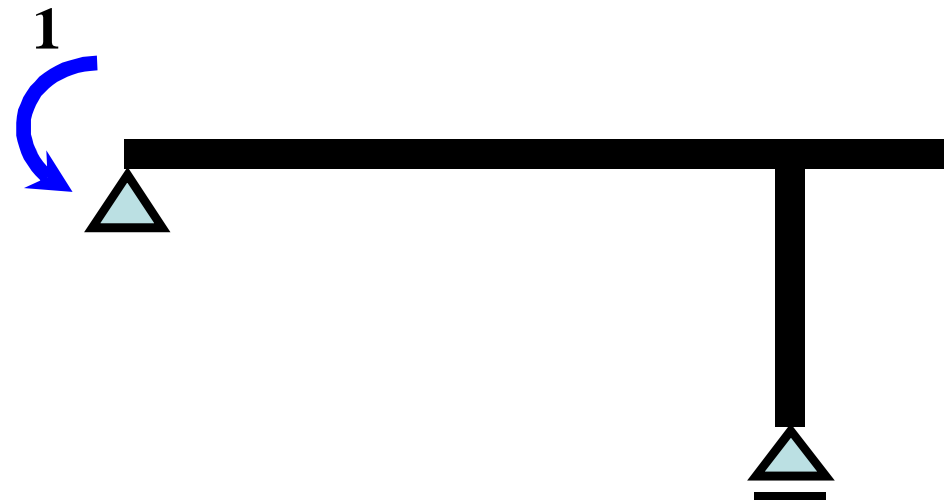
main system



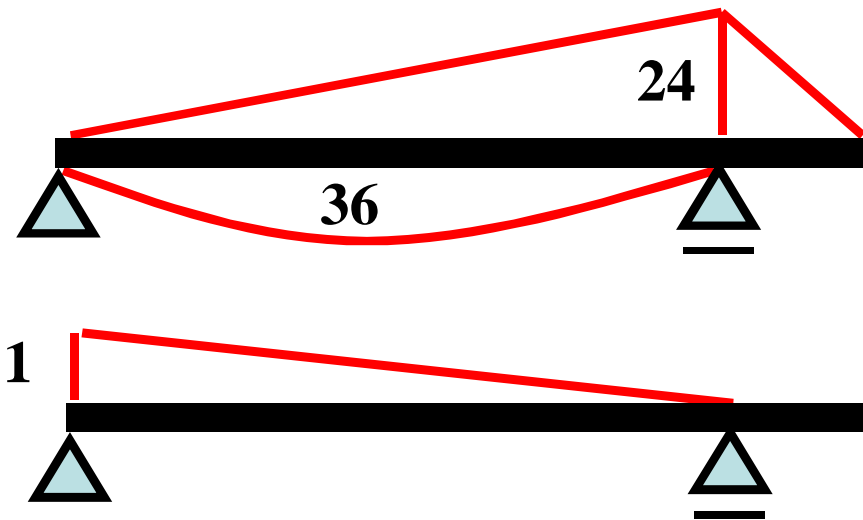
2- draw (M_0) [loads + $X=0$]



3- draw (M_1) [no loads + $X=1$]

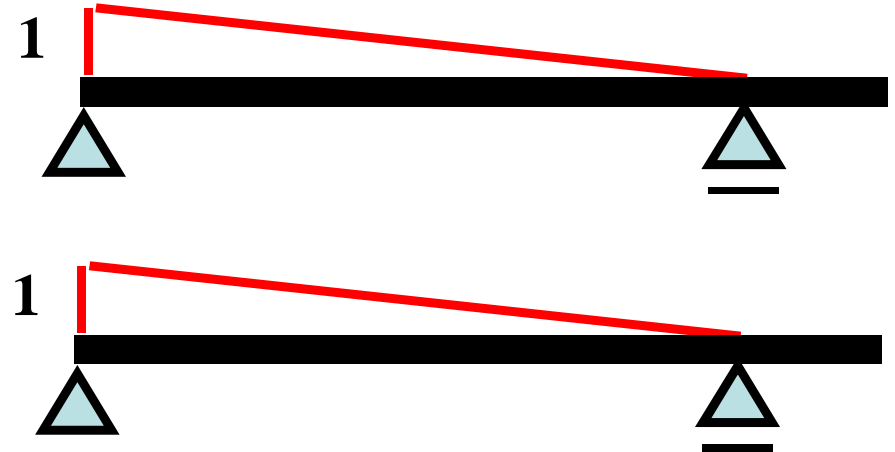


تکامل M_0 مع M_1 $\delta_{10} =$



$$\delta_{10} = \frac{1}{10000} \left[\frac{24 * 1 * 12}{6} - \left(\frac{2}{3} * 36 * 12 * 0.5 \right) \right] = - \frac{96}{10000} m$$

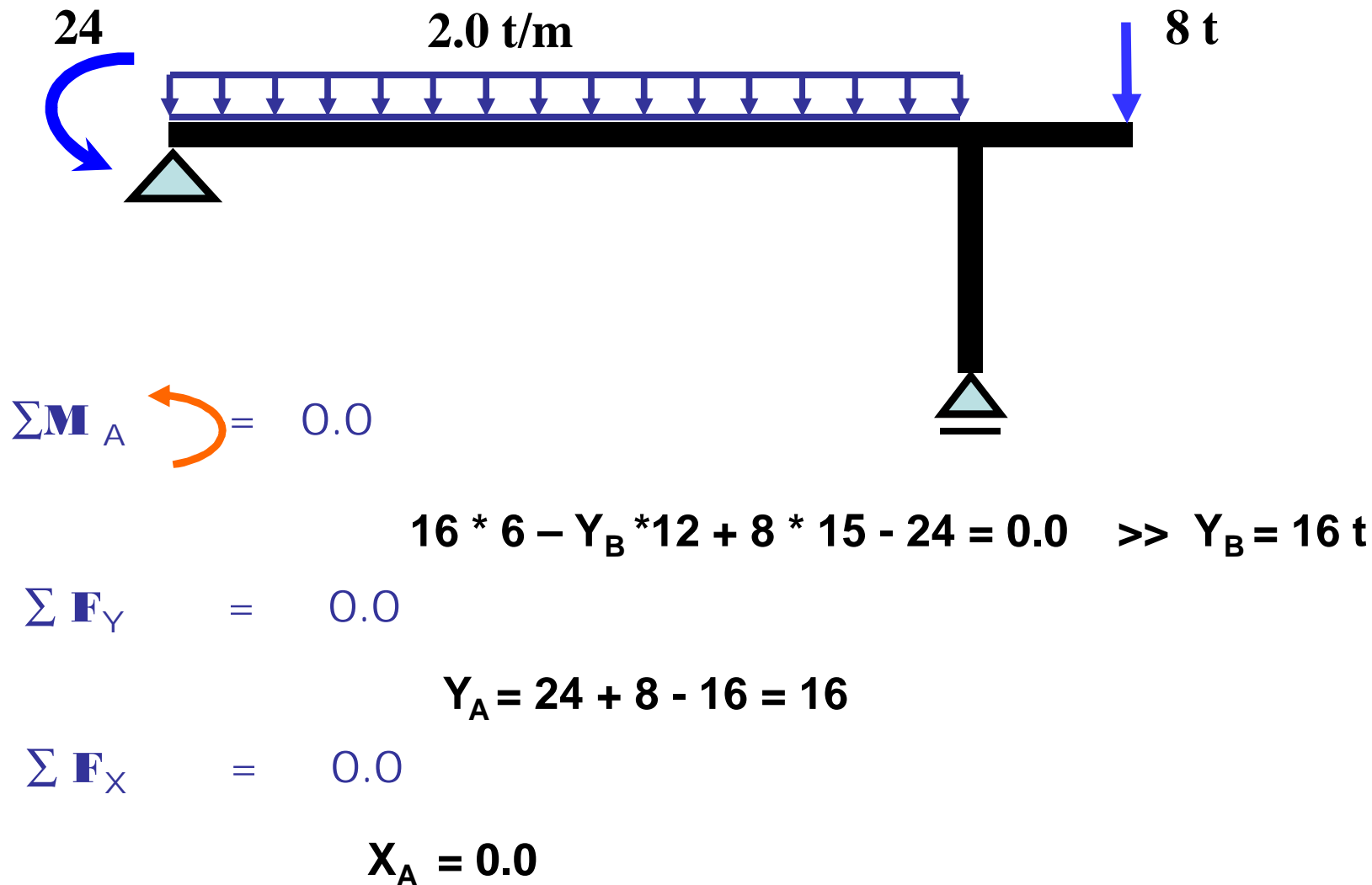
تکامل M_1 مع M_1 $\delta_{11} =$

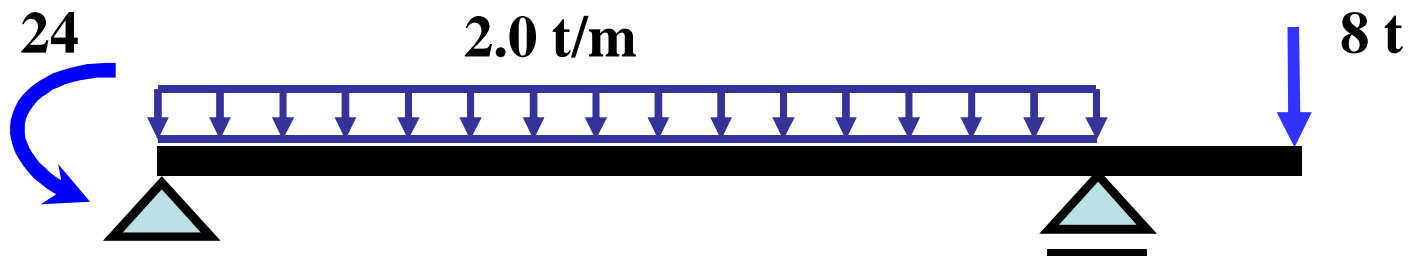


$$\delta_{11} = \frac{1}{10000} \left[\frac{1 * 1 * 12}{3} \right] = \frac{4}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{96}{10000} + X * \frac{4}{10000} = 0.0 \gg X = 24 \text{ t.m}$$

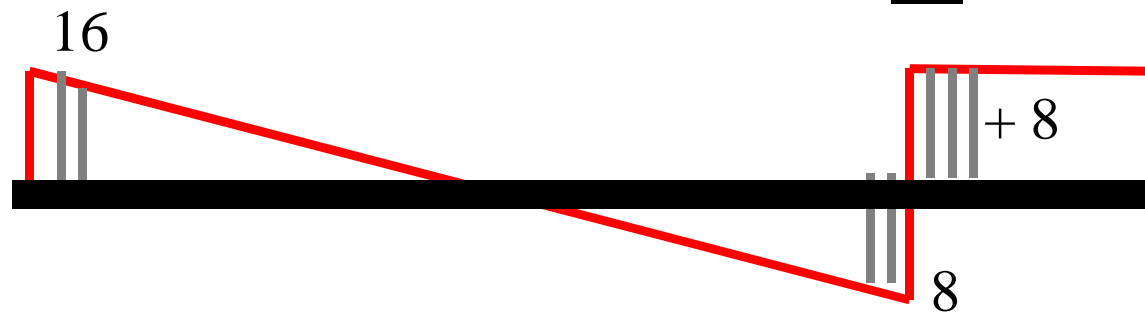




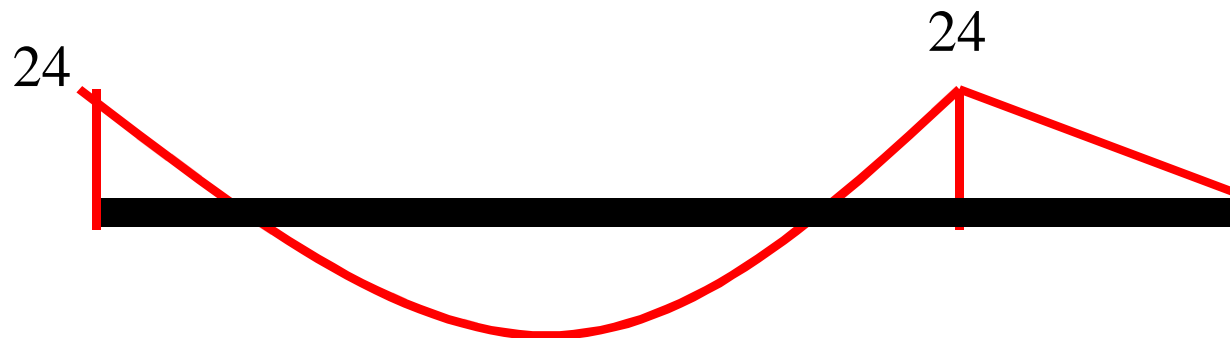
N.F.D



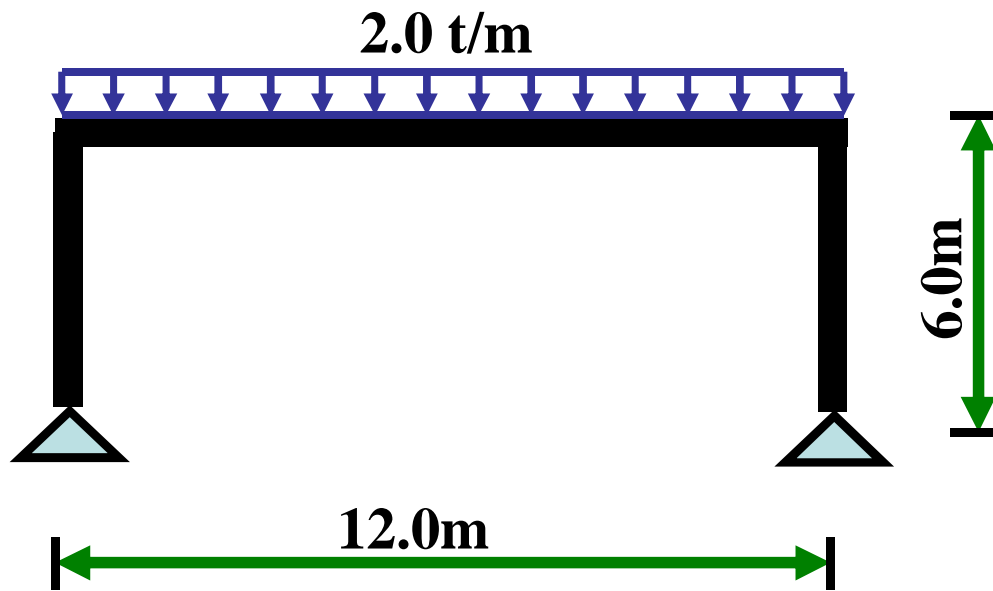
S.F.D



B.M.D

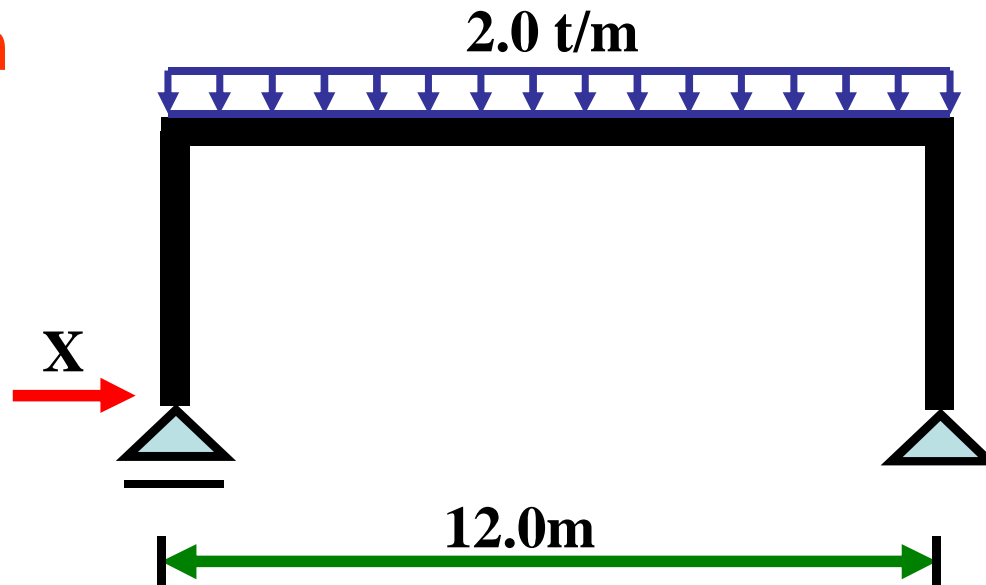


Example 5

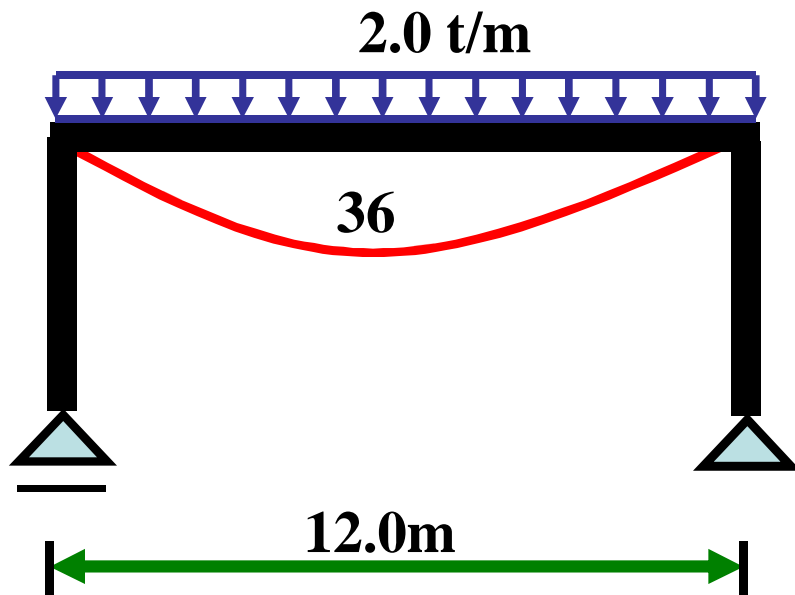
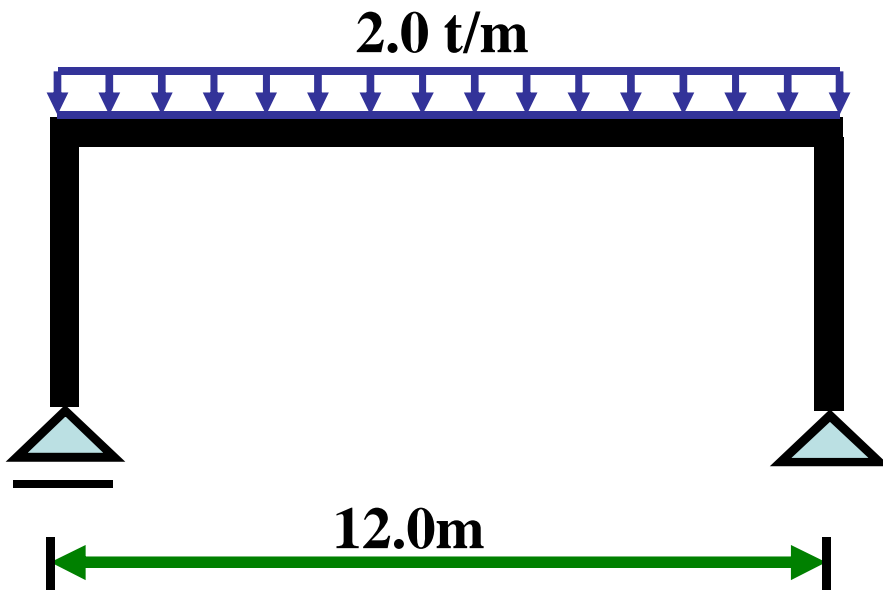


Draw B.M.D , S.F.D, and N.F.D $EI = 10000$

main system

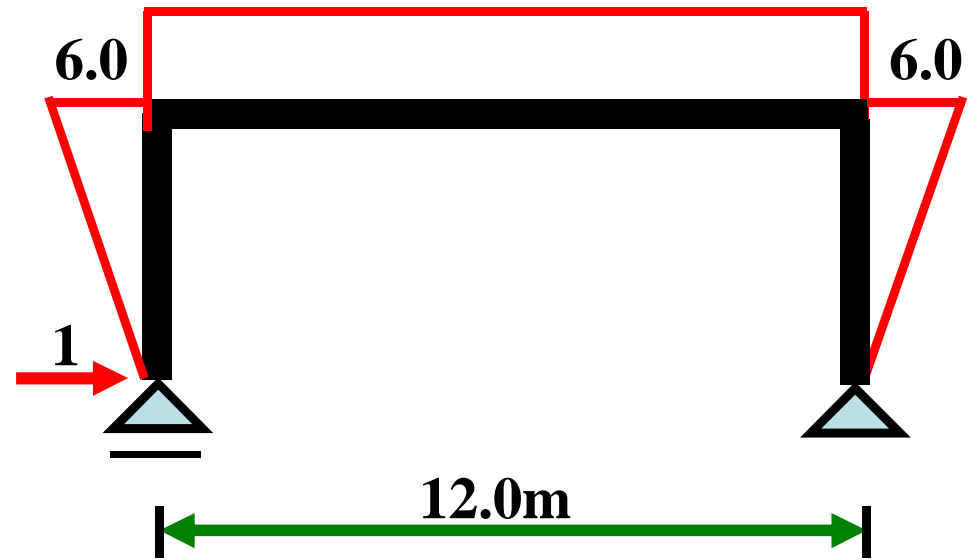
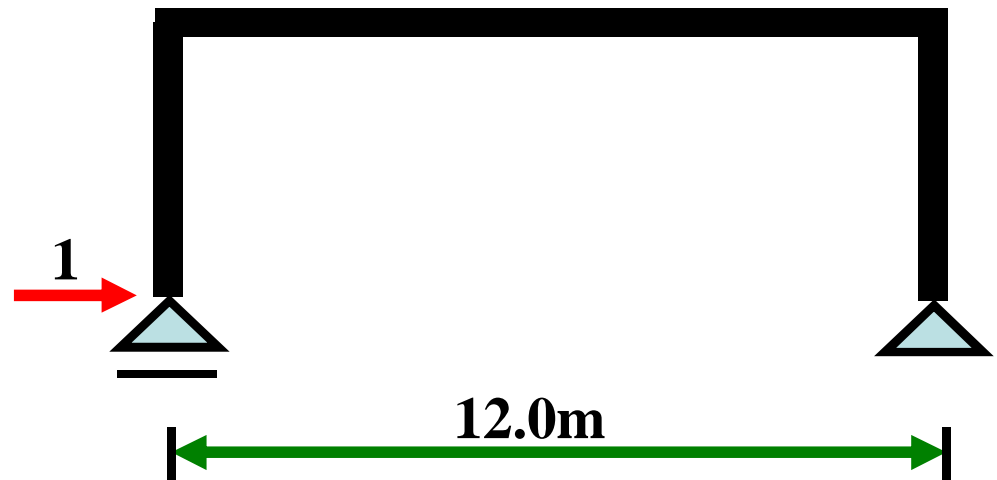


2- draw (M_0) [loads + $X=0$]



M_0

3- draw (M_1) [no loads + $X=1$]



M_1

$\delta_{10} = M_1$ مع M_0 تكامل

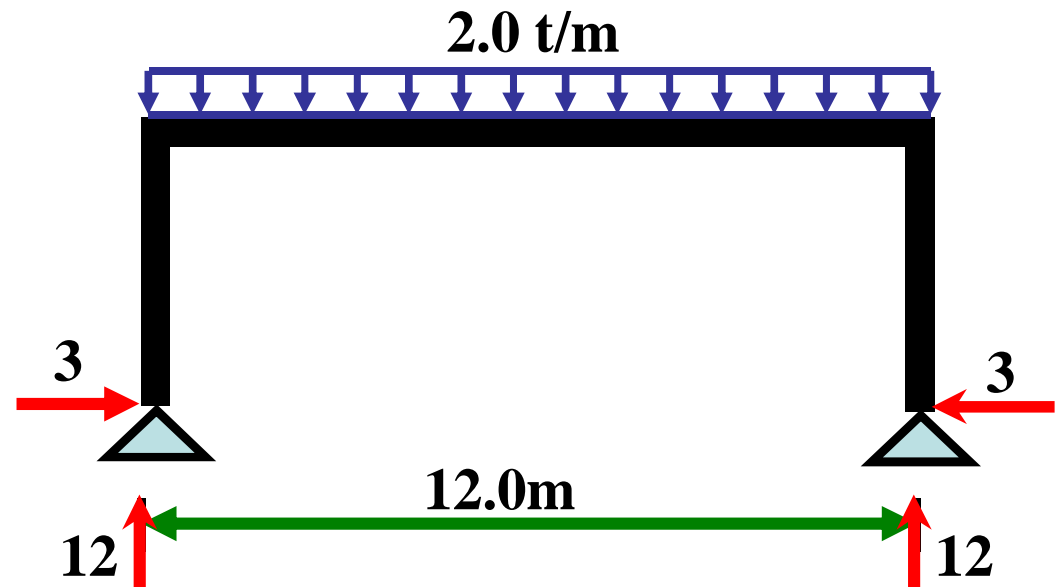
$$\delta_{10} = \frac{1}{10000} \left[- \left(\frac{2}{3} * 36 * 12 * 6 \right) \right] = - \frac{1728}{10000} m$$

$\delta_{11} = M_1$ مع M_1 تكامل

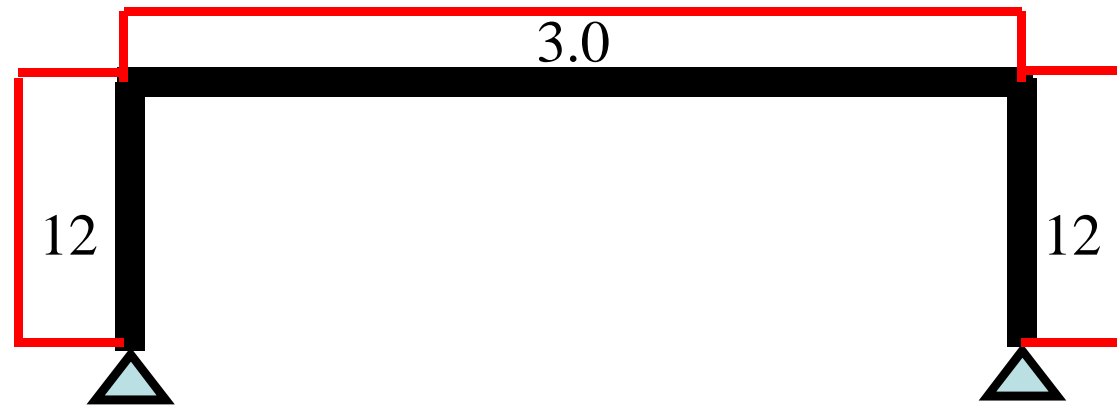
$$\delta_{11} = \frac{1}{10000} \left[\frac{6 * 6 * 6}{3} + \frac{6 * 6 * 6}{3} + 6 * 6 * 12 \right] = \frac{576}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

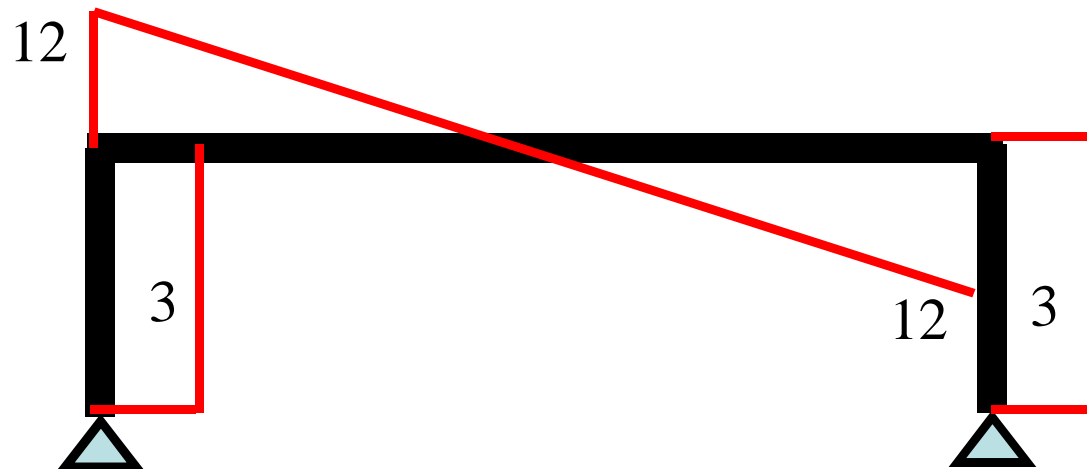
$$- \frac{1728}{10000} + X * \frac{576}{10000} = 0.0 \gg X = 3.0 \text{ t.m}$$



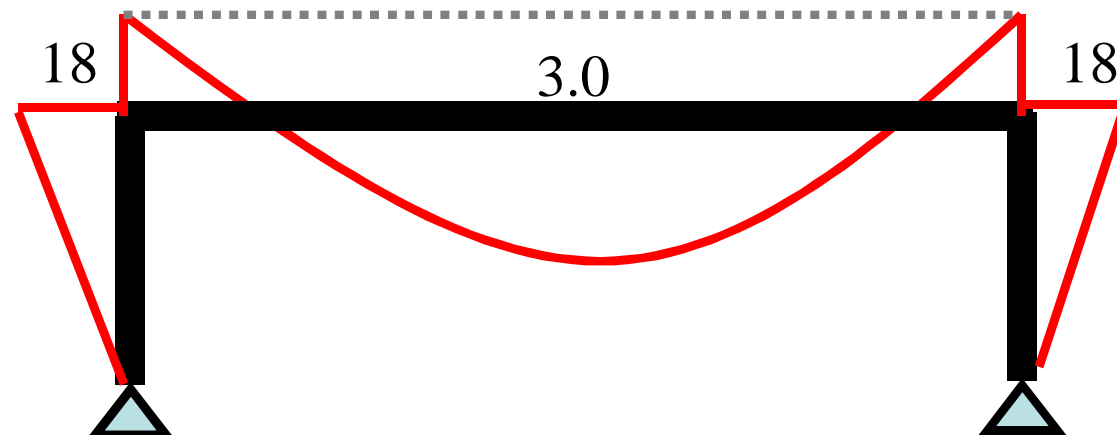
N.F.D



S.F.D



B.M.D



**Consistant Deformation
for n degree of indeterminate**

$$\delta_{10} + X_1 \cdot \delta_{11} = 0.0$$

**Indeterminate
1 degree**

$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} = 0.0$$

$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} = 0.0$$

**Indeterminate
2 degree**

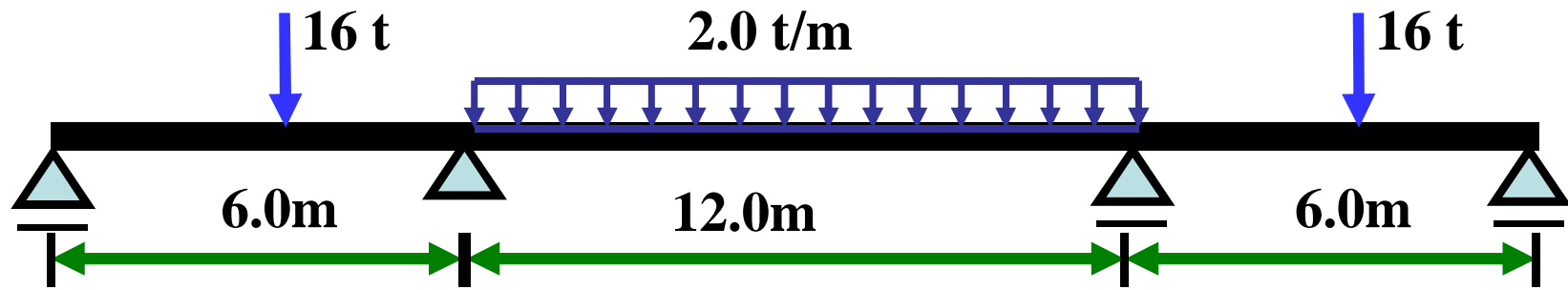
$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} + X_3 \cdot \delta_{13} = 0.0$$

$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} + X_3 \cdot \delta_{23} = 0.0$$

$$\delta_{30} + X_1 \cdot \delta_{31} + X_2 \cdot \delta_{32} + X_3 \cdot \delta_{33} = 0.0$$

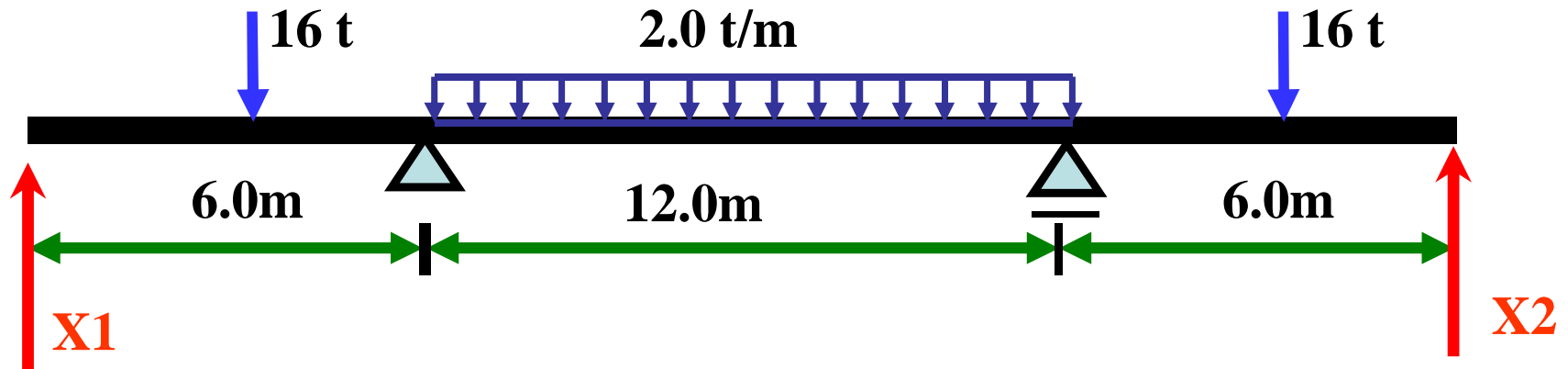
**Indeterminate
3 degree**

Example 1

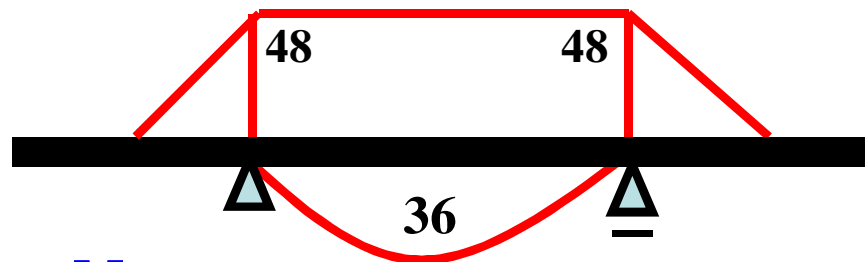
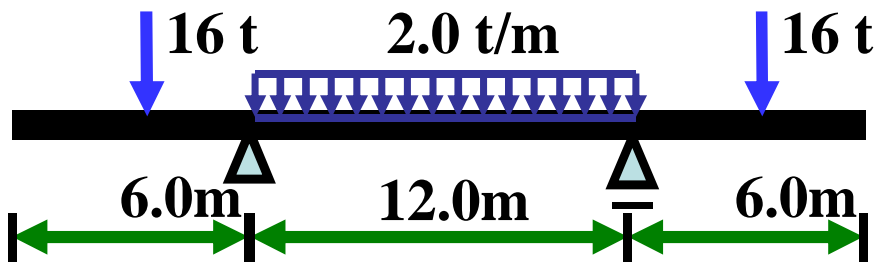


Draw B.M.D , S.F.D, and N.F.D $EI = 10000$

main system



2- draw (M_0)
[loads + $X_1=0$ & $x_2=0$]

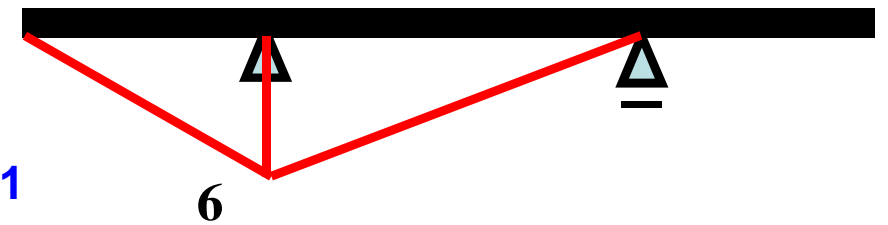


M_0

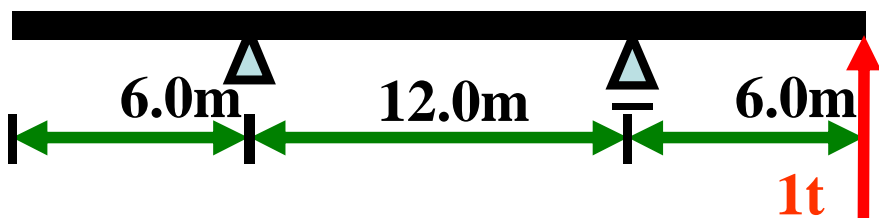
3- draw (M_1)
[no loads + $X_1=1$ + $X_2=0$]



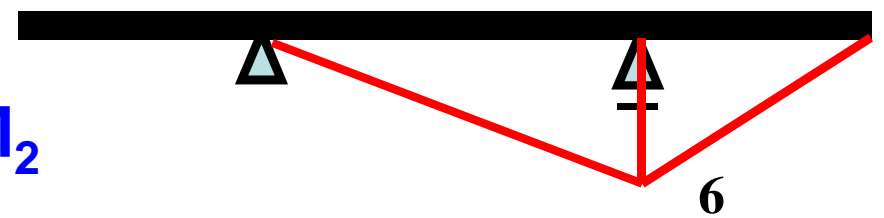
M_1



2- draw (M_2)
[no loads + $X_1=0$ & $x_2=1$]



M_2



$$\delta_{10} = \frac{1}{10000} \left[-\frac{48*6*12}{2} + \left(\frac{2}{3} * 36*12*1/2*6 \right) - \frac{3}{3} * (6*48 + 0.5*(3*48)) \right] = -\frac{1224}{10000} m$$

$$\delta_{20} = \frac{1}{10000} \left[-\frac{48*6*12}{2} + \left(\frac{2}{3} * 36*12*1/2*6 \right) - \frac{3}{3} * (6*48 + 0.5*(3*48)) \right] = -\frac{1224}{10000} m$$

$$\delta_{11} = \frac{1}{10000} \left[\frac{6*6*12}{3} + \frac{6*6*6}{3} \right] = \frac{216}{10000} m$$

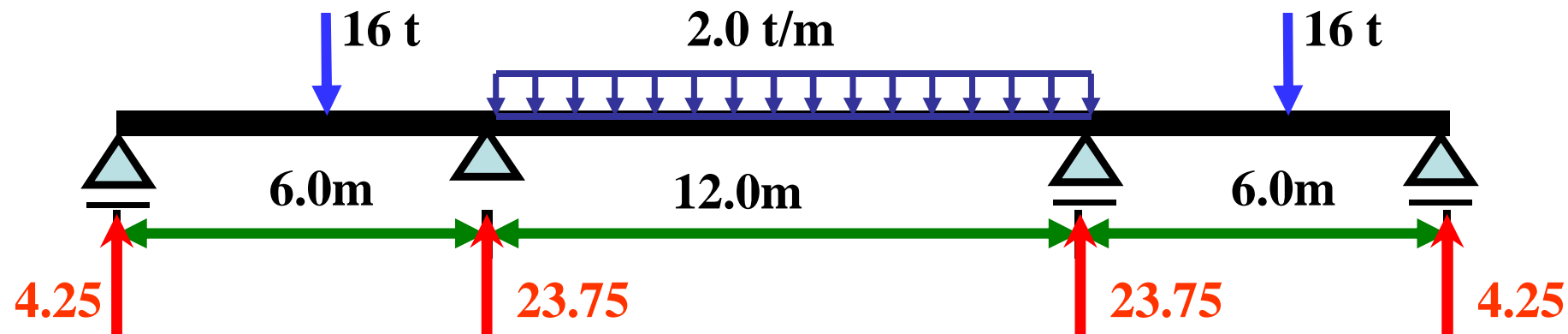
$$\delta_{22} = \frac{1}{10000} \left[\frac{6*6*12}{3} + \frac{6*6*6}{3} \right] = \frac{216}{10000} m$$

$$\delta_{21} = \delta_{12} = \frac{1}{10000} \left[\frac{6*6*12}{6} \right] = \frac{72}{10000} m$$

$$-\frac{1224}{10000} + X_1 \cdot \frac{216}{10000} + X_2 \cdot \frac{72}{10000} = 0.0$$

$$-\frac{1224}{10000} + X_1 \cdot \frac{72}{10000} + X_2 \cdot \frac{216}{10000} = 0.0$$

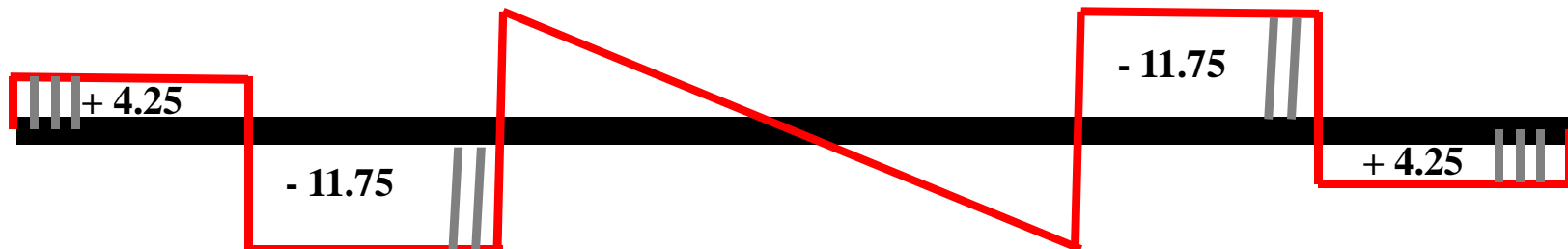
X1= 4.25 ton and X2= 4.25 ton



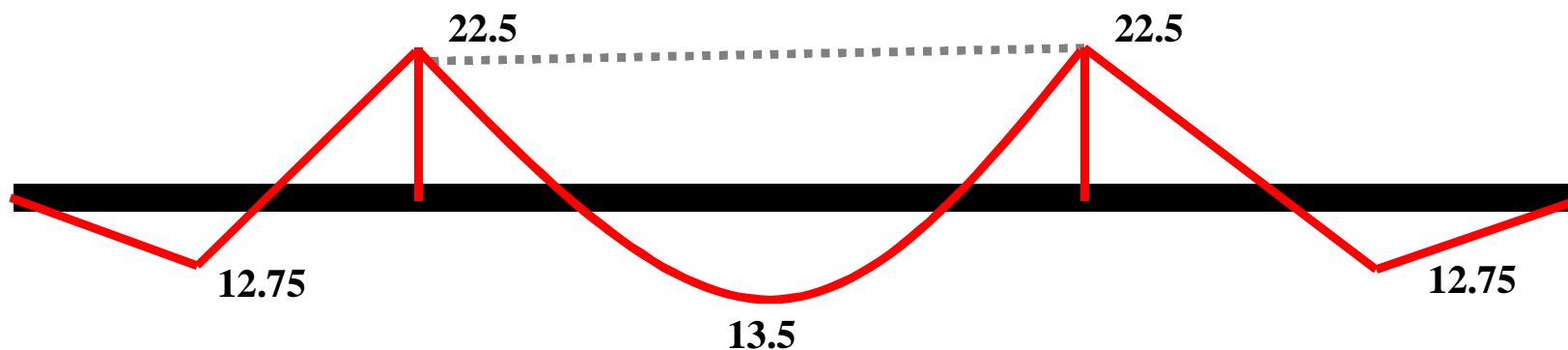
N.F.D

ZERO

S.F.D



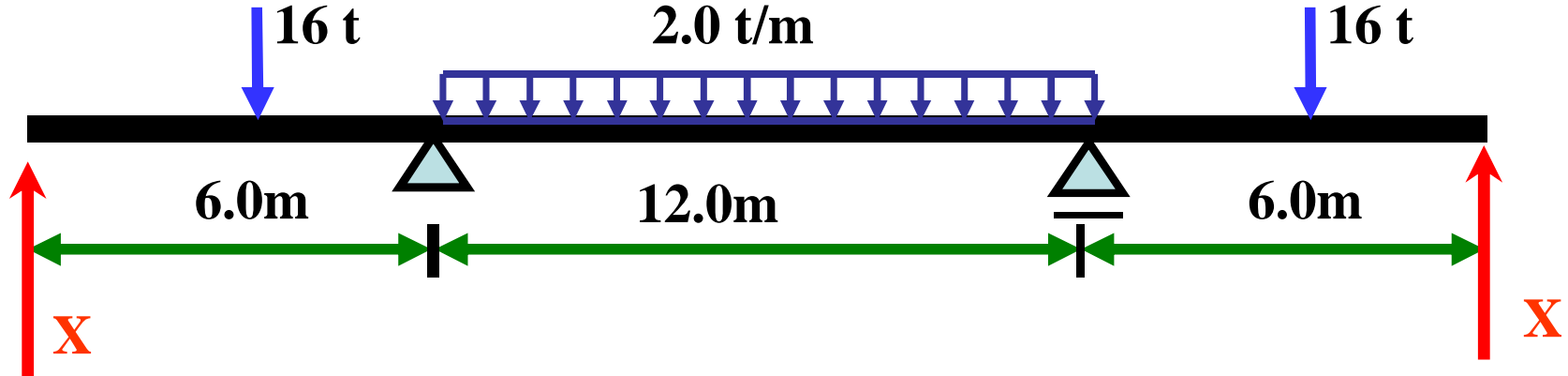
B.M.D



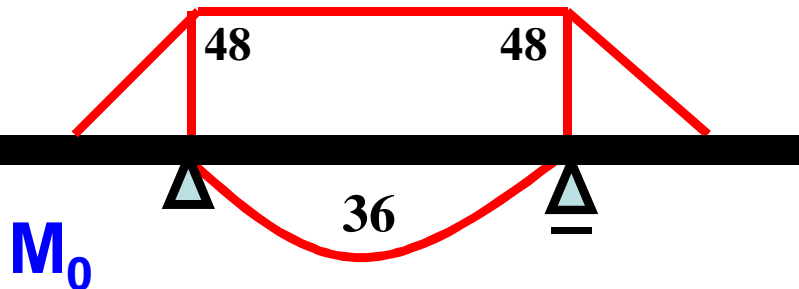
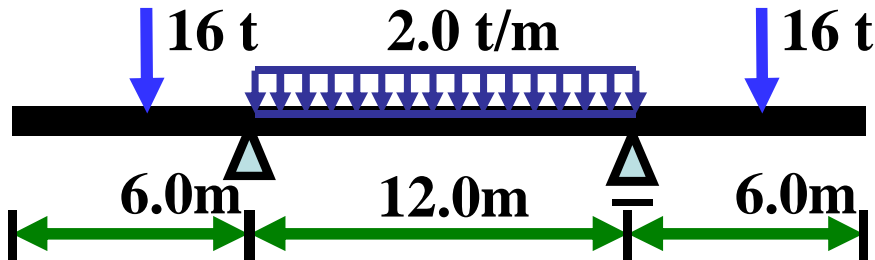
main system

استخدام التماثل

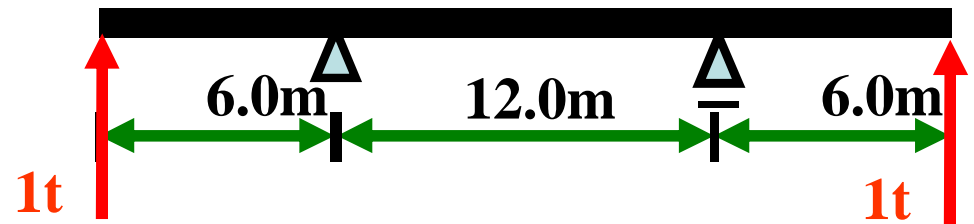
حل اخر:



2- draw (M_0)
[loads + $X=0$]



3- draw (M_1)
[no loads + $X=1$]



$$\delta_{10} = \frac{1}{10000} \left[-48 * 6 * 12 + \left(\frac{2}{3} * 36 * 12 * 6 \right) - 2 * \left(\frac{3}{3} * (6 * 48 + 0.5 * (3 * 48)) \right) \right] = -\frac{2448}{10000} m$$

$$\delta_{11} = \frac{1}{10000} \left[6 * 6 * 12 + \frac{6 * 6 * 6}{3} * 2 \right] = \frac{576}{10000} m$$

$$-\frac{2448}{10000} + X \cdot \frac{576}{10000} = 0.0 \quad \gg \quad X = 4.25t$$

الباقي كما تم في السابق

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Settlement and Temperature

Eng : Aymman abdo

Case of Settlement

$$\delta_{10} = \text{zero}$$

$$\delta_{10} + R \cdot \delta_{11} = \text{settlement}$$

$$\delta_{11} = \text{the same in given loads}$$

Case of Uniform Temperature

$$\delta_{10} = \alpha \cdot \Delta t \cdot \sum A_N$$

$$\delta_{10} + R \cdot \delta_{11} = 0.0$$

$$\delta_{11} = \text{the same in given loads}$$

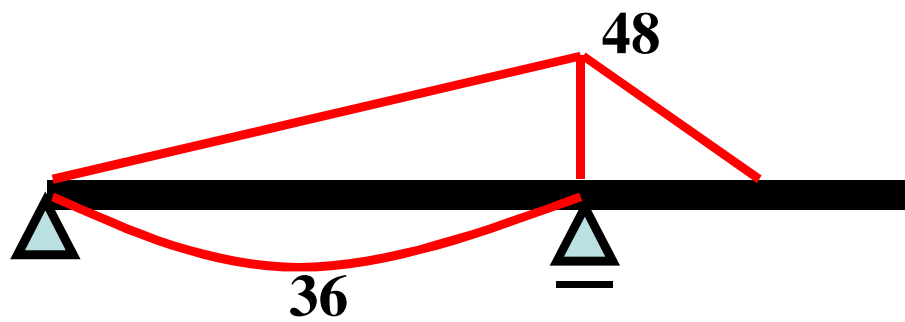
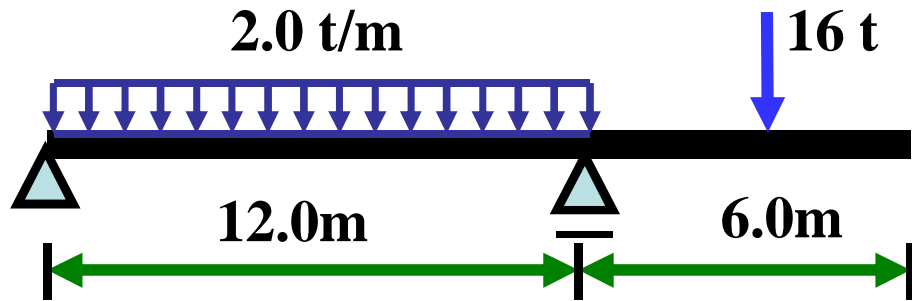
Case of Non Uniform Temperature

$$\delta_{10} = \alpha \cdot \left(\frac{t_1 + t_2}{2}\right) \cdot \sum A_N + \alpha \cdot \left(\frac{t_2 - t_1}{h}\right) \cdot \sum A_M$$

$$\delta_{11} = \text{the same in given loads}$$

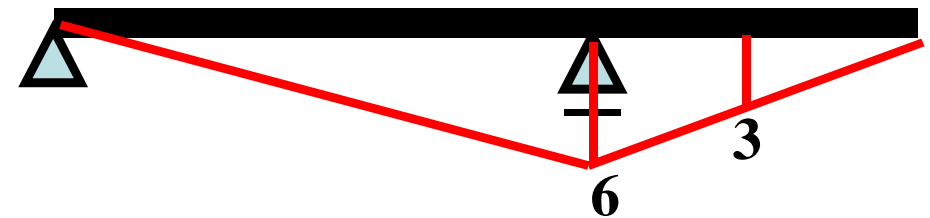
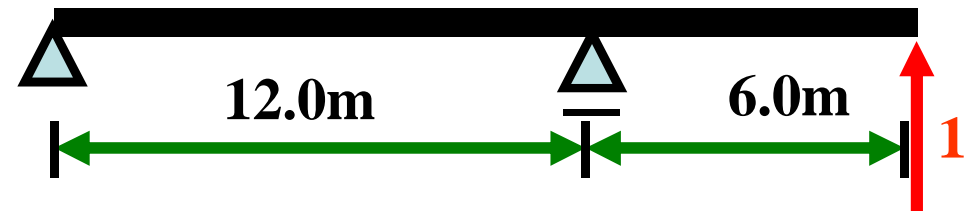
$$\delta_{10} + R \cdot \delta_{11} = 0.0$$

2- draw (M_0) [loads + $X=0$]



M_0

3- draw (M_1) [no loads + $X=1$]



M_1

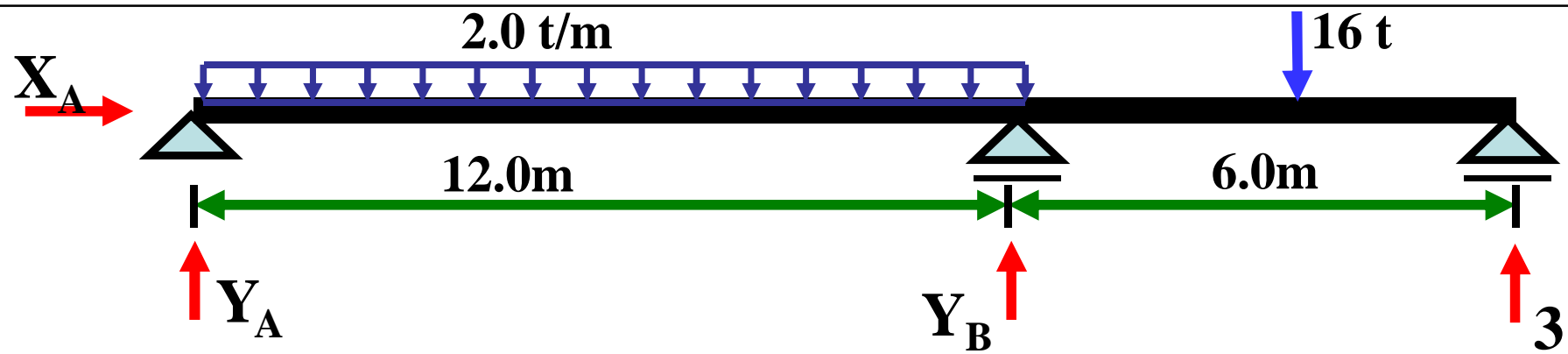
Case of given loads

$$\begin{aligned}\delta_{10} &= \frac{1}{30000} \left[-\frac{48 * 6 * 12}{3} + \left(\frac{2}{3} * 36 * 12 * 1/2 * 6 \right) - \frac{3}{3} * (6 * 48 + 0.5 * (3 * 48)) \right] \\ &= -\frac{648}{30000} m\end{aligned}$$

$$\delta_{11} = \frac{1}{30000} \left[\frac{6 * 6 * 12}{3} + \frac{6 * 6 * 6}{3} \right] = \frac{216}{30000} m$$

$$\delta_{10} + X . \delta_{11} = 0.0$$

$$-\frac{648}{30000} + X * \frac{216}{30000} = 0.0 \gg X = 3 \text{ ton}$$



$$\sum \mathbf{M}_A = 0.0 \quad 24 * 6 + 16 * 15 - Y_B * 12 - 3 * 18 = 0 \quad \Rightarrow Y_B = 27.5 \text{ t}$$

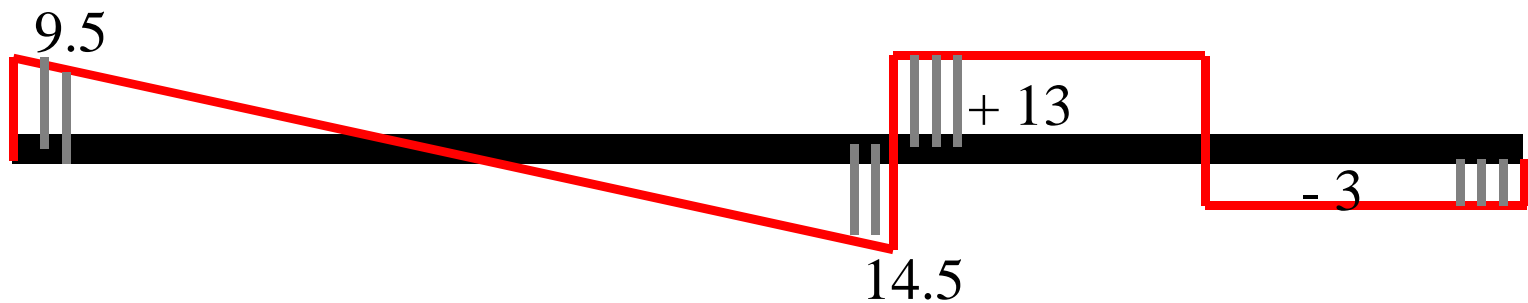
$$\sum \mathbf{F}_Y = 0.0 \quad Y_A = 24 + 16 - 27.5 - 3 = 9.5$$

$$\sum \mathbf{F}_X = 0.0 \quad X_A = 0.0$$

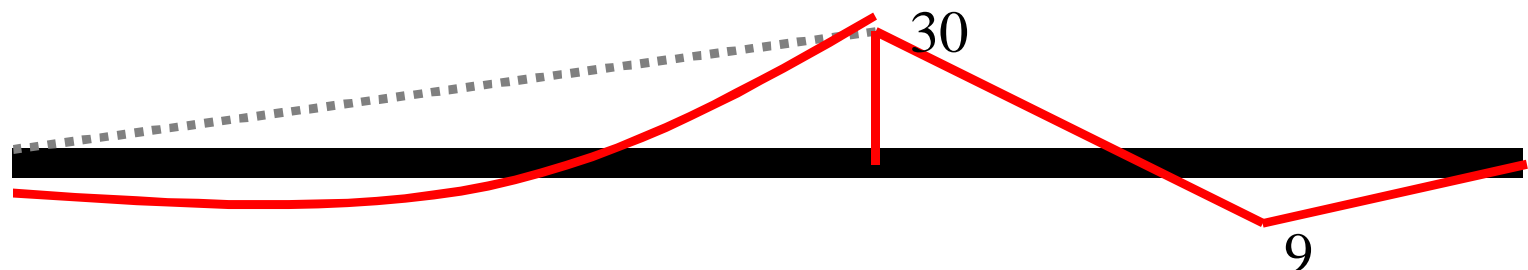
N.F.D

ZERO

S.F.D



B.M.D



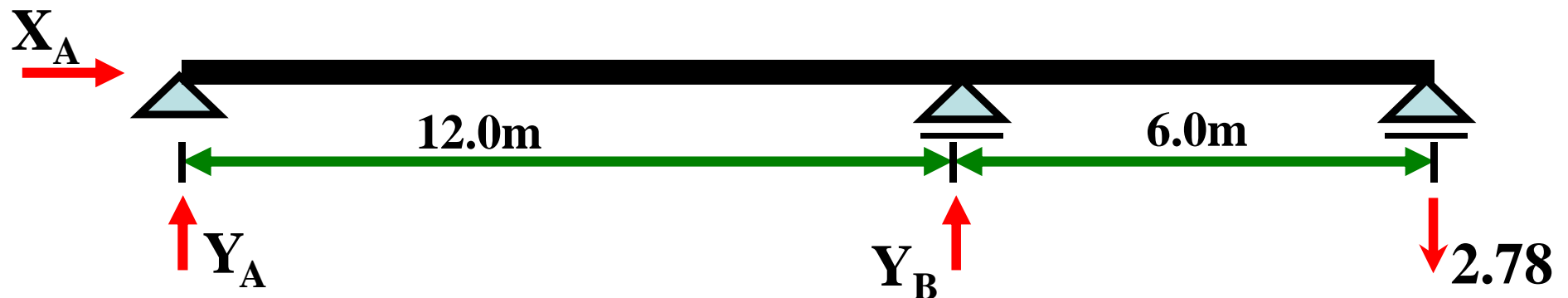
Case of Settlement

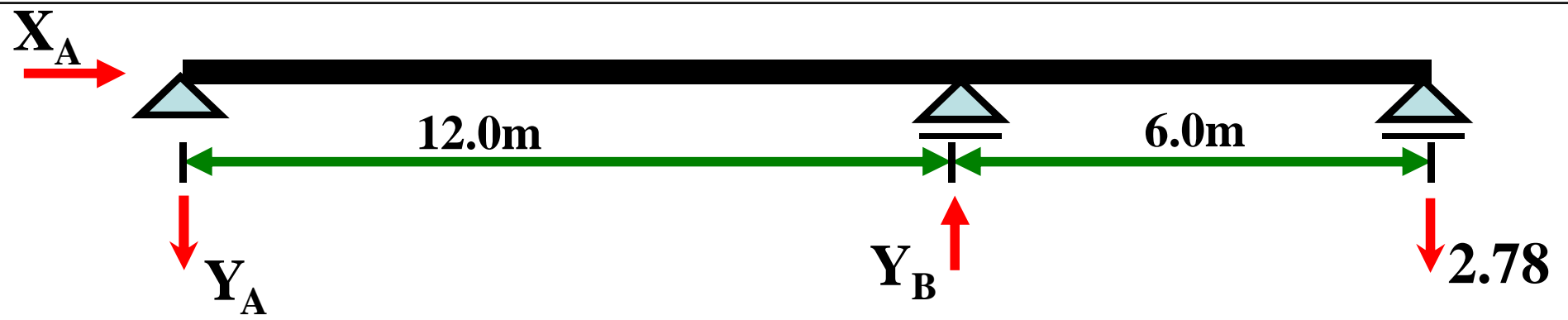
$$\delta_{10} = zero$$

$$\delta_{11} = \frac{216}{10000} m$$

$$\delta_{10} + X \cdot \delta_{11} = -0.02$$

$$0.0 + X * \frac{216}{30000} = -0.02 \gg X = -2.78 ton$$





$$\Sigma \mathbf{M}_A = 0.0$$

$$Y_B * 12 - 2.78 * 18 = 0 \Rightarrow Y_B = 4.17 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

$$Y_A = 4.17 - 2.78 = 1.39$$

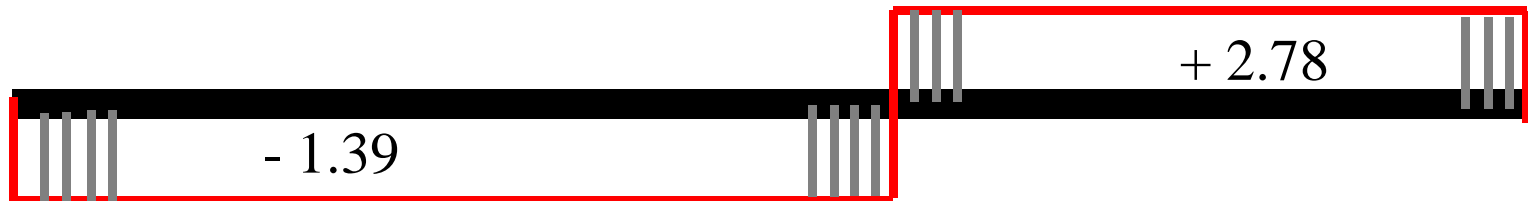
$$\Sigma \mathbf{F}_X = 0.0$$

$$X_A = 0.0$$

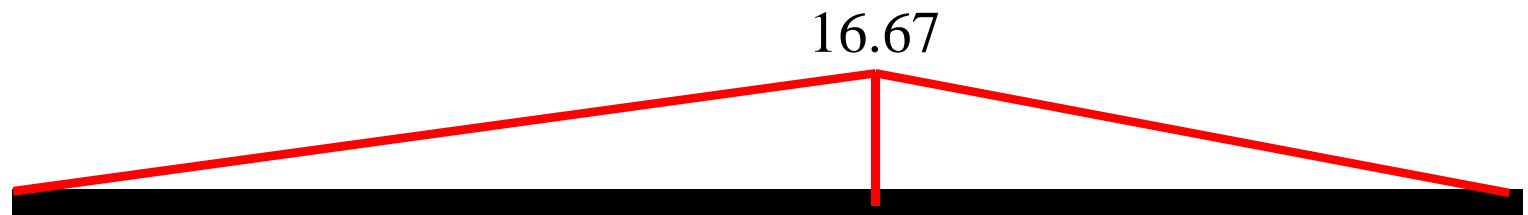
N.F.D

ZERO

S.F.D



B.M.D



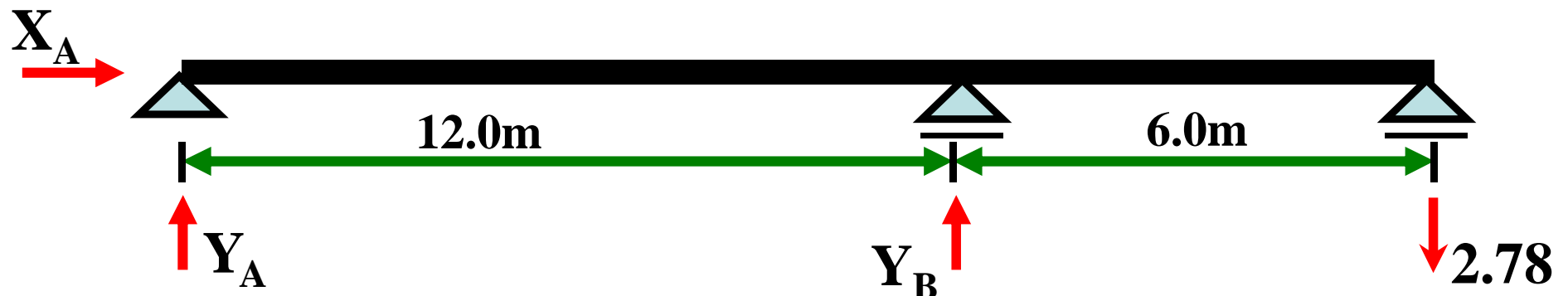
Case of uniform temperature

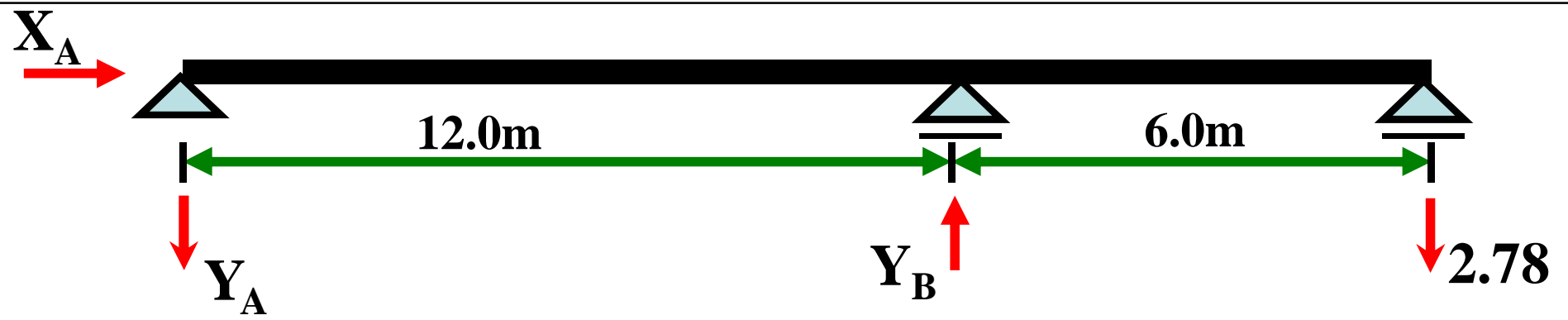
$$\delta_{10} = \text{zero}$$

$$\delta_{11} = \frac{216}{10000} \text{ m}$$

$$\delta_{10} + X \cdot \delta_{11} = -0.02$$

$$0.0 + X * \frac{216}{30000} = -0.02 \gg X = -2.78 \text{ ton}$$





$$\Sigma \mathbf{M}_A = 0.0$$

$$Y_B * 12 - 2.78 * 18 = 0 \Rightarrow Y_B = 4.17 \text{ t}$$

$$\Sigma \mathbf{F}_Y = 0.0$$

$$Y_A = 4.17 - 2.78 = 1.39$$

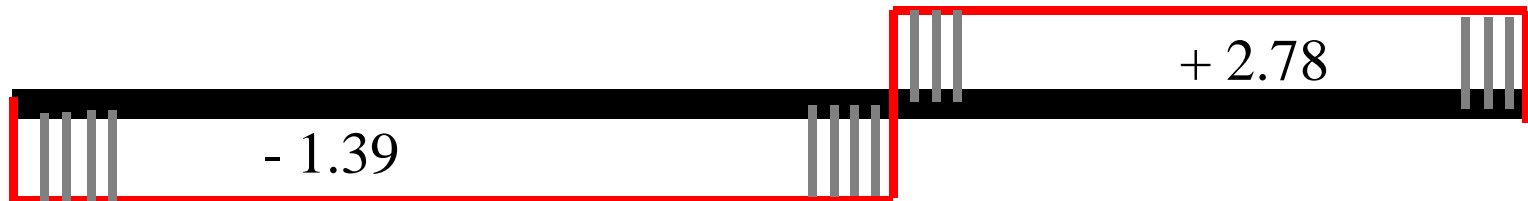
$$\Sigma \mathbf{F}_X = 0.0$$

$$X_A = 0.0$$

N.F.D

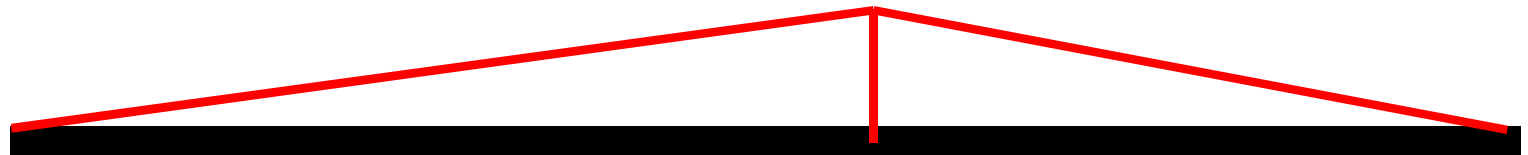
ZERO

S.F.D

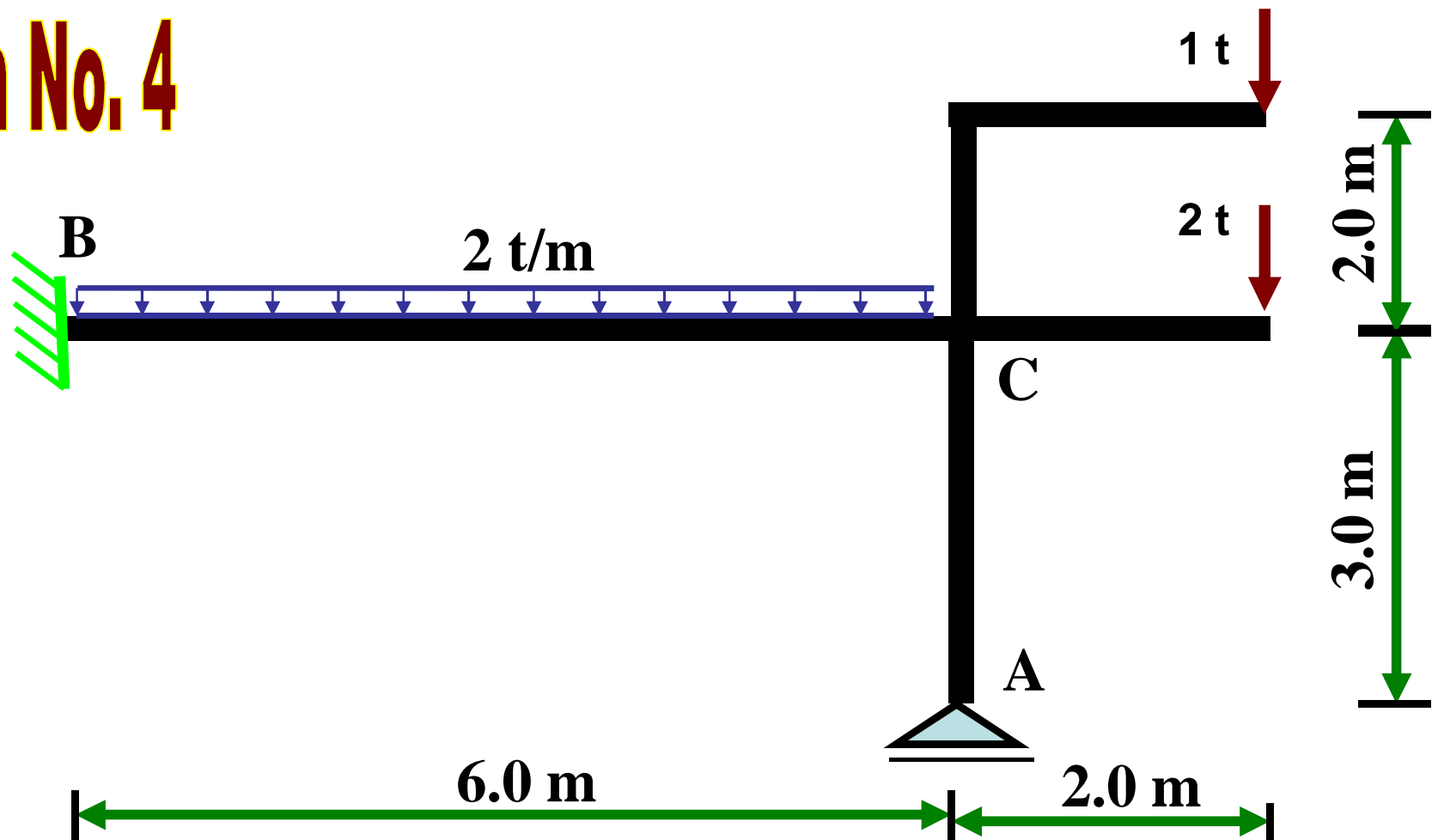


B.M.D

16.67



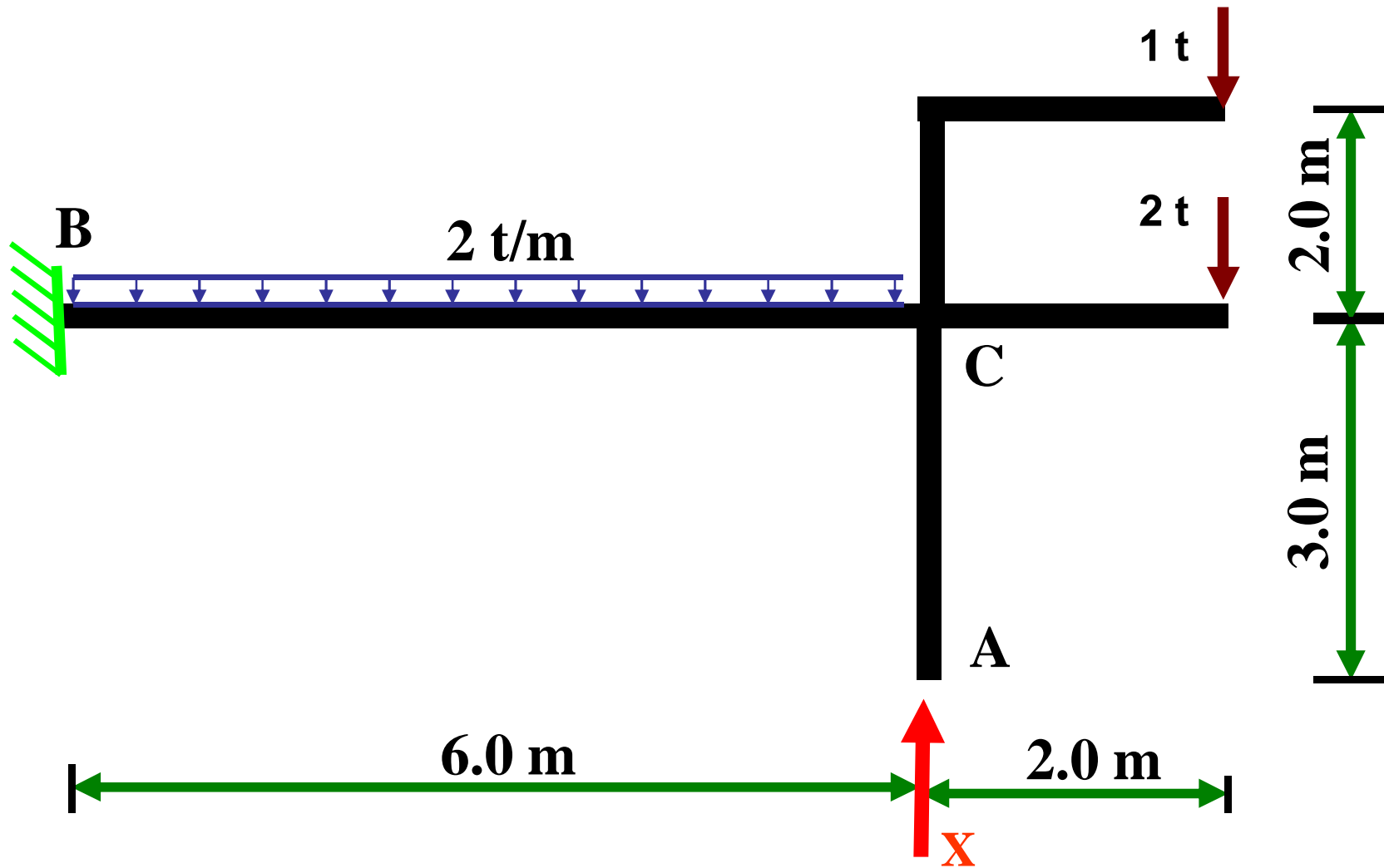
Problem No. 4



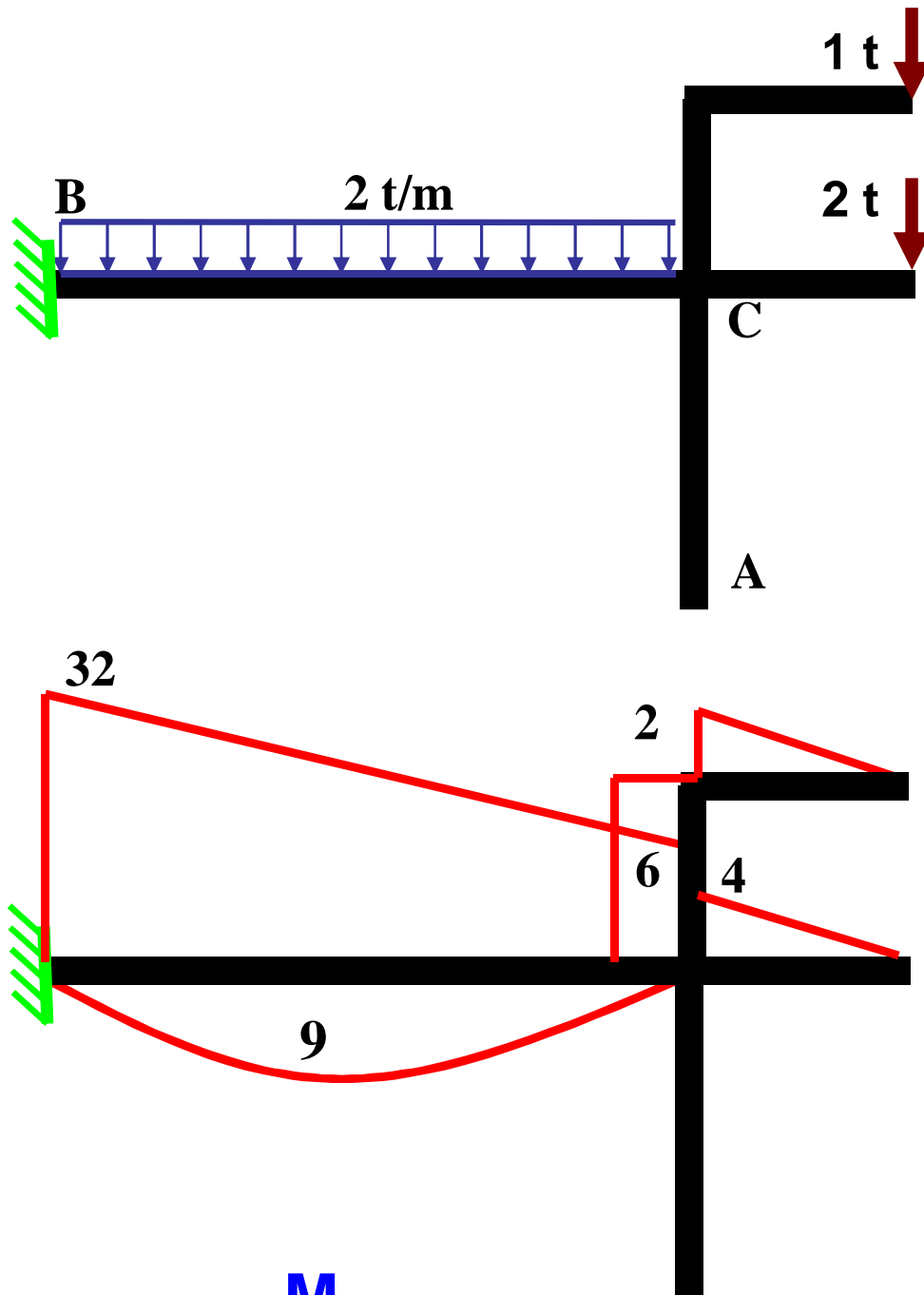
Draw B.M.D , S.F.D, and N.F.D $EI = 6000 \text{ t.m}^2$ $\alpha = 10^{-5}$

Due to (1) given loads.
(2) settlement at A = 2cm.
(3) uniform rise in temperature equals 20°
(4) $t_1=20$, $t_2=40$ and $h = 0.6\text{m}$

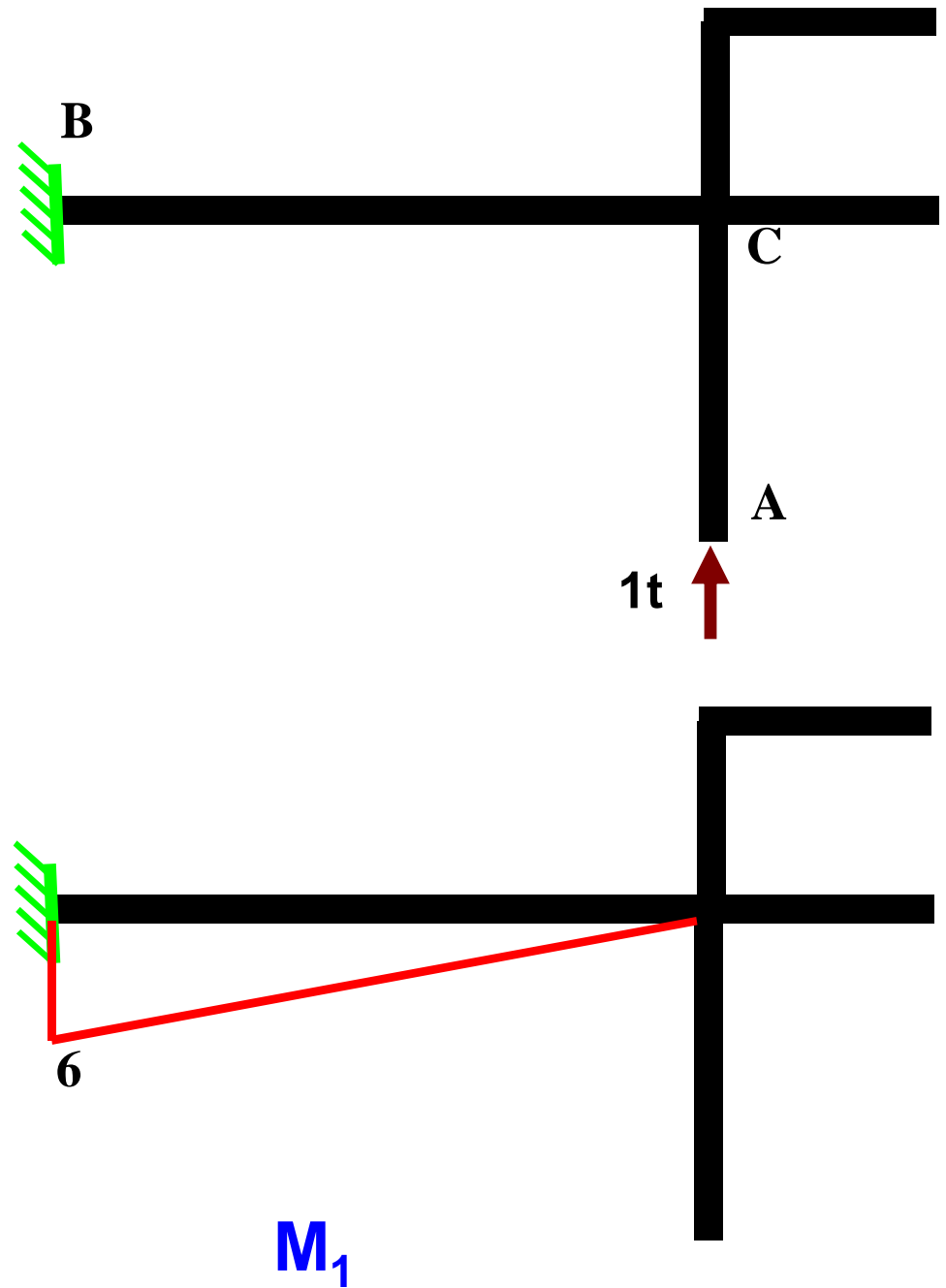
Main System



2- draw (M_0) [loads + $X=0$]



3- draw (M_1) [no loads + $X=1$]



M_0

M_1

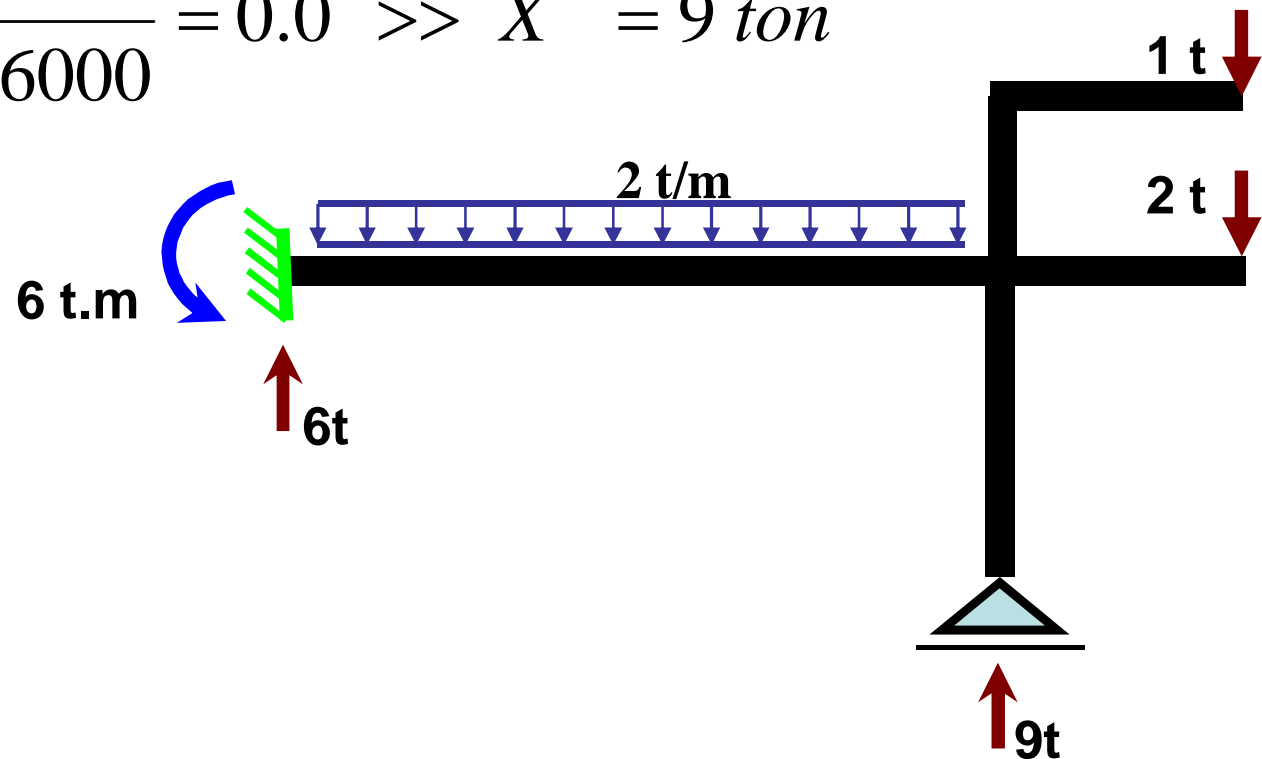
Case of given loads

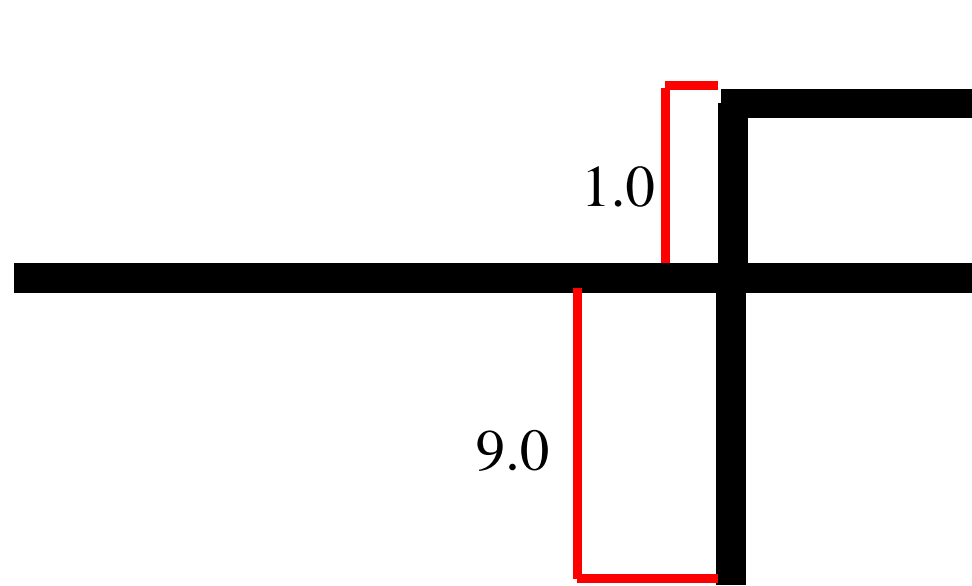
$$\delta_{10} = \frac{1}{6000} \left[-\frac{6}{3} * (6 * 60 + 0.5 * (6 * 6)) + 2/3 * 9 * 6 * 3 \right] = -\frac{648}{6000}$$

$$\delta_{11} = \frac{1}{6000} \left[\frac{6 * 6 * 6}{3} \right] = \frac{72}{6000} m$$

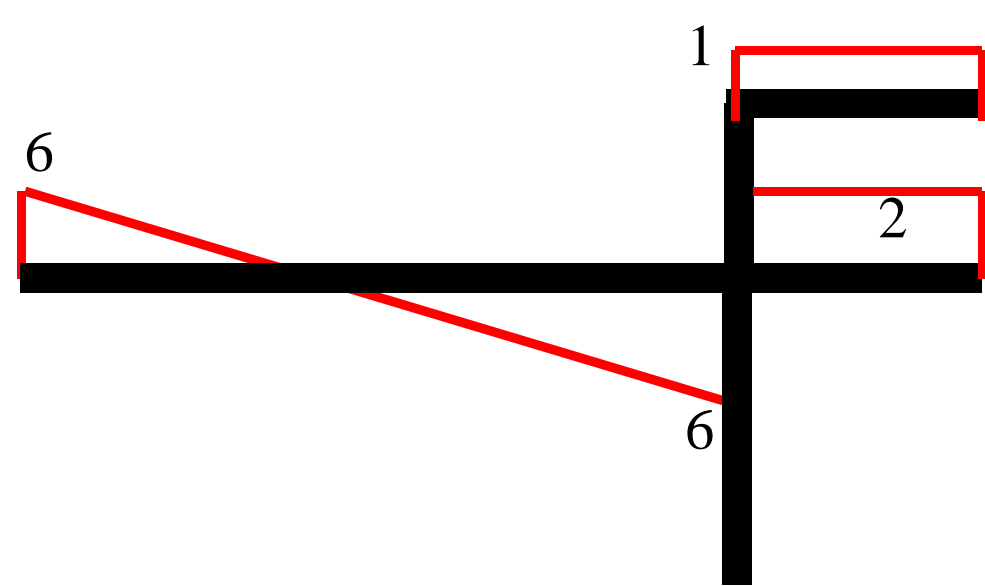
$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{648}{6000} + X * \frac{72}{6000} = 0.0 \gg X = 9 \text{ ton}$$

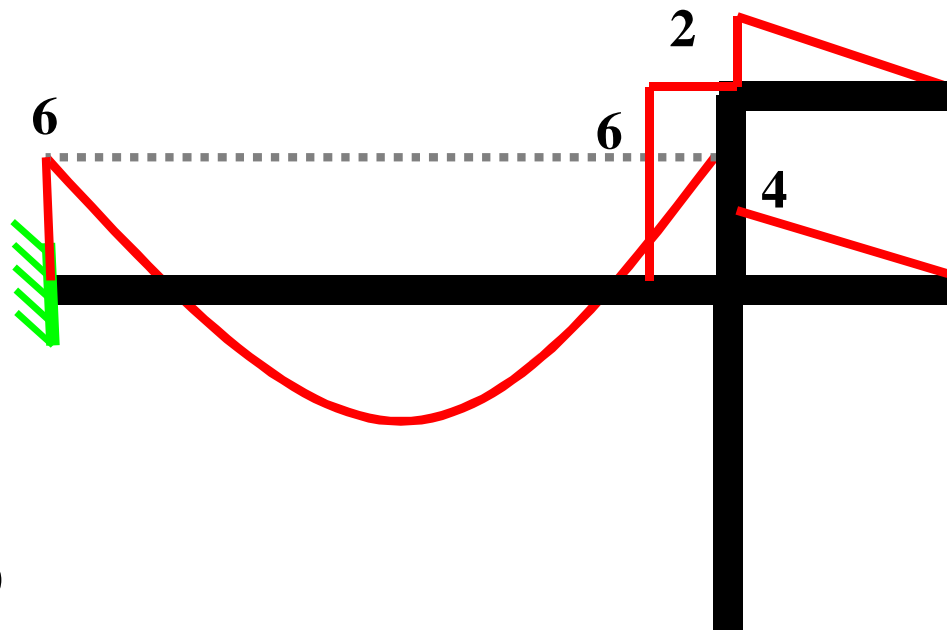




N.F.D



S.F.D



B.M.D

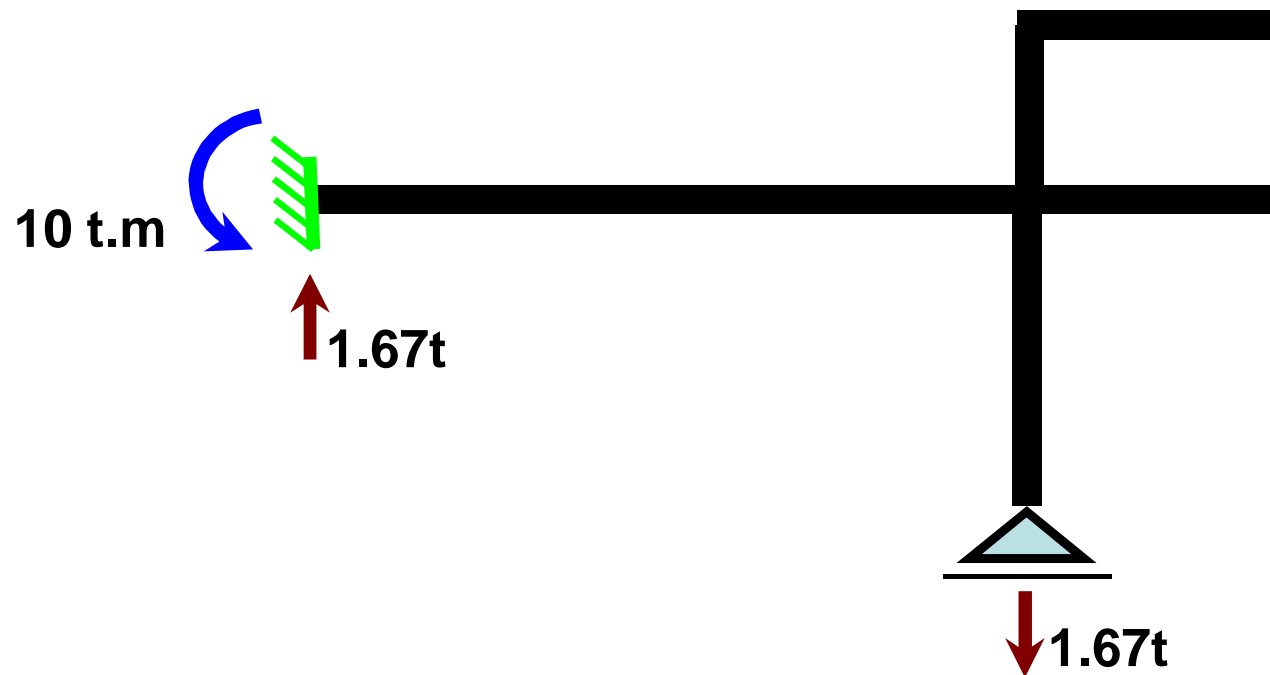
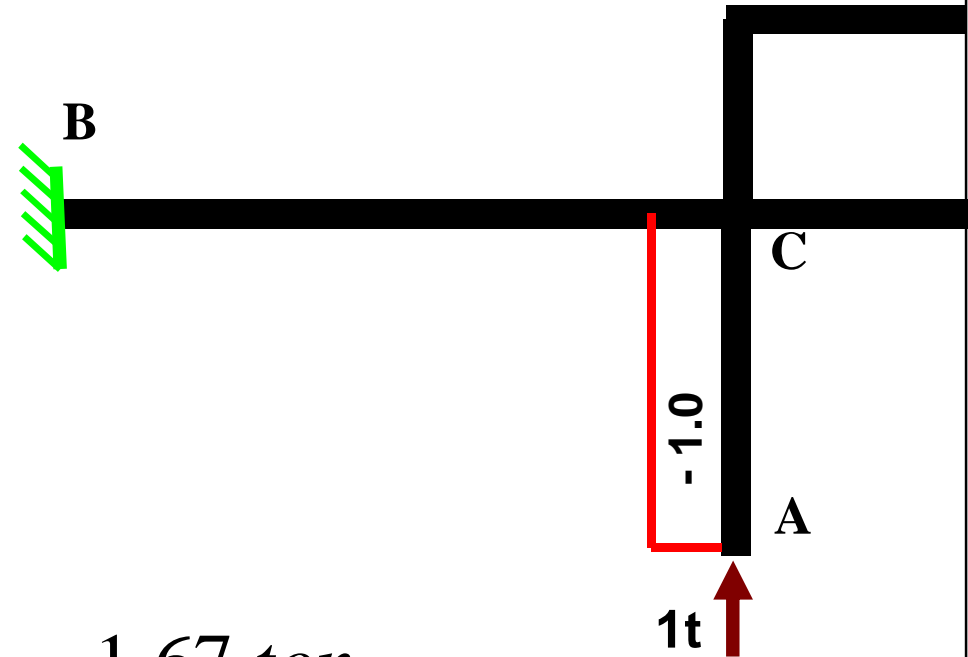
Case of Settlement

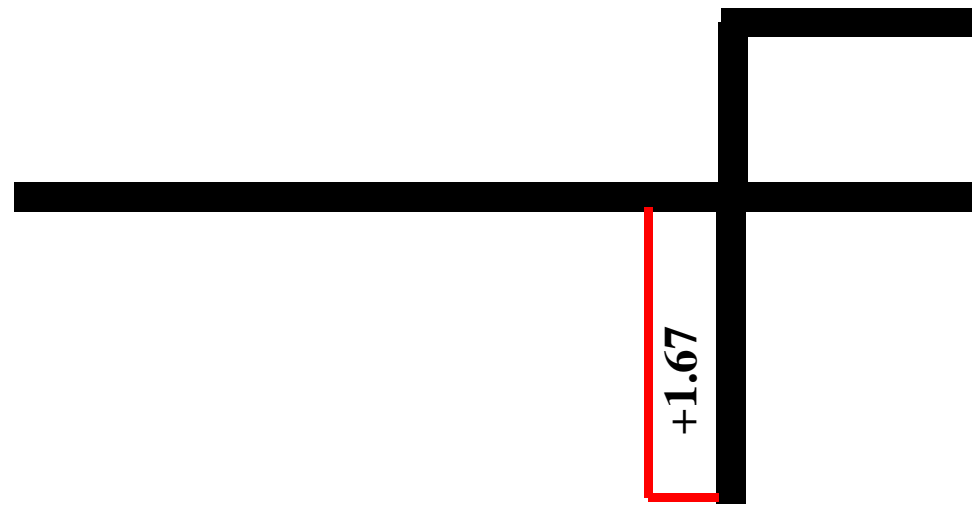
$$\delta_{10} = 0.0$$

$$\delta_{11} = \frac{72}{6000} m$$

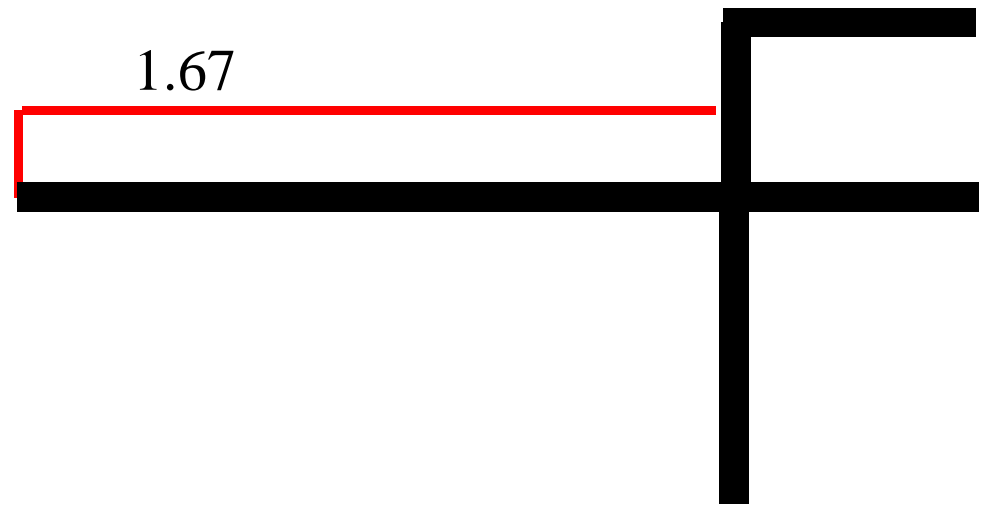
$$\delta_{10} + X \cdot \delta_{11} = -0.02$$

$$0.0 + X \cdot \frac{72}{6000} = -0.02 \Rightarrow X = -1.67 \text{ ton}$$

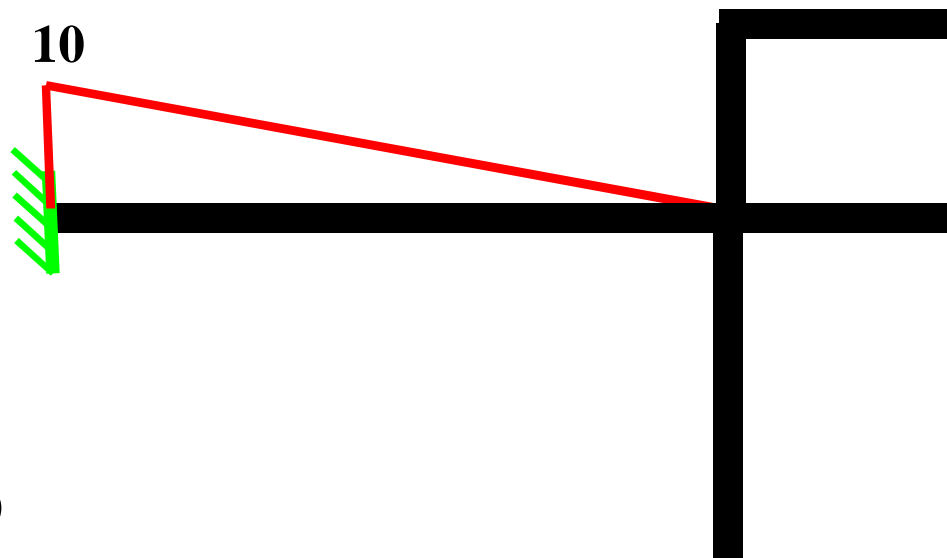




N.F.D



S.F.D



B.M.D

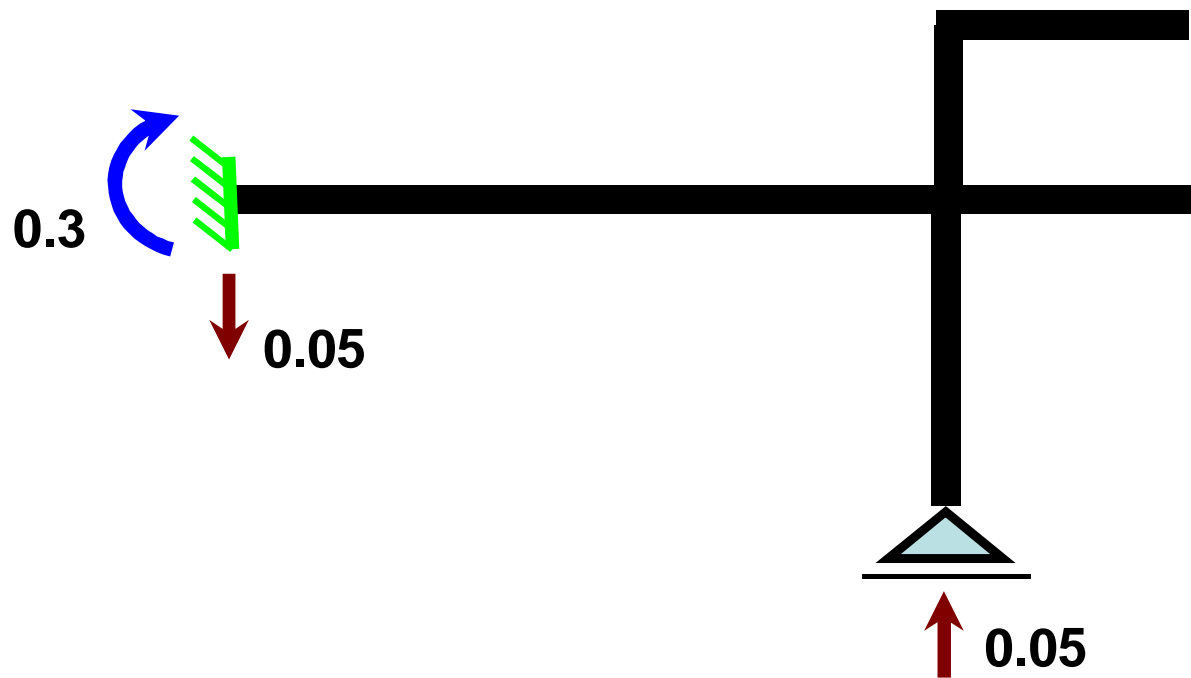
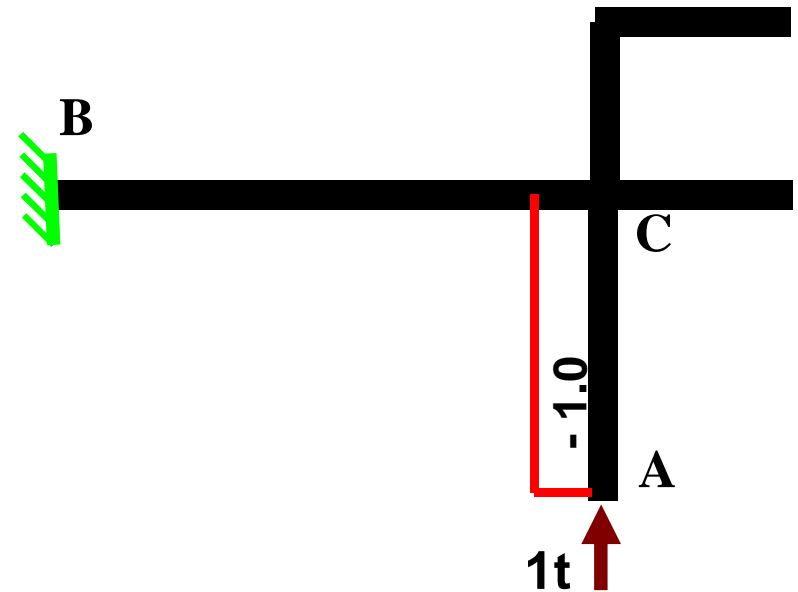
Case of uniform temperature

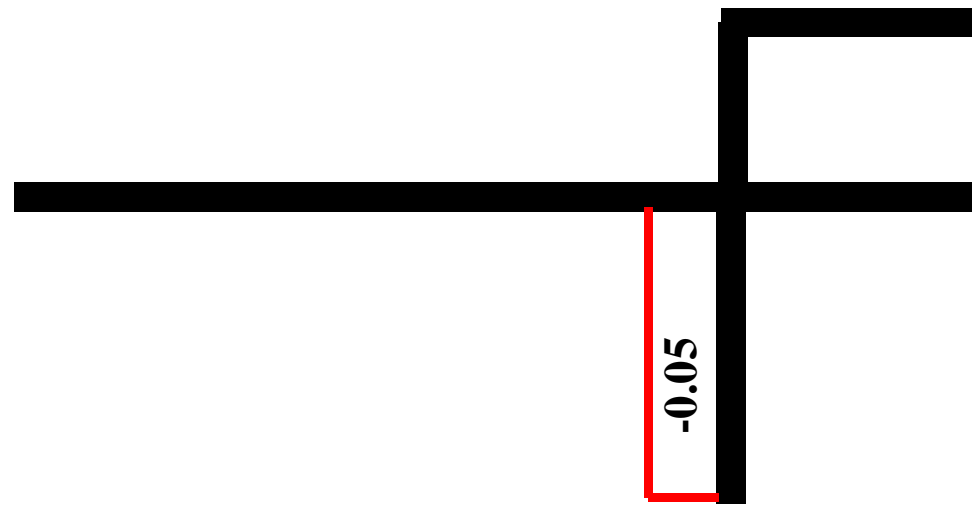
$$\delta_{10} = 10^{-5} * 20 * (-1 * 3) = -60 * 10^{-5}$$

$$\delta_{11} = \frac{72}{6000} m$$

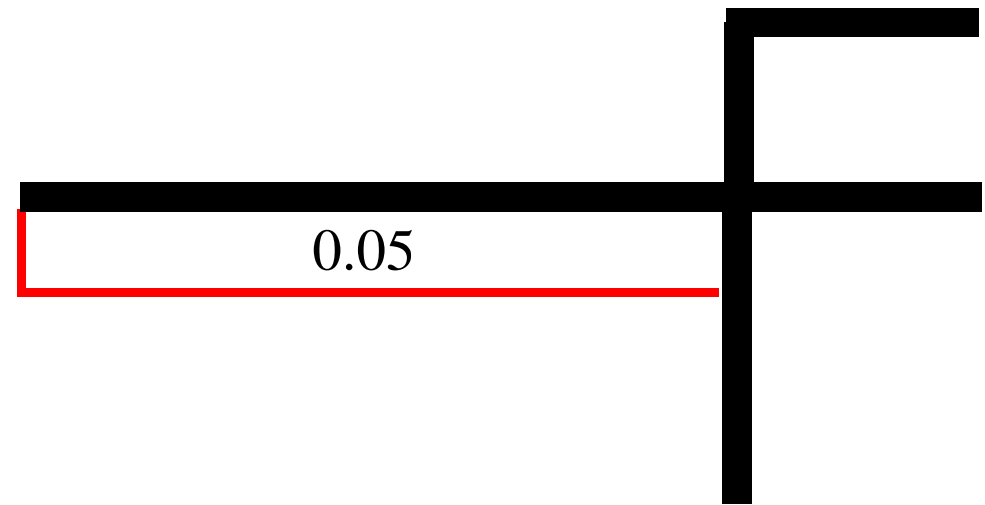
$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-60 * 10^{-5} + X * \frac{72}{6000} = 0.0 \gg X = 0.05 \text{ ton}$$





N.F.D



S.F.D



B.M.D

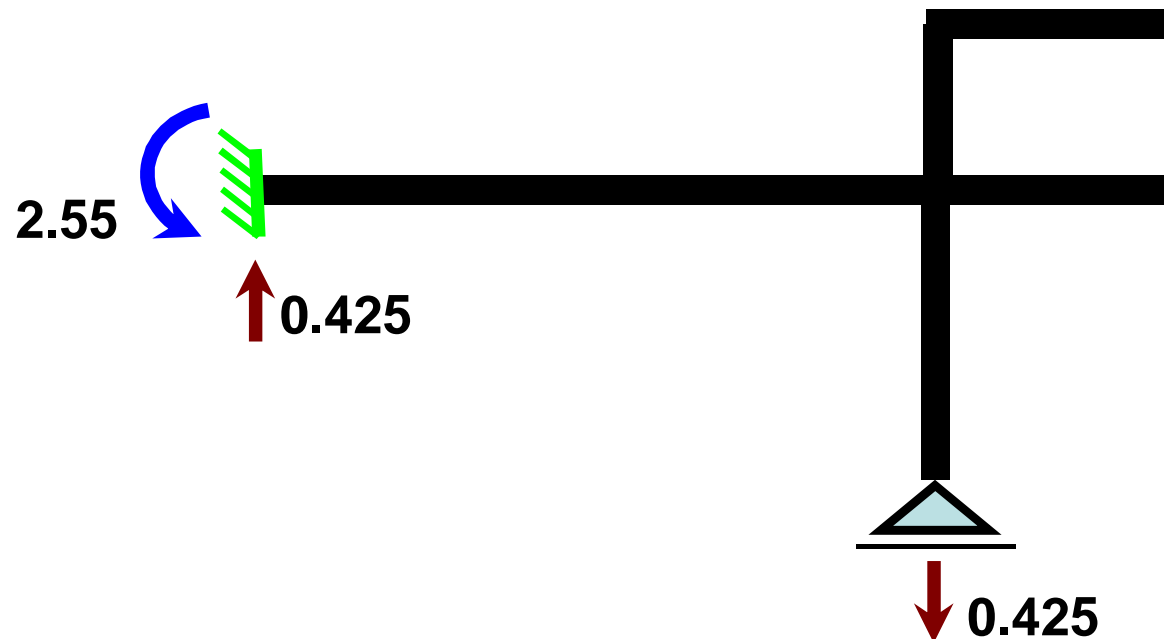
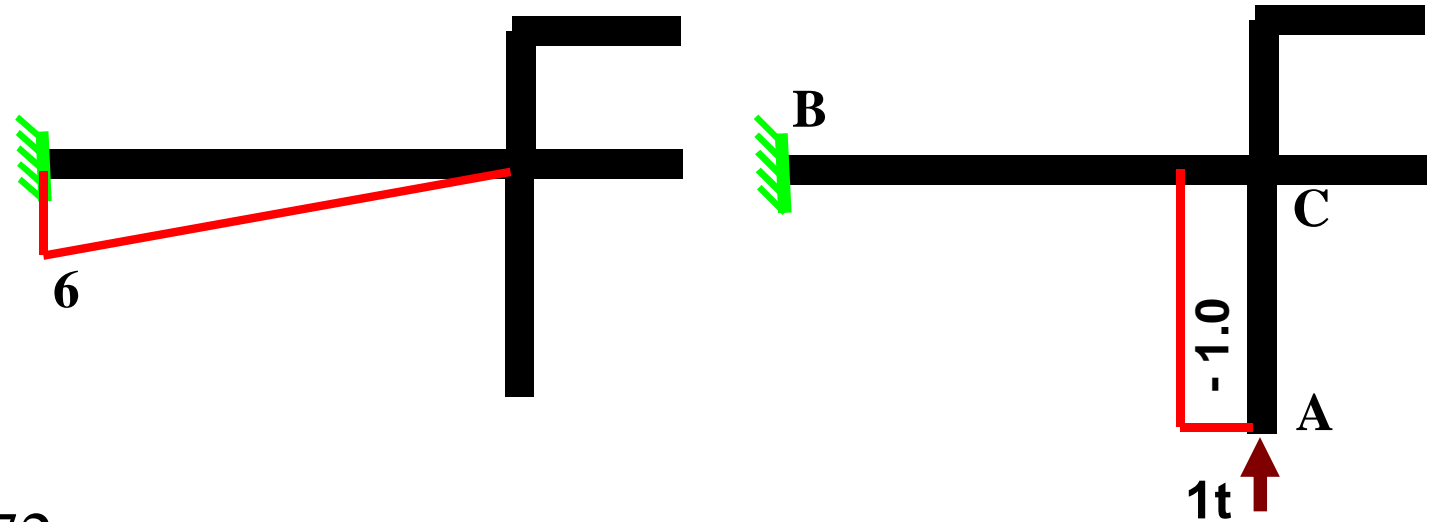
Case of non uniform temperature

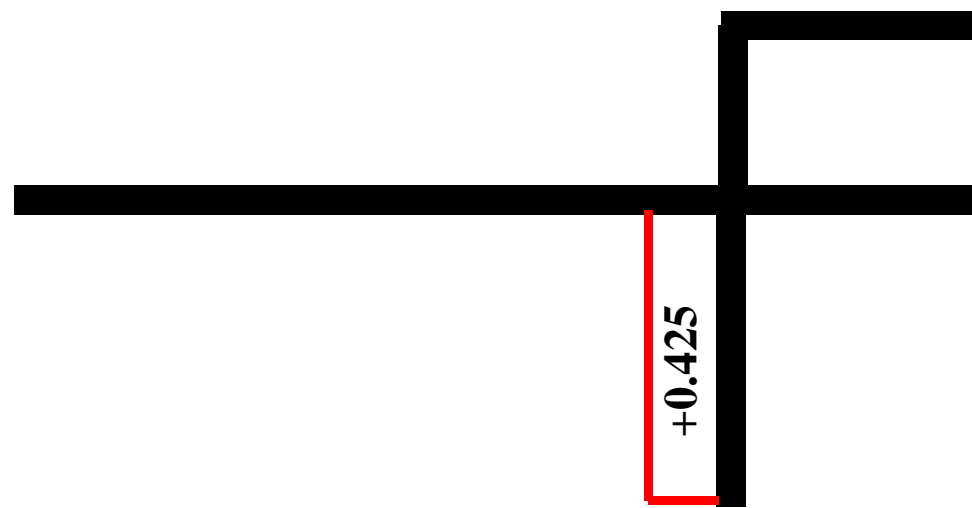
$$\delta_{10} = 10^{-5} * \left(\frac{20+40}{2} * (-1*3) \right) + 10^{-5} * \left(\frac{40-20}{0.6} * (0.5*6*6) \right) = 5.1*10^{-3}$$

$$\delta_{11} = \frac{72}{6000} m$$

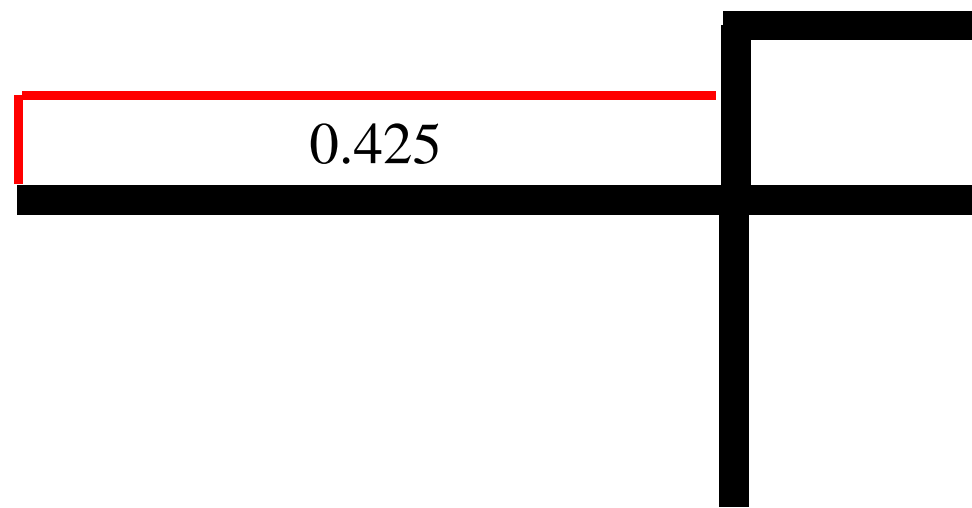
$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$5.1*10^{-3} + X * \frac{72}{6000} = 0.0 \gg X = -0.425 \text{ ton}$$

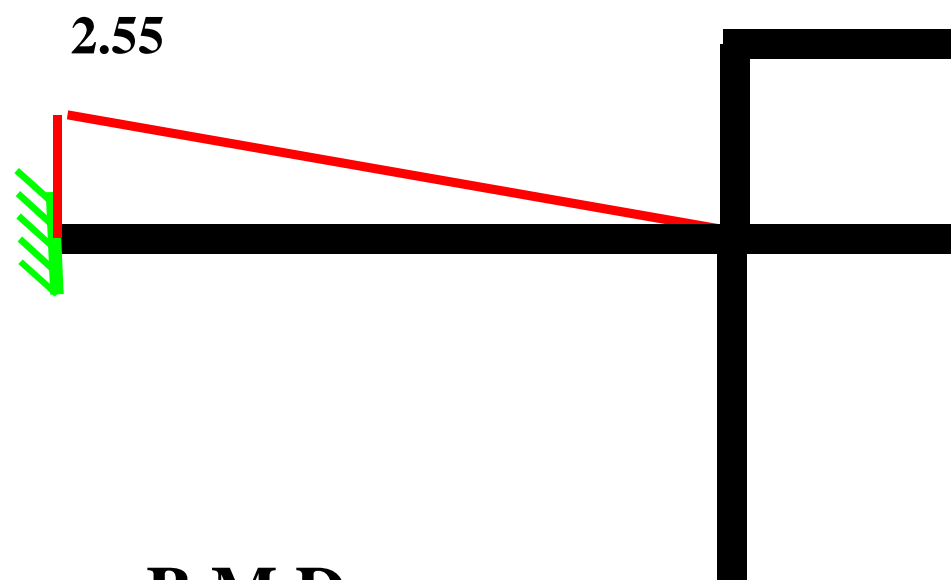




N.F.D



S.F.D

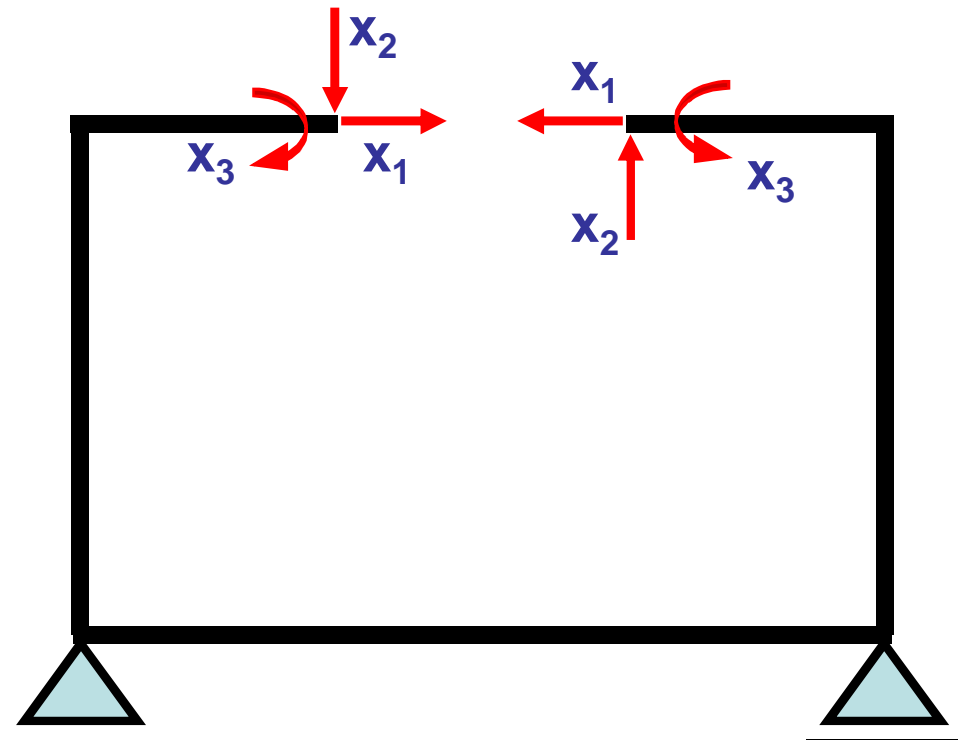
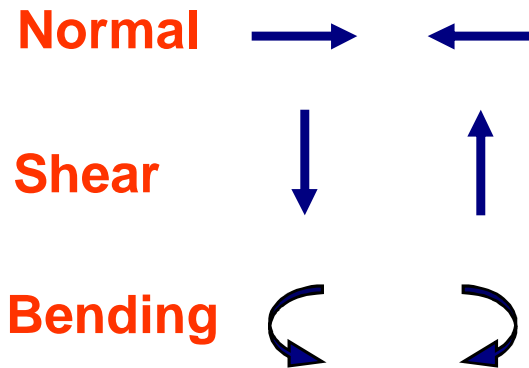


B.M.D

Closed Frame

حالة وجود closed frame

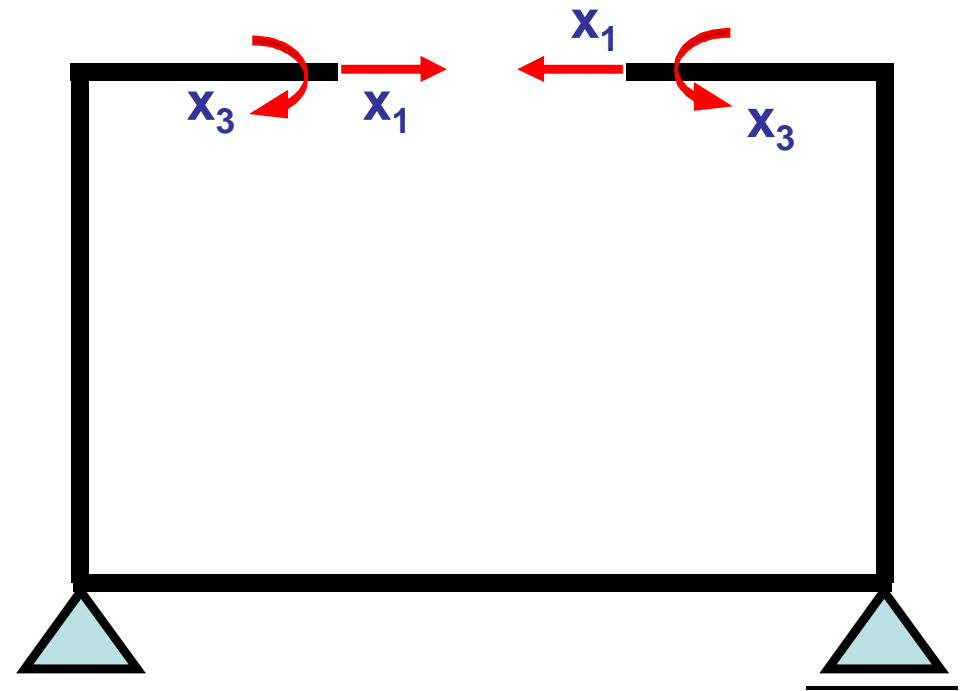
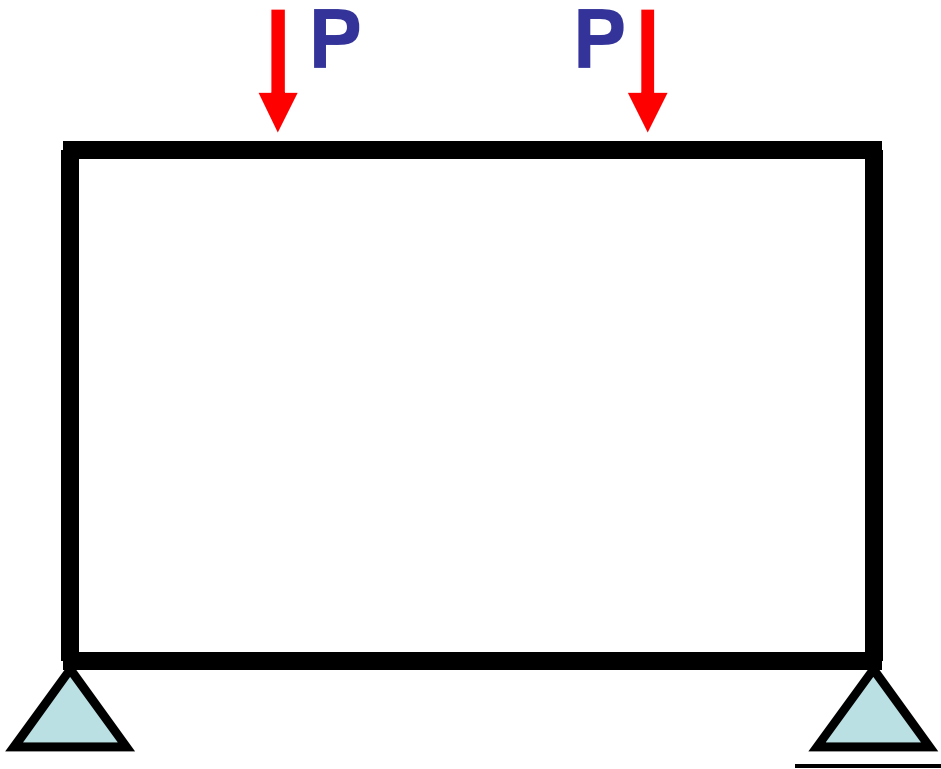
يكون هناك ثلاثة مجاهيل اضافيه وهى



Normal 

Bending 

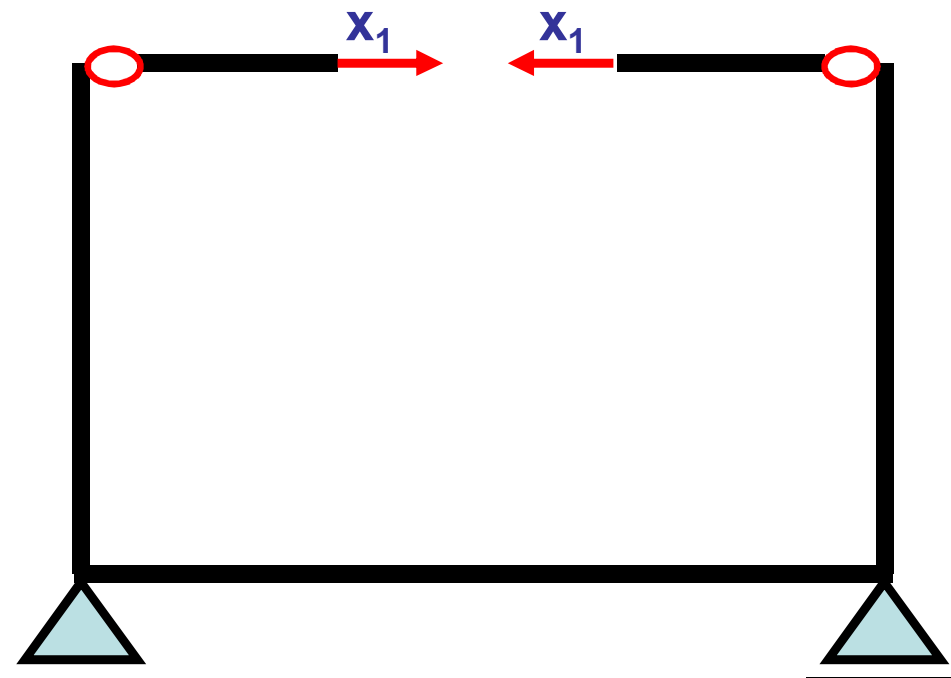
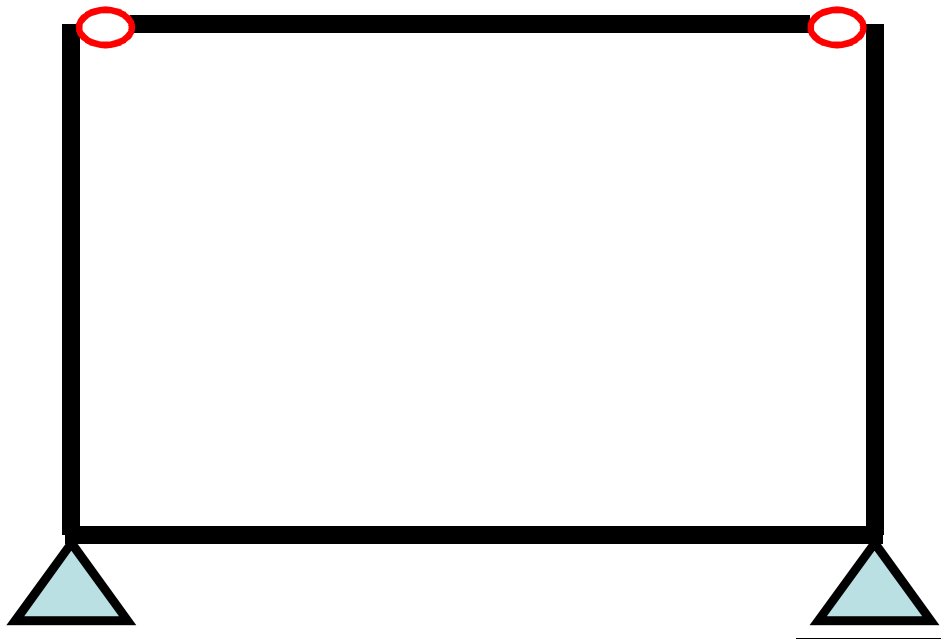
عند خط التماثل يكون الـ (shear = zero)



حالة وجود link member

يكون المجهول الوحيد بها هي الـ (normal force)

Normal → ←

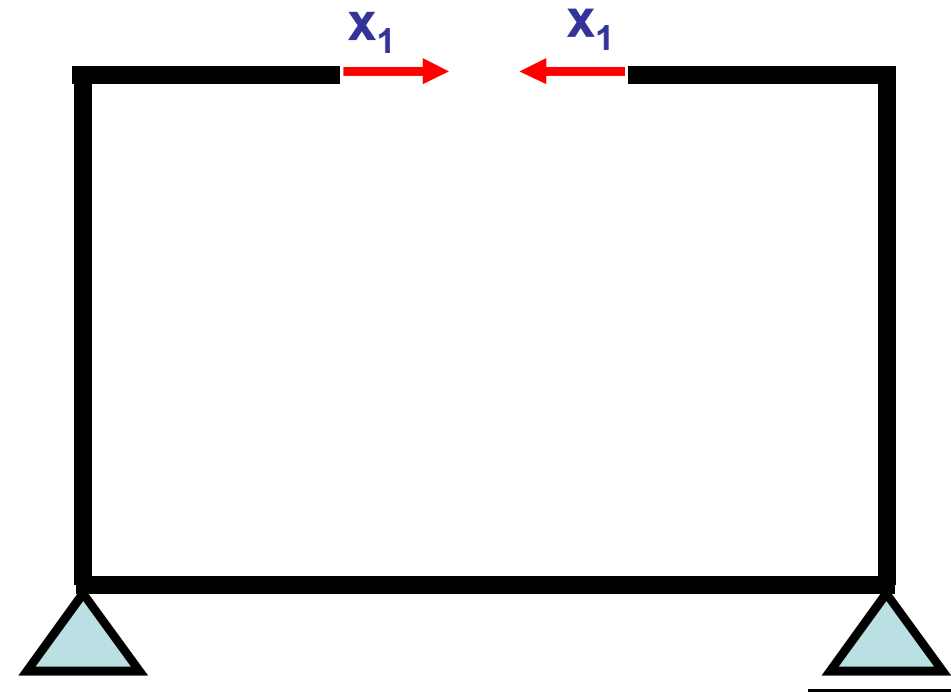
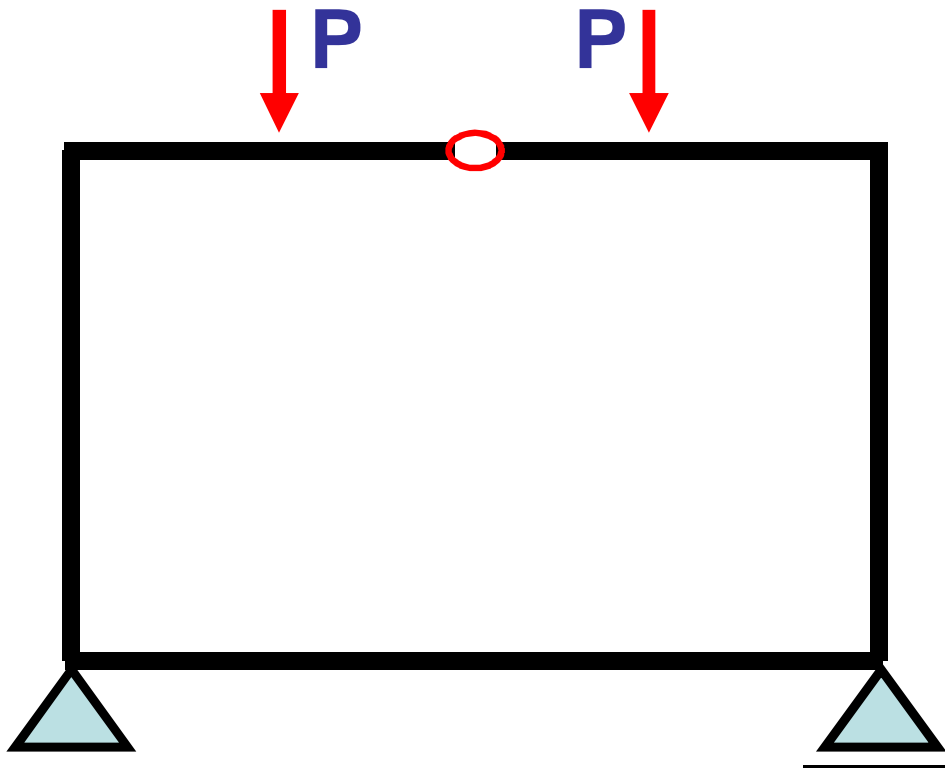


حالة وجود تماثل

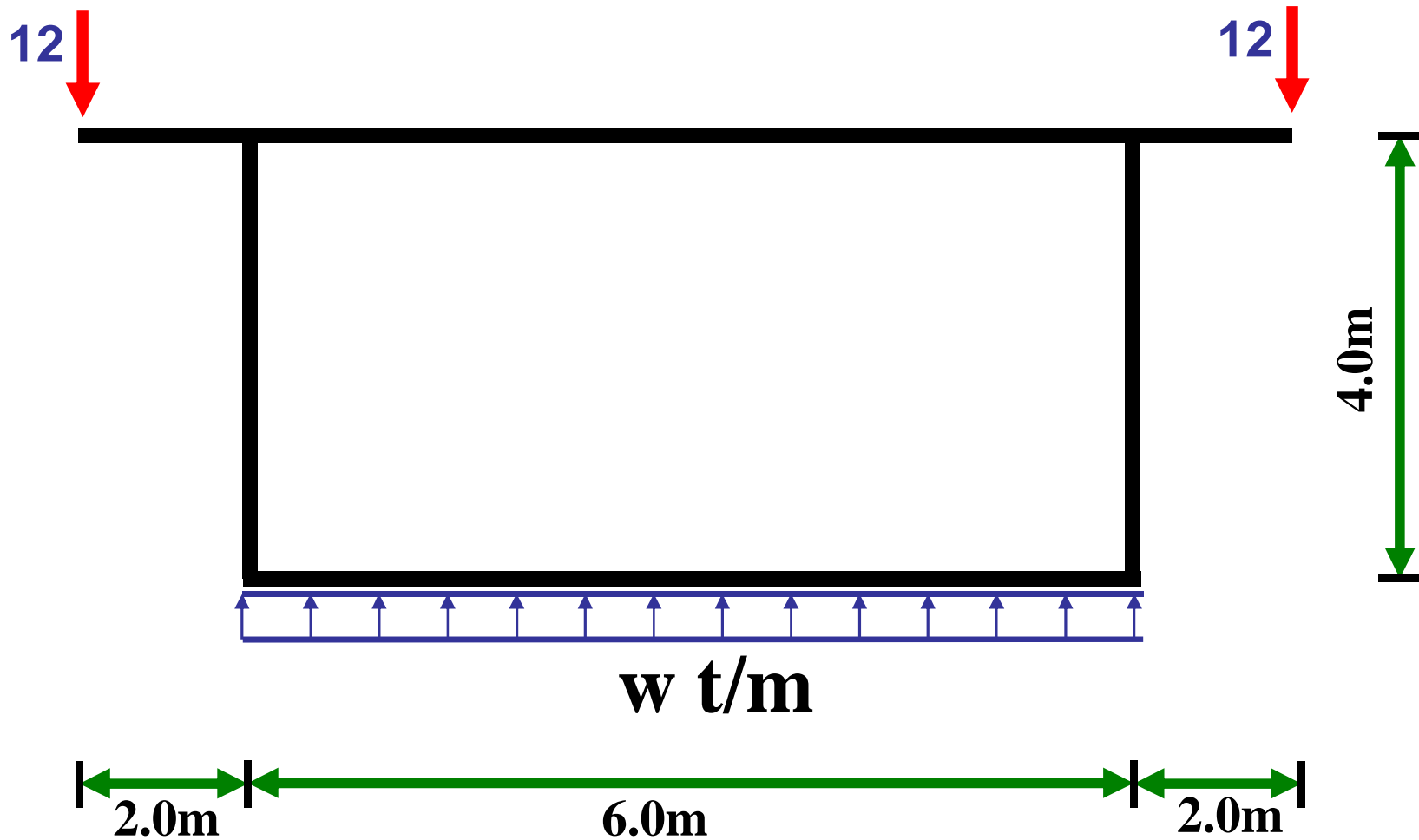
Normal 

Bending 

عند خط التماثل يكون الـ (shear = zero)

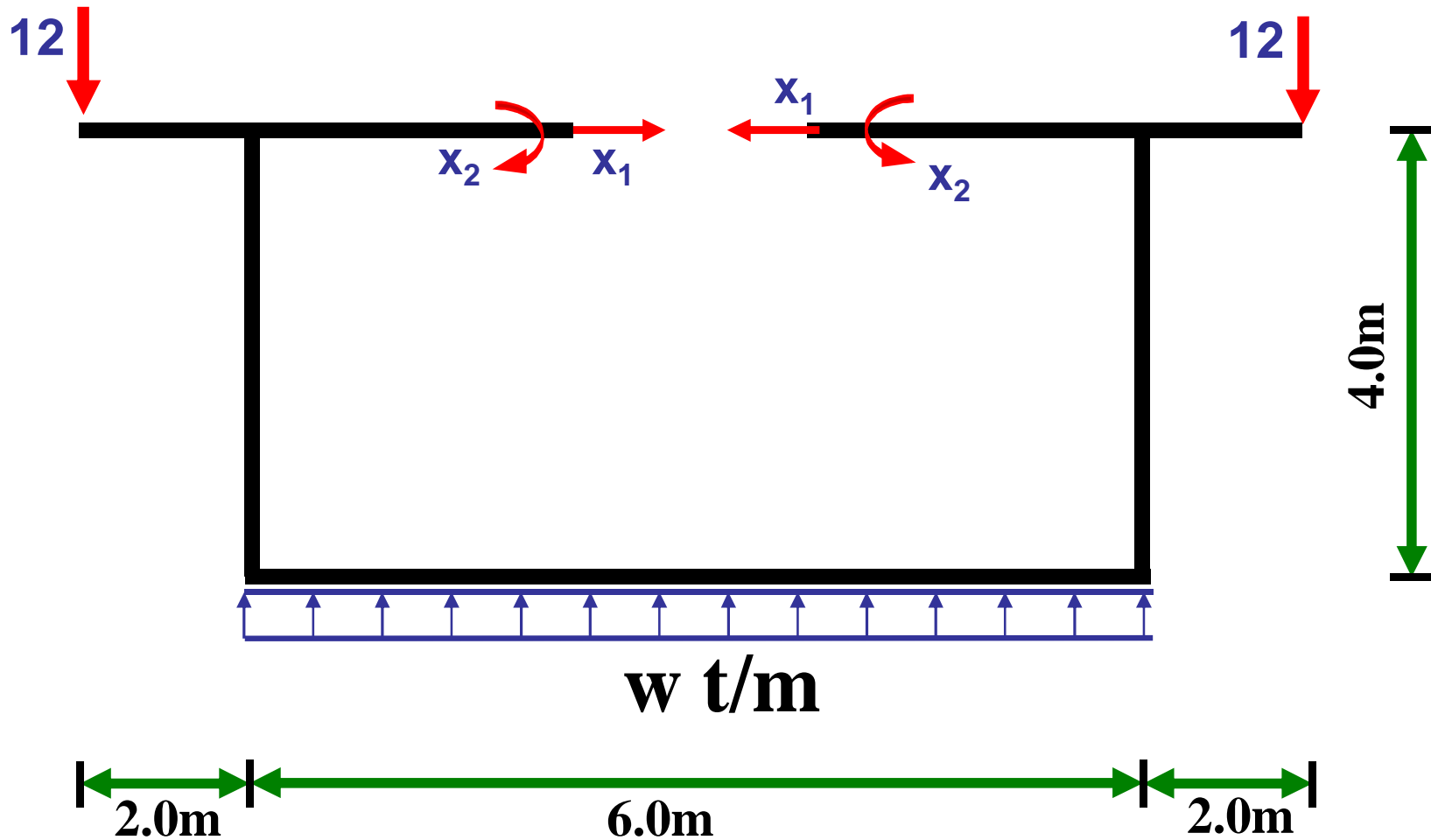


المثال الاول



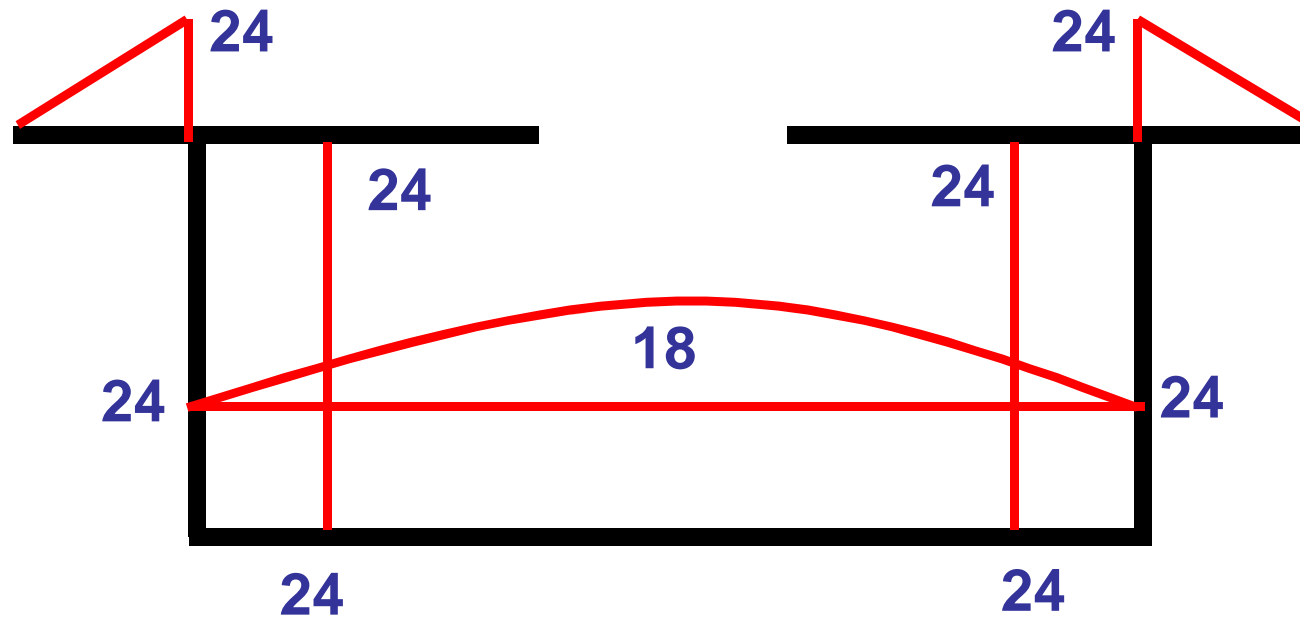
Draw B.M.D , S.F.D, and N.F.D $EI = 6000 \text{ t.m}^2$
Due to (1) given loads

Main System

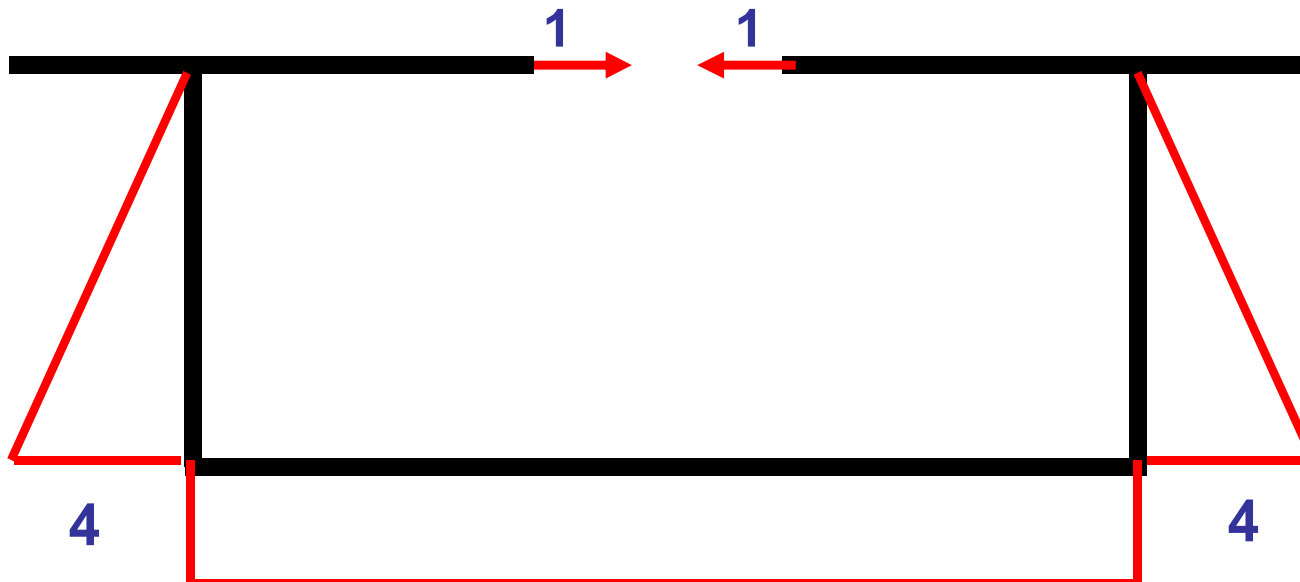


$$\begin{aligned} 6 W &= 24 \\ W &= 4 \text{ t/m} \end{aligned}$$

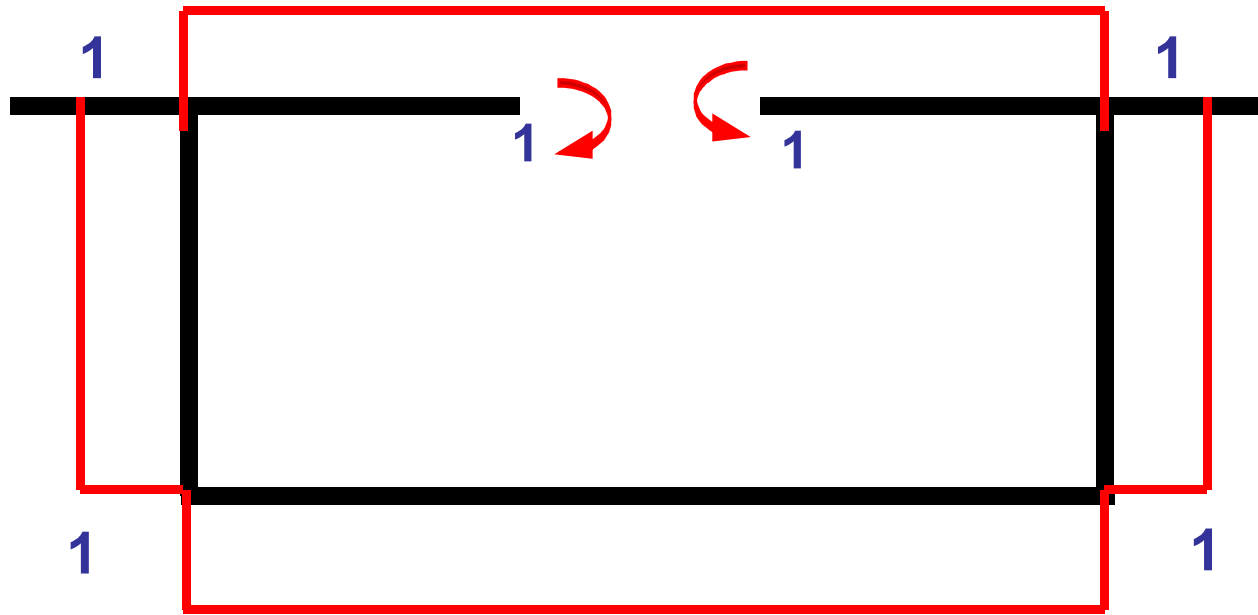
M_0



M_1



M₂



$$\delta_{10} = \frac{1}{10000} \left[-2 * \frac{24 * 4 * 4}{2} - 24 * 4 * 6 - \frac{2}{3} * 18 * 6 * 4 \right] = -\frac{1248}{10000} m$$

$$\delta_{20} = \frac{1}{10000} \left[-2 * 24 * 1 * 4 - 24 * 1 * 6 - \frac{2}{3} * 18 * 6 * 1 \right] = -\frac{408}{10000} m$$

$$\delta_{11} = \frac{1}{10000} \left[\frac{4 * 4 * 4}{3} + 4 * 4 * 6 \right] = \frac{117.33}{10000} m$$

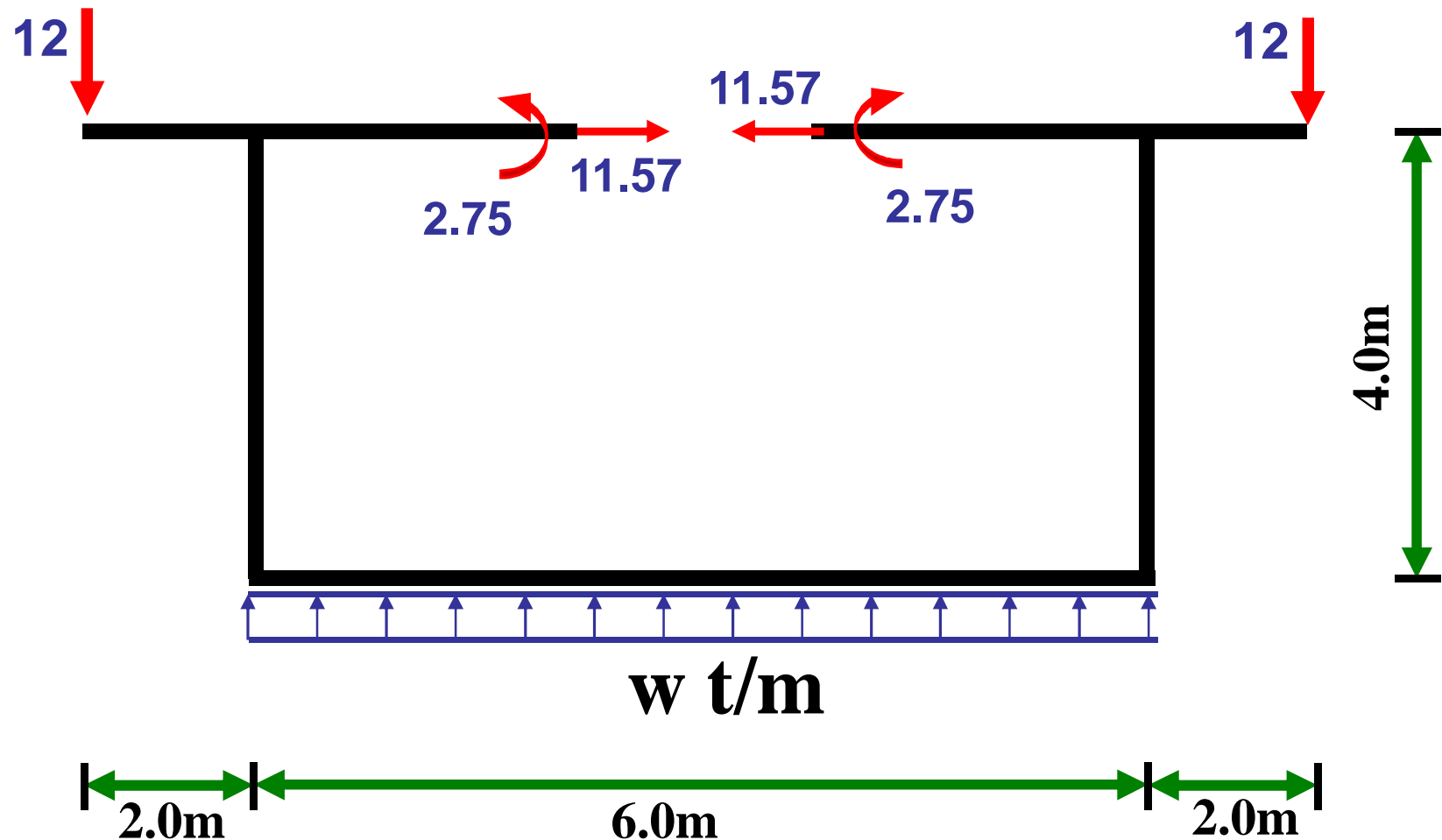
$$\delta_{22} = \frac{1}{10000} \left[1 * 1 * 6 * 2 + 1 * 1 * 4 * 2 \right] = \frac{20}{10000} m$$

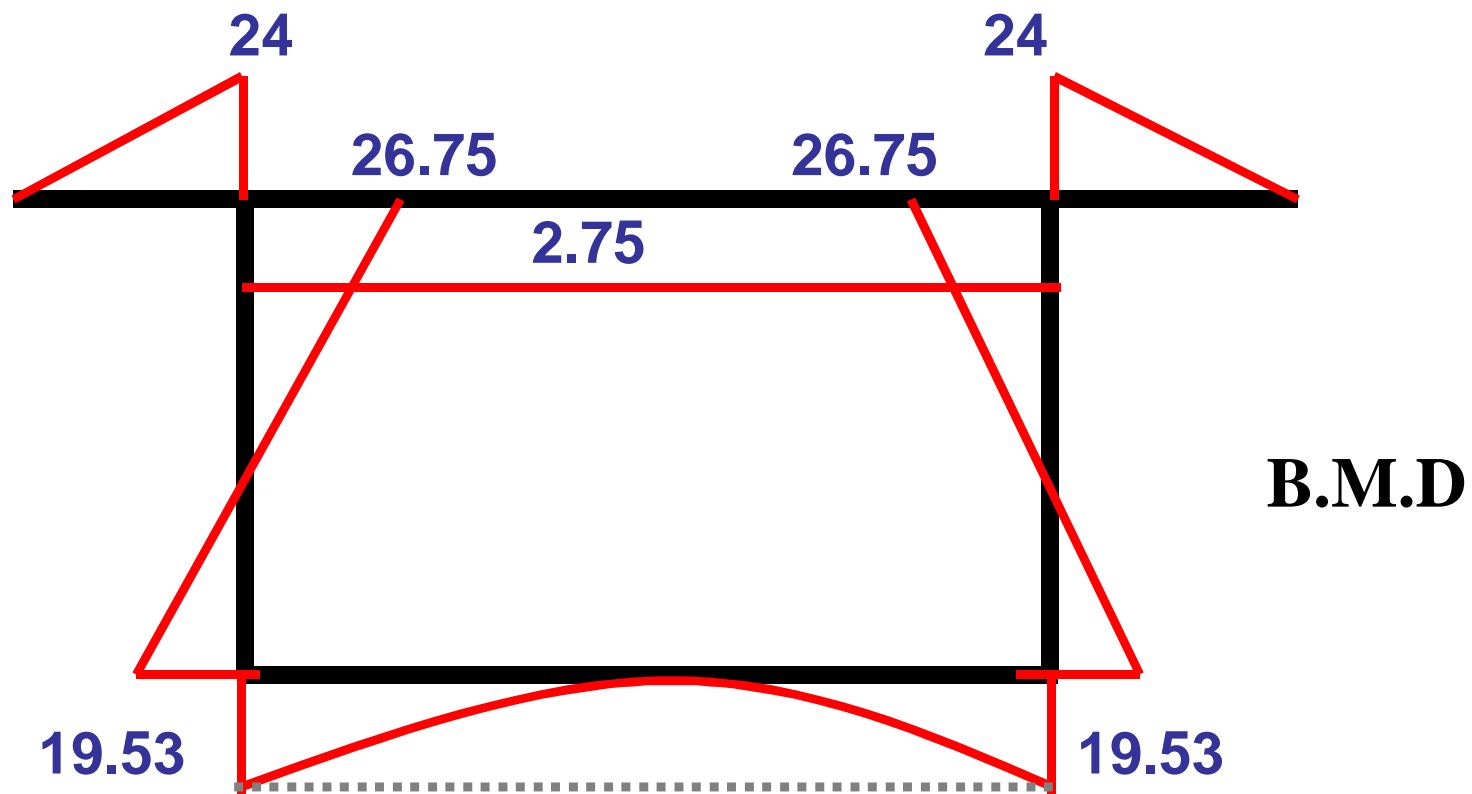
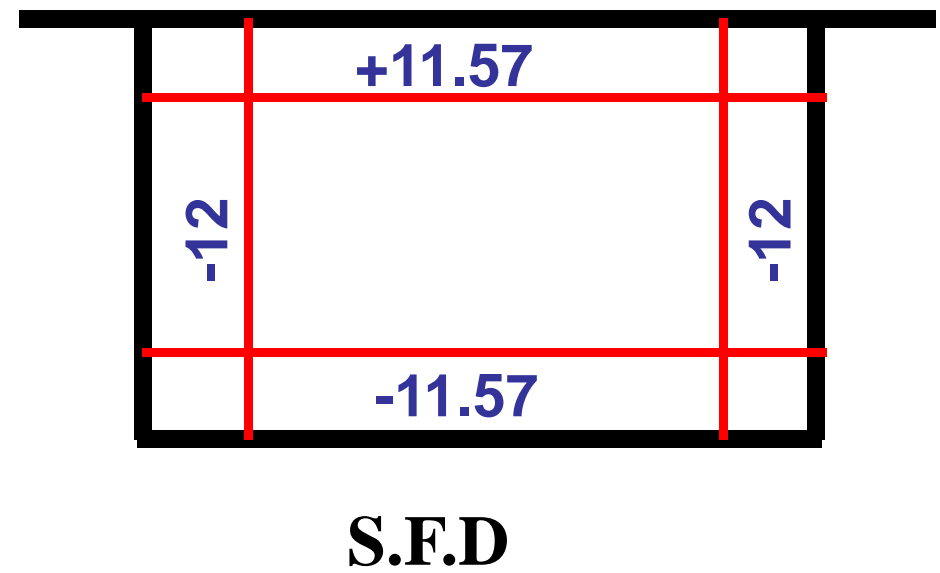
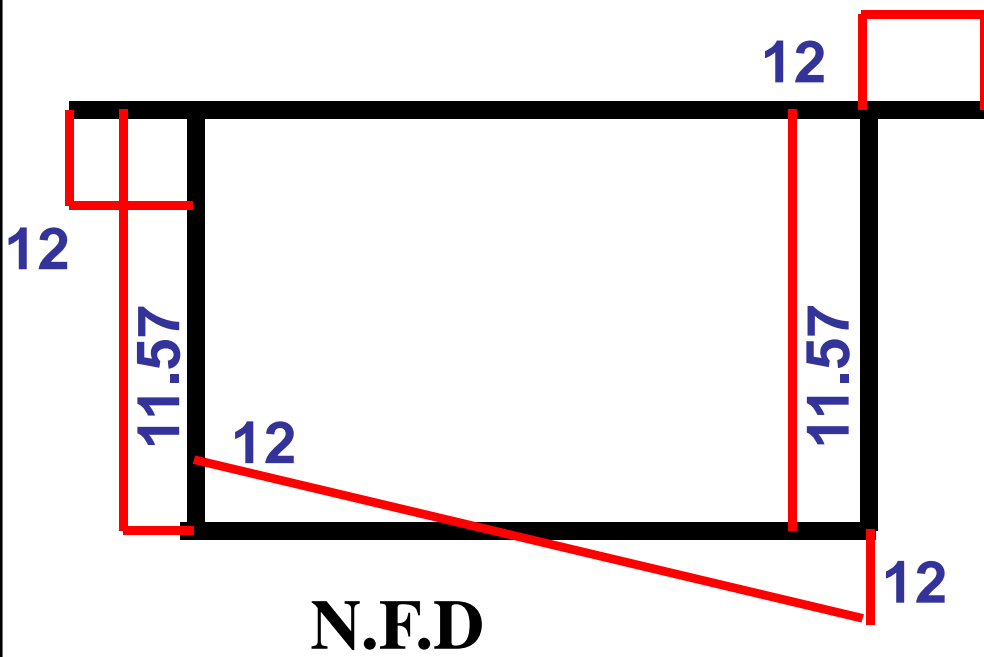
$$\delta_{21} = \delta_{12} = \frac{1}{10000} \left[\frac{1 * 4 * 4}{2} * 2 + 1 * 4 * 6 \right] = \frac{40}{10000} m$$

$$-\frac{1248}{10000} + X_1 \cdot \frac{11733}{10000} + X_2 \cdot \frac{40}{10000} = 0.0$$

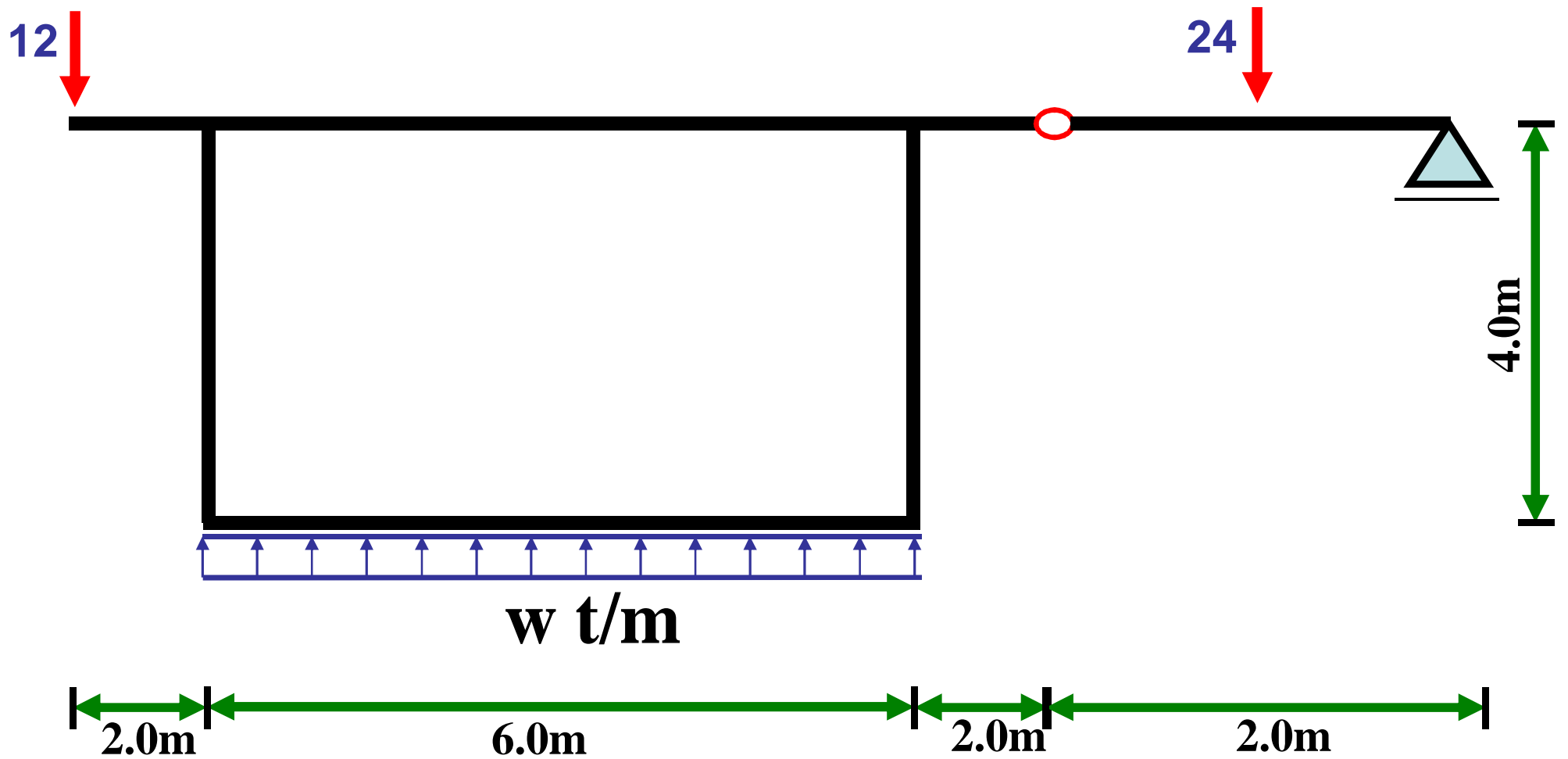
$$-\frac{408}{10000} + X_1 \cdot \frac{40}{10000} + X_2 \cdot \frac{20}{10000} = 0.0$$

X1= 11.57 ton and X2= -2.75 ton

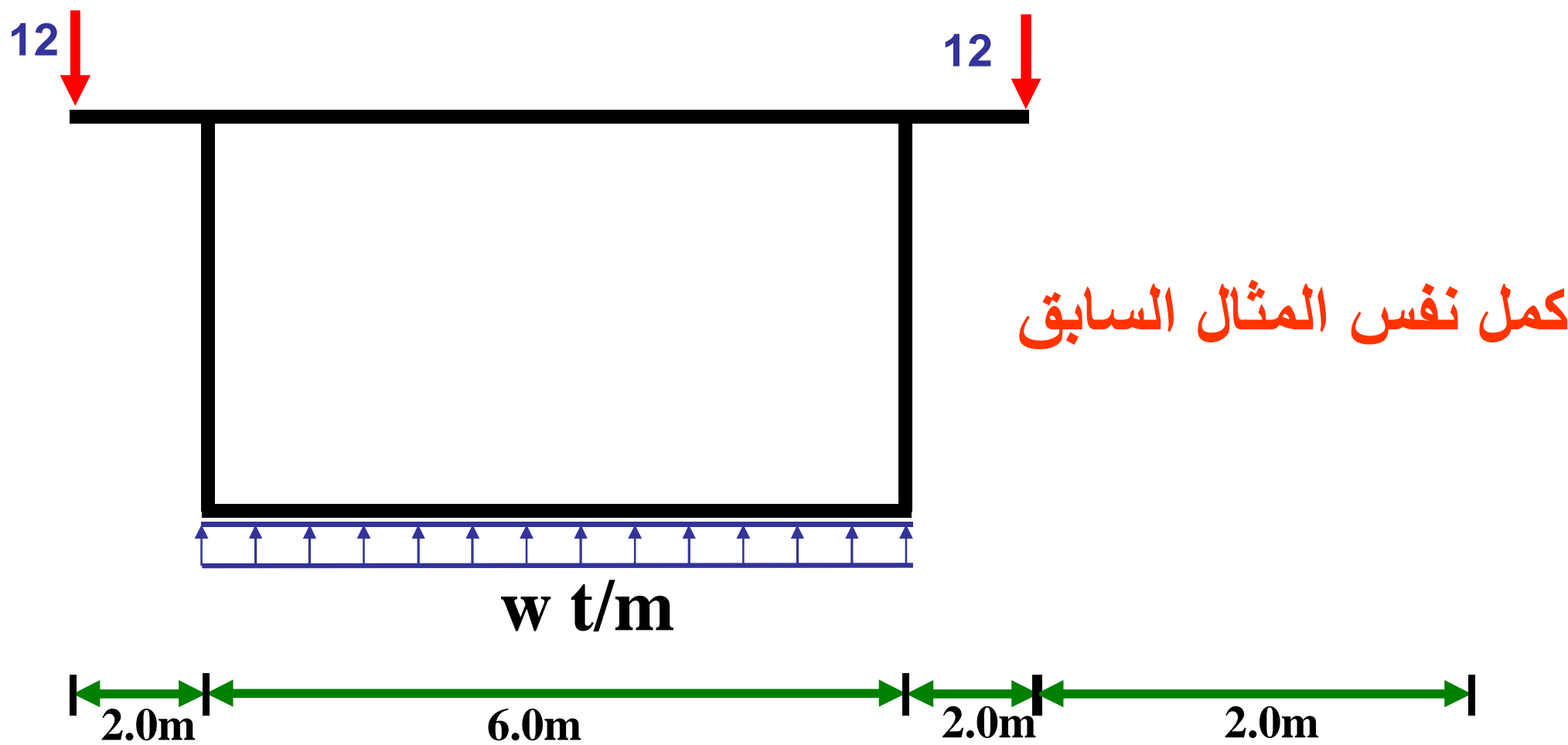




المثال الثاني



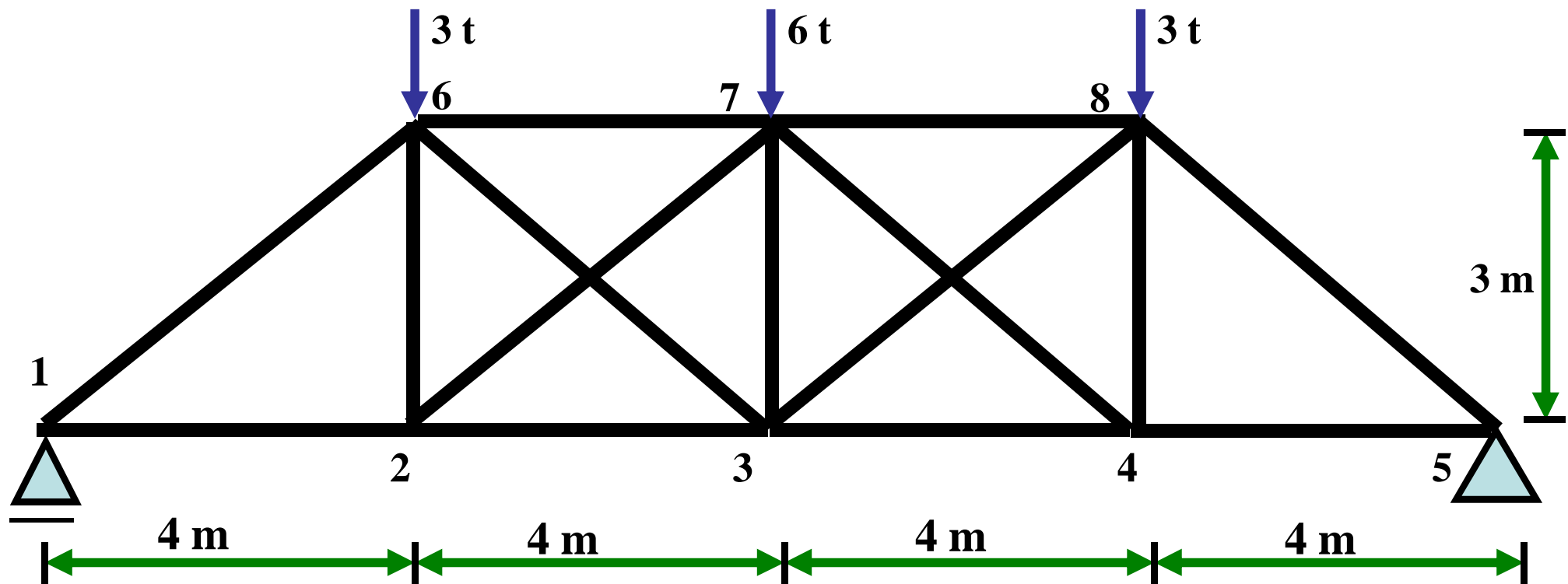
Draw B.M.D , S.F.D, and N.F.D $EI = 6000 \text{ t.m}^2$
Due to (1) given loads



Consistant Deformation

Truss

Example one



For the shown truss determine find the forces in all members due to:

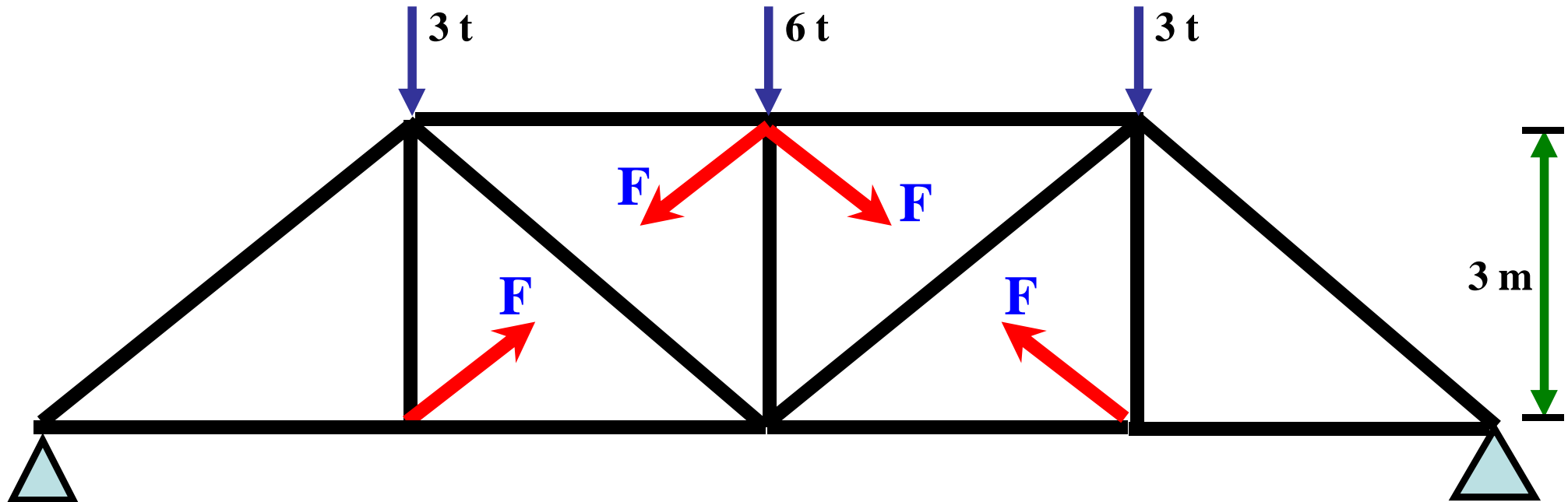
1- Given Load.

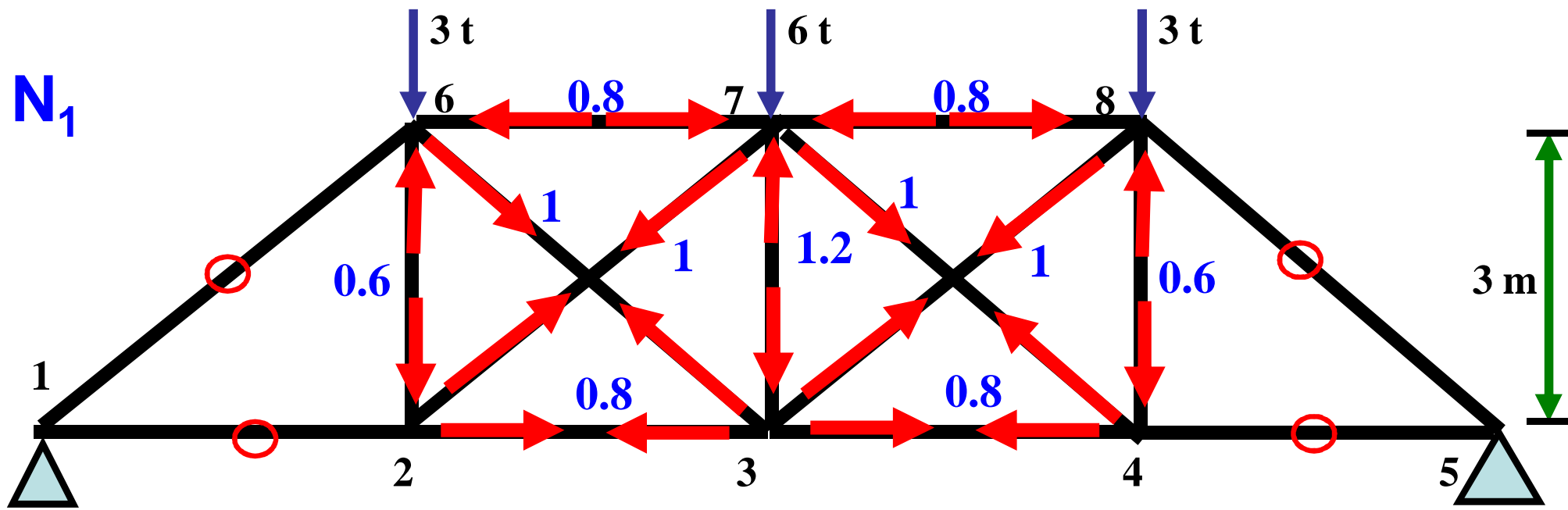
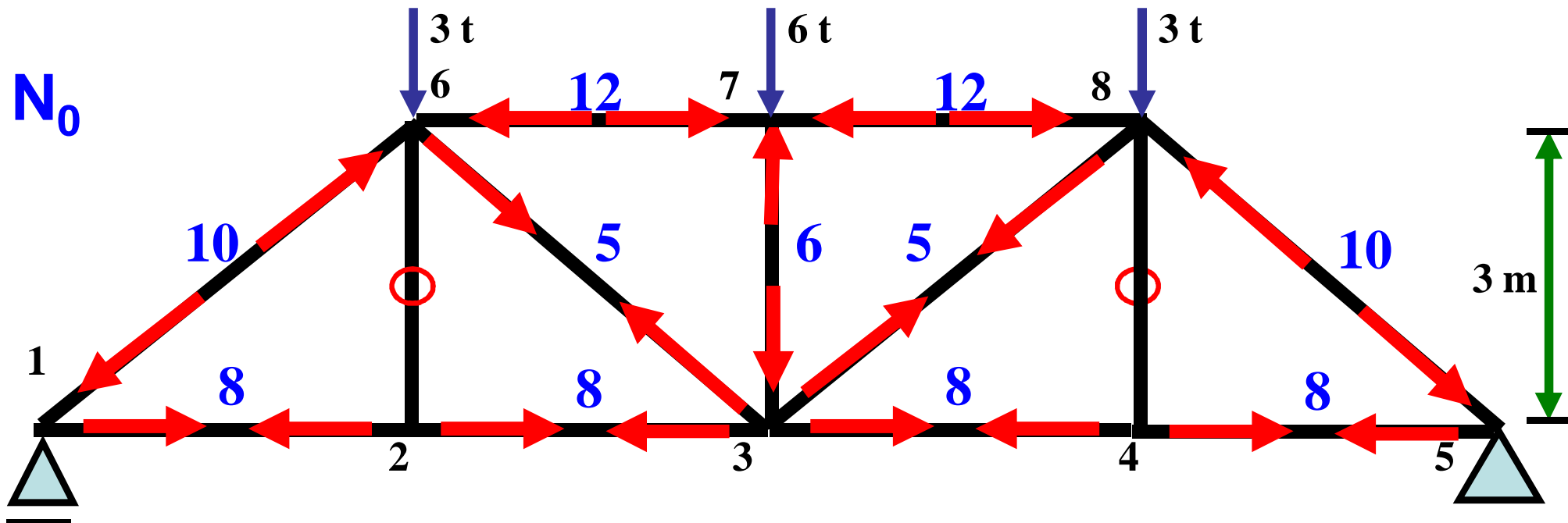
2- Settlement at (point 5)=1.5 cm.

3- drop In Temperature 30° .

$$EA = 40000 \text{ ton} \quad \alpha = 10^{-5}$$

main system





<div><div>temperature</div><div>setteltment</div><div>Given loads</div></div>											
member	L	N ₀	N ₁		N ₀ N ₁ L	N ₁ N ₁ L	N ₁ L		N _F	N _F	N _F
2-6	3	0	-0.6		0	1.08	-1.8				
3-7	3	-6	-1.2		21.6	4.32	-3.6				
4-8	3	0	-0.6		0	1.08	-1.8				
1-2	4	8	0		0	0	0				
2-3	4	8	-0.8		-25.6	2.56	-3.2				
3-4	4	8	-0.8		-25.6	2.56	-3.2				
4-5	4	8	0		0	0	0				
6-7	4	-12	-0.8		38.4	2.56	-3.2				
7-8	4	-12	-0.8		38.4	2.56	-3.2				
1-6	5	-10	0		0	0	0				
2-7	5	0	1		0	5	5				
3-8	5	5	1		25	5	5				
3-6	5	5	1		25	5	5				
4-7	5	0	1		0	5	5				
5-8	5	-10	0		0	0	0				
389 ∑					97.2	36.72	0.0				

Case of given loads

$$\delta_{10} = \frac{\sum N_0 N_1 L}{EA} = \frac{97.2}{40000}$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{36.72}{40000}$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$\frac{97.2}{40000} + F * \frac{36.72}{40000} = 0.0 \gg F = -2.65 \text{ ton}$$

Given loads

$$N_F = N_0 + F * N_1 = N_0 - 2.65 N_1$$

يتم عمل الحسابات في الجدول فقط

Case of Temperature

$$\delta_{10} = \alpha . \Delta t . \sum (N1.L) = 0.0$$

$$\delta_{11} = \text{given load} = \frac{36.72}{40000}$$

$$\delta_{10} + X . \delta_{11} = 0.0$$

$$0 + F * \frac{36.72}{40000} = 0.0 \gg F = 0$$

Temperature

$$N_F = F * N_1 = 0$$

يتم عمل الحسابات في الجدول فقط

Case of settelment

$$\delta_{10} = 0.0$$

$$\delta_{11} = \text{given load} = \frac{36.72}{40000}$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$0 + F * \frac{36.72}{40000} = 0.0 \gg F = 0$$

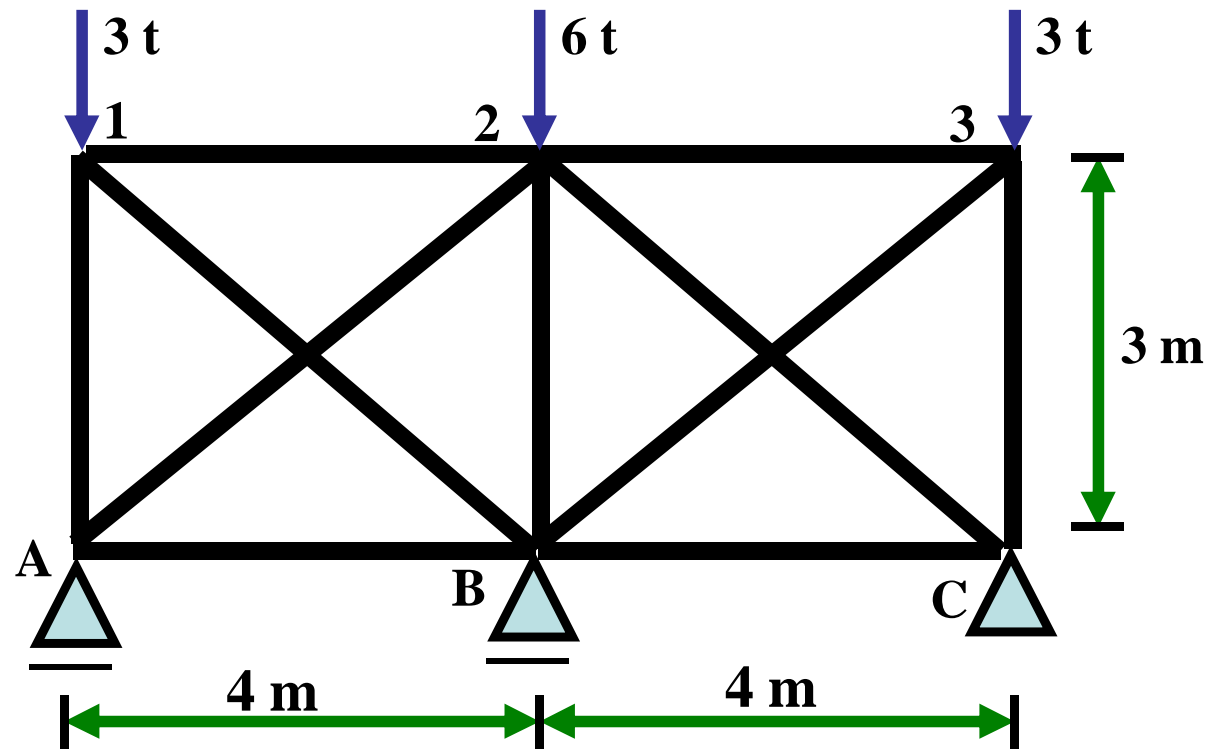
Temprature

$$N_F = F * N_1 = 0$$

يتم عمل الحسابات في الجدول فقط

لاحظ انه عندما يكون المنشأ خارجي determinate فانه لا يتأثر بالحرارة ولا الـ settlement

Example Two

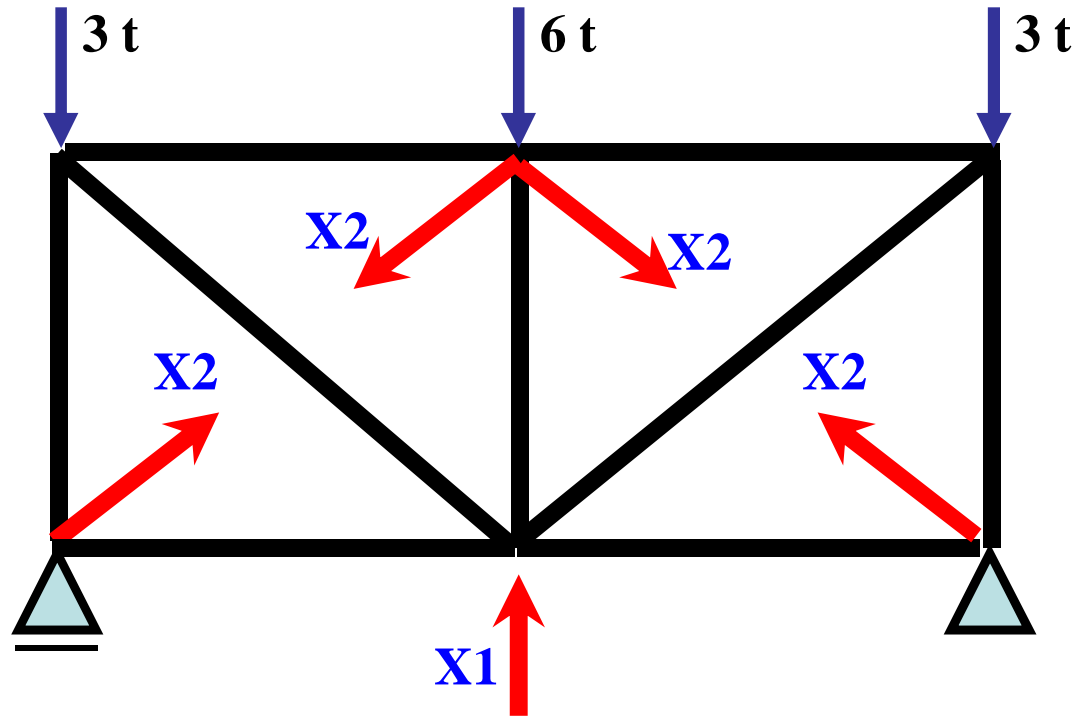


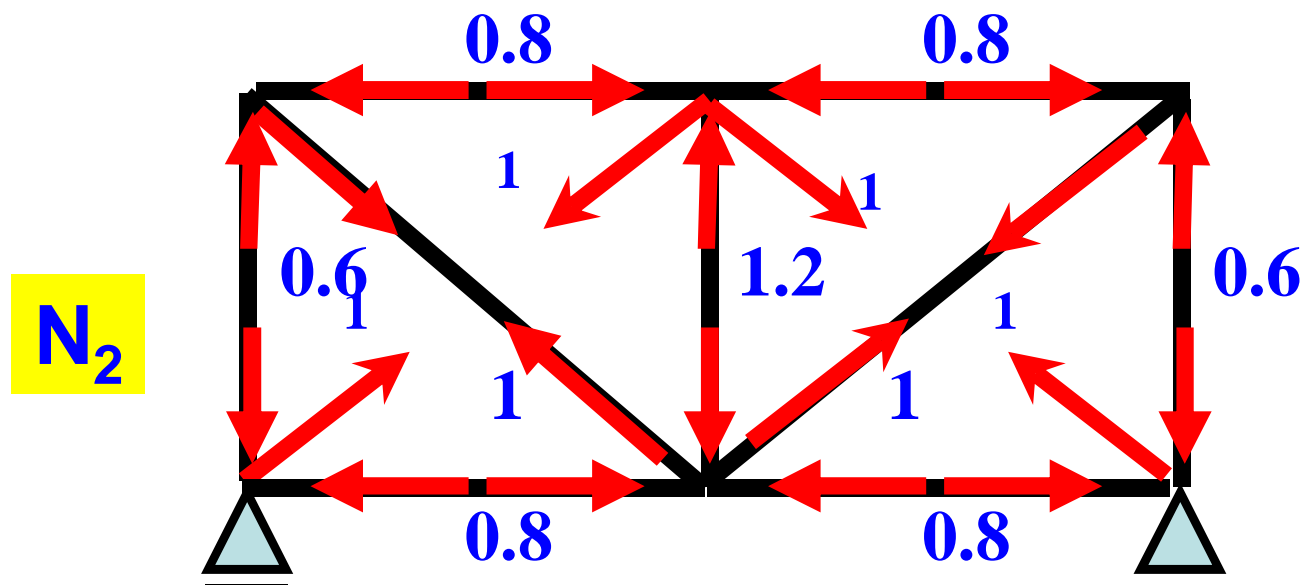
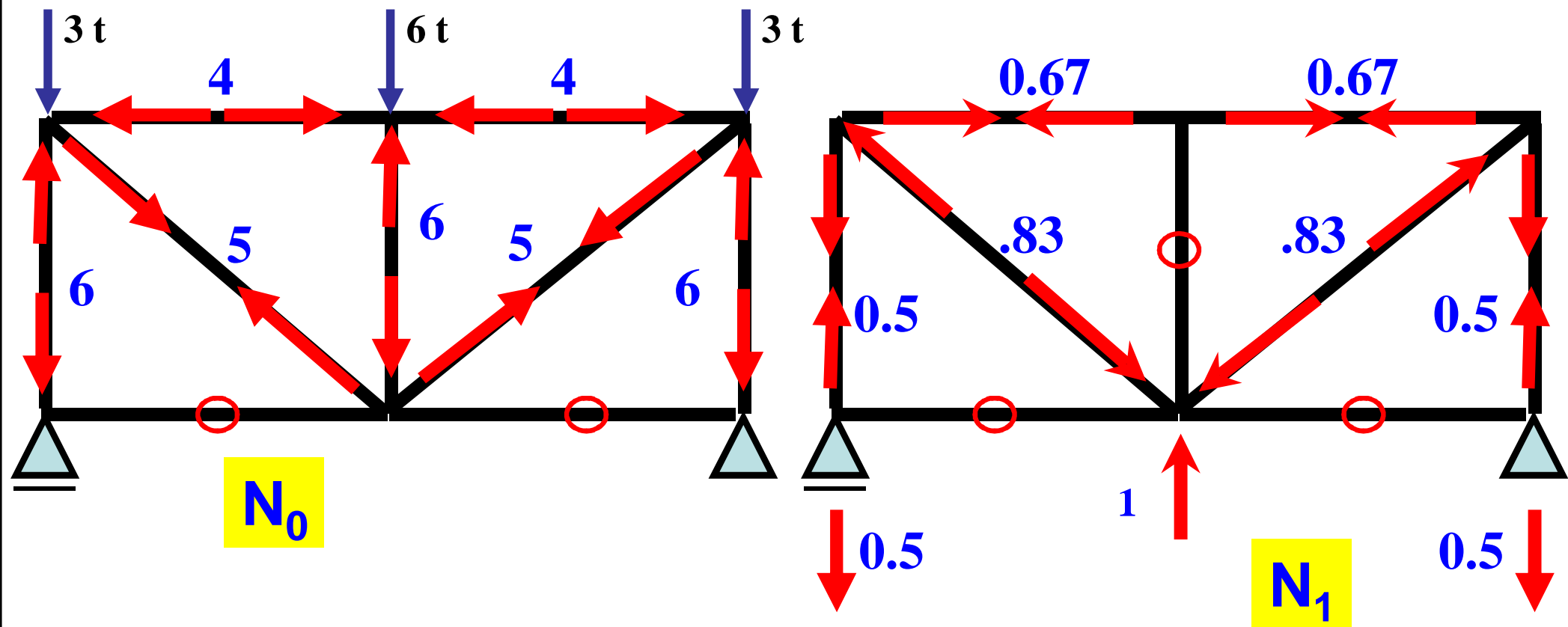
For the shown truss determine find the forces in all members due to:

- 1- Given Load.
- 2- Settlement at B=1.5 cm.
- 3- drop In Temperature 30° In Chord AB2.

$$EA=10000 \text{ ton} \quad \alpha =10^{-5}$$

main system





temprature

setteltment

Given loads

member	L	N ₀	N ₁	N ₂	N ₀ N ₁ L	N ₀ N ₂ L	N ₁ N ₁ L	N ₂ N ₂ L	N ₂ N ₁ L	N ₁ L	N ₂ L	N _F	N _F	N _F
AB	4	0	0	-0.8	0	0	0	2.56	0	0	-3.2			
BC	4	0	0	-0.8	0	0	0	2.56	0	0	-3.2			
1-2	4	-4	0.67	-0.8	-10.67	12.8	1.77	2.56	-2.13	2.67	-3.2			
2-3	4	-4	0.67	-0.8	-10.67	12.8	1.77	2.56	-2.13	2.67	-3.2			
A-1	3	-6	0.5	-0.6	-15	10.8	0.75	1.08	-0.9	1.5	-1.8			
B-2	3	-6	0	-1.2	-15	21.6	0	4.32	0	0	-3.6			
C-3	3	-6	0.5	-0.6	-15	10.8	0.75	1.08	-0.9	1.5	-1.8			
A-2	5	0	0	1	0	0	0	5	0	0	5			
B-3	5	5	-.83	1	-20.83	25	3.47	5	-4.166	-4.15	5			
B-1	5	5	-.83	1	-20.83	25	3.47	5	-4.166	-4.15	5			
C-2	5	0	0	1	0	0	0	5	0	0	5			
Σ					-108	118.8	12	36.75	-14.4	0	0			

Case of given loads

$$\delta_{10} = \frac{\sum N_0 N_1 L}{EA} = -\frac{108}{10000}$$

$$\delta_{20} = \frac{\sum N_0 N_2 L}{EA} = \frac{118.8}{10000}$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{12}{10000}$$

$$\delta_{22} = \frac{\sum N_2 N_2 L}{EA} = \frac{36.75}{10000}$$

$$\delta_{12} = \frac{\sum N_1 N_2 L}{EA} = -\frac{14.4}{10000}$$

$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} = 0.0$$

$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} = 0.0$$

$$>> x_1=9.66 \text{ \& } x_2=0.55$$

Given loads

$$N_F = N_0 + 9.66 N_1 + 0.55 N_2$$

يتم عمل الحسابات في الجدول فقط

Case of Temperature

$$\delta_{10} = 10^{-5} * -30 * 0 = 0$$

$$\delta_{20} = 10^{-5} * -30 * 0 = 0$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{12}{10000}$$

$$\delta_{22} = \frac{\sum N_2 N_2 L}{EA} = \frac{36.75}{10000}$$

$$\delta_{12} = \frac{\sum N_1 N_2 L}{EA} = -\frac{14.4}{10000}$$

$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} = 0.0$$

$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} = 0.0$$

>> x1=0 & x2=0

Temperature

$$N_F = 0$$

يتم عمل الحسابات في الجدول فقط

Case of settelment

$$\delta_{10} = 0.0$$

$$\delta_{20} = 0.0$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{12}{10000}$$

$$\delta_{22} = \frac{\sum N_2 N_2 L}{EA} = \frac{36.75}{10000}$$

$$\delta_{12} = \frac{\sum N_1 N_2 L}{EA} = -\frac{14.4}{10000}$$

$$\delta_{10} + X_1 \cdot \delta_{11} + X_2 \cdot \delta_{12} = -0.015$$

$$\delta_{20} + X_1 \cdot \delta_{21} + X_2 \cdot \delta_{22} = 0.0$$

$$>> x1=-23.6 \text{ \& } x2=-9.25$$

Temprature

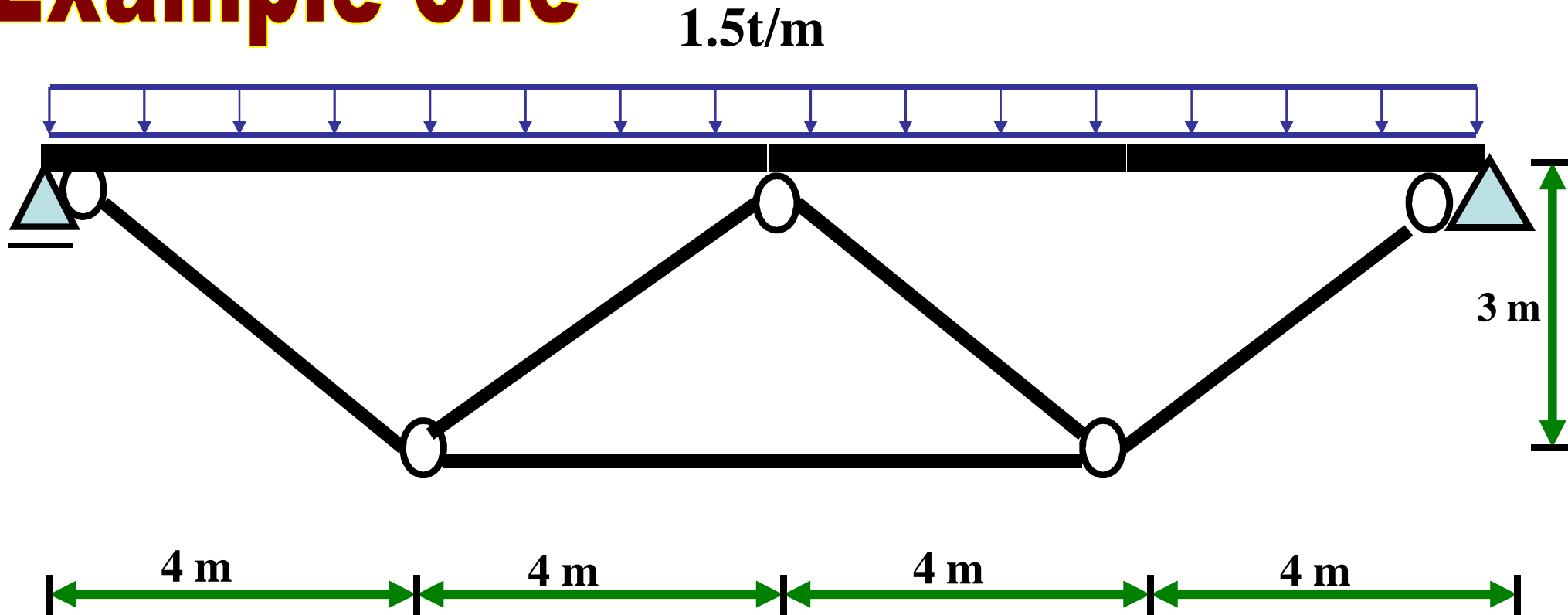
$$N_F = N_0 - 23.6N_1 - 9.25N_2$$

يتم عمل الحسابات فى الجدول فقط

Consistant Deformation

Trussed beam

Example one



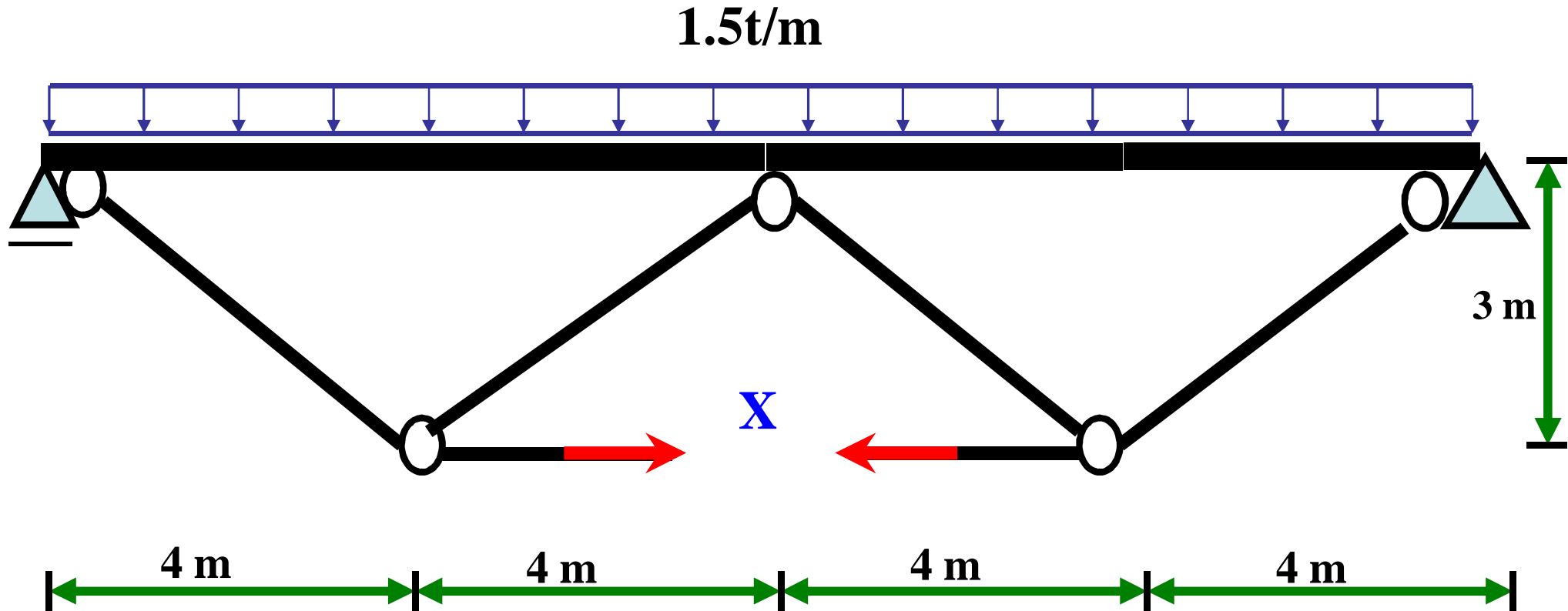
For the shown truss determine find the forces in all members due to:

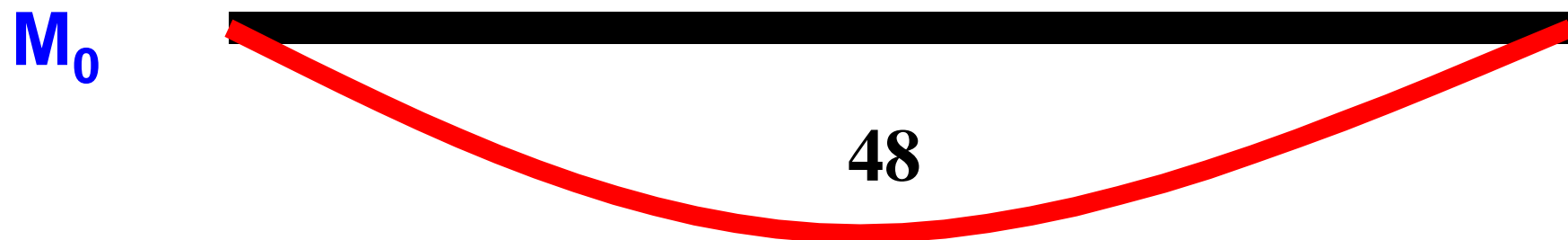
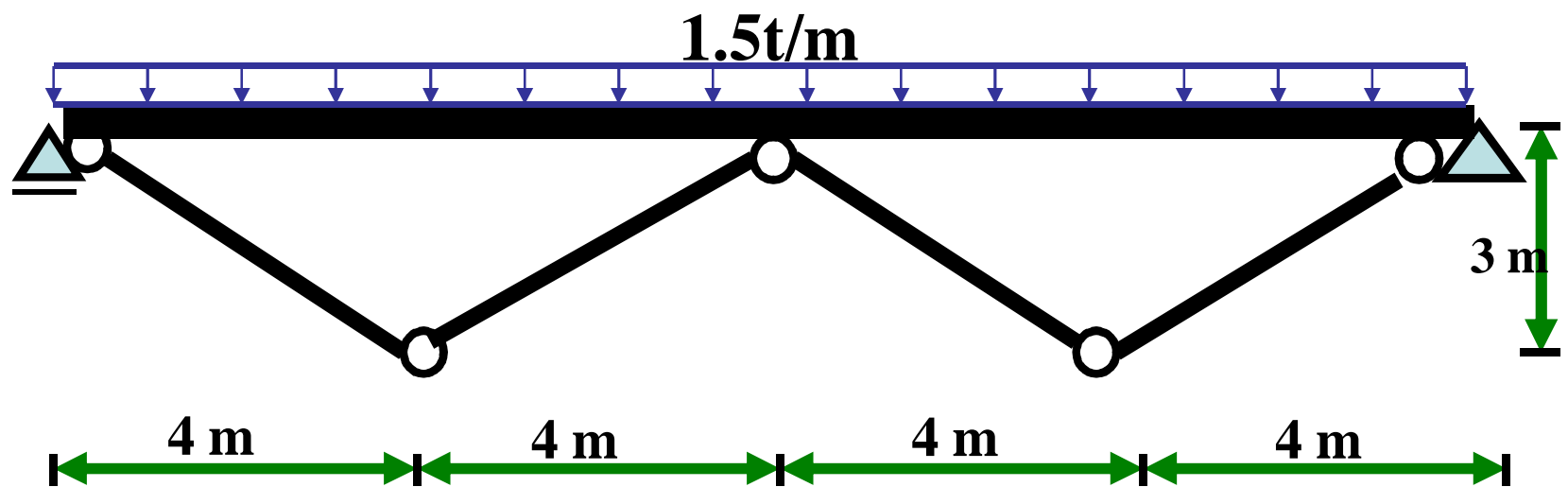
$$EI_{\text{beam}} = 6000 \text{ ton}$$

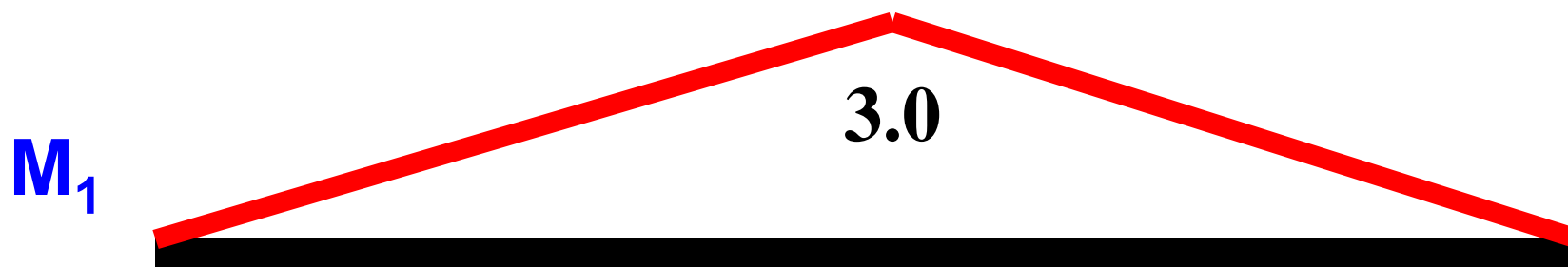
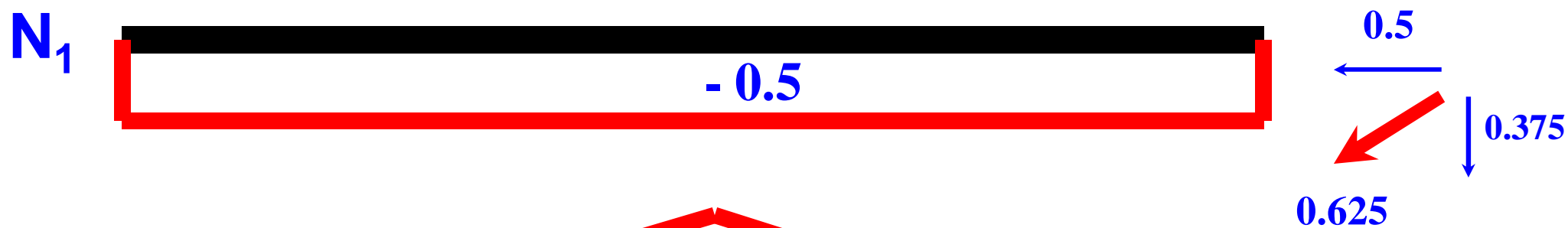
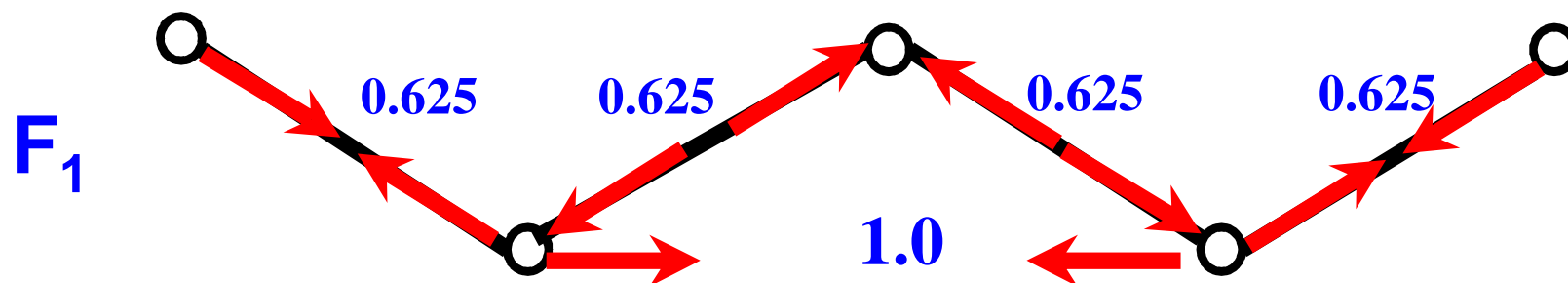
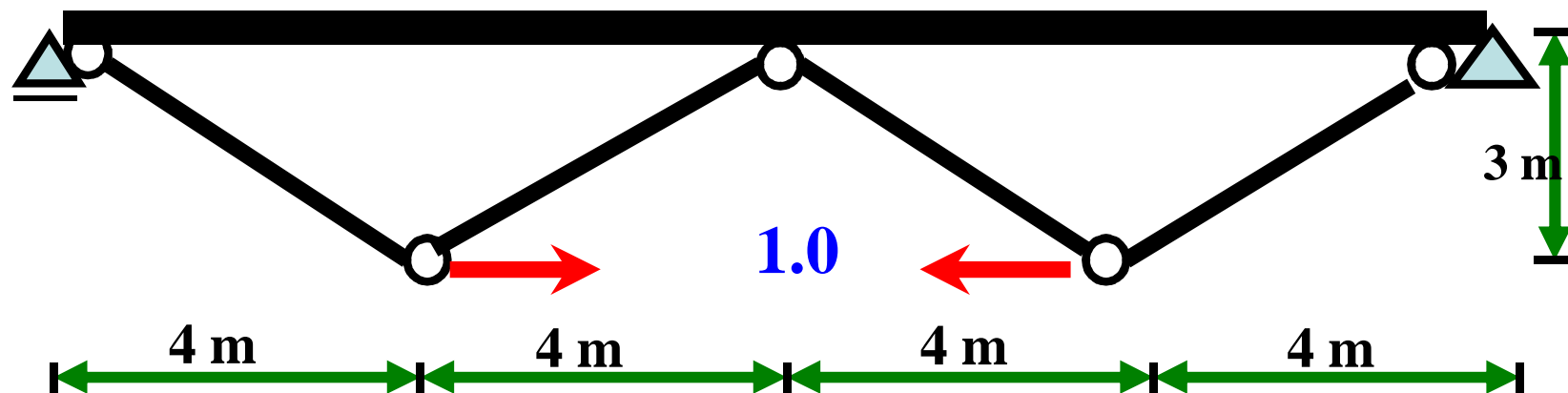
$$EA_{\text{beam}} = 2400 \text{ ton}$$

$$EA_{\text{link}} = 4000 \text{ ton}$$

main system







Case of given loads

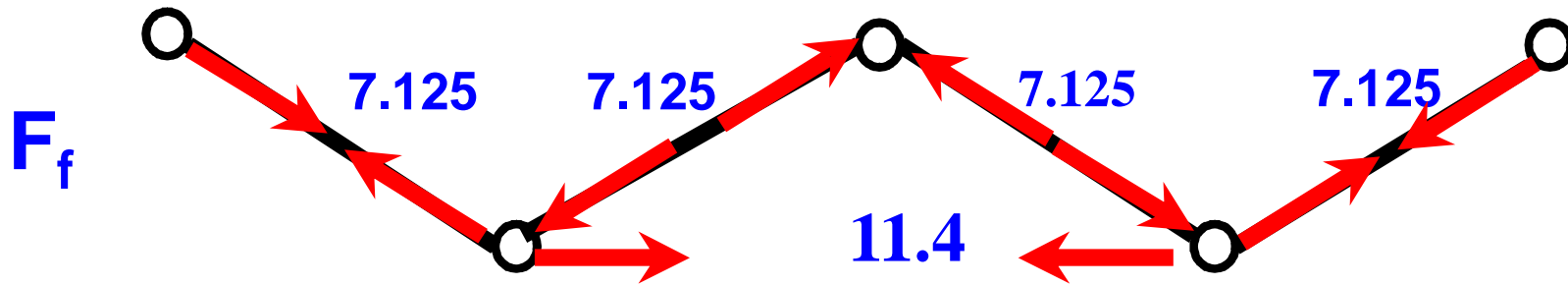
$$\begin{aligned}\delta_{10} &= \int \frac{M_0 M_1}{EI} + \frac{\sum N_0 N_1 L}{EA} + \frac{\sum F_0 F_1 L}{EA} \\ &= \frac{1}{6000} \left[-\frac{2}{3} * 48 * 8 * 5/8 * 3 * 2 \right] + 0.0 + 0.0 = -0.16\end{aligned}$$

$$\begin{aligned}\delta_{11} &= \int \frac{M_1 M_1}{EI} + \frac{\sum N_1 N_1 L}{EA} + \frac{\sum F_1 F_1 L}{EA} \\ &= \frac{1}{6000} \left[\frac{3 * 3 * 8}{3} * 2 \right] + \frac{1}{2400} [0.5 * 0.5 * 16] + \frac{1}{4000} [0.625^2 * 5 * 4 + 1 * 8] = 0.0136\end{aligned}$$

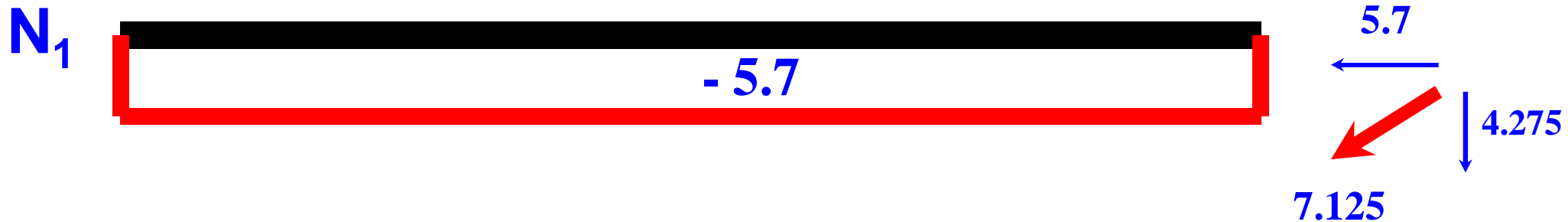
$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

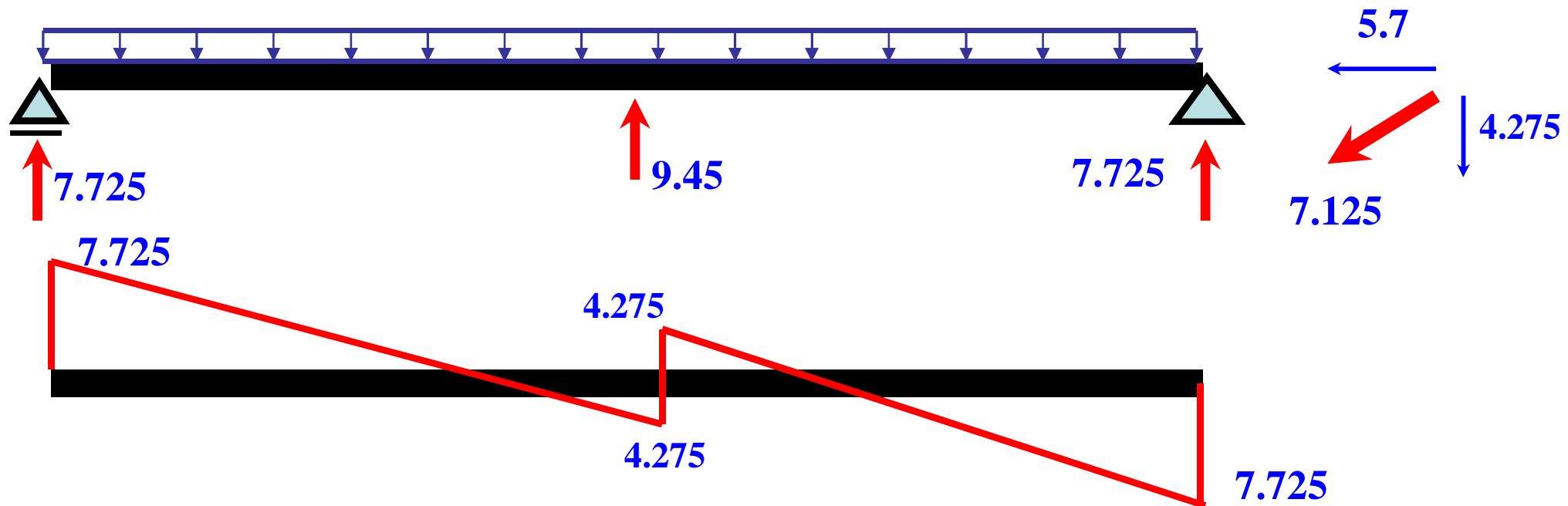
$$-0.16 + X * 0.0136 = 0.0 \gg X = 11.74 \text{ ton}$$

$$F_F = F_0 + 11.4F_1 = 0 + 11.4F_1 = 11.4F_1$$

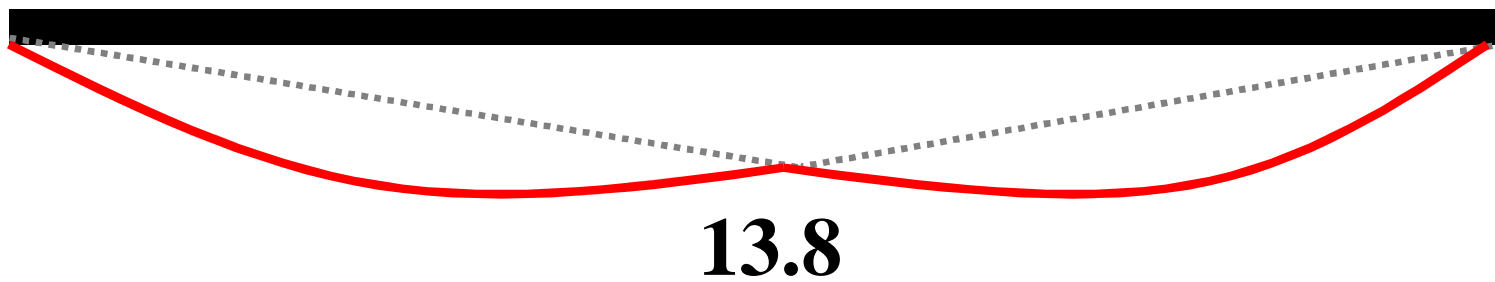


$$N_F = N_0 + 11.4N_1 = 0 + 11.4N_1 = 11.4N_1$$





$$M_F = M_0 + 11.4M_1$$

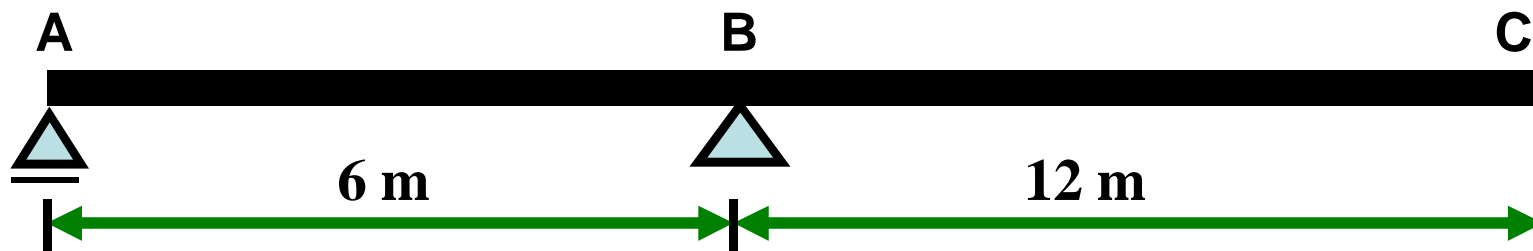


Consistant Deformation

Forced deformation

Forced deformation

هي تحويل الهبوط المعطى الى المكان المأخوذ في الـ main system بطريقة الـ virtual work

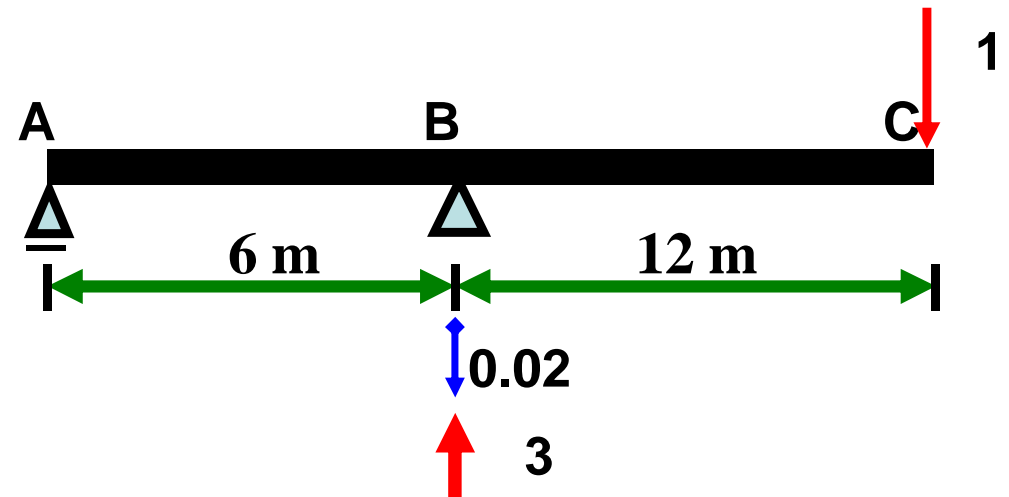


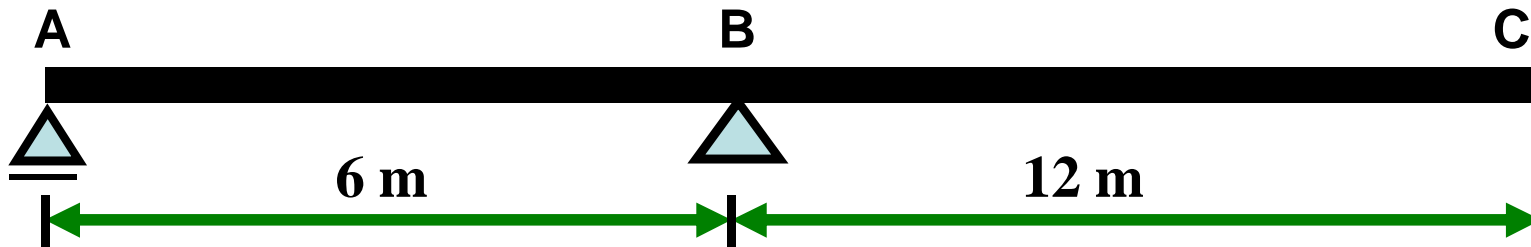
Given settlement at b = 2cm

Required deflection at C

$$\Delta_c * 1 - 3 * 0.02 = 0.0$$

$$\Delta_c = 0.06$$



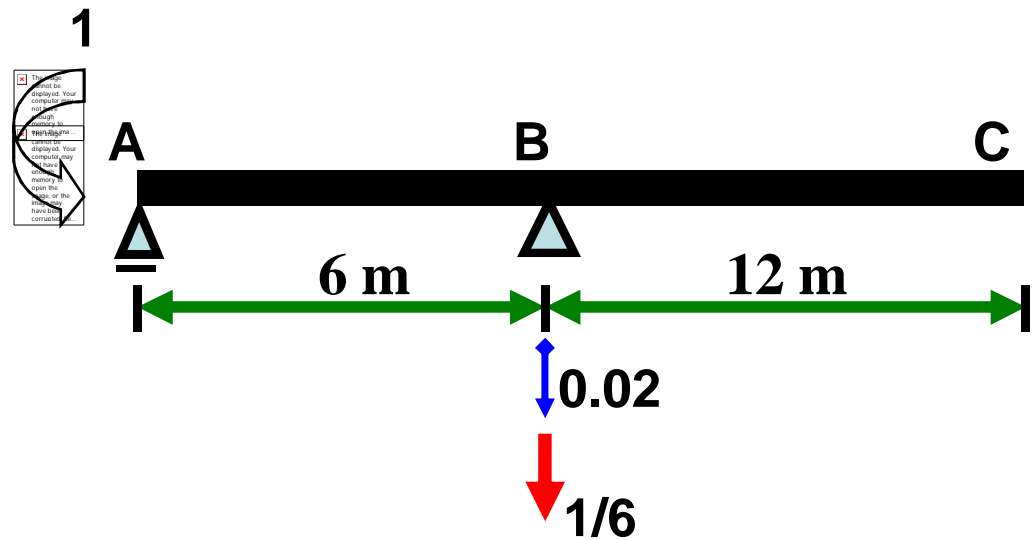


Given settlement at b = 2cm

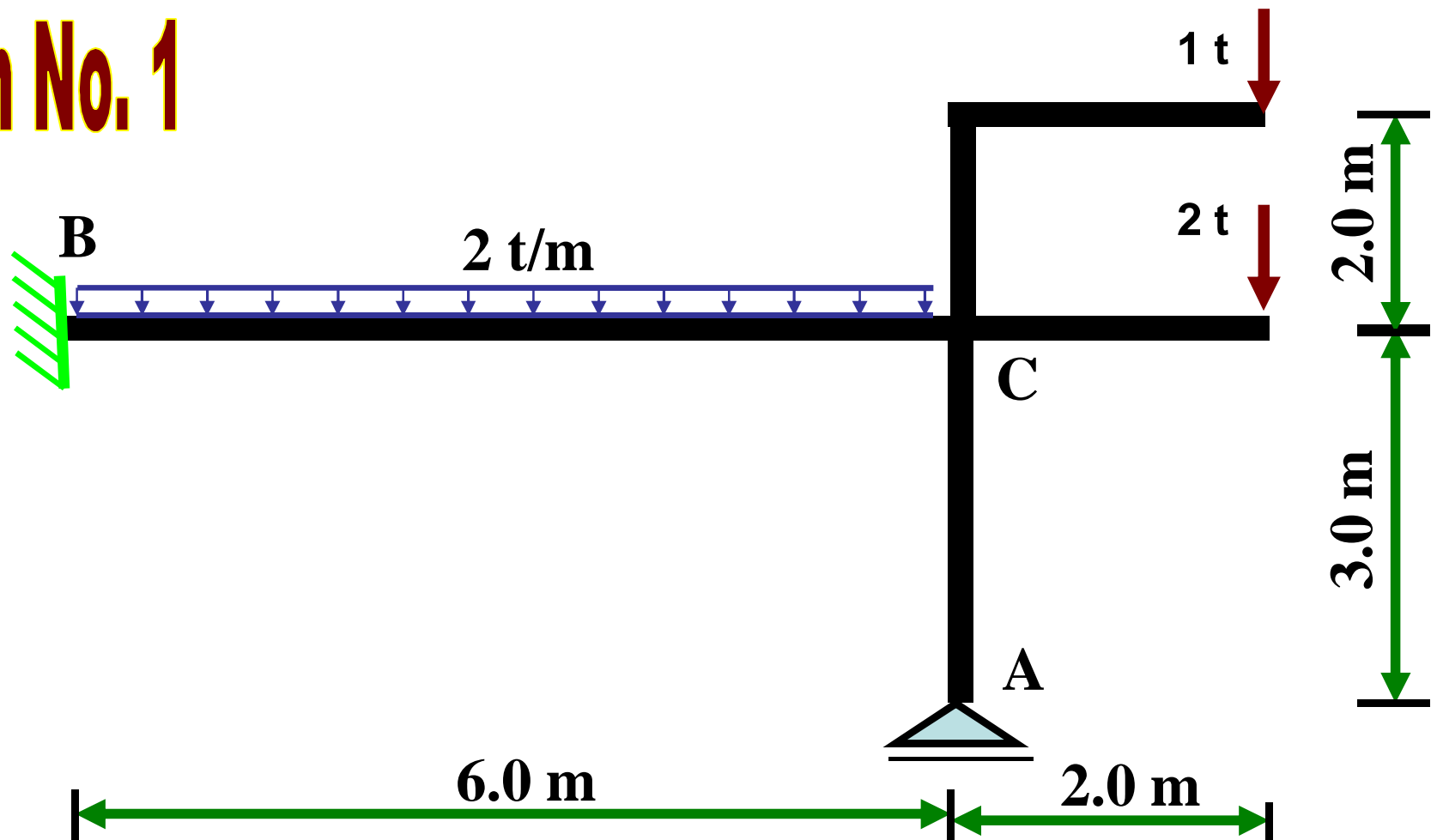
Required slope at A

$$\theta_A * 1 - 1/6 * 0.02 = 0.0$$

$$\theta_A = 0.0033$$



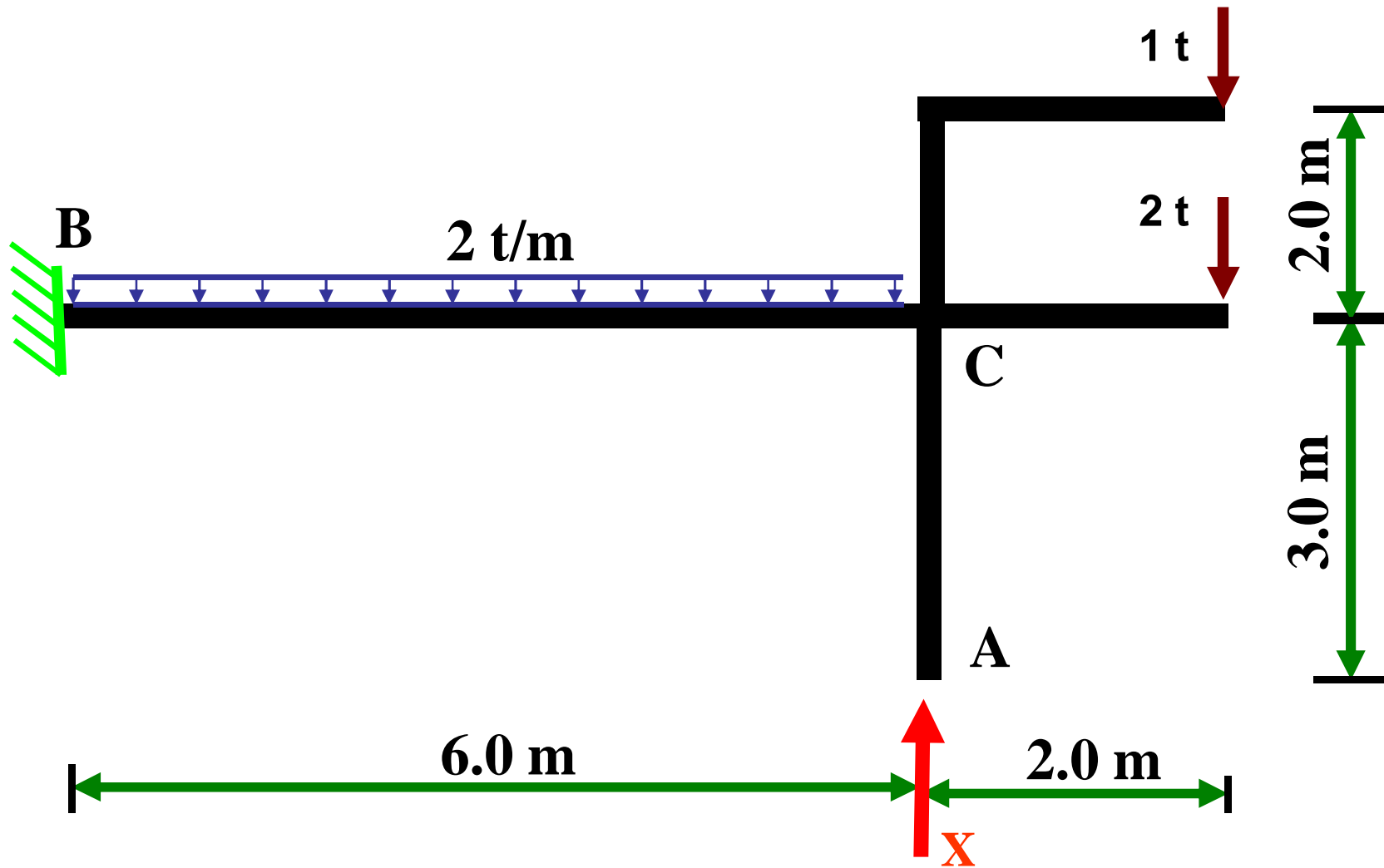
Problem No. 1



Draw B.M.D , S.F.D, and N.F.D $EI = 6000 \text{ t.m}^2$

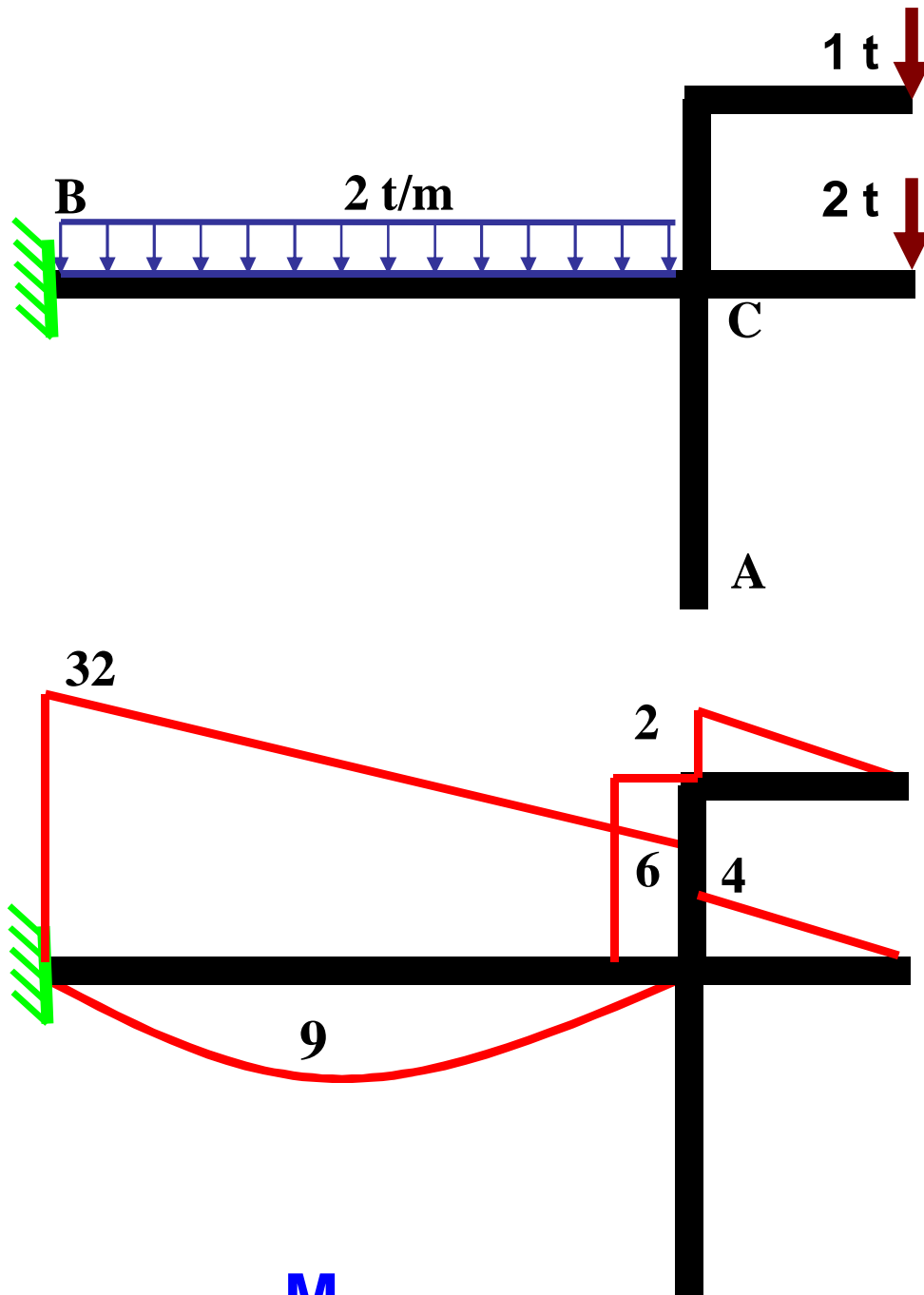
Due to (1) given loads.
(2) rotation at B = 0.003 rad clockwise.

Main System

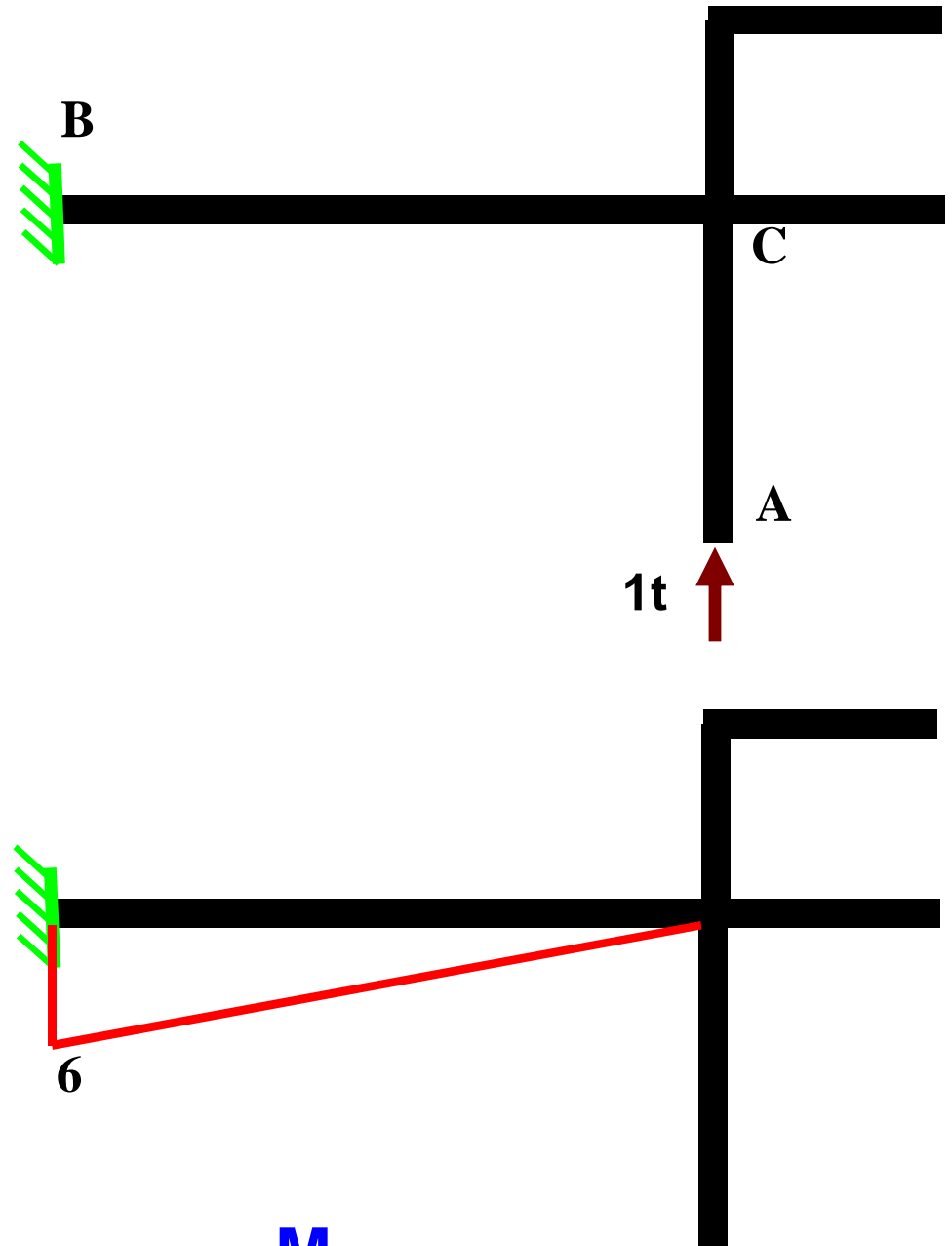


2- draw (M0) [loads + X=0]

3- draw (M1) [no loads + X=1]



M_0



M_1

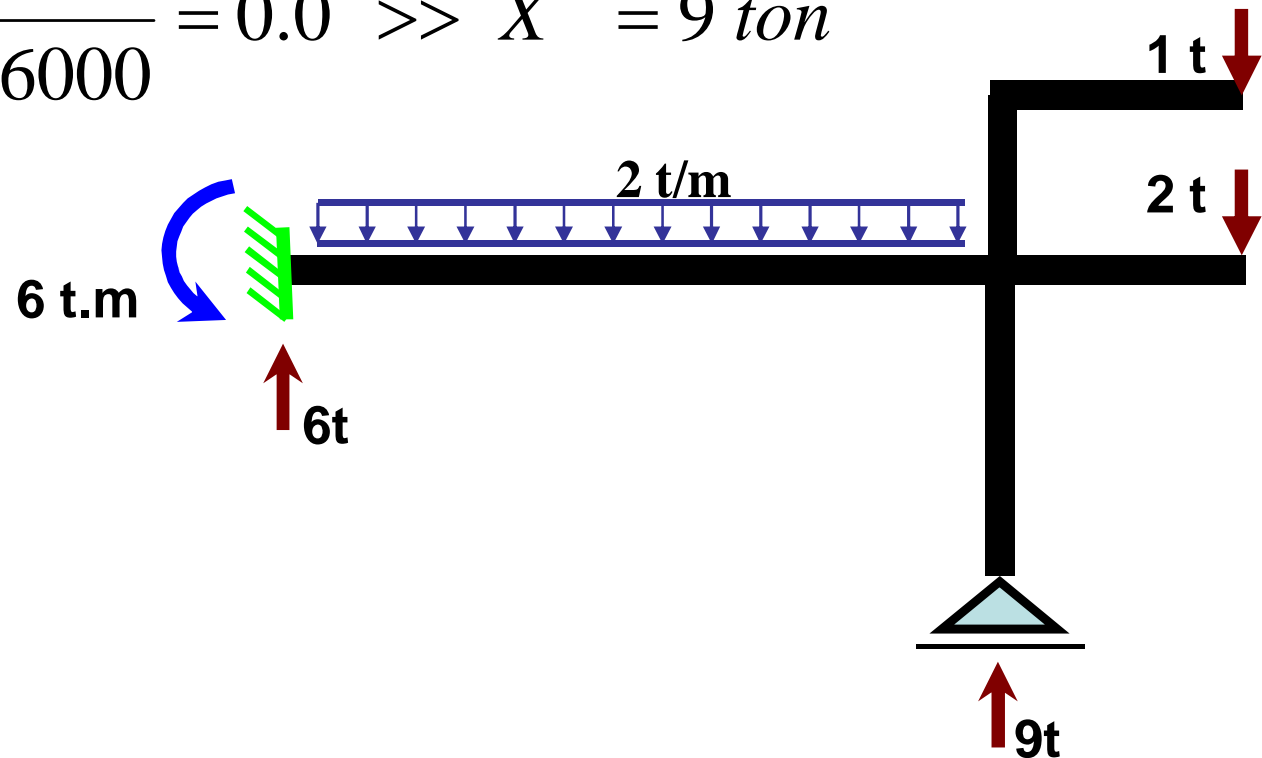
Case of given loads

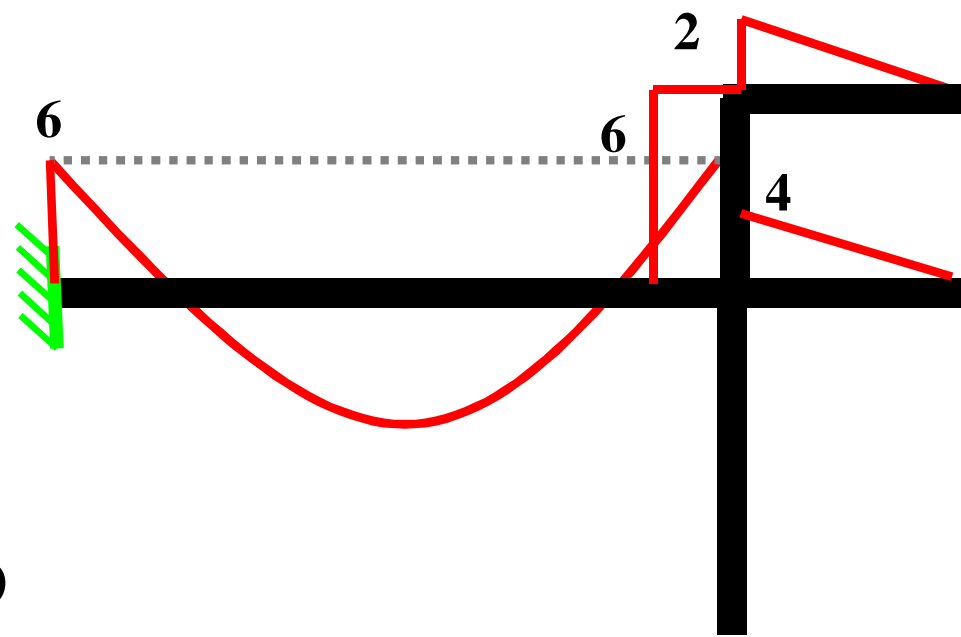
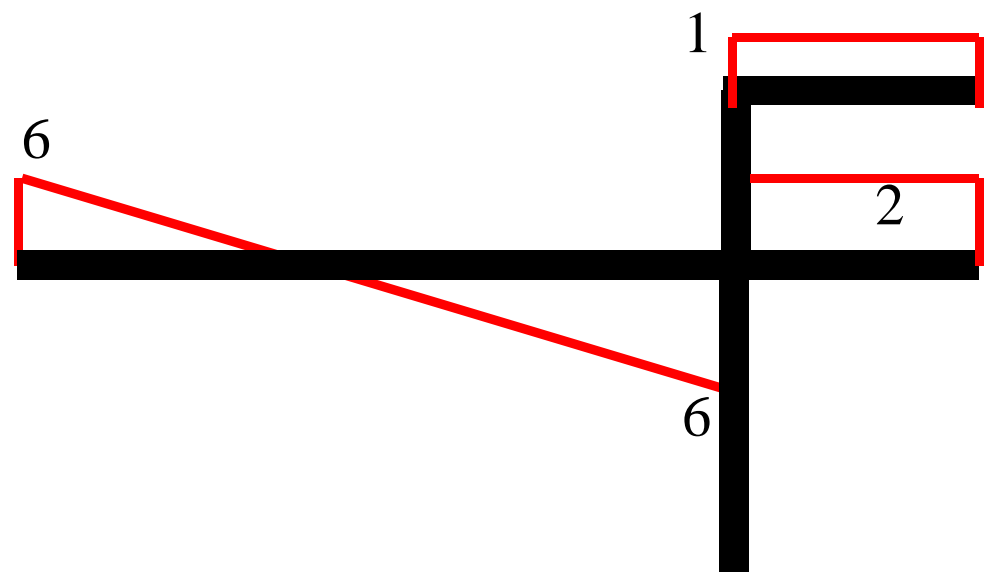
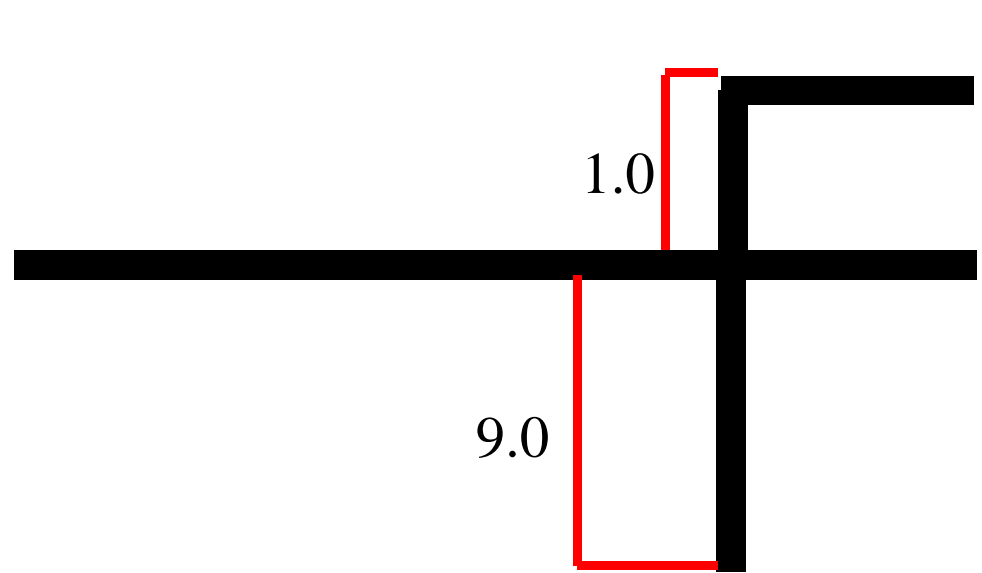
$$\delta_{10} = \frac{1}{6000} \left[-\frac{6}{3} * (6 * 60 + 0.5 * (6 * 6)) + 2/3 * 9 * 6 * 3 \right] = -\frac{648}{6000}$$

$$\delta_{11} = \frac{1}{6000} \left[\frac{6 * 6 * 6}{3} \right] = \frac{72}{6000} m$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{648}{6000} + X * \frac{72}{6000} = 0.0 \gg X = 9 \text{ ton}$$



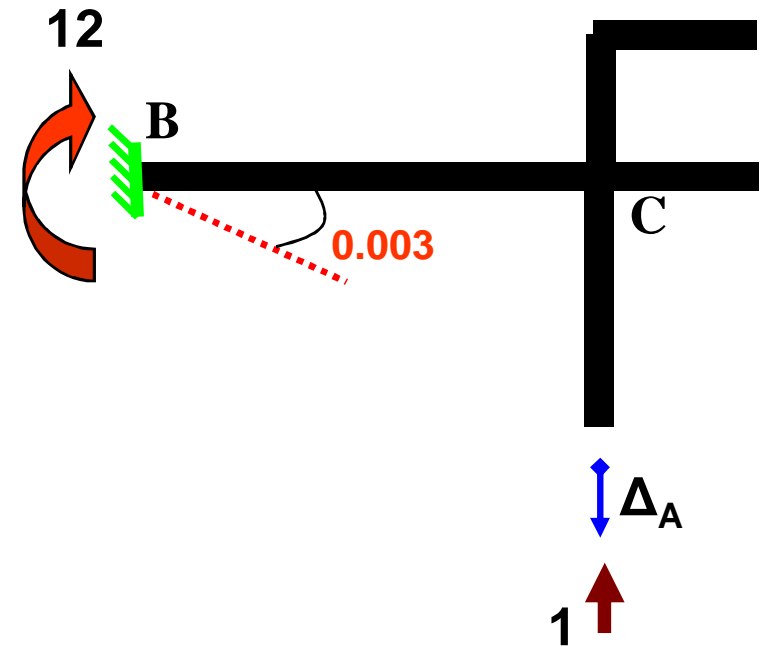


Case of Settlement

$$\delta_{10} = 0.0$$

$$\delta_{11} = \frac{72}{6000} m$$

$$\delta_{10} + X \cdot \delta_{11} = \Delta_A$$



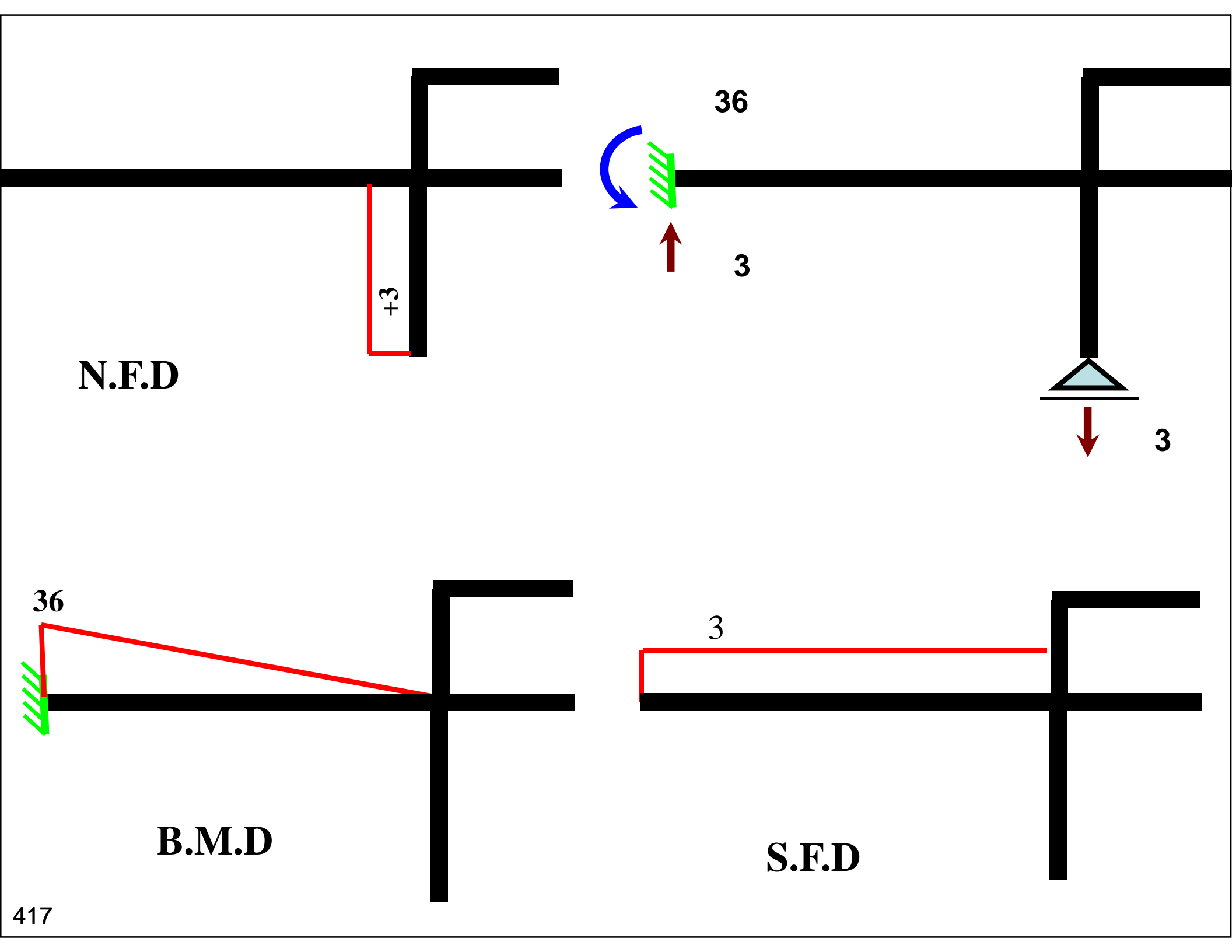
لاحظ ان المعطى هو θ_B والموجود فى المعادلة Δ_A
 فيتم نقل الدوران عند B الى هبوط عند A بطريقة virtual work

$$0.003 * 12 + 1 * \Delta_A = 0.0$$

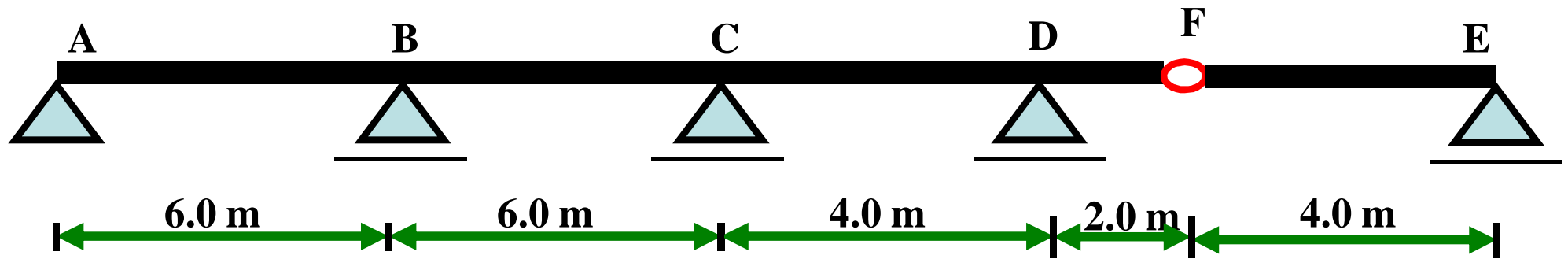
$$\Delta_A = -0.036$$

$$0.0 + X \cdot \frac{72}{6000} = -0.036$$

$$X \cdot \frac{72}{6000} = -3 \text{ ton}$$



Problem No. 1



Draw B.M.D , S.F.D, and N.F.D $EI = 5000 \text{ t.m}^2$

Due to

- (1) settlement at A 2.5 cm .**
- (2) settlement at B 1.0 cm .**
- (3) settlement at C 1.5 cm .**
- (4) settlement at E 3.0 cm .**

Main System



الجزء الـ determinate لا يتأثر
بالهبوط وبالتالي يتم حذف الهبوط عند E

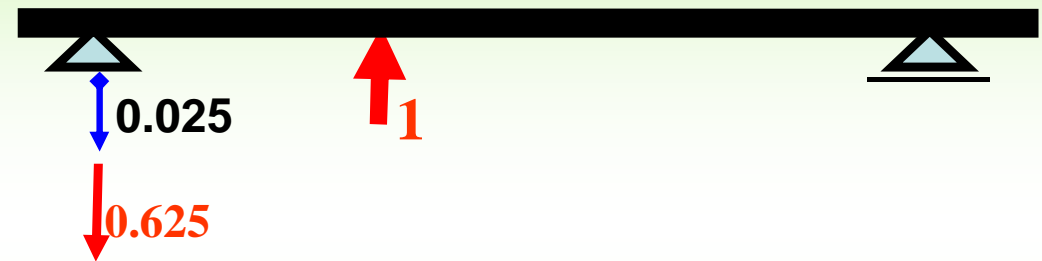
Direct settelment

$$\Delta B = -1.0 \text{ cm}$$

$$\Delta C = -1.5 \text{ cm}$$

Forced deformation settelment

نقل الهبوط عند A الى B, C



$$0.625 \cdot 0.025 + 1 \cdot \Delta B = 0$$

$$\Delta B = -0.0156$$

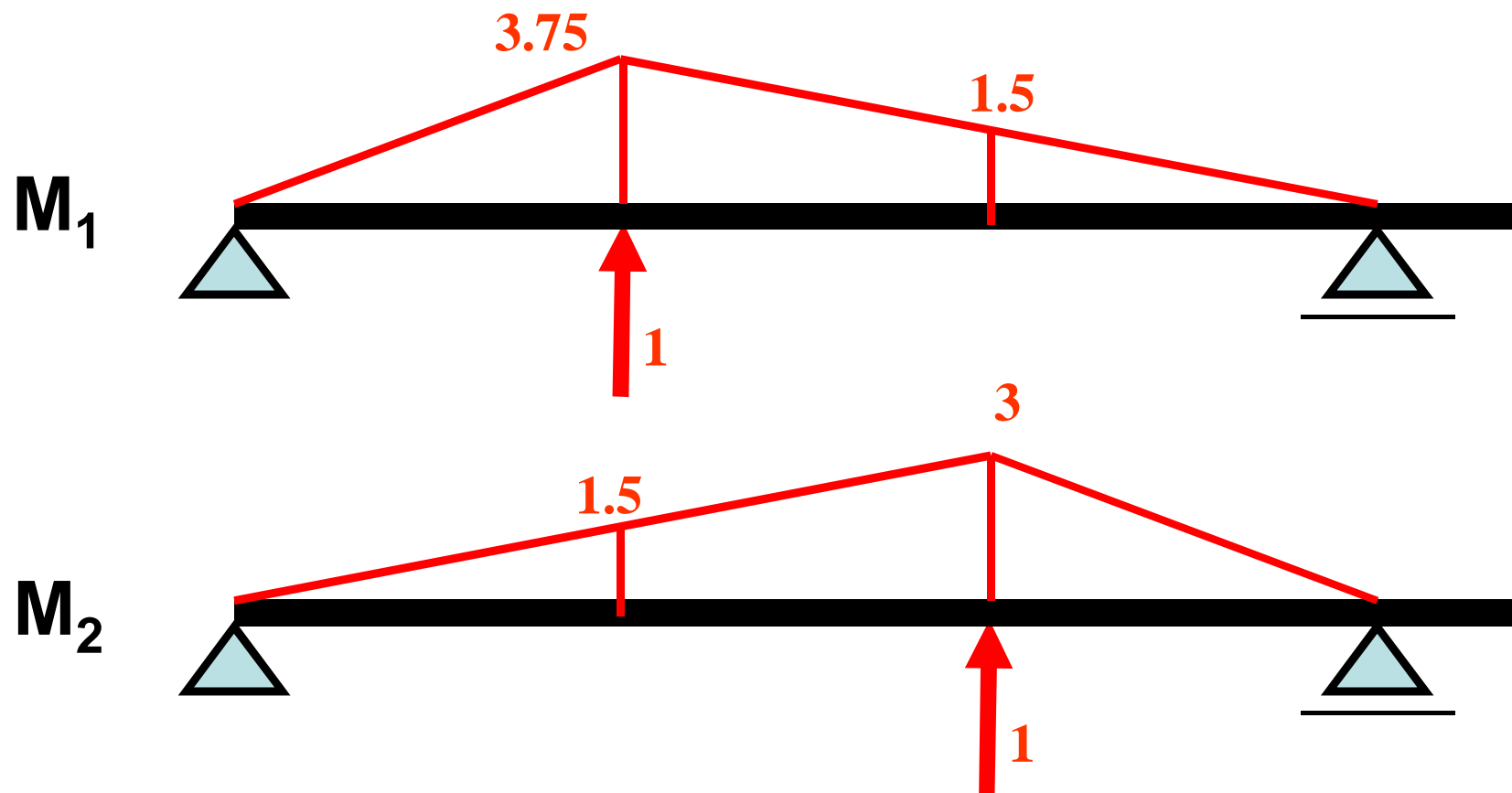


$$0.25 \cdot 0.025 + 1 \cdot \Delta C = 0$$

$$\Delta C = -0.00625$$

$$\Delta B = -0.01 - 0.0156 = -0.0256 \text{ m}$$

$$\Delta C = -0.015 - 0.00625 = -0.02125 \text{ m}$$



$$\delta_{12} = \frac{1}{5000} \left[\frac{3.75 * 1.5 * 6}{3} + \frac{3.0 * 1.5 * 4}{3} + \frac{6}{3} [1.5 * 3.75 + 3 * 1.5 + 0.5 * [1.5 * 1.5 + 3 * 3.75]] \right]$$

$$= 0.0102 \text{ m}$$

$$\delta_{11} = \frac{1}{5000} [3.75 * 3.75 * 6 / 3 + 3.75 * 3.75 * 10 / 3] = 0.015 \text{ m}$$

$$\delta_{22} = \frac{1}{5000} \left[\frac{3 * 3 * 12}{3} + \frac{3 * 3 * 4}{3} \right] = 0.0096 \text{ m}$$

$$\delta_{10} + X_1 \cdot \delta_{11} + + X_2 \cdot \delta_{12} = \Delta_B$$

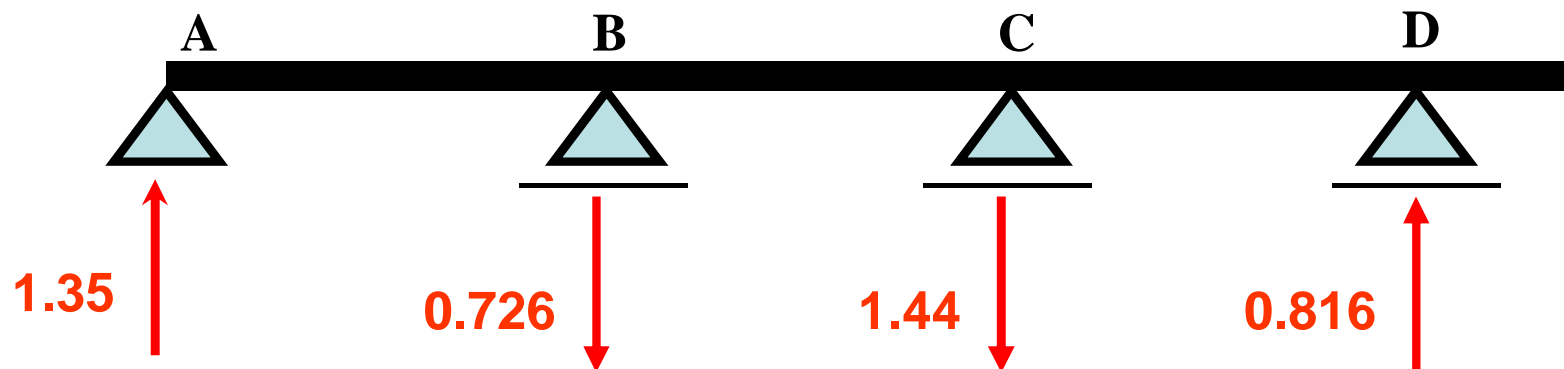
$$\delta_{20} + X_1 \cdot \delta_{21} + + X_2 \cdot \delta_{22} = \Delta_C$$

$$0.0 + X_1 \cdot 0.015 + X_2 \cdot 0.0102 = -0.0256$$

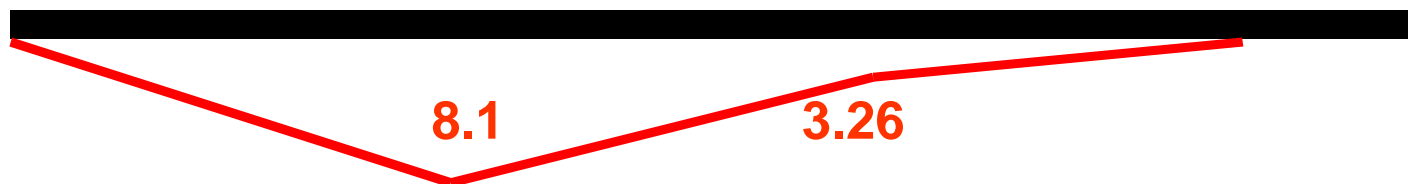
$$0.0 + X_1 \cdot 0.0102 + X_2 \cdot 0.0096 = -0.02125$$

$$\mathbf{X1 = -0.726}$$

$$\mathbf{X2 = -1.44}$$

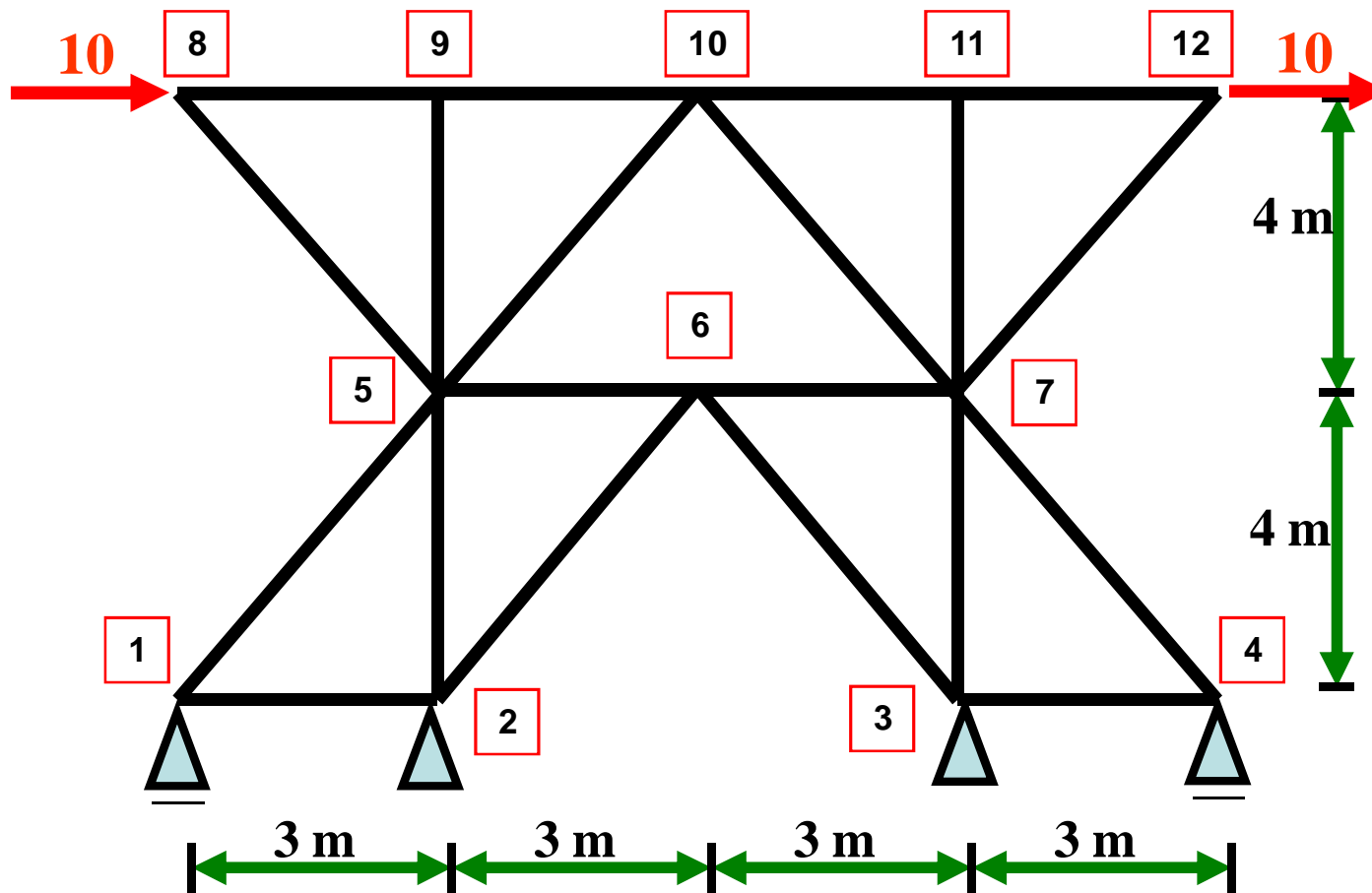


ZERO



Examples

Examples one



For the shown truss determine find the forces in all members due to:

- 1- Given Load.
- 3- drop In Temperature 30°in member 8 — 12 .

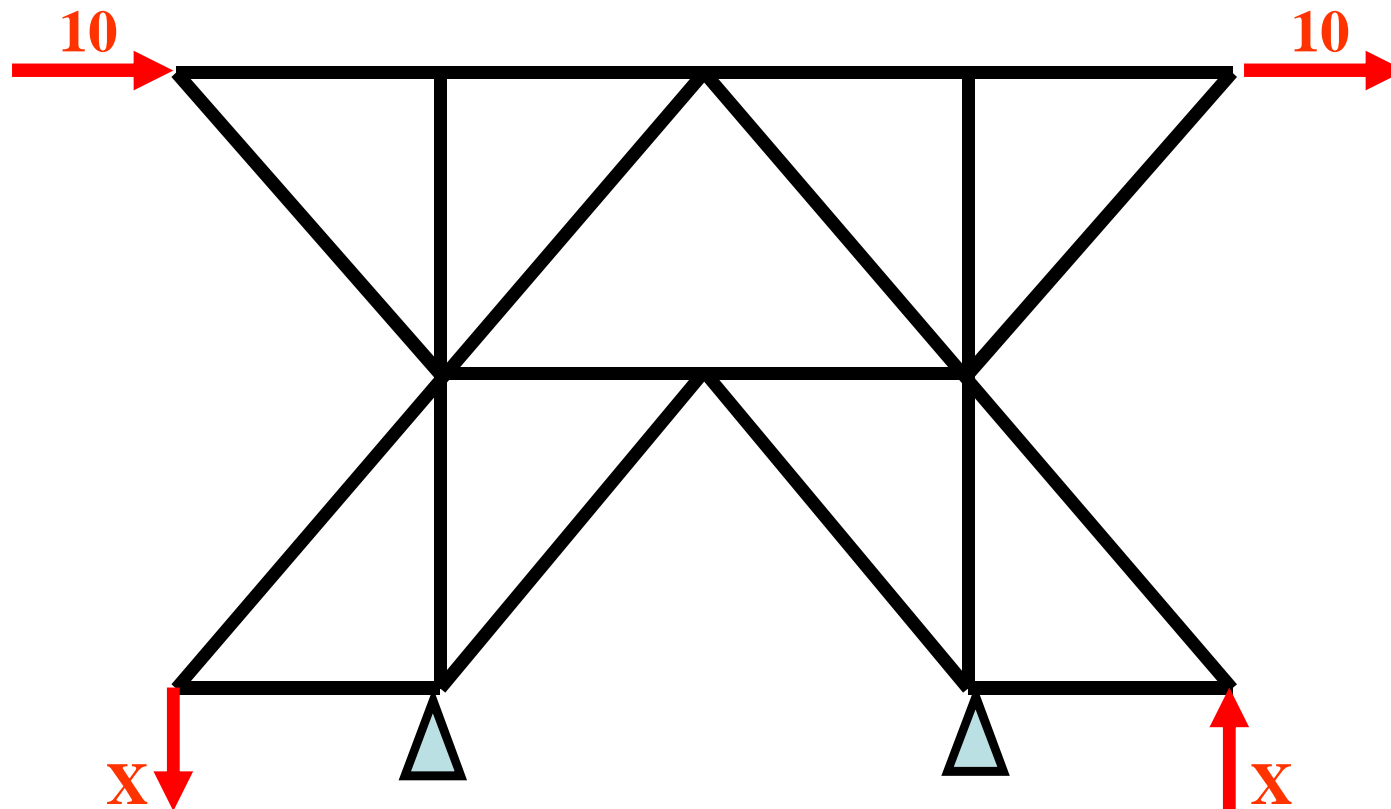
$EA=10000 \text{ ton}$ $\alpha =10^{-5}$

$$b = 20, J = 12, r = 6$$

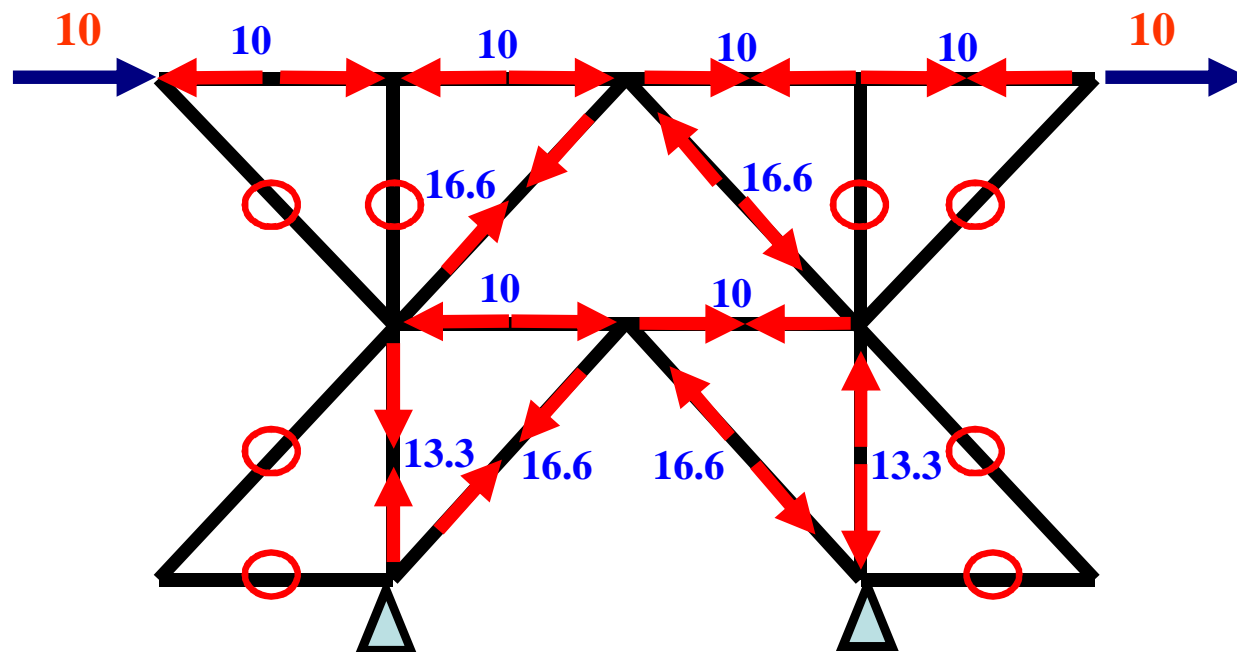
$$b + r = 26 \text{ unknowns}$$

$$2j = 24 \text{ equations}$$

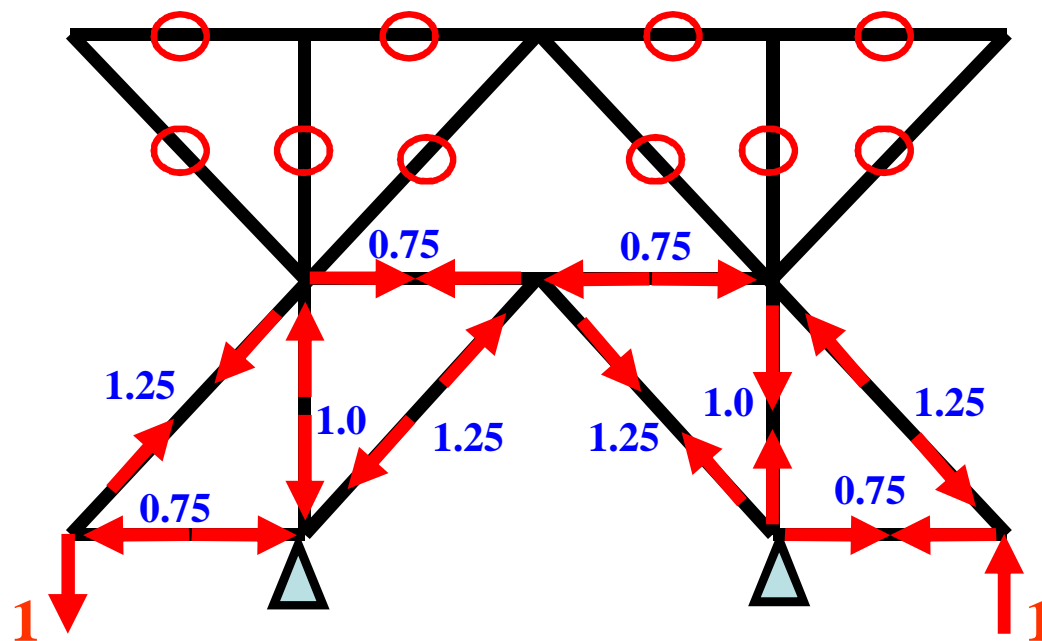
عدد المجاهيل الزيادة 2 وبسبب التماثل العكسي يتم فرضه مجهول واحد



N_0



N_1



 The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been deleted. Restart your computer and delete the image. If the image still appears, you may have to delete the image and reinsert it.

temperature

 The image cannot be displayed. Your computer may not have enough memory to open the image, or the image may have been deleted. Restart your computer and delete the image. If the image still appears, you may have to delete the image and reinsert it.

Given loads

member	L	N ₀	N ₁		N ₀ N ₁ L	N ₁ N ₁ L	N ₁ L	N _F	N _F
1-2	3	0	-0.75		0	1.6875	-2.25		0
3-4	3	0	0.75		0	1.6875	2.25		0
5-6	3	-10	0.75		-22.5	1.6875	2.25		0
6-7	3	10	- 0.75		-22.5	1.6875	- 2.25		0
8-9	3	-10	0		0	0	0		0
9-10	3	-10	0		0	0	0		0
10-11	3	10	0		0	0	0		0
11-12	3	10	0		0	0	0		0
2-5	4	13.3	-1		-53.2	4	-4		0
5-9	4	0	0		0	0	0		0
3-7	4	-13.3	1		-53.2	4	4		0
7-11	4	0	0		0	0	0		0
1-5	5	0	1.25		0	7.8125	6.25		0
2-6	5	16.6	-1.25		-104.16	7.8125	-6.25		0
3-6	5	-16.6	1.25		-104.16	7.8125	6.25		0
4-7	5	0	-1.25		0	7.8125	-6.25		0
5-8	5	0	0		0	0	0		0
5-10	5	16.6	0		0	0	0		0
7-10	5	-16.6	0		0	0	0		0
7-12	5	0	0		0	0	0		0
428 ∑					-359.72	46	0		

Case of given loads

$$\delta_{10} = \frac{\sum N_0 N_1 L}{EA} = -\frac{359.72}{10000}$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{49}{10000}$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

$$-\frac{359.72}{10000} + X * \frac{49}{10000} = 0.0 \gg X = 7.35 \text{ ton}$$

Given loads

$$N_F = N_0 + 7.35 * N_1$$

يتم عمل الحسابات في الجدول فقط

Case of Temperature

$$\delta_{10} = \alpha \cdot \Delta t \cdot \sum (N_1 \cdot L) = 0.0$$

$$\delta_{11} = \frac{\sum N_1 N_1 L}{EA} = \frac{49}{10000}$$

$$\delta_{10} + X \cdot \delta_{11} = 0.0$$

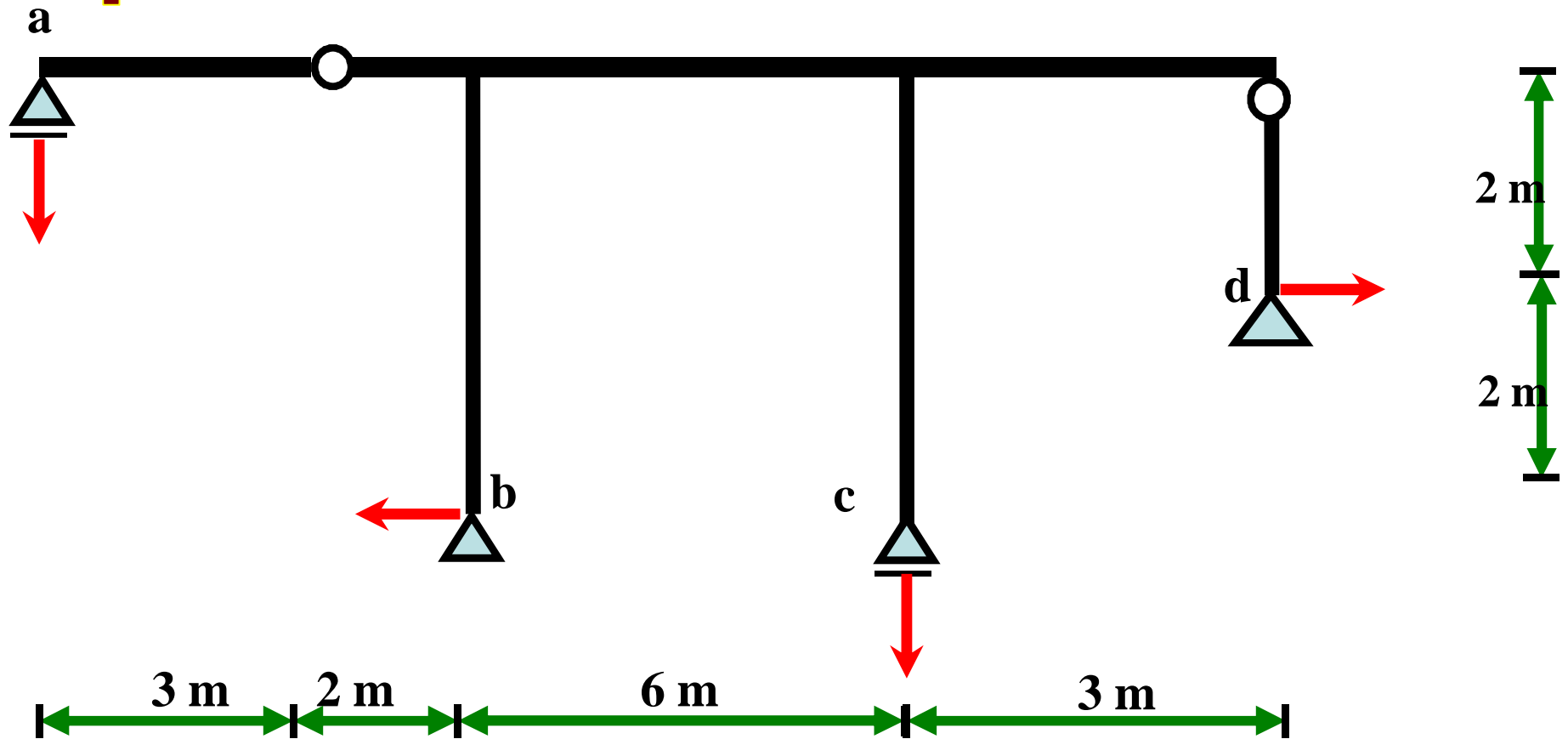
$$0 + F * \frac{36.72}{40000} = 0.0 \gg F = 0$$

Temperature

$$N_F = F * N_1 = 0$$

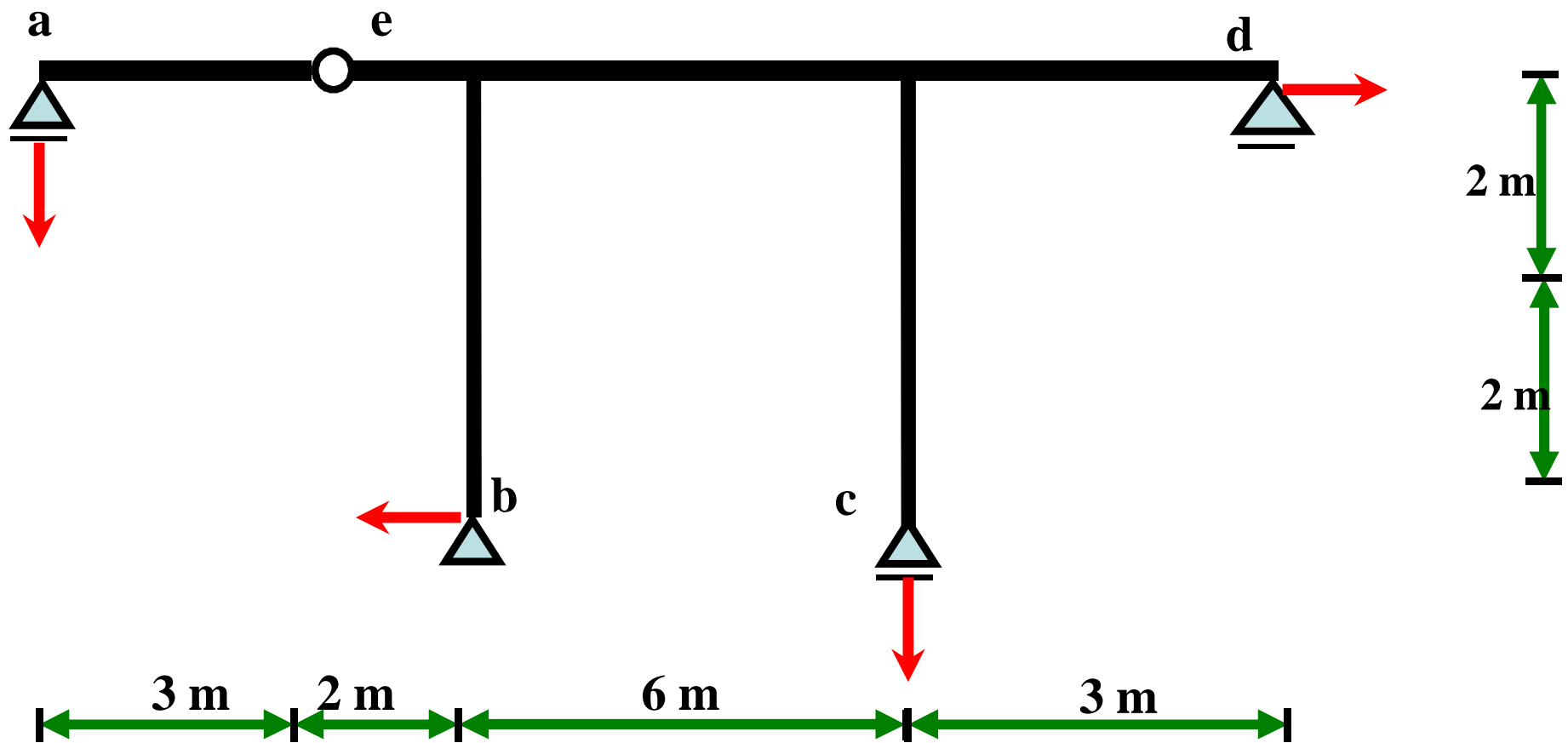
يتم عمل الحسابات في الجدول فقط

Examples two



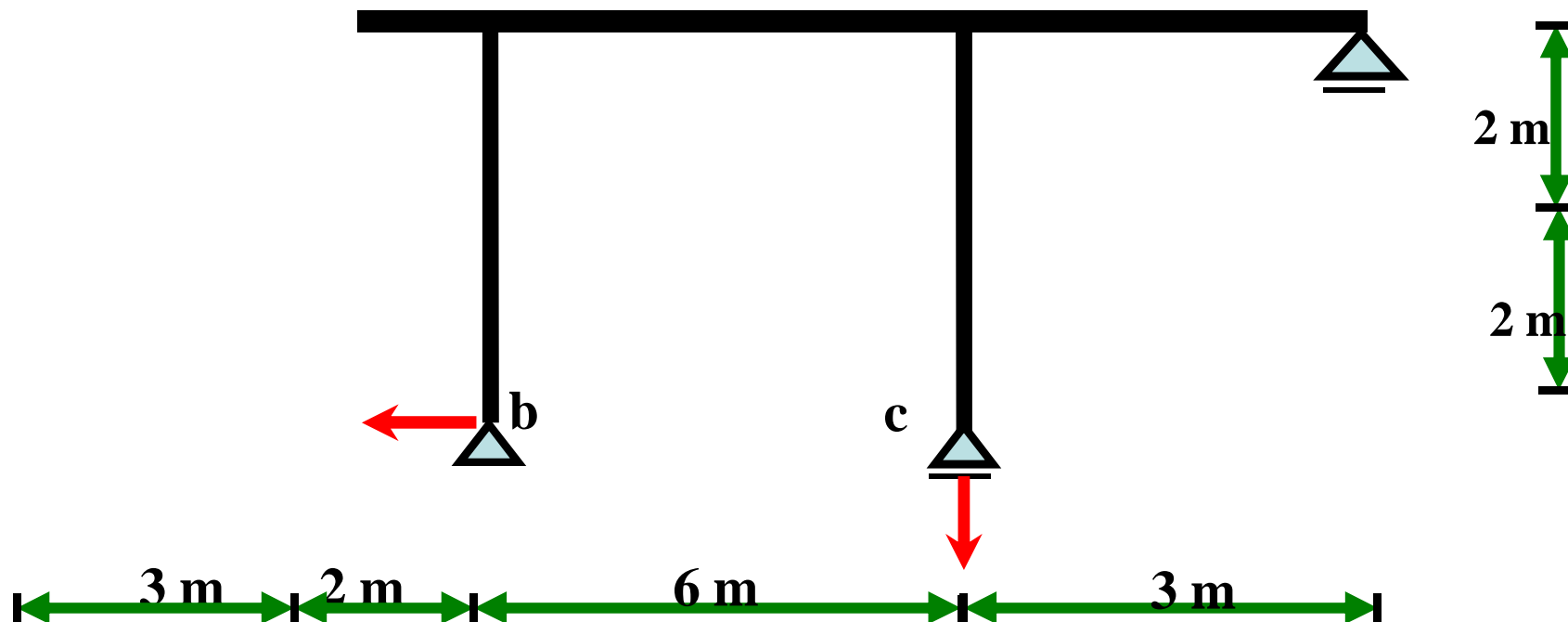
Draw B.M.D , S.F.D, and N.F.D $EI = 6000 \text{ t.m}^2$

Due to (1) settlement at A = 1cm.
and left movement at B = 2cm.
and settlement at C = 3cm.
and right movement at D = 2cm.

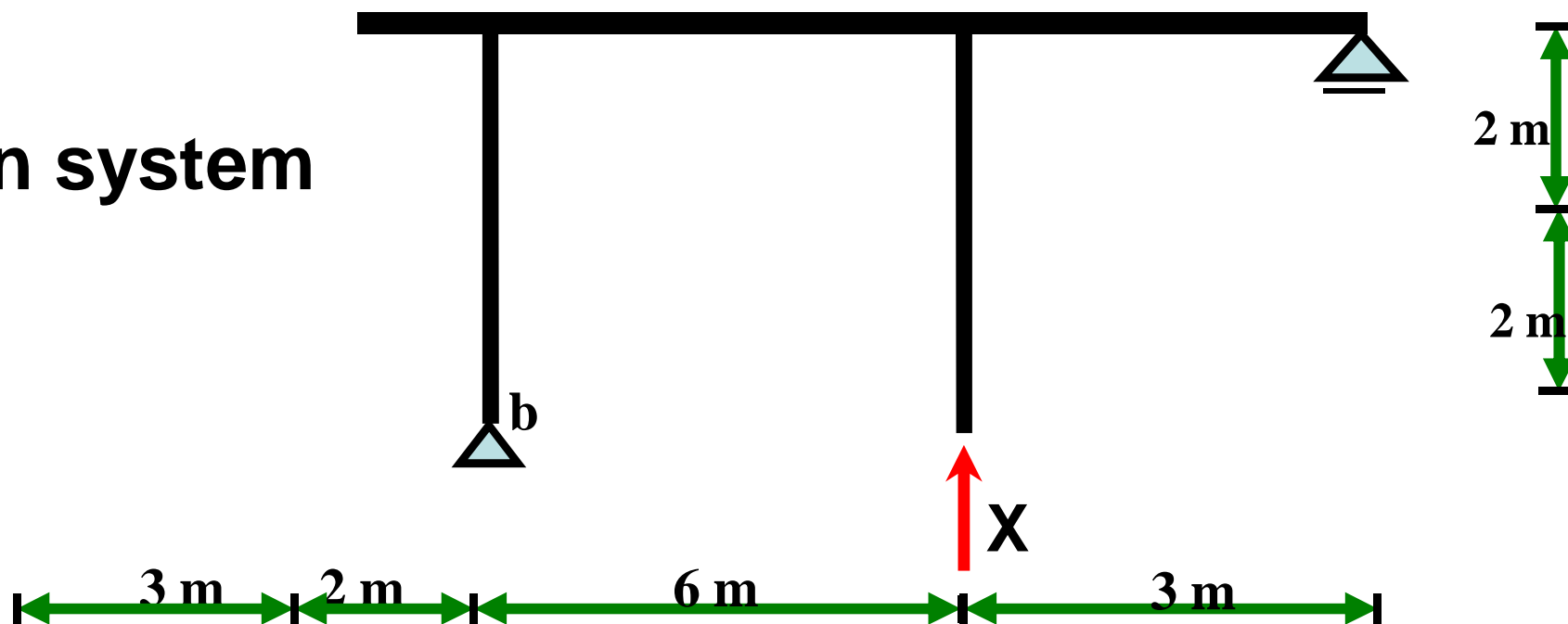


Part (a e) >> determinate
الهبوط لا يؤثر في الـ determinate structure

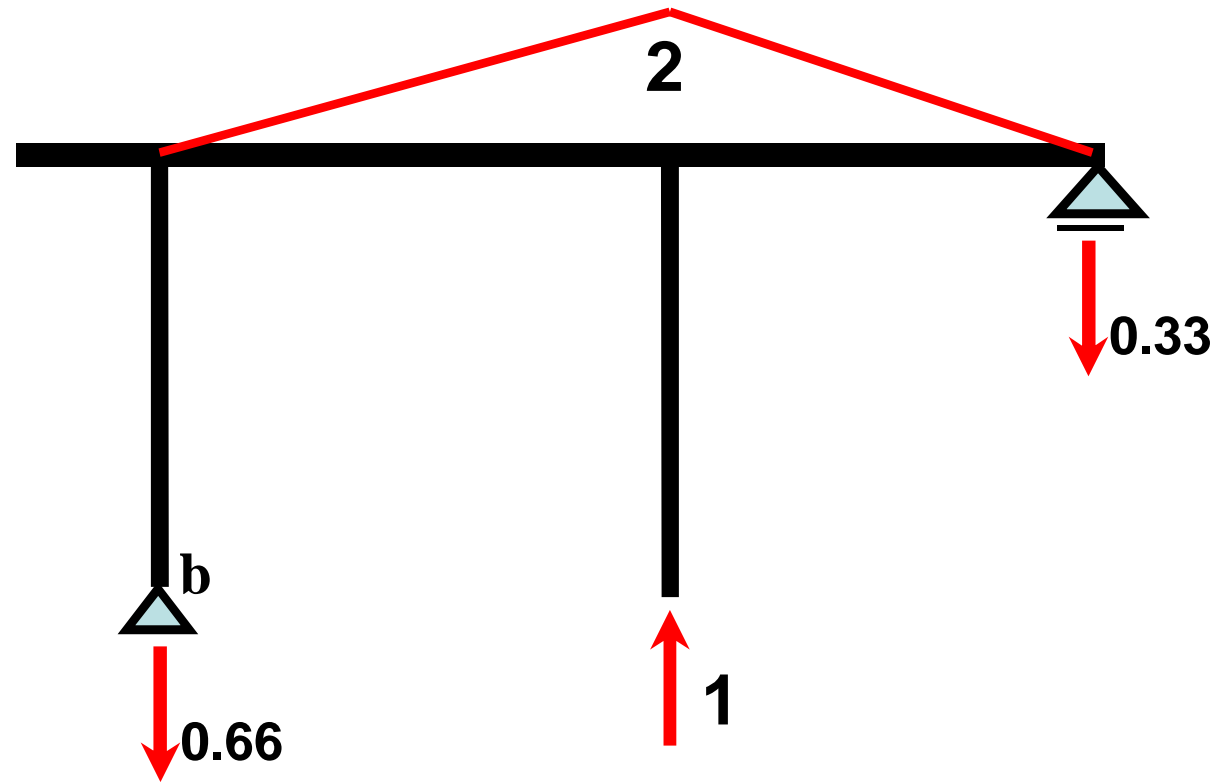
الحركة عند d افقية وهى الحركة الاساسيه للـ roller وبالتالي لا تؤثر



Main system



M_1



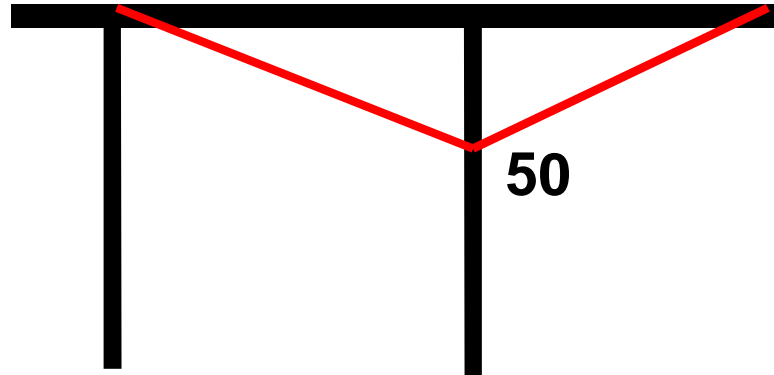
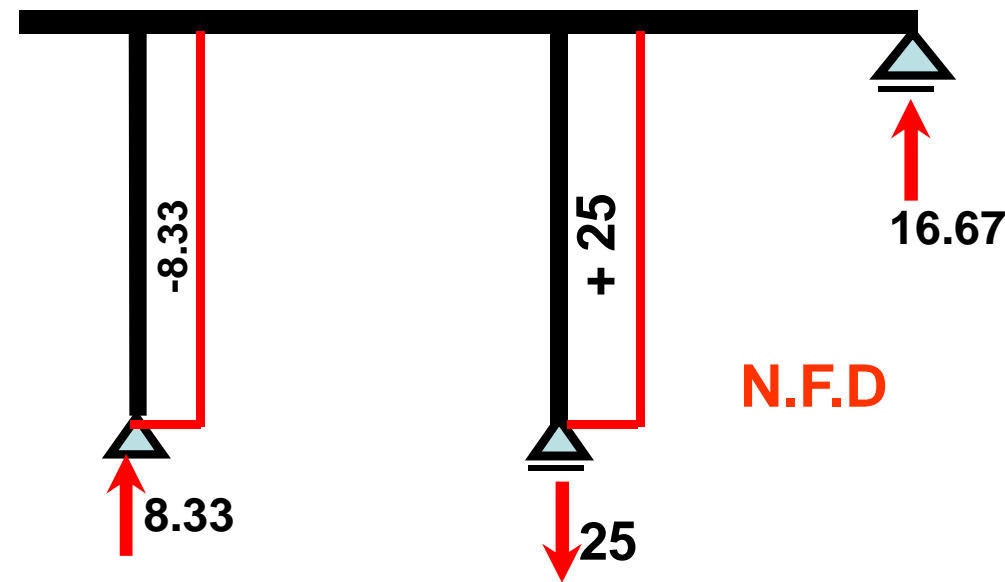
$$\delta_{11} = \frac{1}{10000} \left[\frac{2 * 2 * 3}{3} + \frac{2 * 2 * 6}{3} \right] = 12 * 10^{-4}$$

$$\delta_{10} = 0.0$$

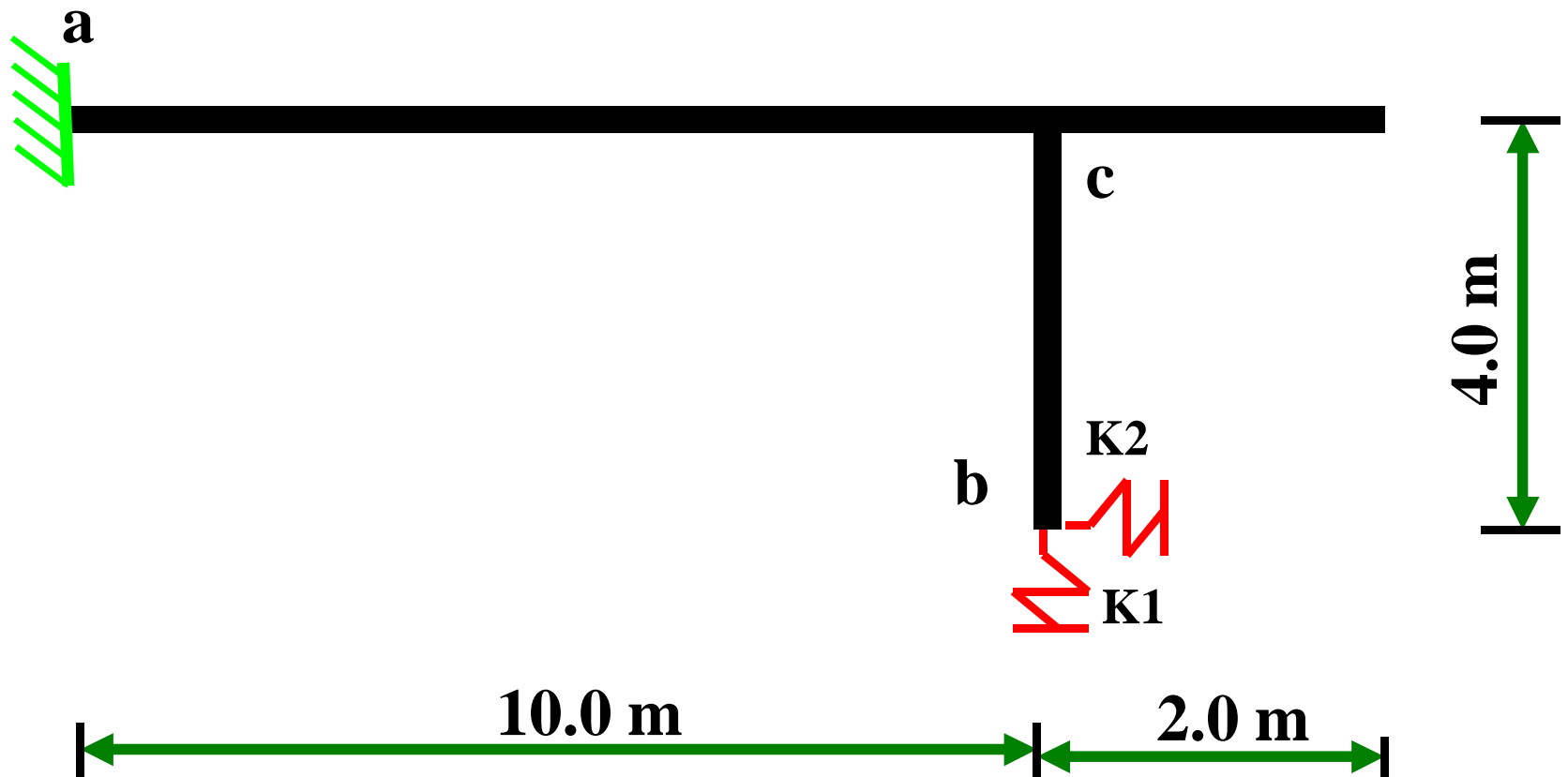
$$\delta_{11} = 12 * 10^{-4}$$

$$\delta_{10} + X \cdot \delta_{11} = \Delta_c$$

$$0 + X * 12 * 10^{-4} = 0.03 \gg X = -25 t$$



Examples three



Draw B.M.D , S.F.D, and N.F.D $EI = 10000 \text{ t.m}^2$

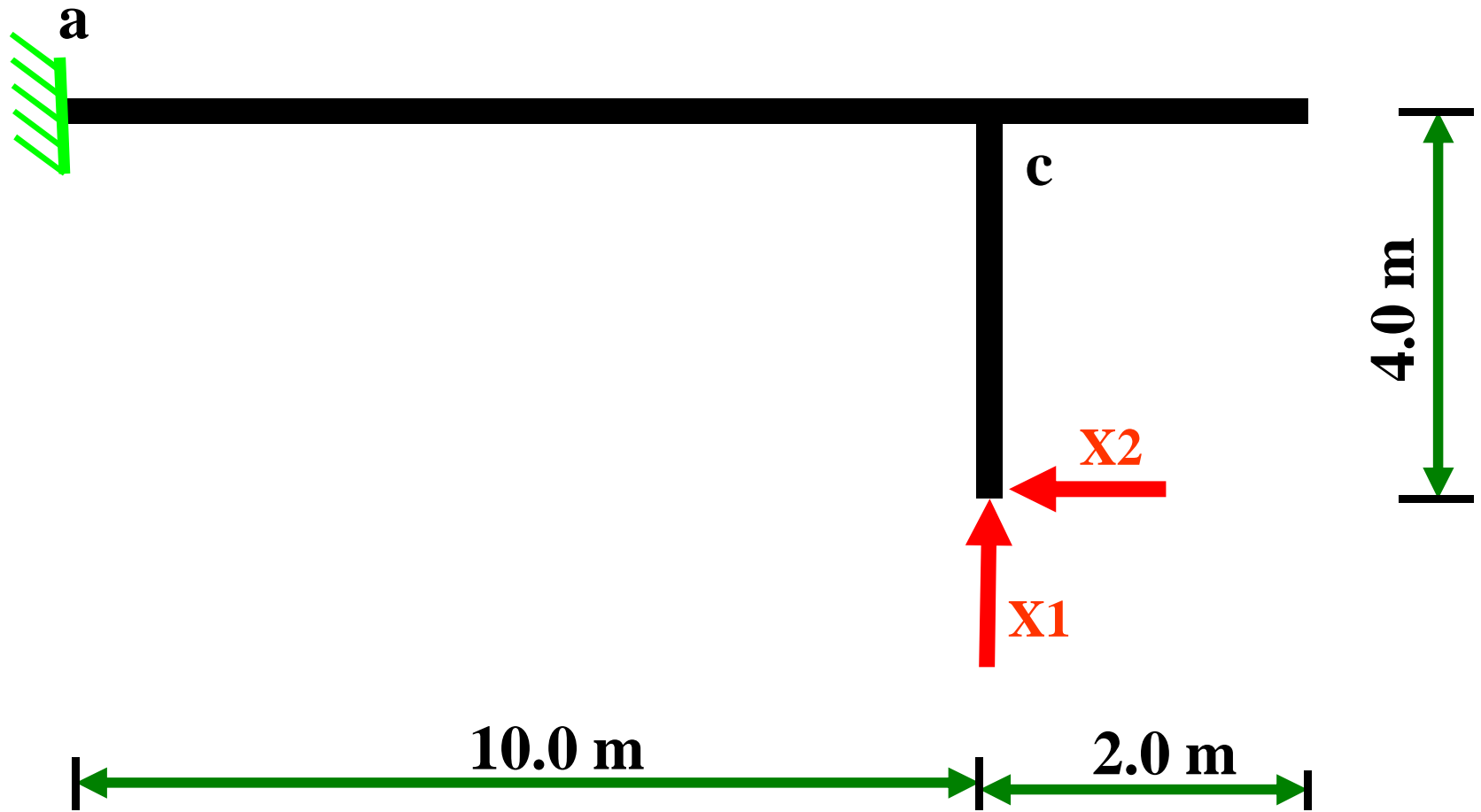
Due to

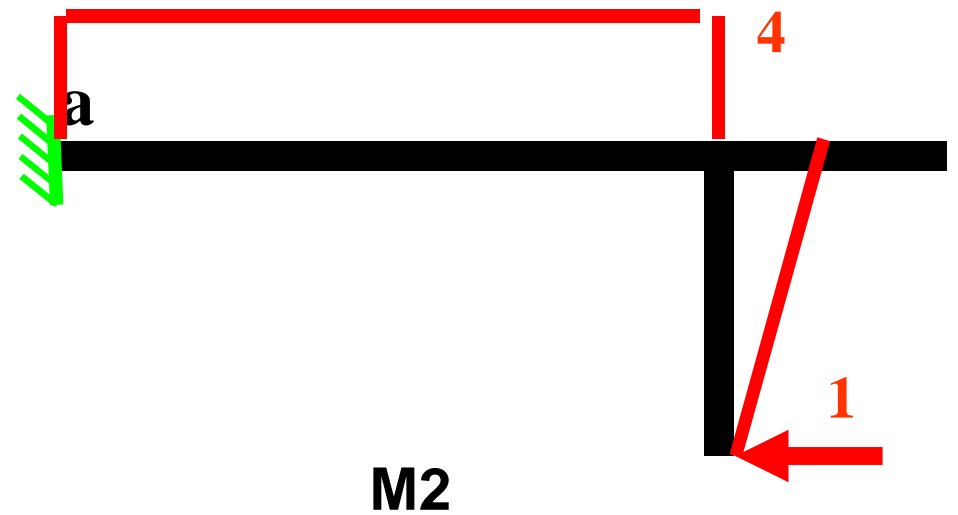
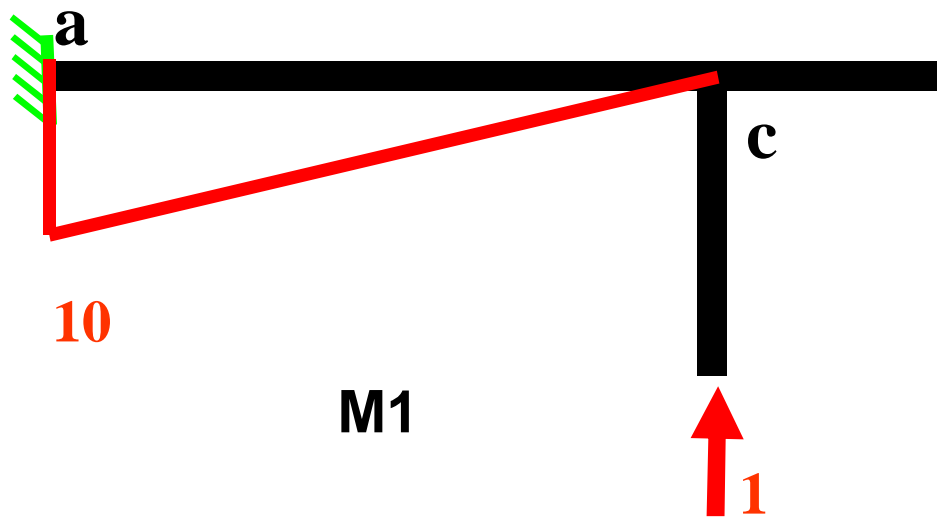
Anti Clockwise Rotation at A = 0.001 rad.

$K1 = 400 \text{ t/m}$

$K2 = 200 \text{ t/m}$

Main System

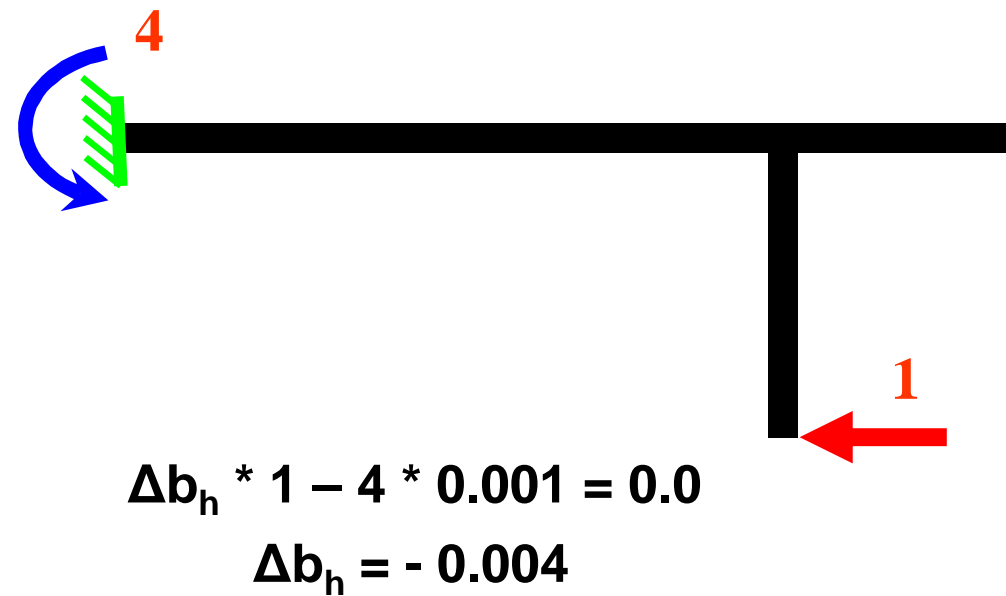
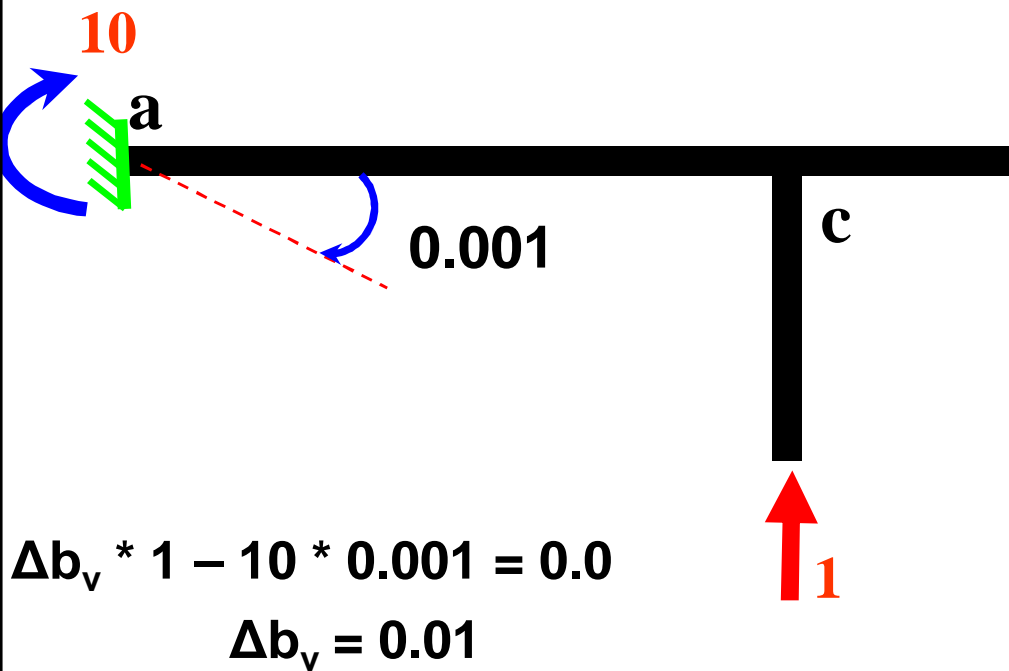




$$\delta_{11} = \frac{1}{10000} \left[\frac{10 * 10 * 10}{3} \right] = 0.033$$

$$\delta_{22} = \frac{1}{10000} \left[\frac{4 * 4 * 4}{3} + 4 * 4 * 10 \right] = 0.018$$

$$\delta_{12} = \frac{1}{10000} \left[\frac{10 * 4 * 10}{2} \right] = 0.02$$

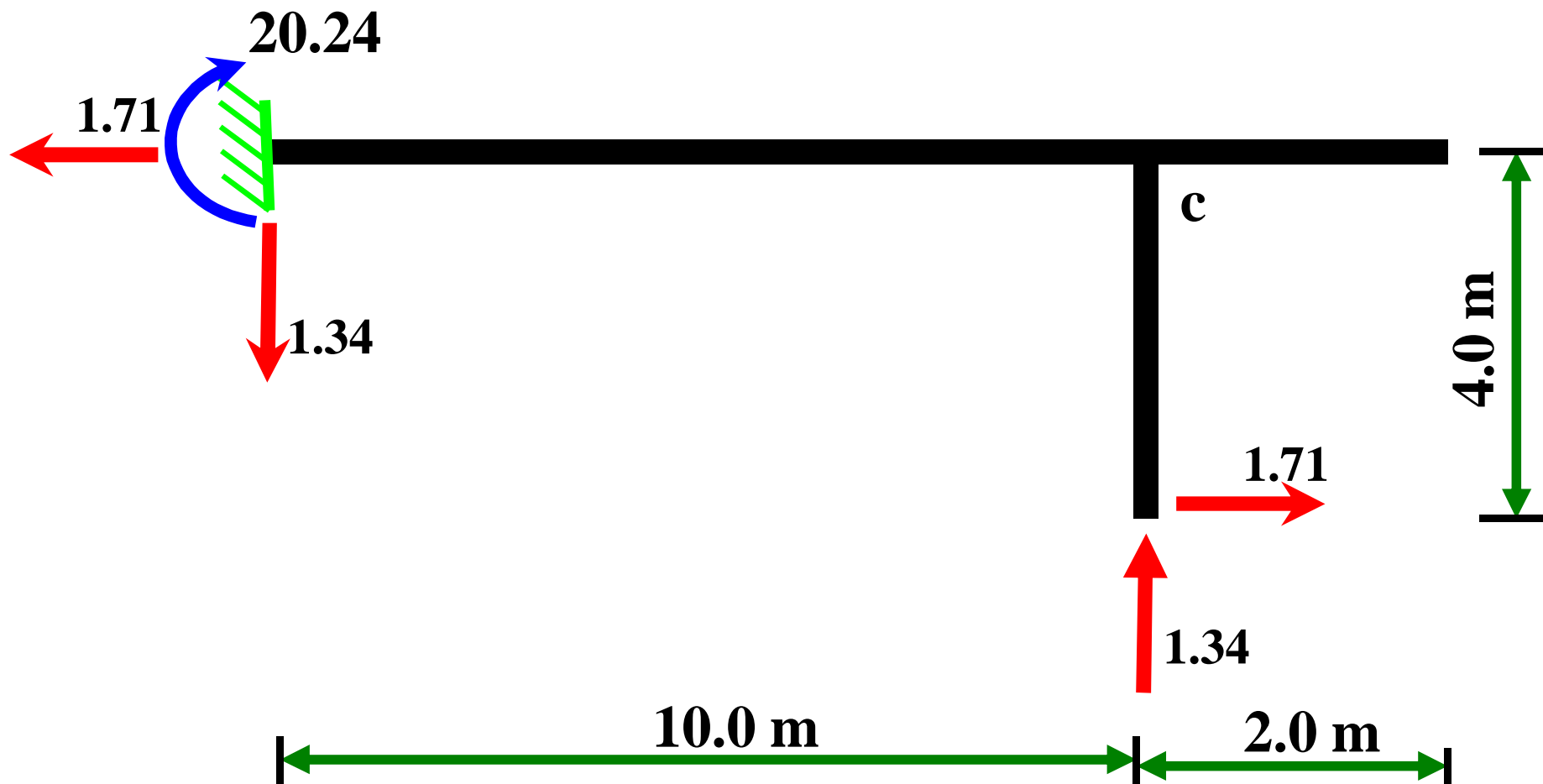


$$0.0 + X_1 * (0.033 + \frac{1}{400}) + X_2 * 0.02 = 0.0$$

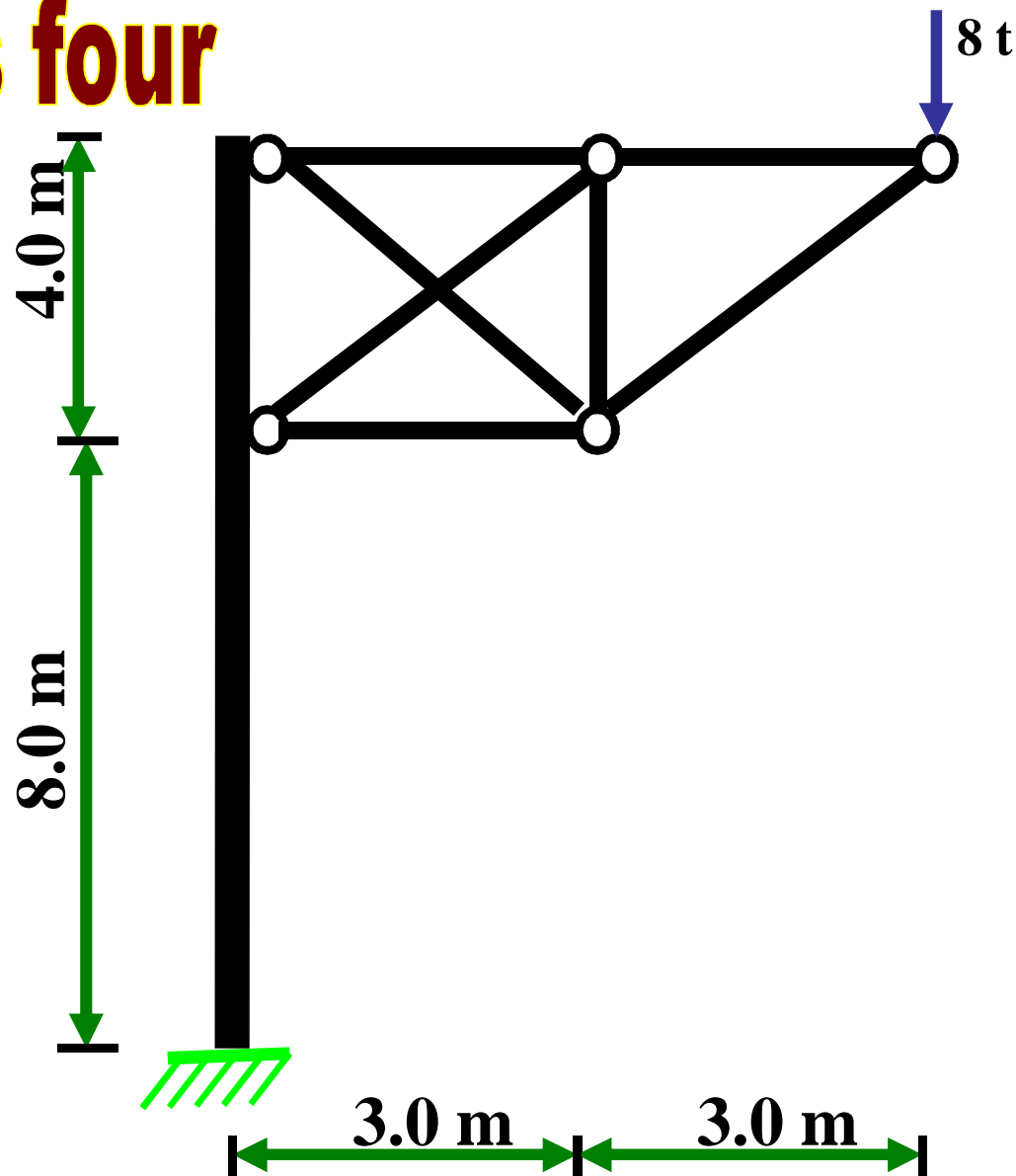
$$0.0 + X_1 * 0.02 + X_2 * (0.018 + \frac{1}{200}) = 0.0$$

$$X1=1.34$$

$$X2=-1.71$$



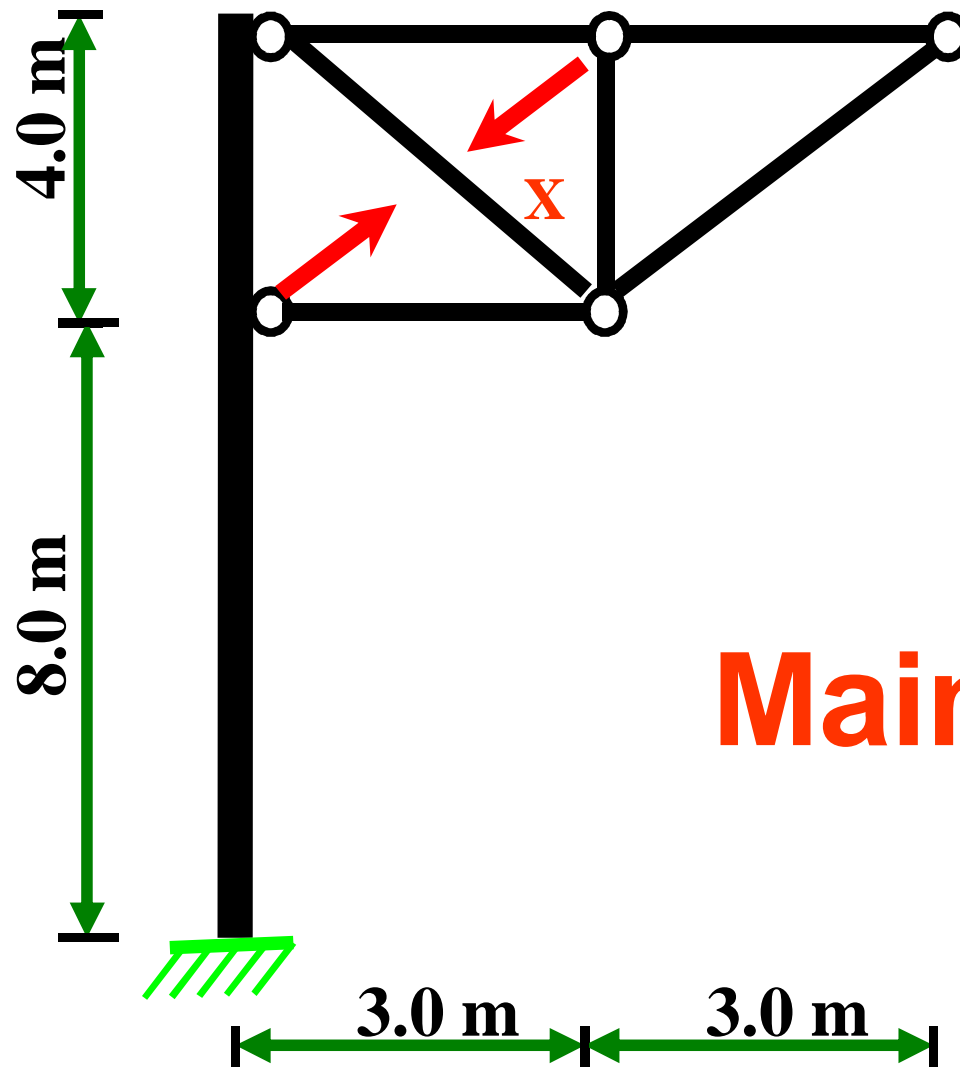
Examples four



For the shown truss determine find the forces in all members due to:

$EI_{\text{column}} = 6000 \text{ ton}$

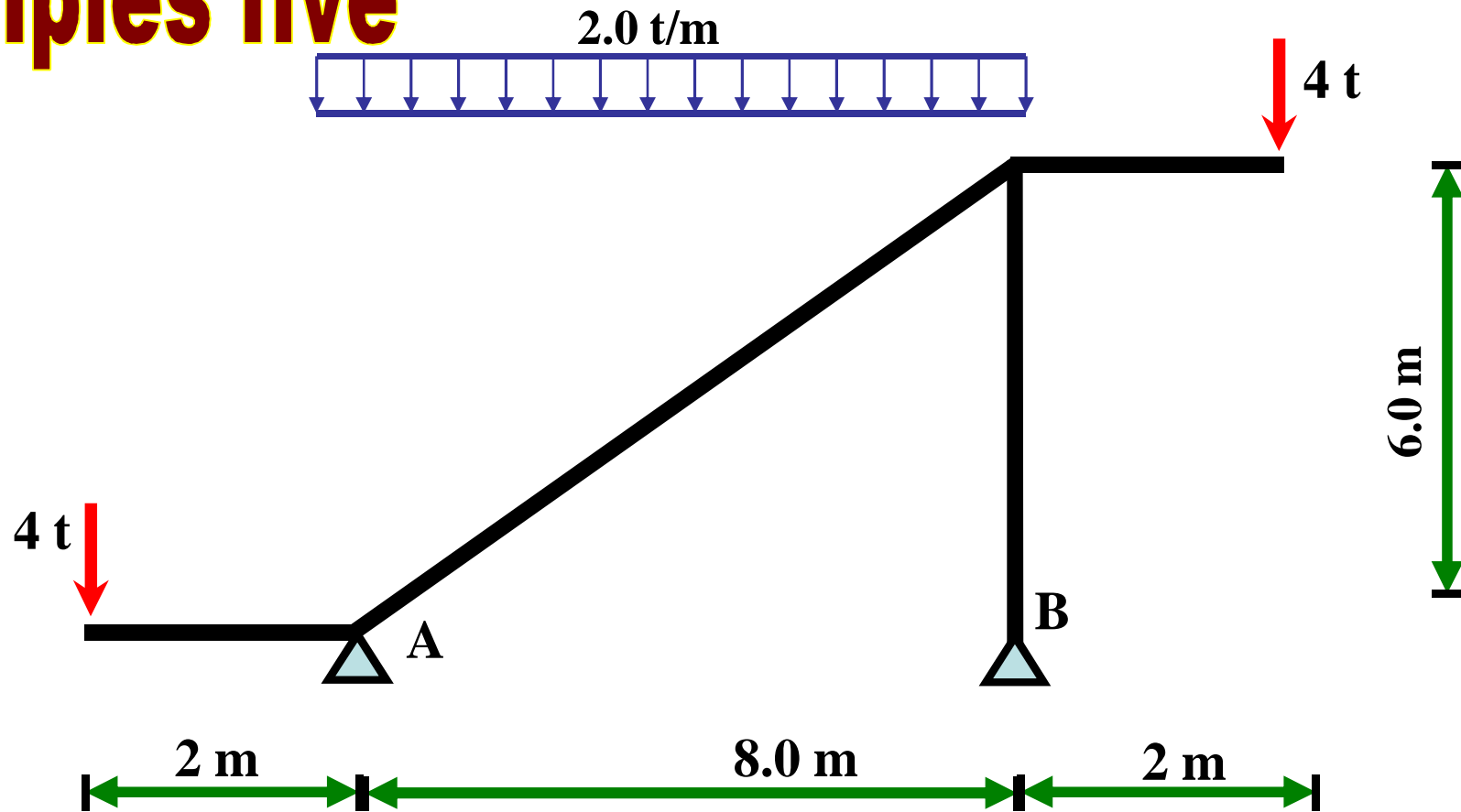
$EA_{\text{link}} = 4000 \text{ ton}$



Main system

continue

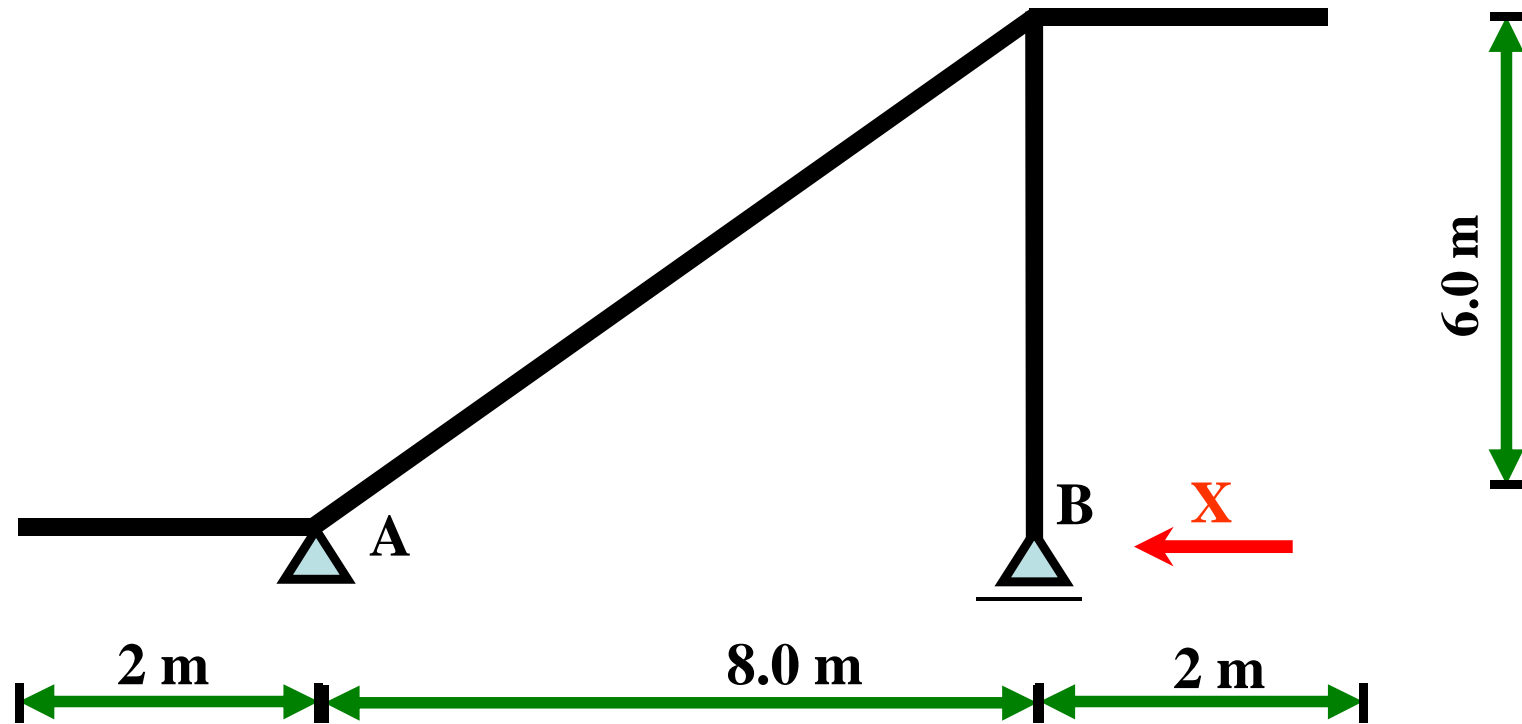
Examples five



Draw B.M.D , S.F.D, and N.F.D $EI = 8000 \text{ t.m}^2$ & $EA = 20000 \text{ t.m}^2$

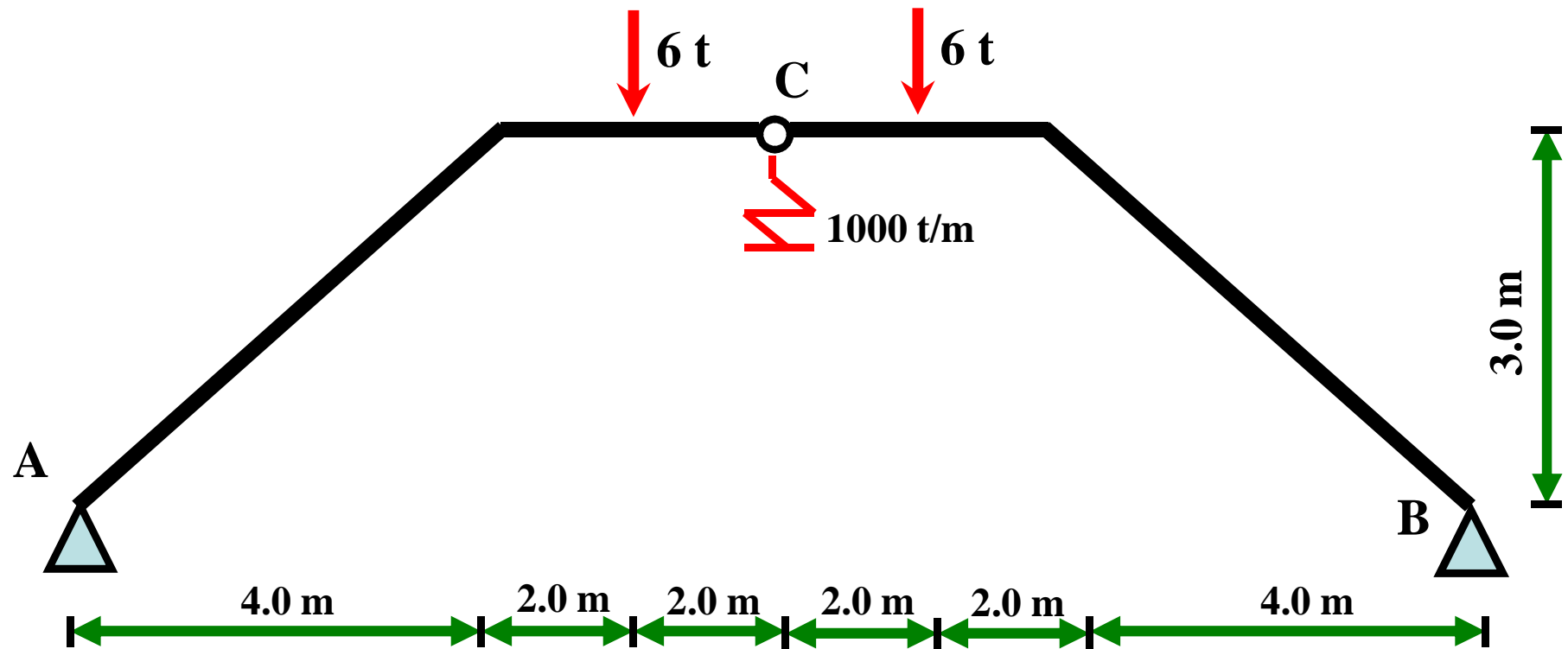
Due to (1) given loads
(2) uniform rise in temperature in member AD 50°

Main system



continue

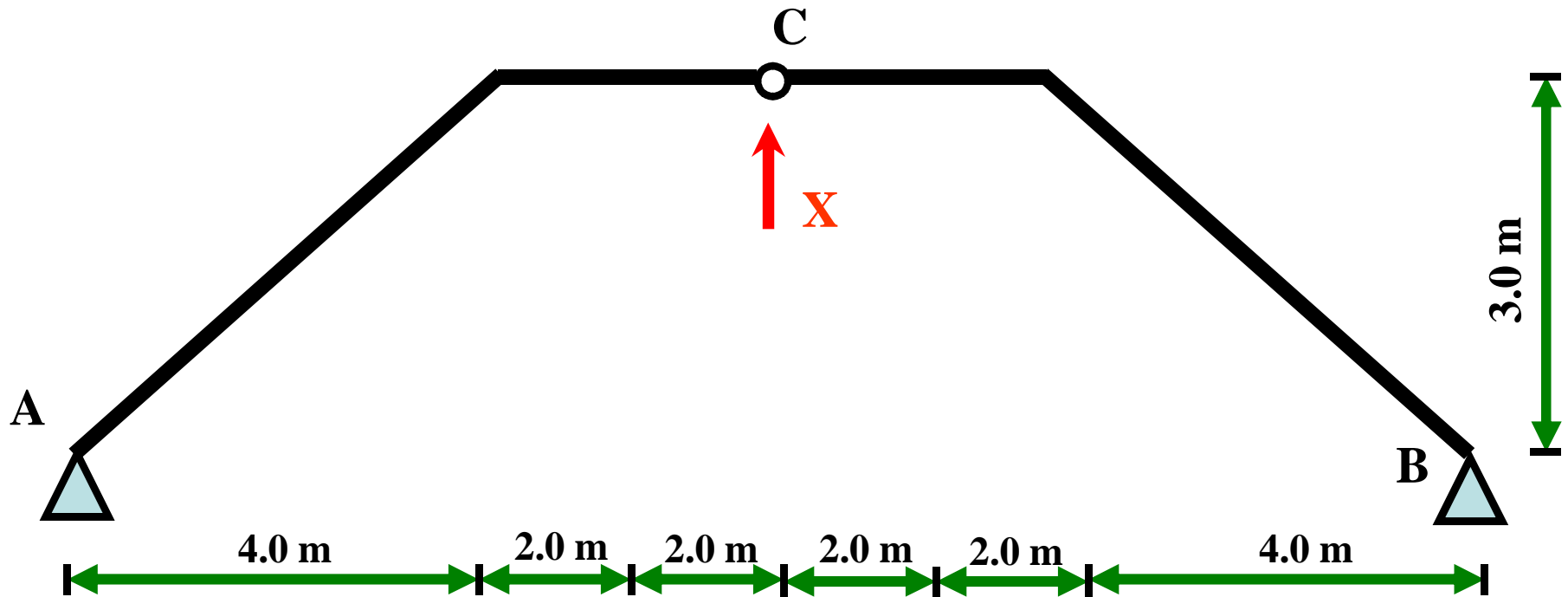
Examples six



Draw B.M.D , S.F.D, and N.F.D $EI = 10000 \text{ t.m}^2$

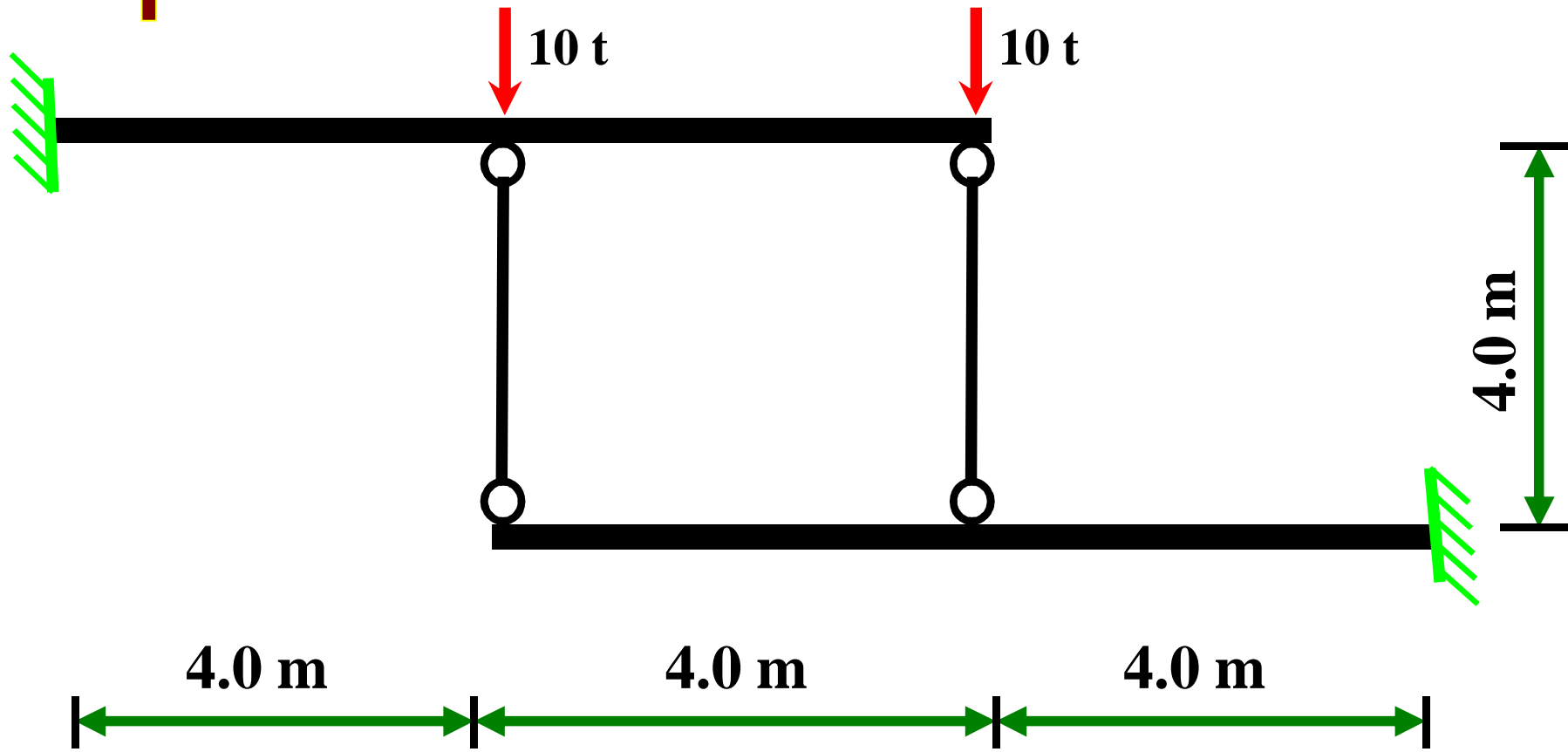
Due to (1) given loads
(2) uniform rise in temperature 30°

Main system



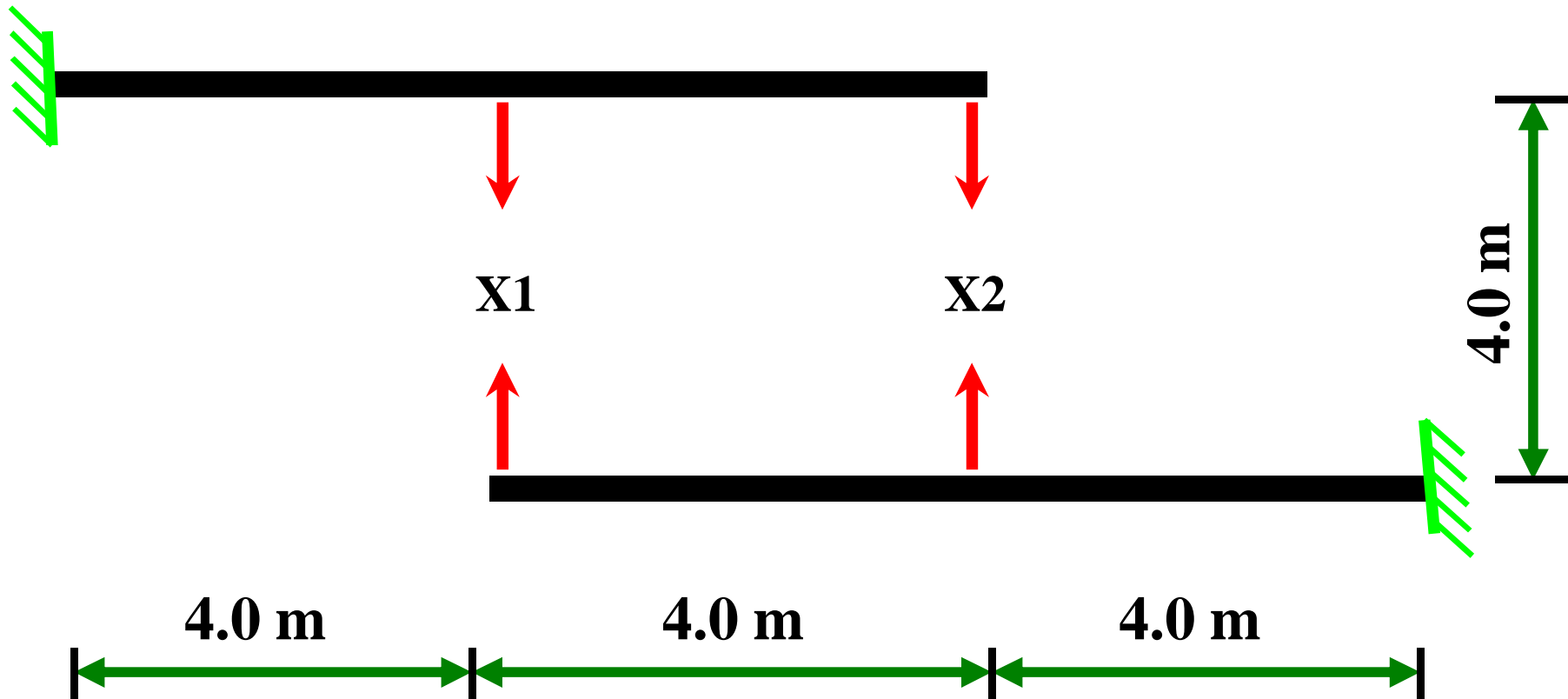
continue

Examples seven



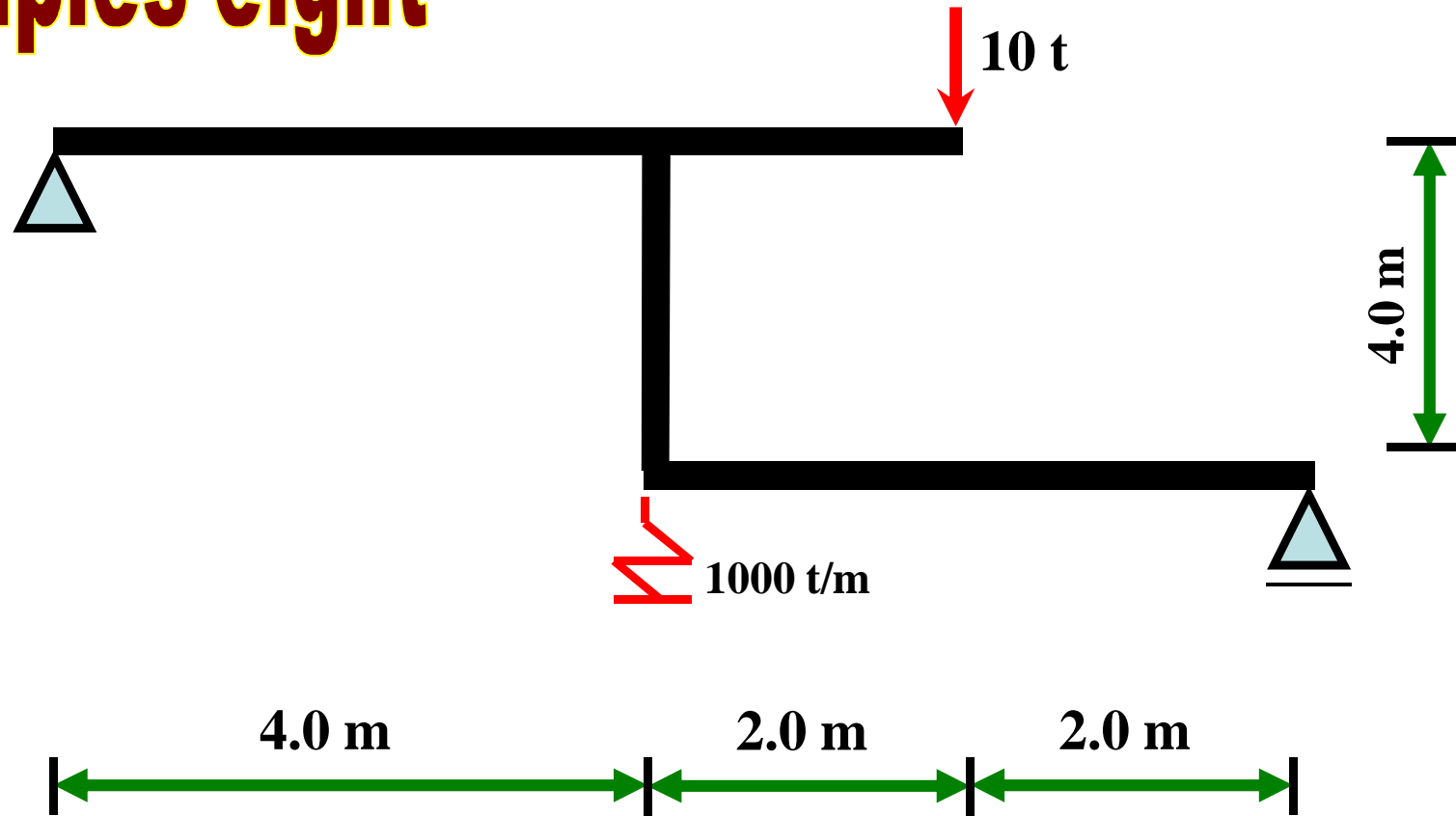
Draw B.M.D , S.F.D, and N.F.D $EI = 10000 \text{ t.m}^2$

Main system



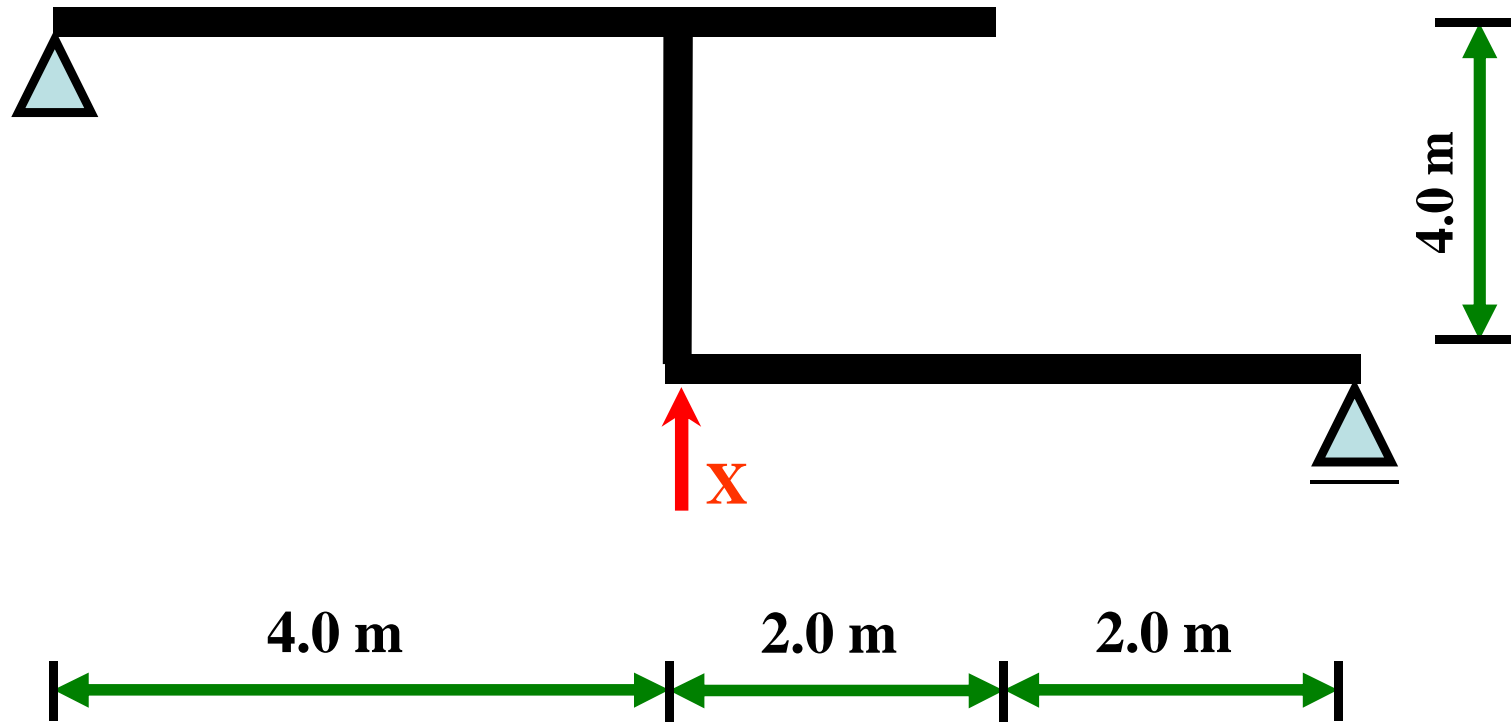
continue

Examples eight



Draw B.M.D , S.F.D, and N.F.D $EI = 10000 \text{ t.m}^2$

Main system



continue

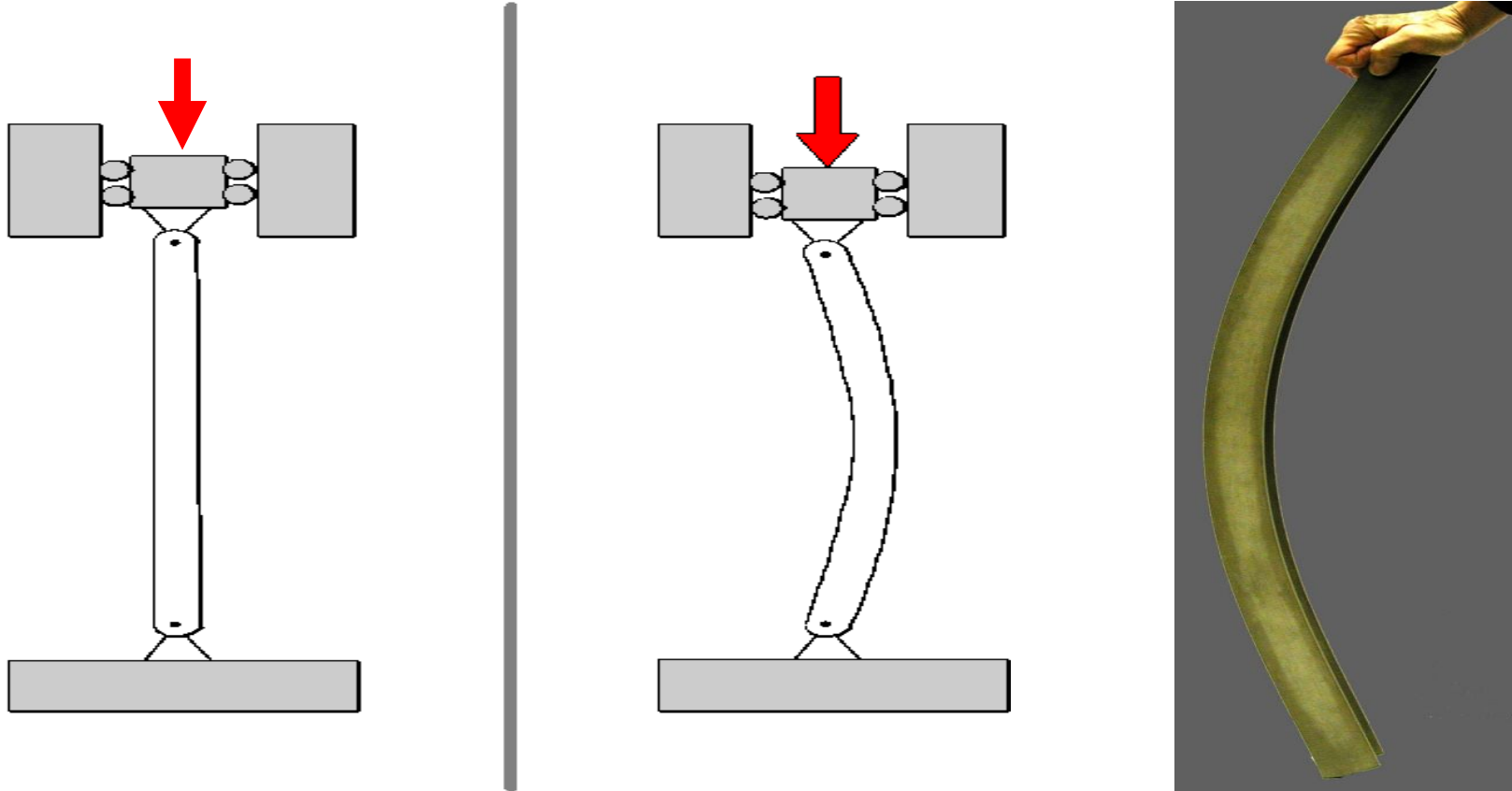
بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Buckling in Columns

Subjected to Normal only

Eng : Aymman abdo

• عندما يكون ابعاد العمود صغيرة بالنسبة لإرتفاعه فإن تعرض العمود لأحمال ضغط كبيره تسبب حدوث إنبعاج (تقوس) فى العمود.



• لاحظ أن الحمل فى البداية ماشى مع الـ **axis** للعمود وبالتالي يتولد **normal** فقط أما فى الحالة الثانية بعد حدوث التقوس يكون هناك ترحيل بمسافة (**y**) تسبب وجود عزوم

$$M = P \cdot y$$



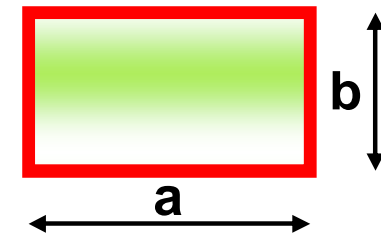
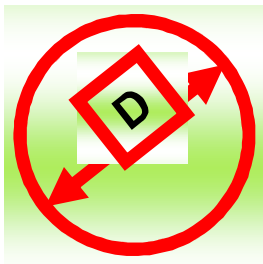
بعض المصطلحات المهمة جدا في الحسابات

1- INERTIA

1- تكون معطاه في بعض المسائل.

I_x and I_y

2- يتم حسابها في بعض المسائل ويكون للأشكال السهلة.



$$I_x = I_y = \frac{\pi}{64} D^4$$

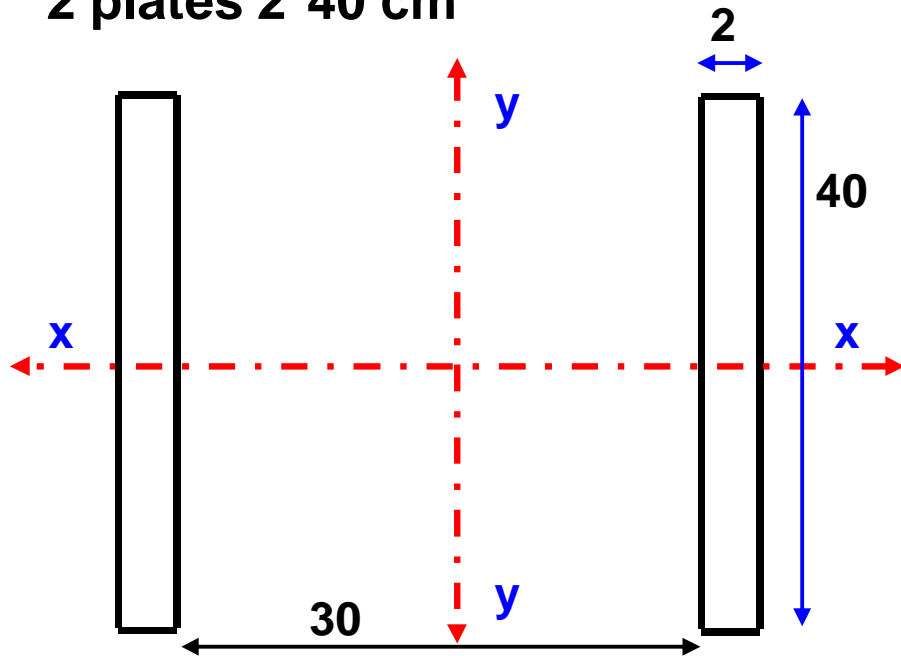
$$I_x = \frac{a * b^3}{12}$$

$$I_y = \frac{b * a^3}{12}$$

3- فى بعض المسائل يتم اعطاء ال-inertia لجزء من القطاع ويتم تركيب جزء اخر عليه ليتم اضافة فى الحسابات.

تجميع شكلين معا

2 plates 2*40 cm



$$A = 2 * (2 * 40) = 160 \text{ cm}^2$$

$$I_x = 2 * \left(\frac{2 * (40)^3}{12} \right)$$

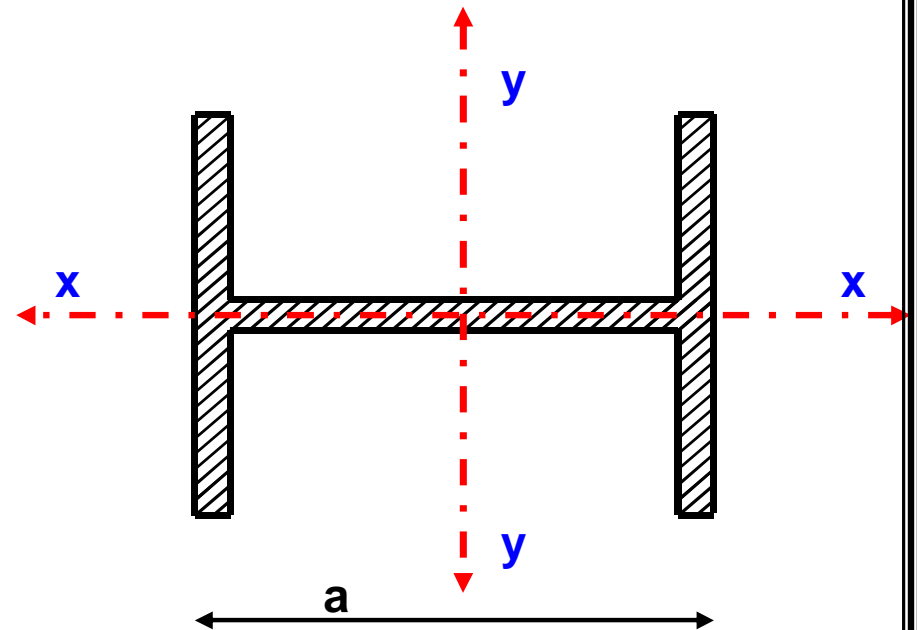
$$I_y = 2 * \left(\frac{40 * (2)^3}{12} + 80 * (16)^2 \right)$$

for BFIB No 30

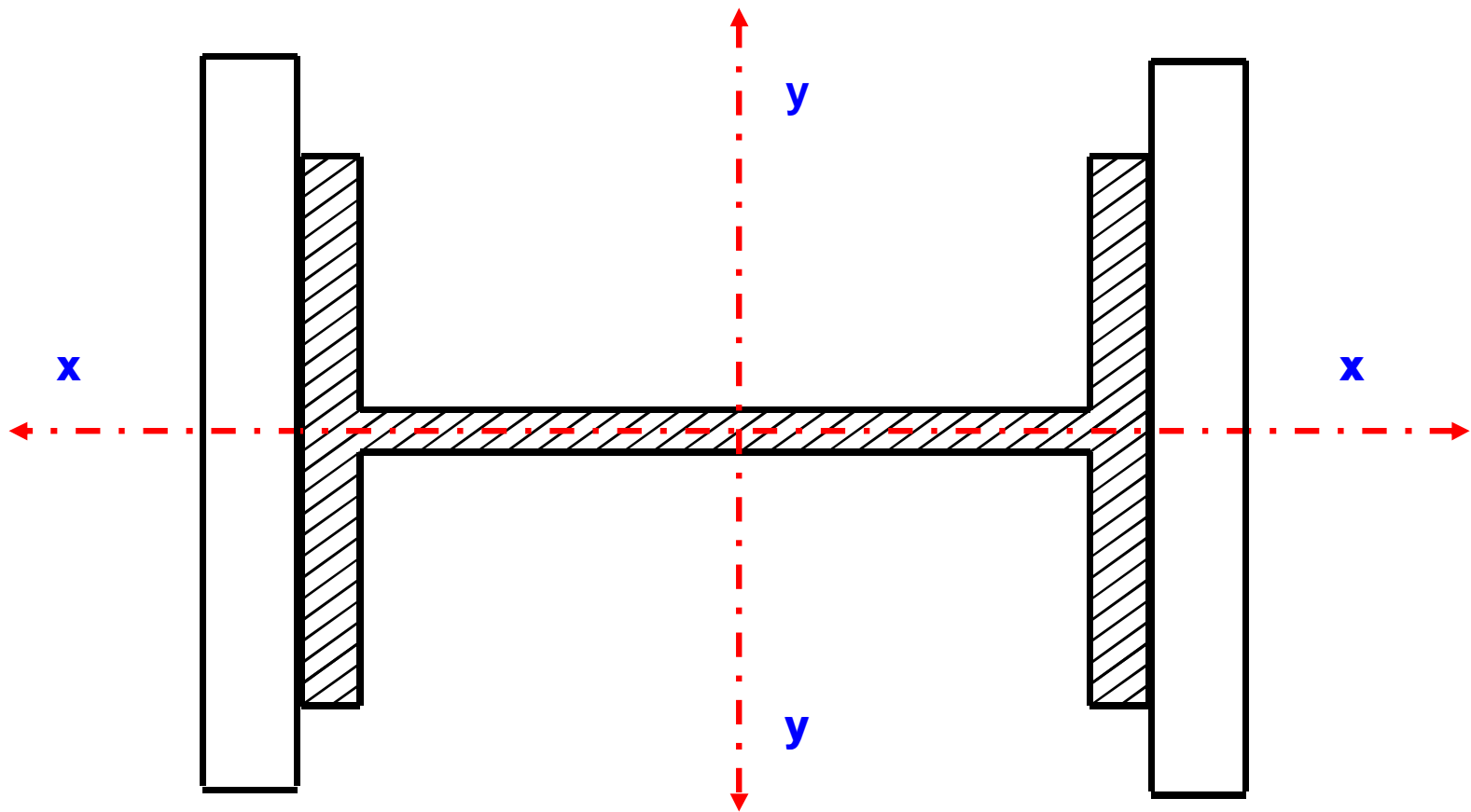
$$A = 100 \text{ cm}^2$$

$$I_x = 50000 \text{ cm}^4$$

$$I_y = 100000 \text{ cm}^4$$



معنى No 30 ان المسافة $a=30$ عادة لا توضع على الرسم يتم معرفتها من اسم ال-I beam



$$A = 100 + 160 = 260 \text{ cm}^2$$

$$I_x = 50000 + 2 * \left(\frac{2 * (40)^3}{12} \right)$$

$$I_y = 100000 + 2 * \left(\frac{40 * (2)^3}{12} + 80 * (16)^2 \right)$$

تجميع شكلين معا

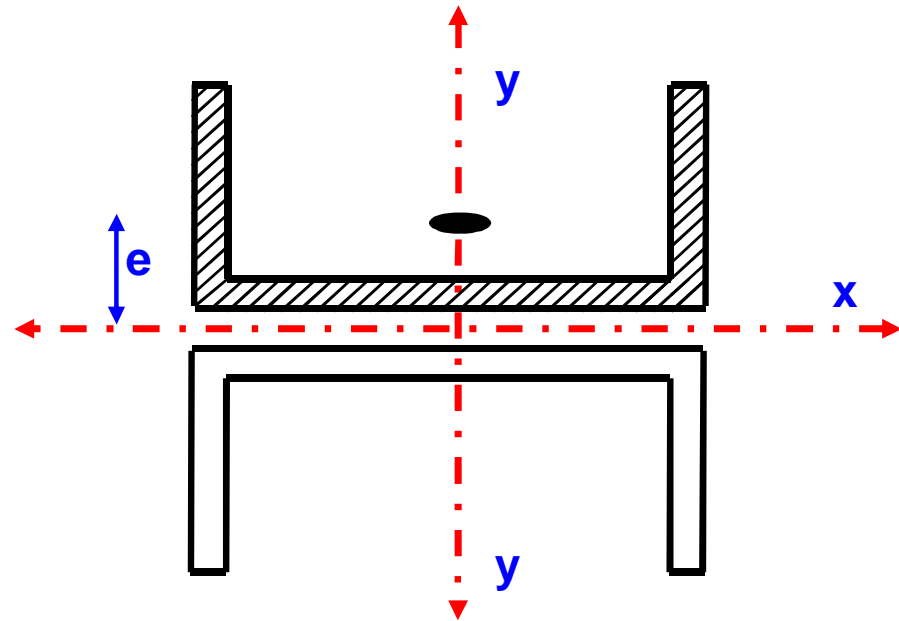
for CH No 10

$$A = 13.5$$

$$I_x = 206 \text{ cm}^4$$

$$I_y = 29.3 \text{ cm}^4$$

$$e = 1.55$$



$$A = 13.5 * 2 = 27 \text{ cm}^2$$

$$I_x = 2 * (206 + 13.5 * (1.55)^2) = 477$$

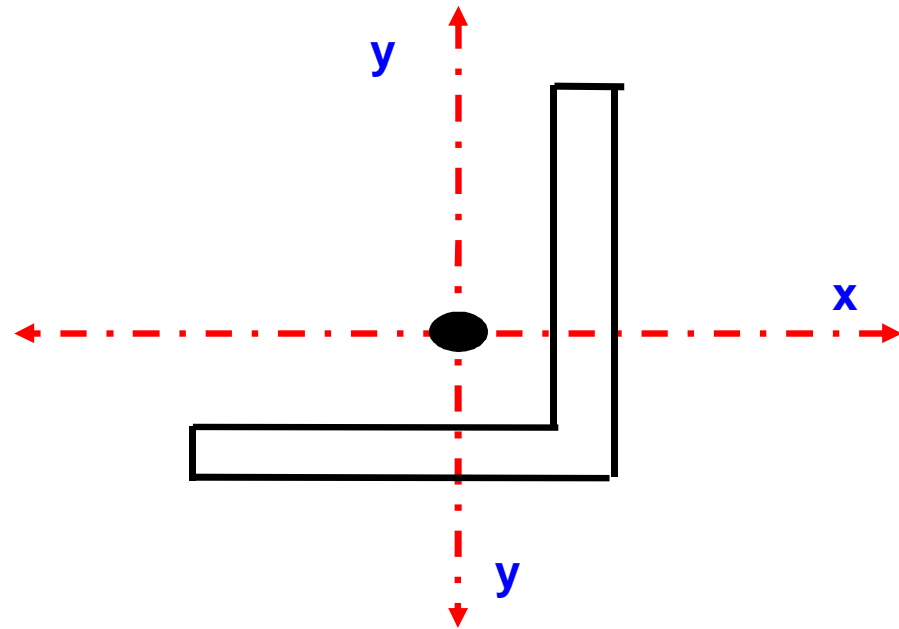
$$I_y = 2 * (29) = 58 \text{ cm}^4$$

Minimum Inertia

for L sec No 10

$$I_x = I_y = 472 \text{ cm}^4$$

$$I_{xy} = 312 \text{ cm}^4$$

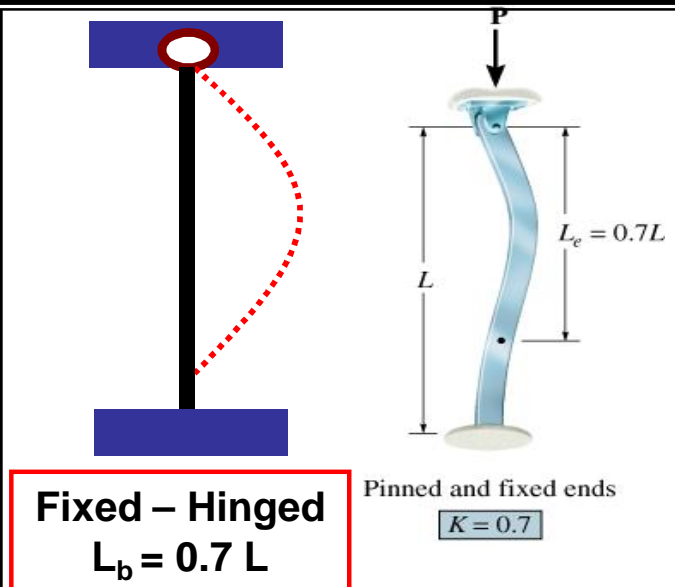
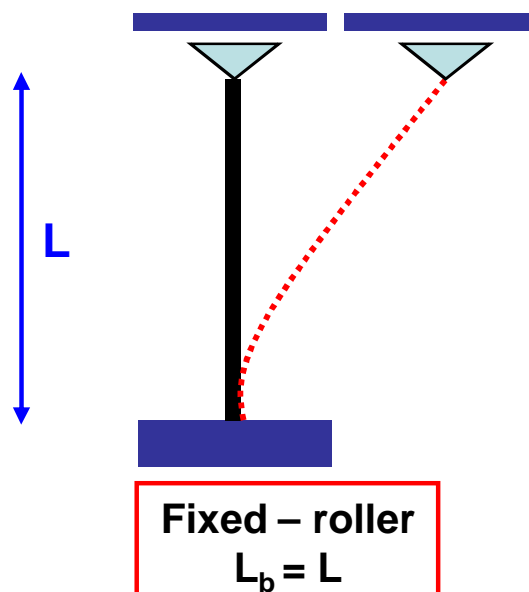
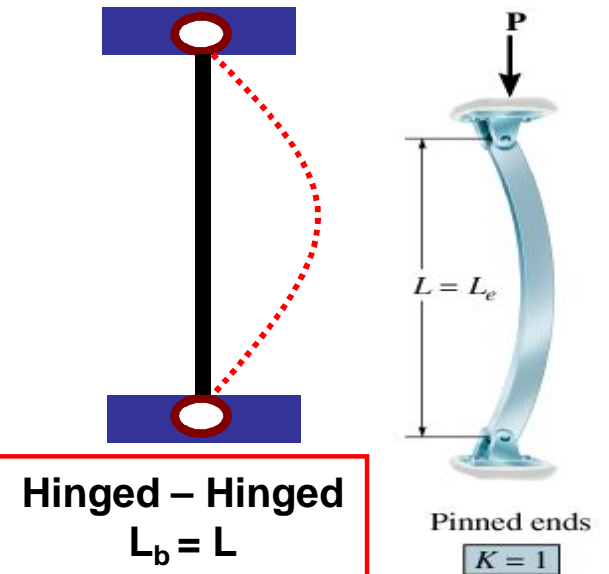
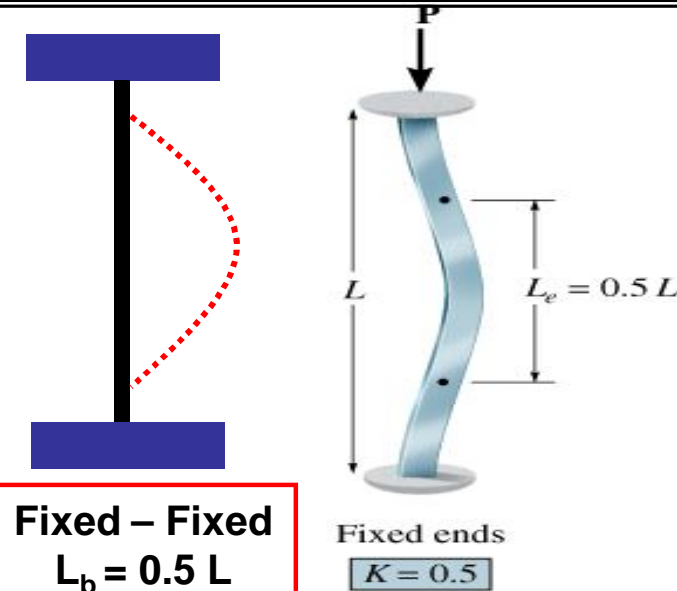
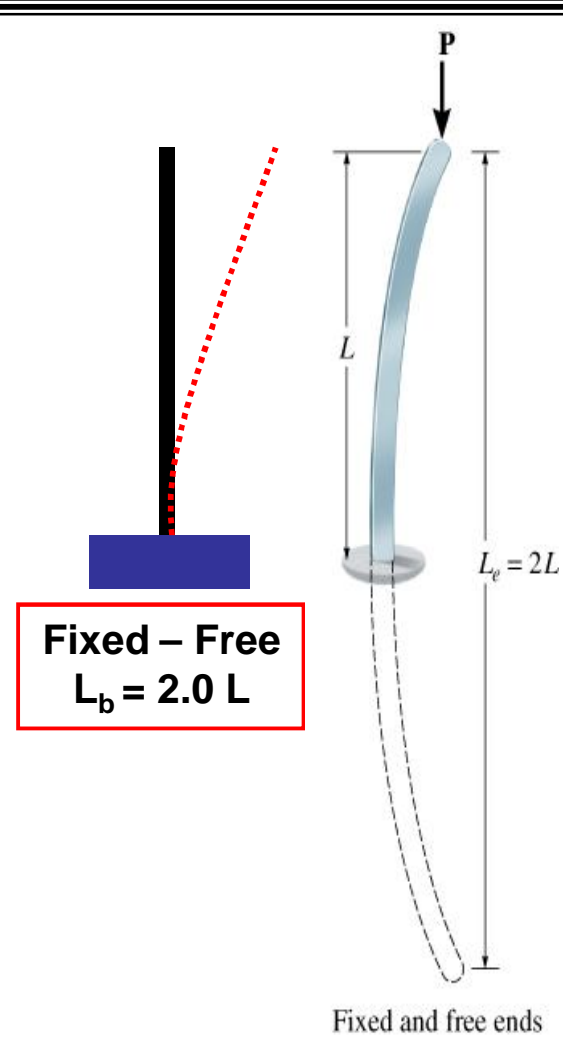


$$I_{(\min)} = \left(\frac{I_x + I_y}{2} \right) - \sqrt{\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2}$$

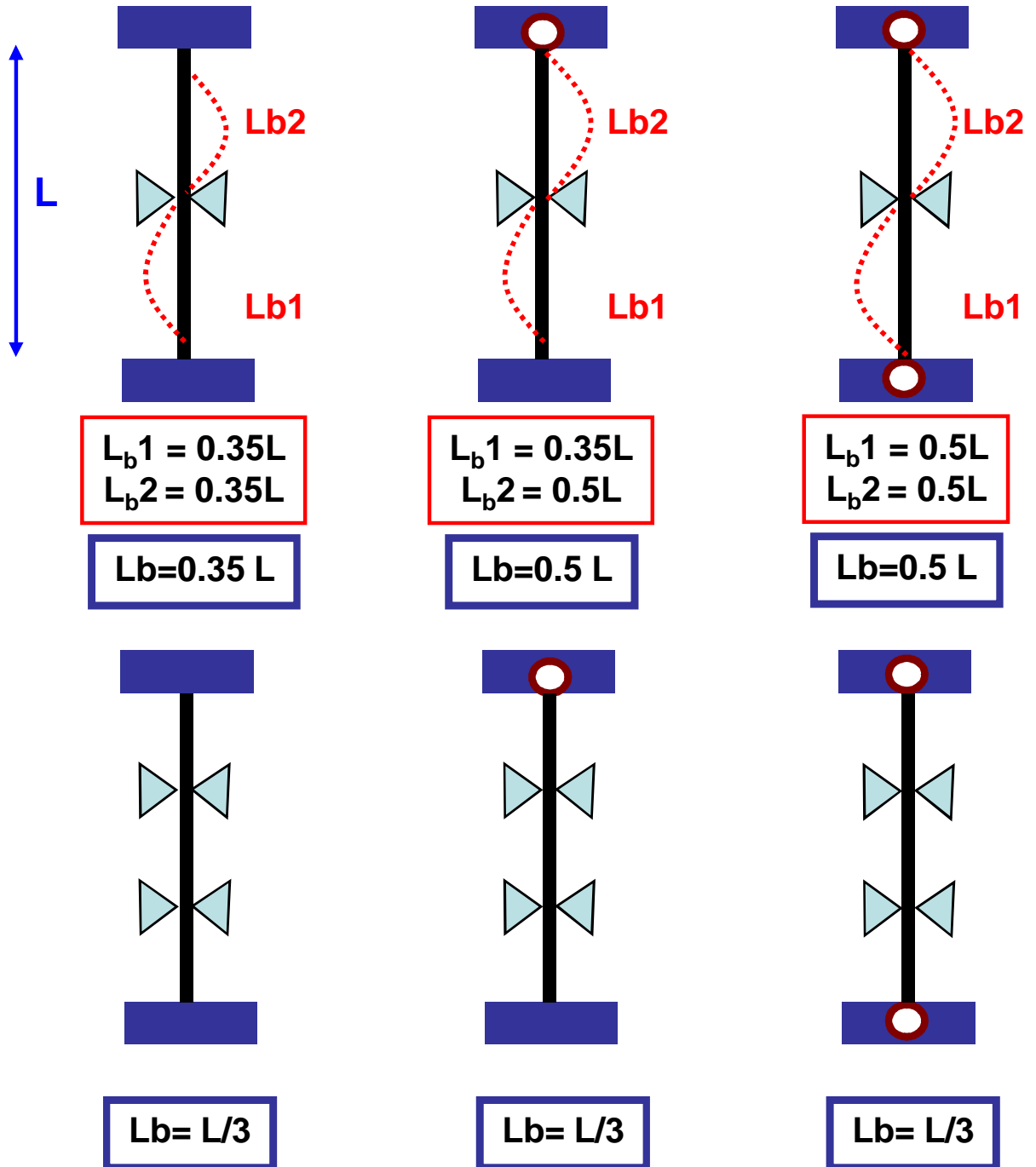
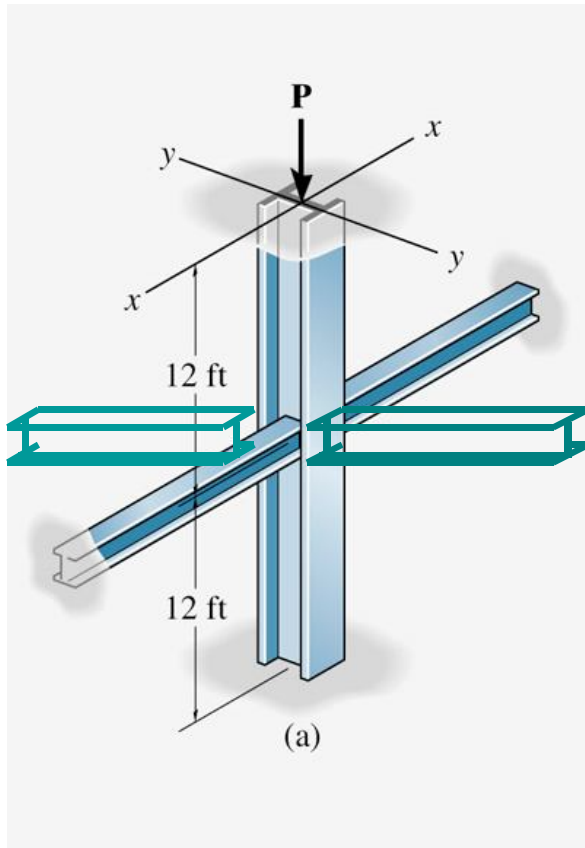
$$I_{(\min)} = \left(\frac{472 + 472}{2} \right) - \sqrt{\left(\frac{472 - 472}{2} \right)^2 + (312)^2} = 160 \text{ cm}^2$$

2- Buckling Length

الحالة الاولى عندما يكون العمود غير مقيد من البدايه للنهاية نجد ان الـ buckling length متساوى حول المحورين



الحالة الثانية عندما يكون العمود مقيد في الاتجاهين نجد ان الـ **buckling length** متساوى أيضا حول المحورين

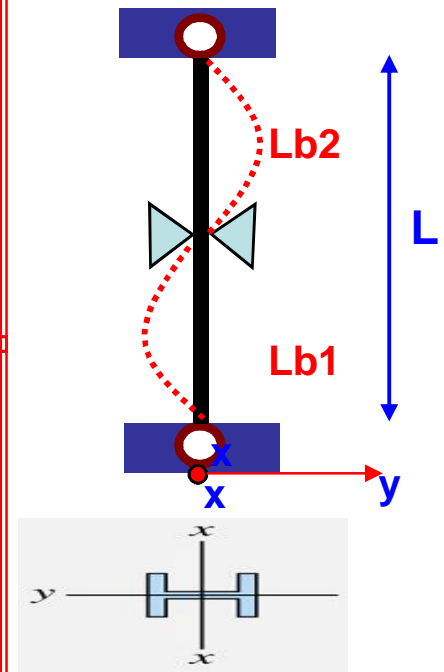


الحالة الثالثة عندما يكون العمود مقيد في اتجاه واحد نجد ان الـ buckling length مختلف حول المحورين

مع المحور الذي يظهر نقطه
على العمود (X)

$$L_b x = 0.5 L$$

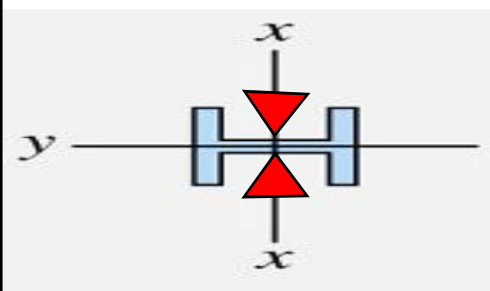
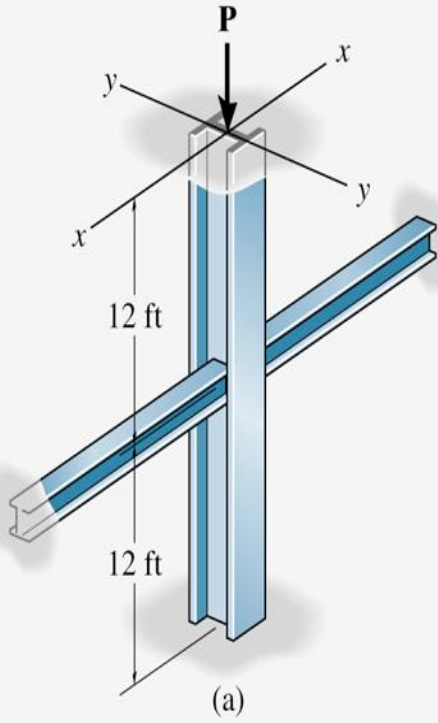
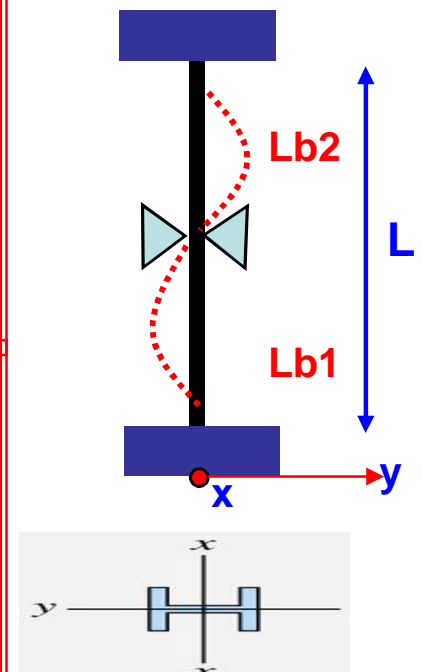
مع المحور الاخر
على العمود (y)

$$L_b y = L$$


مع المحور الذي يظهر نقطه
على العمود (X)

$$L_b x = 0.7 * 0.5 L = 0.35 L$$

مع المحور الاخر
على العمود (y)

$$L_b y = 0.5 L$$


Critical Buckling Load of Column

first step (Radius of gyration)

$$i_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

second step (slenderness ratio)

$$\lambda = \frac{L_b}{i_{\min}}$$

The critical buckling load (P_{cr}) known as euler buckling load as:

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2}$$

where:

E = Young Modulus of Elasticity.

I = Minimum moment of Inertia.

L = Buckling length of column.

Permissible Buckling Load of Column

For steel 37

$$\text{if } \lambda \geq 104 \quad \text{-----} \quad \sigma_{pb} = \frac{6000}{\lambda^2} \quad (t/cm^2)$$

$$\text{if } \lambda \leq 104 \quad \text{-----} \quad \sigma_{pb} = 1.1 - 0.00005 \lambda^2 \quad (t/cm^2)$$

$$P_{pb} = \sigma_{pb} * A$$

For steel 44

$$\text{if } \lambda \geq 96 \quad \text{-----} \quad \sigma_{pb} = \frac{6000}{\lambda^2} \quad (t/cm^2)$$

$$\text{if } \lambda \leq 96 \quad \text{-----} \quad \sigma_{pb} = 1.3 - 0.00007 \lambda^2 \quad (t/cm^2)$$

$$P_{pb} = \sigma_{pb} * A$$

For steel 52

$$\text{if } \lambda \geq 104 \quad \text{-----} \quad \sigma_{pb} = \frac{7500}{\lambda^2} \quad (t/cm^2)$$

$$\text{if } \lambda \leq 104 \quad \text{-----} \quad \sigma_{pb} = 1.4 - 0.000065 \lambda^2 \quad (t/cm^2)$$

$$P_{pb} = \sigma_{pb} * A$$

طريقة الحل

First step

(Calculate Buckling Length)

Second step

(Calculate Minimum Inertia)

Third step (Radius of gyration)

$$i_{\min} = \sqrt{\frac{I_{\min}}{A}}$$

Fourth step (Slenderness ratio)

$$\lambda = \frac{L_b}{i_{\min}}$$

Fifth step (Critical Load)

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2}$$

six step (Permissible Load) by (σ_{pb} & λ)

$$P_{pb} = \sigma_{pb} * A$$

seven step (Factor of safety)

$$n = \frac{P_{cr}}{P_{pb}}$$

Example one

given

for IBE 30

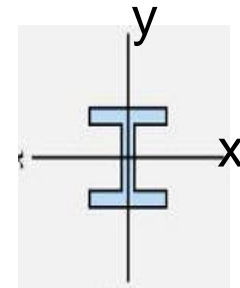
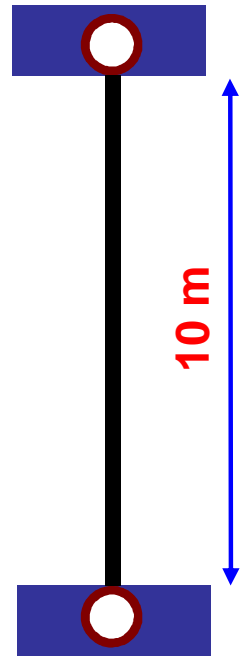
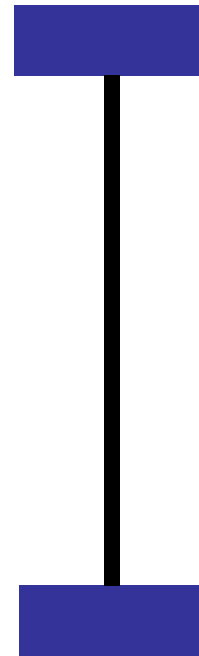
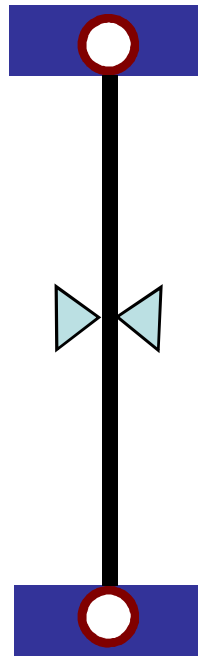
$$I_x = 25700$$

$$I_y = 9010$$

$$E = 2100$$

$$A = 154$$

$$st \ 52$$



Required P_{cr} P_{pb} and n in the three cases

(a) Hinged – Hinged case.

(b) Fixed -- Fixed case.

(c) Hinged – Hinged and restricted in x direction only

Case (a)

First step

$$(L_b = 1000 \text{ cm})$$

Second step

$$(I_{min} = 9010)$$

Third step (Radius of gyration)

$$i_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{9010}{154}} = 7.65 \text{ cm}$$

Fourth step (Slenderness ratio)

$$\lambda = \frac{L_b}{i_{min}} = \frac{1000}{7.65} = 130.7$$

Fifth step (Critical Load)

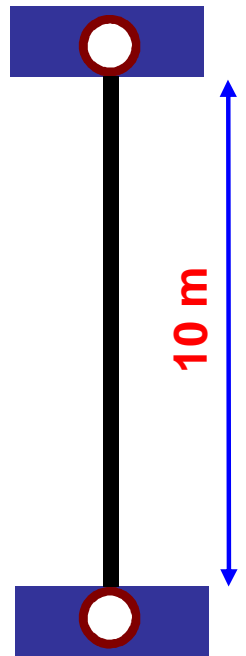
$$P_{cr} = \frac{\pi^2 E I_{min}}{L_b^2} = \frac{\pi^2 * 2100 * 9010}{(1000)^2} = 186.75 \text{ t}$$

six step (Permissible Load) by (st52 & $\lambda = 130$)

$$\sigma_{pb} = \frac{7500}{\lambda^2} = \frac{7500}{(130.7)^2} = 0.438 \text{ t/cm}^2 \quad \text{---} \quad P_{pb} = \sigma_{pb} * A = 0.438 * 154 = 67.45 \text{ t}$$

seven step (Factor of safety)

$$n = \frac{P_{cr}}{P_{pb}} = \frac{186.75}{67.45} = 2.77$$



Case (b)

First step

$$(L_b = 500 \text{ cm})$$

Second step

$$(I_{min} = 9010)$$

Third step (Radius of gyration)

$$i_{min} = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{9010}{154}} = 7.65 \text{ cm}$$

Fourth step (Slenderness ratio)

$$\lambda = \frac{L_b}{i_{min}} = \frac{500}{7.65} = 65.35$$

Fifth step (Critical Load)

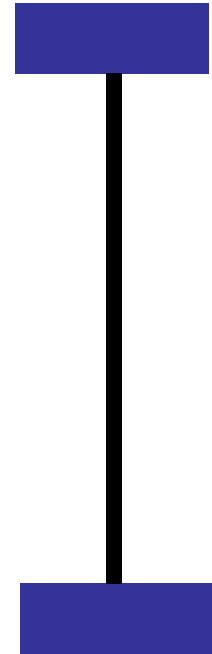
$$P_{cr} = \frac{\pi^2 E I_{min}}{L_b^2} = \frac{\pi^2 * 2100 * 9010}{(500)^2} = 747 \text{ t}$$

six step (Permissible Load) by (st52 & $\lambda = 65$)

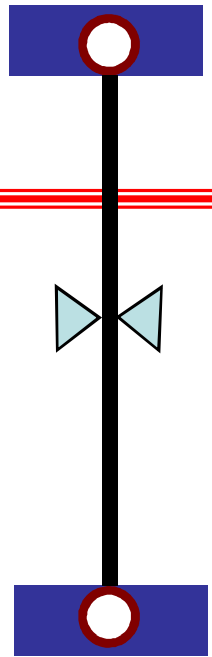
$$\sigma_{pb} = 1.4 - 0.000065\lambda^2 = 1.122 \text{ t/cm}^2 \quad \text{---} \quad P_{pb} = 1.122 * 154 = 172.85 \text{ t}$$

seven step (Factor of safety)

$$n = \frac{P_{cr}}{P_{pb}} = \frac{747}{172.85} = 4.32$$



Case (c)



$$(Lbx = 1000 \text{ cm})$$

$$(Ix = 25760)$$

$$ix = \sqrt{\frac{Ix}{A}} = \sqrt{\frac{25760}{154}} = 12.93 \text{ cm}$$

$$\lambda x = \frac{L_{bx}}{i_x} = \frac{1000}{12.93} = 77.32$$

$$P_{crx} = \frac{\pi^2 E I_x}{L_b x^2} = \frac{\pi^2 * 2100 * 25760}{(1000)^2} = 534 \text{ t}$$

$$(st52 \text{ \& } \lambda = 77)$$

$$\sigma_{pb} = 1.0114 \text{ t/cm}^2 \rightarrow P_{pb} = 1.0114 * 154 = 155.75 \text{ t}$$

$$n = \frac{P_{cr}}{P_{pb}} = \frac{534}{155.75} = 3.428$$

$$(Lby = 500 \text{ cm})$$

$$(Iy = 9010)$$

$$iy = \sqrt{\frac{Iy}{A}} = \sqrt{\frac{9010}{154}} = 7.65 \text{ cm}$$

$$\lambda y = \frac{L_{by}}{i_y} = \frac{500}{7.65} = 65.36$$

$$P_{cry} = \frac{\pi^2 E I_y}{L_b y^2} = \frac{\pi^2 * 2100 * 9010}{(500)^2} = 747 \text{ t}$$

$$(st52 \text{ \& } \lambda = 65.36)$$

$$\sigma_{pb} = 1.1224 \text{ t/cm}^2 \rightarrow P_{pb} = 1.1224 * 154 = 172.85 \text{ t}$$

$$n = \frac{P_{cr}}{P_{pb}} = \frac{747}{172.85} = 4.32$$

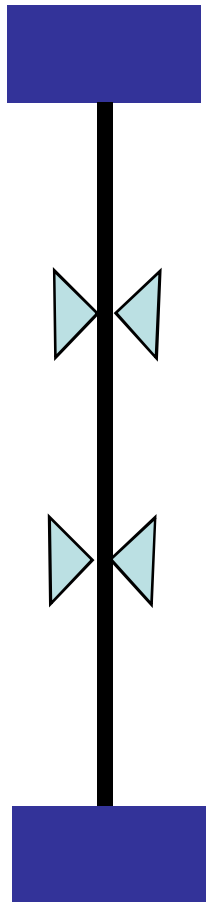
$$>> P_{cr} = 534$$

$$>> P_{pb} = 155$$

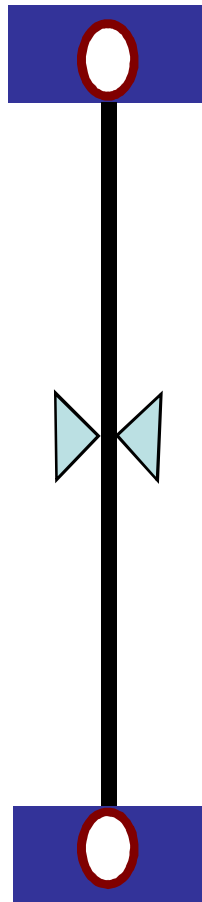
Example two

Using Euler formula find the critical buckling load for each case of the following member and draw the deformed shape.

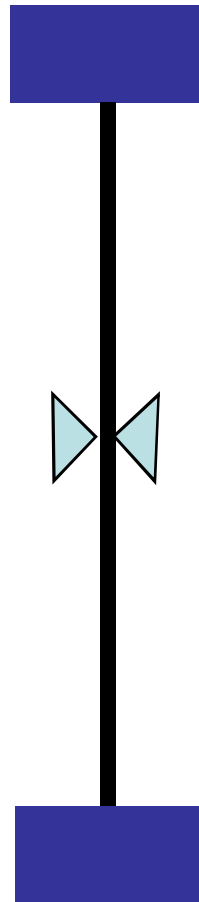
$I = 300 \text{ cm}^4$ and $L = 600 \text{ cm}$



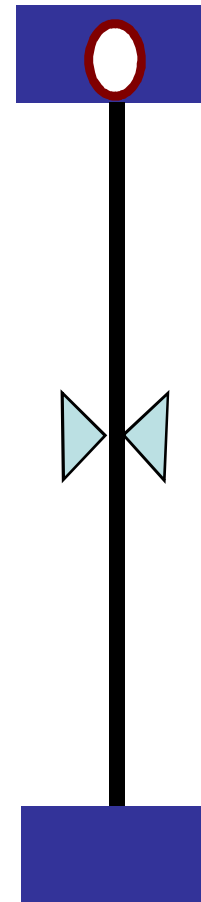
Case d



Case c



Case b



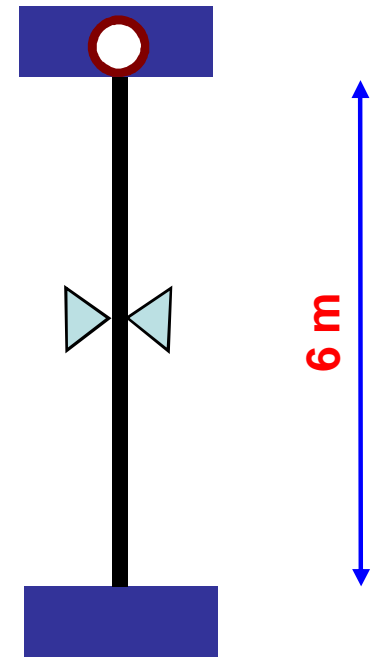
Case a

6 m

Case a

$$(L_b = 300 \text{ cm})$$

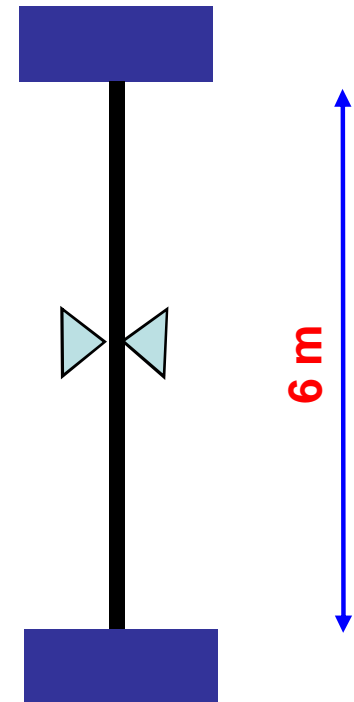
$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2} = \frac{\pi^2 * 2000 * 300}{(300)^2} = 65.8 \text{ ton}$$



Case b

$$(L_b = 0.7 * 300 = 210 \text{ cm})$$

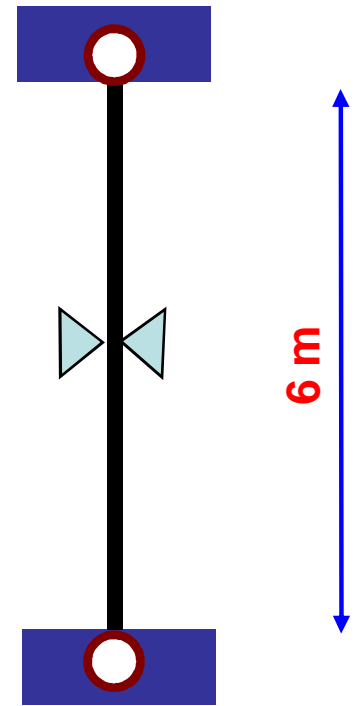
$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2} = \frac{\pi^2 * 2000 * 300}{(210)^2} = 134 \text{ ton}$$



Case c

($L_b = 300 \text{ cm}$)

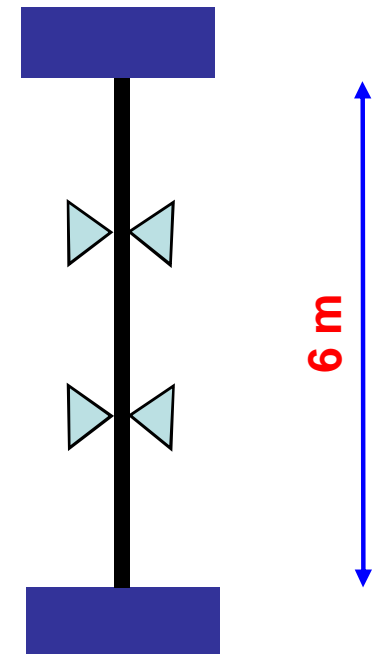
$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2} = \frac{\pi^2 * 2000 * 300}{(300)^2} = 65.8 \text{ ton}$$



Case d

($L_b = 200 \text{ cm}$)

$$P_{cr} = \frac{\pi^2 E I_{\min}}{L_b^2} = \frac{\pi^2 * 2000 * 300}{(200)^2} = 148 \text{ ton}$$



Example three

given

for IBE30

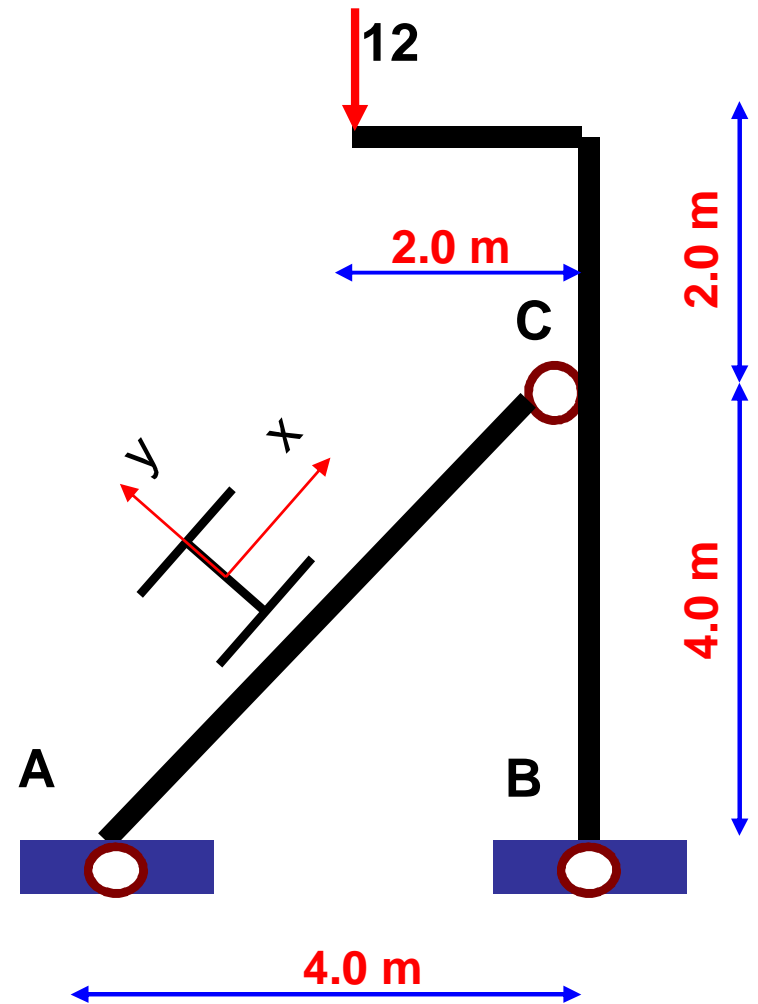
$$I_x = 5740$$

$$I_y = 988$$

$$E = 2100$$

$$A = 53.4$$

st 37



(a) Required Ppb in member AC

(b) Actual F. O. S

(c) Increase the permissible load to the twice without increase dimension of section

$$\Sigma M_B = 0.0$$

$$F \cos 45^\circ * 4 = 12 * 2$$

$$F = 8.5 \text{ ton}$$

$$(L_b = 565 \text{ cm})$$

$$(I_{\min} = 988)$$

$$i = \sqrt{\frac{I}{A}} = \sqrt{\frac{988}{53.4}} = 4.3 \text{ cm}$$

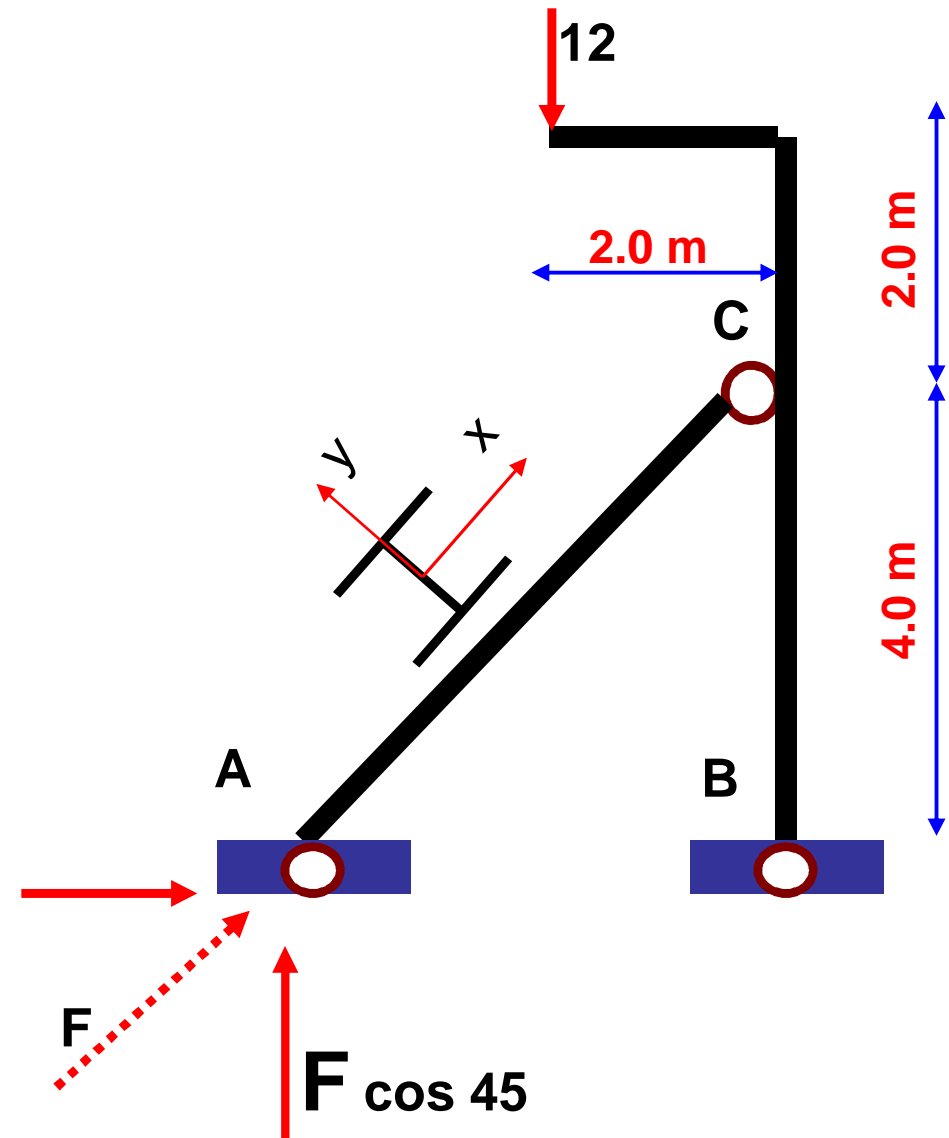
$$\lambda = \frac{L_b}{i_x} = \frac{565}{4.3} = 131 > 104$$

$$P_{cr} = \frac{\pi^2 E I}{L_b^2} = \frac{\pi^2 * 2100 * 988}{(565)^2} = 20.37 \text{ ton}$$

$$(st37 \text{ \& } \lambda = 131)$$

$$\sigma_{pb} = \frac{6000}{131^2} t/cm^2 \rightarrow P_{pb} = 0.3496 * 53.4 = 18.67 \text{ t}$$

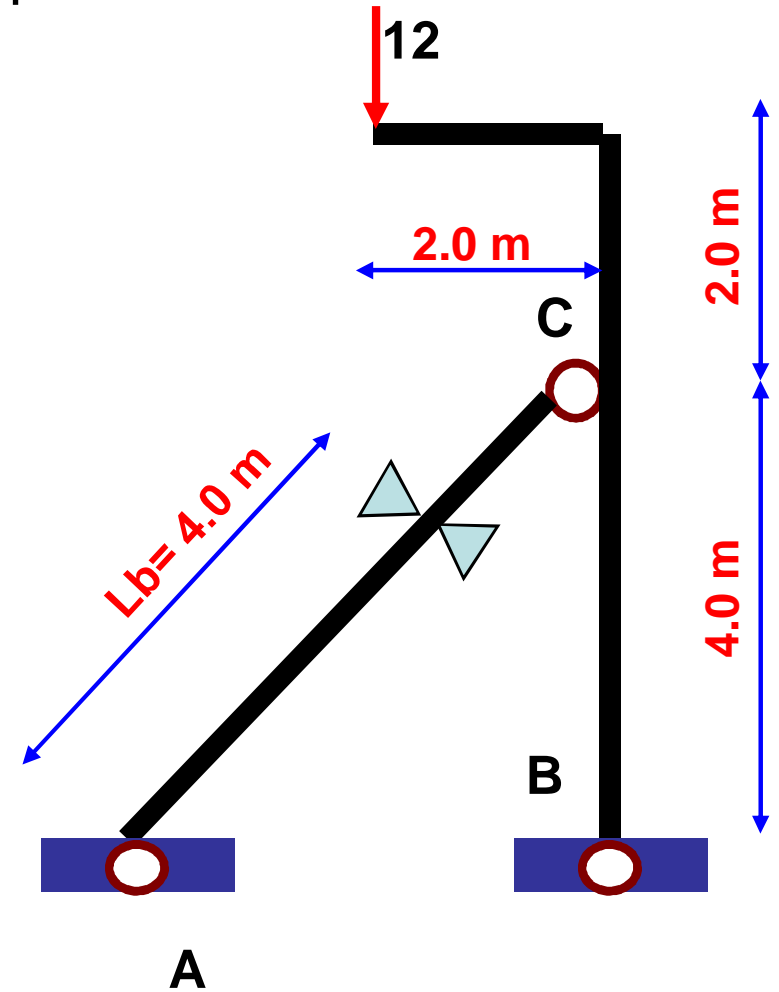
$$n_{actual} = \frac{20.37}{8.5} = 2.4$$



$$P_{pb} = 2 * 18.67 = 37.35 \gg \sigma_{pb} = \frac{37.35}{53.4} = 0.7$$

$$\sigma_{pb} = \frac{6000}{\lambda^2} = 0.7 \rightarrow \lambda = 92.6$$

$$\lambda = \frac{L_b}{4.3} = 92.6 \text{ --- } L_b = 400 \text{ cm}$$



يتم ترك خواص القطاع كما هي

ويتم مضاعفة الـ P_{pb} وبالمراجع يتم حساب L_b

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Buckling in Columns

Normal and Moment

Eng : Ayman abdo

حالة ان يكون العمود معرض لـ normal and moment

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} + \frac{M_x}{I_x} * y + \frac{M_y}{I_y} * x \leq \sigma_{pc}$$

σ_{pc} (permissible compression)

for **st37**---- $\sigma_{pc} = 1.1 \text{ t/cm}^2$

for **st44**---- $\sigma_{pc} = 1.3 \text{ t/cm}^2$

for **st52**---- $\sigma_{pc} = 1.4 \text{ t/cm}^2$

σ_{pb} (permissible buckling)

$L_b = \dots\dots$

$i_{\min} = \dots\dots$

$\lambda = \dots\dots$

$\Sigma_{pb} = \dots\dots$

خواص القطاع

A, I_x, I_y
given

خواص القطاع

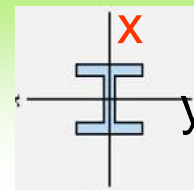
x, y
معدة على القطاع

في المسائل لا يوجد الا عزم واحد فقط من العزمين M_x, M_y

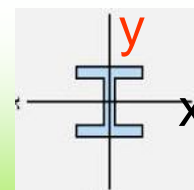
P {normal force in the column}

M_x {normal force in the column}

M_y {normal force in the column}



عندما يكون المحور x هو العمودي



عندما يكون المحور y هو العمودي

افكار المسائل

النوعية الثانية

معطى حمل مجهول على المنشأ
والمطلوب عمل (max load)

1- calculate - P , (M_x or M_y)
بدلالة المجهول

2- σ_{\max} = as a function of P
بدلالة المجهول

3- $\sigma_{\max} = \sigma_{pc}$

>>>> $P = \text{----}$

النوعية الأولى

معطى كل الاحمال على المنشأ
والمطلوب عمل (check)

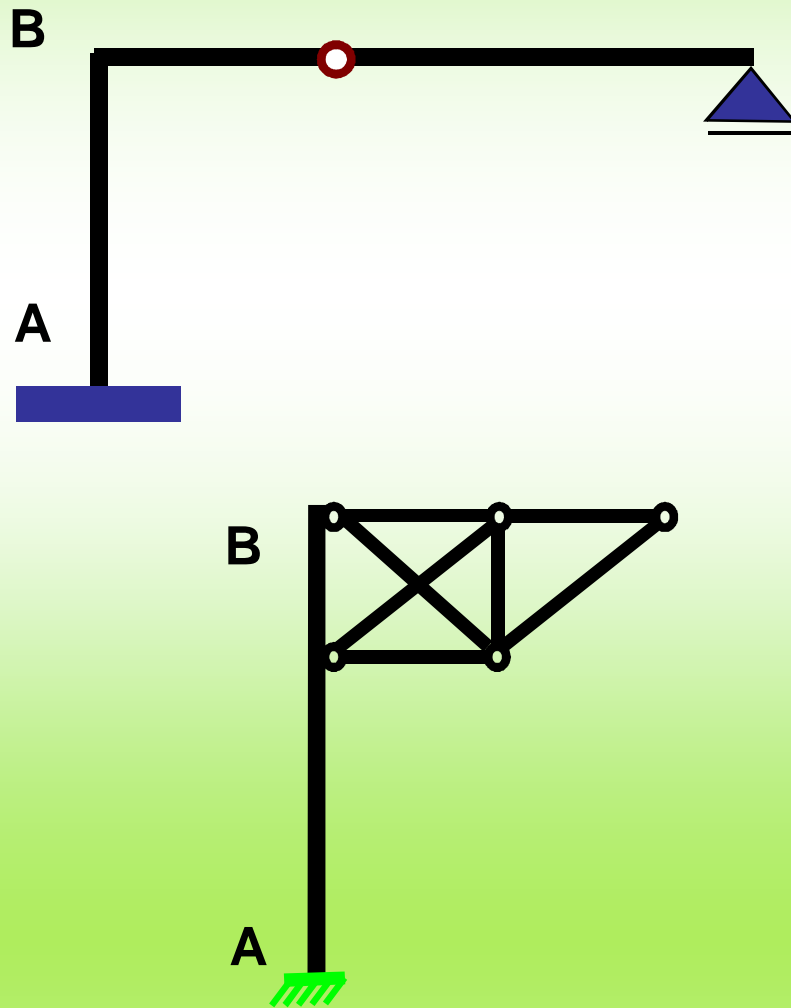
1- calculate ----- P , (M_x or M_y)

2- σ_{\max} = from the equation=-----

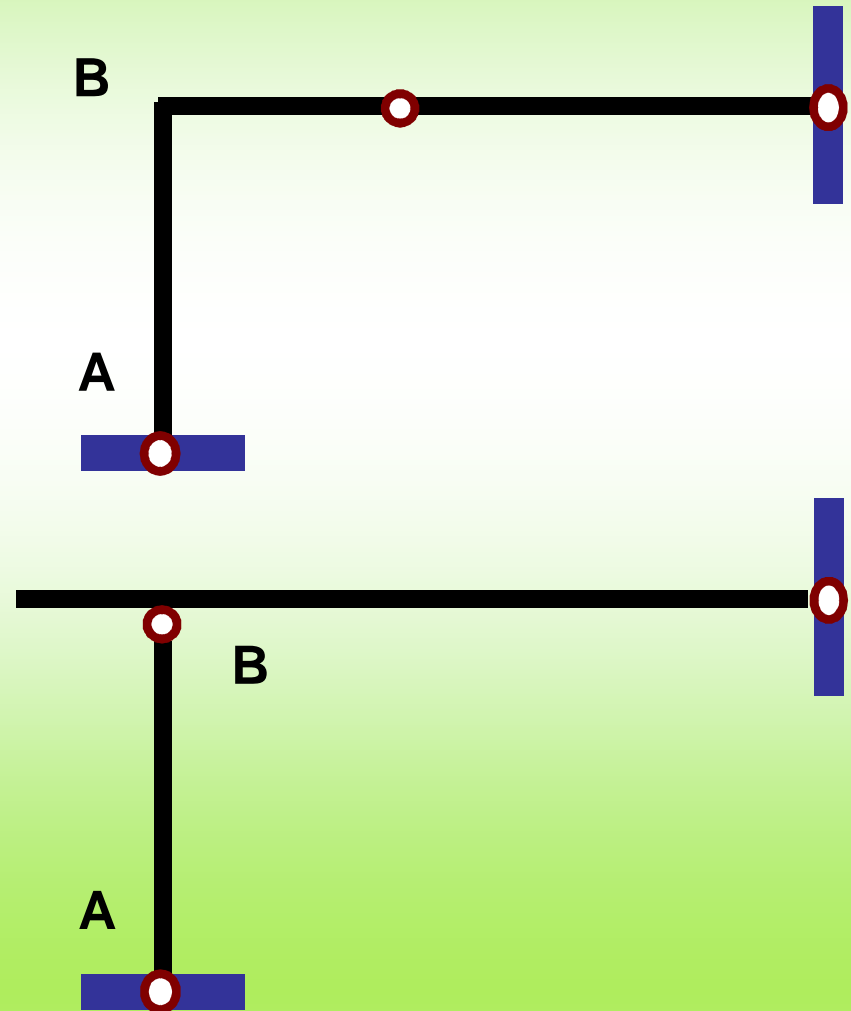
3- if $\sigma_{\max} \leq \sigma_{pc}$ safe

4- if $\sigma_{\max} > \sigma_{pc}$ unsafe

Buckling length



في المسائل اذا كان العمود مثبت على الارض
بـ fixed فيتم اخذه (fixed – free)



في المسائل اذا كان العمود مثبت على الارض
بـ hinge فيتم اخذه (hinged – hinged)

Example one

given

for IBE32

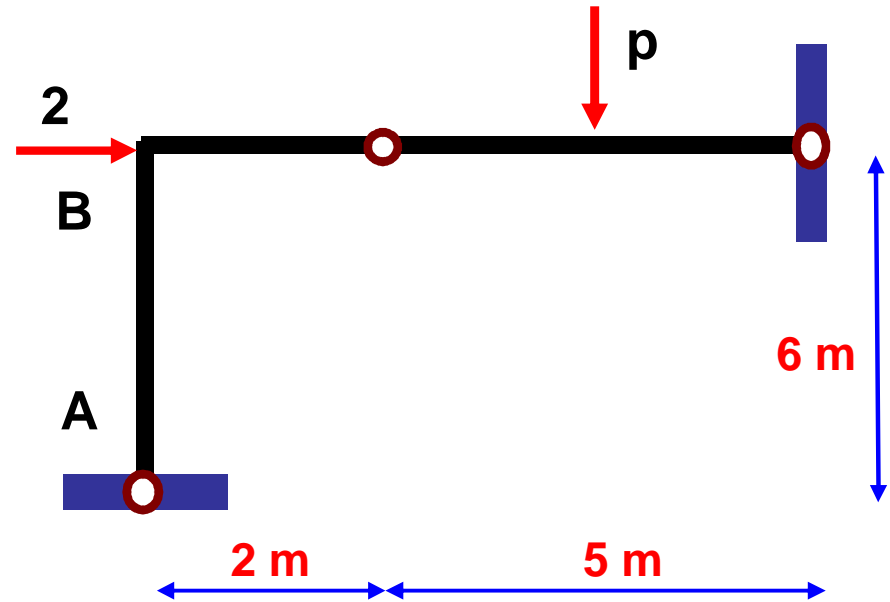
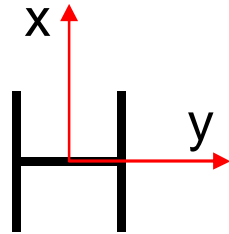
$$I_x = 45000$$

$$I_y = 10800$$

$$E = 2000$$

$$A = 190$$

$$st \ 44$$



Required for column AB :

(1) max safe load P.

(2) critical load P_{cr} .

$$\sum \mathbf{M}_{dr} = 0.0$$

$$P * 2.5 - Y_c * 5 = 0.0 \Rightarrow Y_c = 0.5 P$$

$$\sum \mathbf{F}_Y = 0.0$$

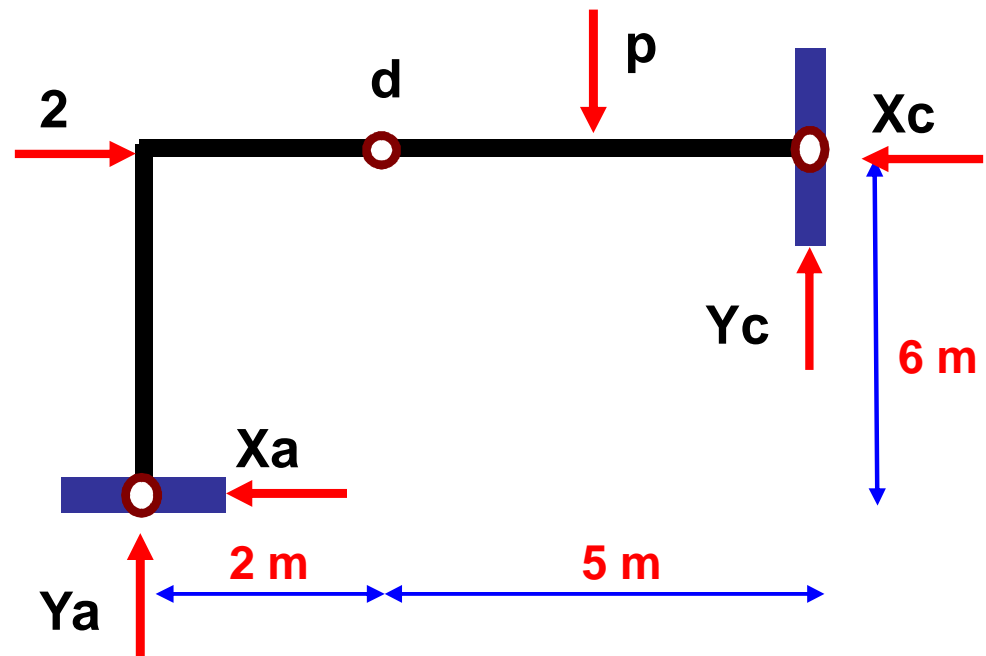
$$Y_A = 0.5 P$$

$$\sum \mathbf{M}_A = 0.0$$

$$P * 4.5 - 0.5 P * 7 - X_c * 6 = 0.0 \Rightarrow X_c = 2 + P/6$$

$$\sum \mathbf{F}_X = 0.0$$

$$X_A = 2.0$$



الـ normal فى العمود A-B ($N = 0.5 P t$)

محور x لاعلى -- M_x هى اللى شغالة - ($M_x = 2 * 600 = 1200 t.cm$)

العمود A-B مثبت بـ hinge على الارض (hinged - hinged) ---- $L_b = 600cm$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} + \frac{M_x}{I_x} * y$$

($L_b = 600 \text{ cm}$)

$$i = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{10800}{190}} = 7.54 \text{ cm}$$

$$\lambda = \frac{L_b}{i_x} = \frac{600}{7.54} = 79.6 < 96$$

$$P_{cr} = \frac{\pi^2 E I}{L_b^2} = \frac{\pi^2 * 2000 * 10800}{(600)^2} = 592 \text{ ton}$$

$$\sigma_{pb} = 1.3 - 0.00007 \lambda^2 = 0.856 \text{ t/cm}^2$$

for st44---- $\sigma_{pc} = 1.3 \text{ t/cm}^2$

خواص القطاع

A=190

I_x=45000

y = 16

($M_x = 1200 \text{ t.cm}$)

($N = 0.5 P \text{ t}$)

$$\sigma_{\max} = \frac{0.5P}{190} * \frac{1.3}{0.856} + \frac{1200}{45000} * 16 = 1.3$$

$$>>>> P_{\max} = 32.87 \text{ ton}$$

Example two

given

for IBE32

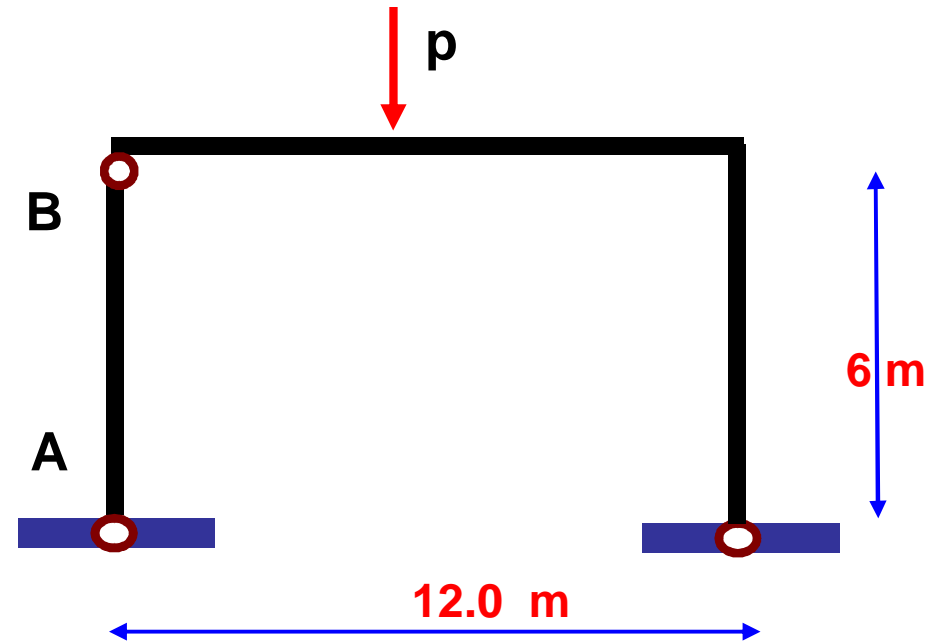
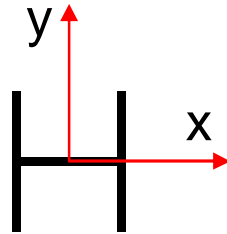
$$I_x = 4670$$

$$I_y = 13900$$

$$E = 2000$$

$$A = 254$$

$$st \ 44$$



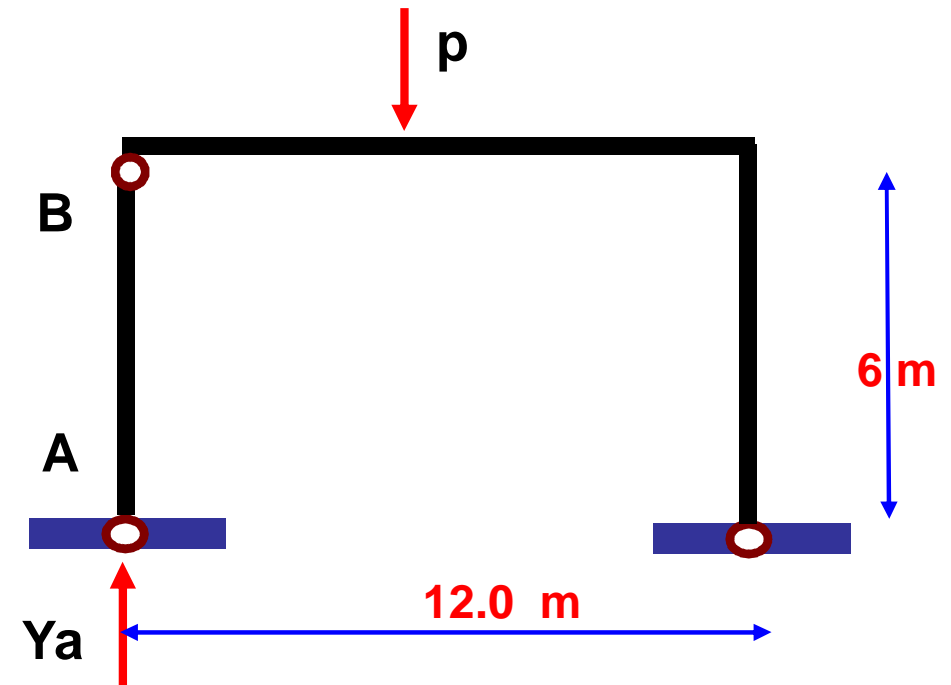
Required in column AB:

(1) max safe load P.

(2) critical load P_{cr} .

$$\sum F_Y = 0.0$$

$$Y_A = 0.5 P$$



$$(N = 0.5 P t)$$

$$(M_y = 0.0 \text{ for link member})$$

$$\text{for st44} \text{----} \sigma_{pc} = 1.3 \text{ t/cm}^2$$

العمود A-B مثبت بـ hinge على الارض (hinged – hinged) ---- $L_b = 600 \text{ cm}$

خواص القطاع

$$A = 254$$

$$I_y = 13900$$

$$x = 16$$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} + \frac{My}{I_y} * x = \sigma_{pc}$$

$$(Lb = 600 \text{ cm})$$

$$i = \sqrt{\frac{I_m}{A}} = \sqrt{\frac{4670}{254}} = 4.28 \text{ cm}$$

$$\lambda = \frac{Lb}{i} = \frac{600}{4.28} = 140 < 96$$

$$P_{cr} = \frac{\pi^2 E I}{Lb^2} = \frac{\pi^2 * 2000 * 4670}{(600)^2} = 256 \text{ ton}$$

$$\sigma_{pb} = \frac{6000}{140^2} = 0.306 \text{ t / cm}^2$$

$$\sigma_{\max} = \frac{0.5P}{254} * \frac{1.3}{0.306} = 1.3$$

$$>>>> P_{\max} = 77 \text{ ton}$$

Example three

for *BF1B* No 30

$$A = 100 \text{ cm}^2$$

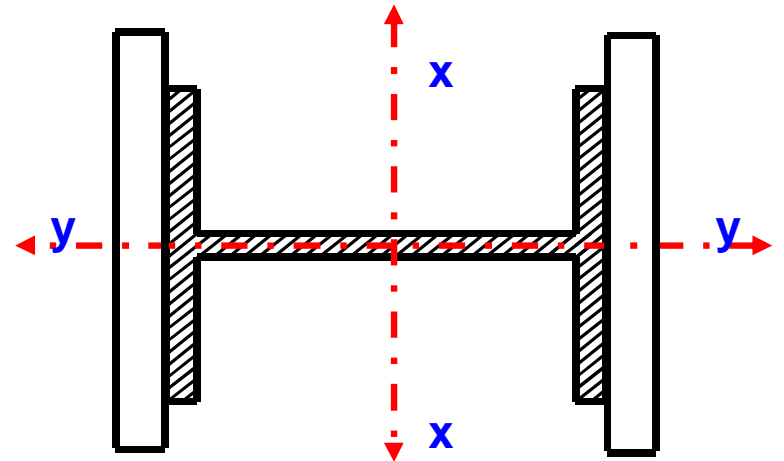
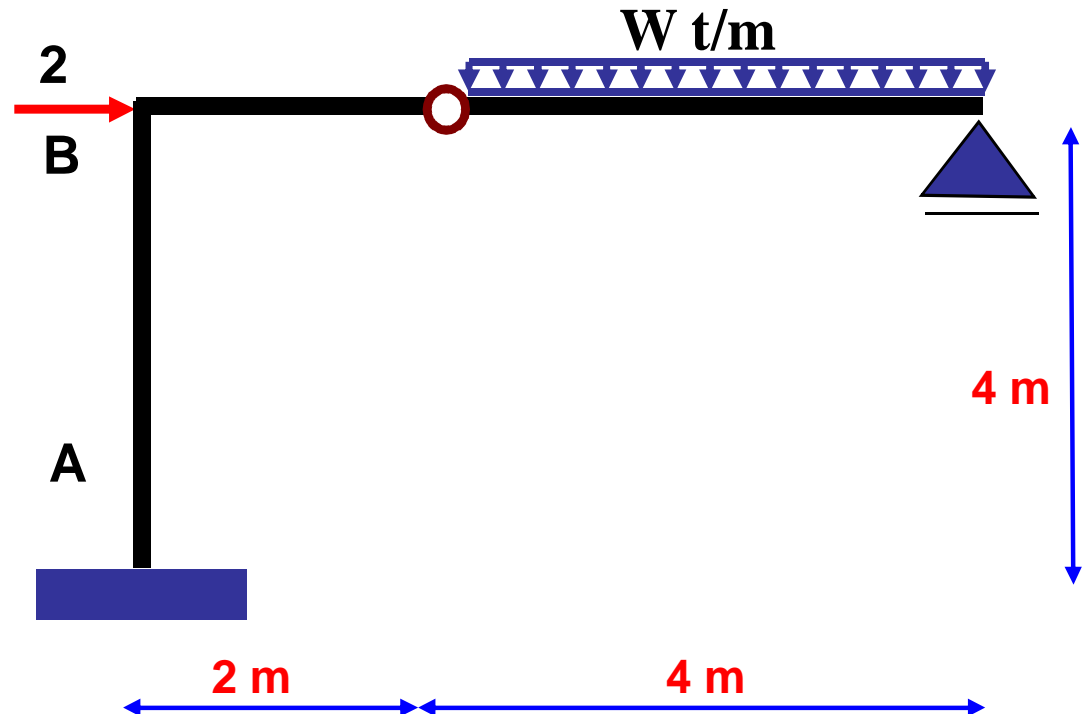
$$I_x = 5000 \text{ cm}^4$$

$$I_y = 10000 \text{ cm}^4$$

2 plates 2*40 cm

$$E = 20000$$

st 52



Required for column AB :

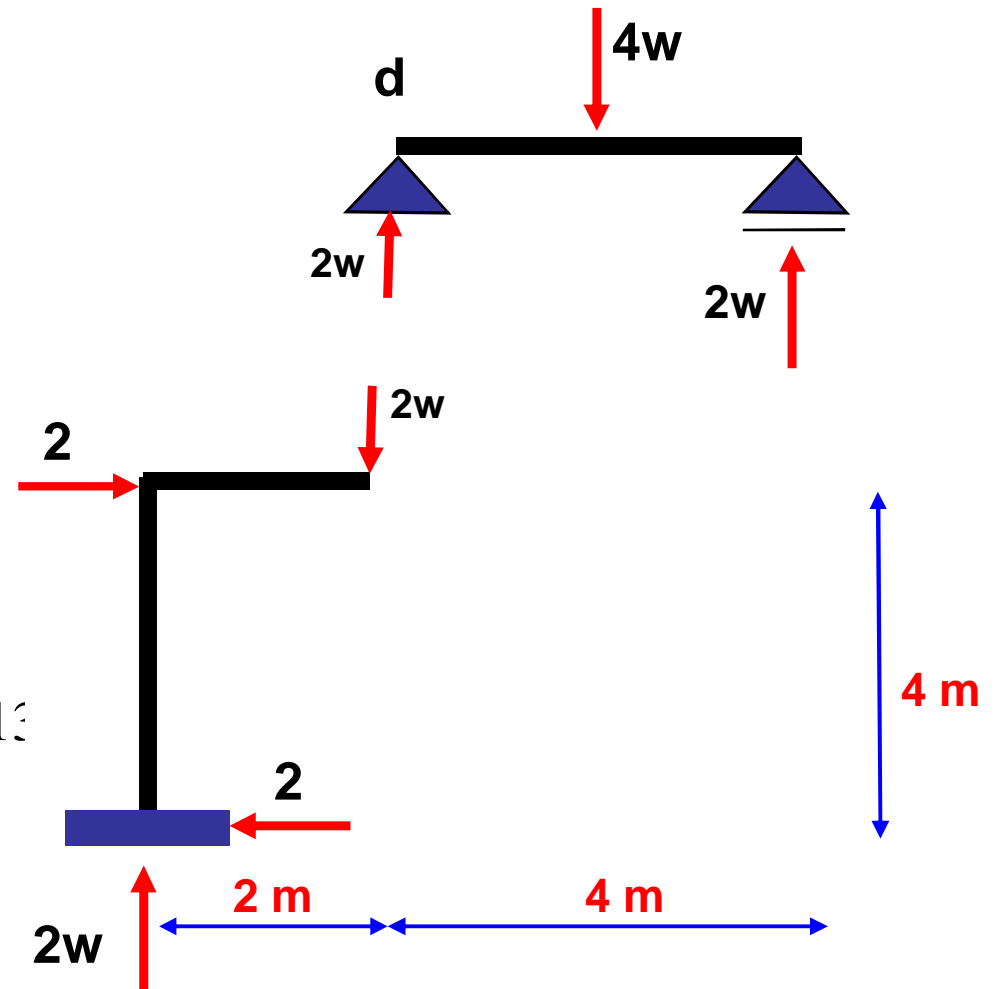
(1) W max.

(2) critical load P_{cr} .

$$A = 100 + 160 = 260 \text{ cm}^2$$

$$I_x = 5000 + 2 * \left(\frac{2 * (40)^3}{12} \right) = 27333$$

$$I_y = 10000 + 2 * \left(\frac{40 * (2)^3}{12} + 80 * (16)^2 \right) = 51013$$



الـ normal فى العمود A-B ($N = 2w t$)

محور x لاعلى -- M_x هى اللى شغالة - ($M_x = 2 * 400 + 2w * 200 = 800 + 400w$)

العمود A-B مثبت بـ fixed على الارض (fixed - free) ---- $L_b = 2 * 400 = 800 \text{ cm}$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} + \frac{M_x}{I_x} * y$$

$$(L_b = 800 \text{ cm})$$

$$i = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{27333}{260}} = 32.42 \text{ cm}$$

$$\lambda = \frac{L_b}{i_x} = \frac{800}{32.42} = 24.7 < 85$$

$$P_{cr} = \frac{\pi^2 E I}{L_b^2} = \frac{\pi^2 * 2000 * 27333}{(800)^2} = 843 \text{ ton}$$

$$\sigma_{pb} = 1.4 - 0.000065 \lambda^2 = 1.36 \text{ t/cm}^2$$

for st52---- $\sigma_{pc} = 1.4 \text{ t/cm}^2$

خواص القطاع

A=260

I_x=27333

y = 15+2=17

($M_x = 800 + 400w$)

($N = 2w \text{ t}$)

$$\sigma_{\max} = \frac{2w}{260} * \frac{1.4}{1.36} + \frac{(800 + 400w)}{27333} * 17 = 1.4$$

$$>>>> w_{\max} = 3.5 \text{ t/m}$$

Example four

given

for IBE30

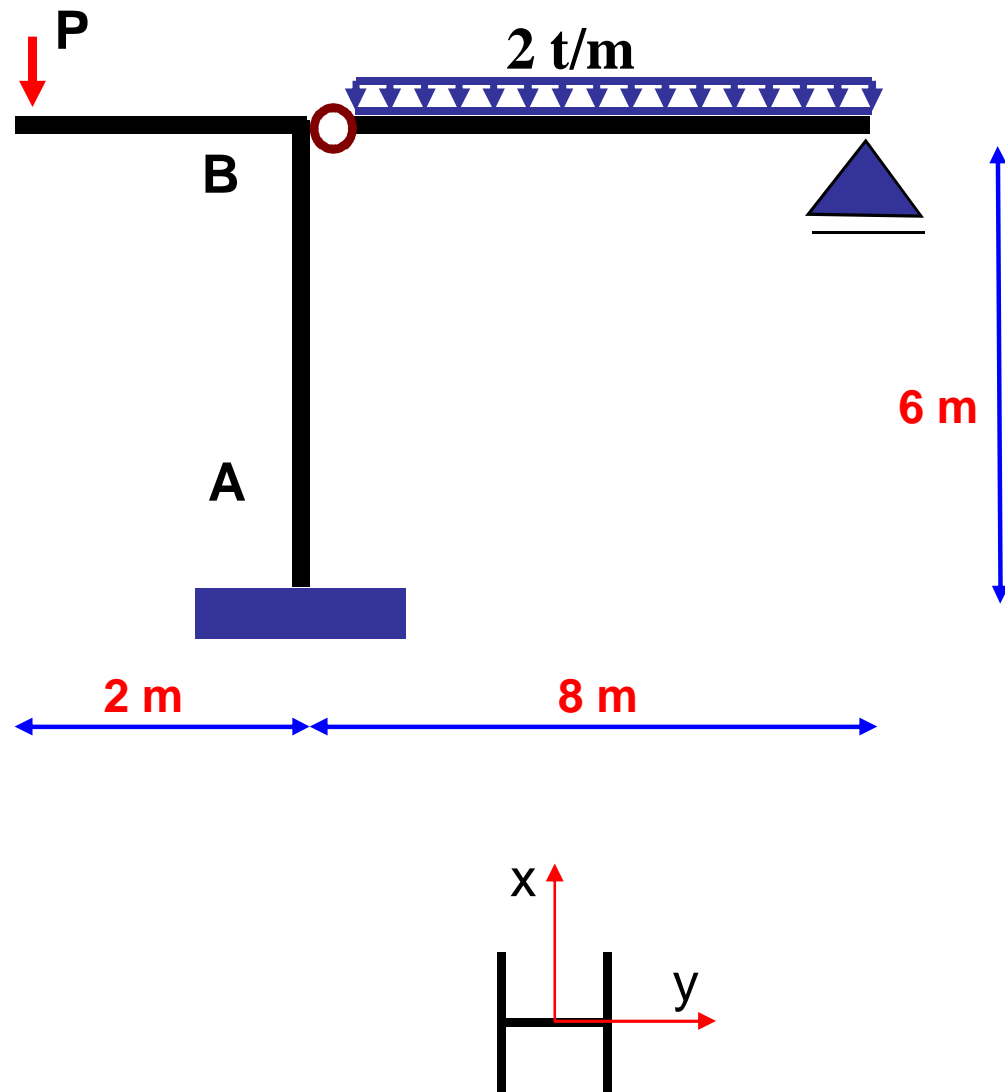
$$I_x = 80000$$

$$I_y = 120000$$

$$E = 2100$$

$$A = 300$$

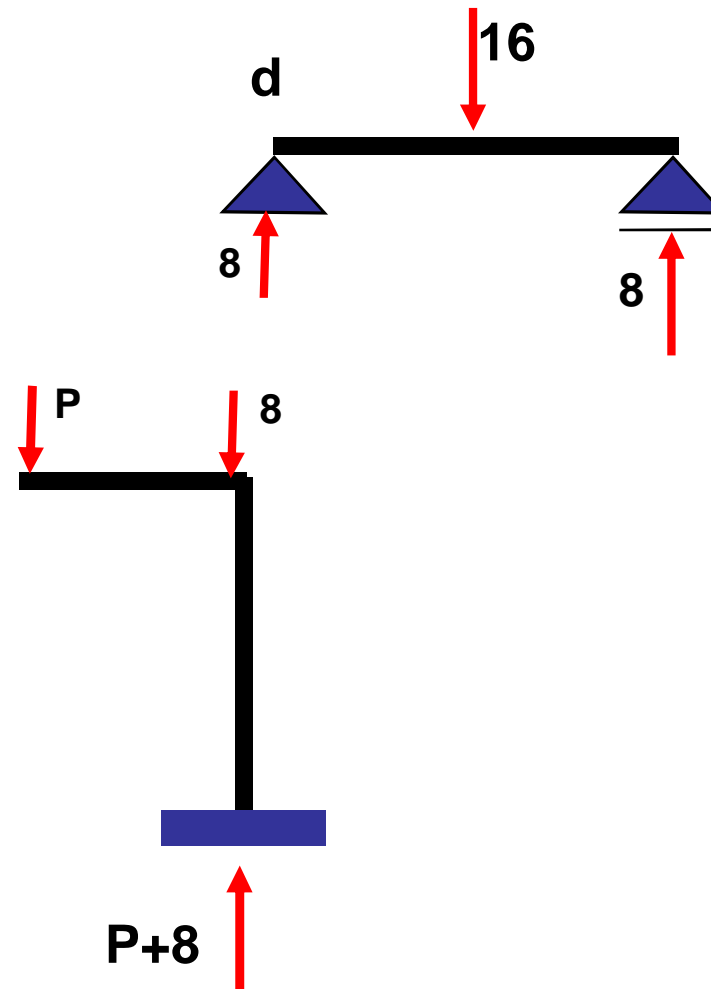
st 37



Required for column AB :

(1) P_{\max} .

(2) critical load P_{cr} .



الـ normal فى العمود A-B ($N = (P+8)$)

محور x لاعلى -- M_x هى اللى شغالة - ($M_x = 200 \cdot P = 200P$)

العمود A-B مثبت بـ fixed على الارض (fixed – free) ---- $L_b = 2 \cdot 600 = 1200 \text{ cm}$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} + \frac{Mx}{Ix} * y$$

$$(L_b = 1200 \text{ cm})$$

$$i = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{80000}{300}} = 16.33 \text{ cm}$$

$$\lambda = \frac{L_b}{i_x} = \frac{1200}{16.33} = 73.44 < 104$$

$$P_{cr} = \frac{\pi^2 E I}{L_b^2} = \frac{\pi^2 * 2100 * 80000}{(1200)^2} = 115 \text{ ton}$$

$$\sigma_{pb} = 1.1 - 0.00005 \lambda^2 = 0.83 \text{ t/cm}^2$$

for st37---- $\sigma_{pc} = 1.1 \text{ t/cm}^2$

خواص القطاع

A=300

Ix=80000

y = 15

(Mx = 200P)

(N = 8+P)

$$\sigma_{\max} = \frac{(8+P)}{300} * \frac{1.1}{0.83} + \frac{200P}{120000} * 15 = 1.1$$

$$>>>> P_{\max} = 52 \text{ t/m}$$

Example five

Final 2009

given

for IBE32

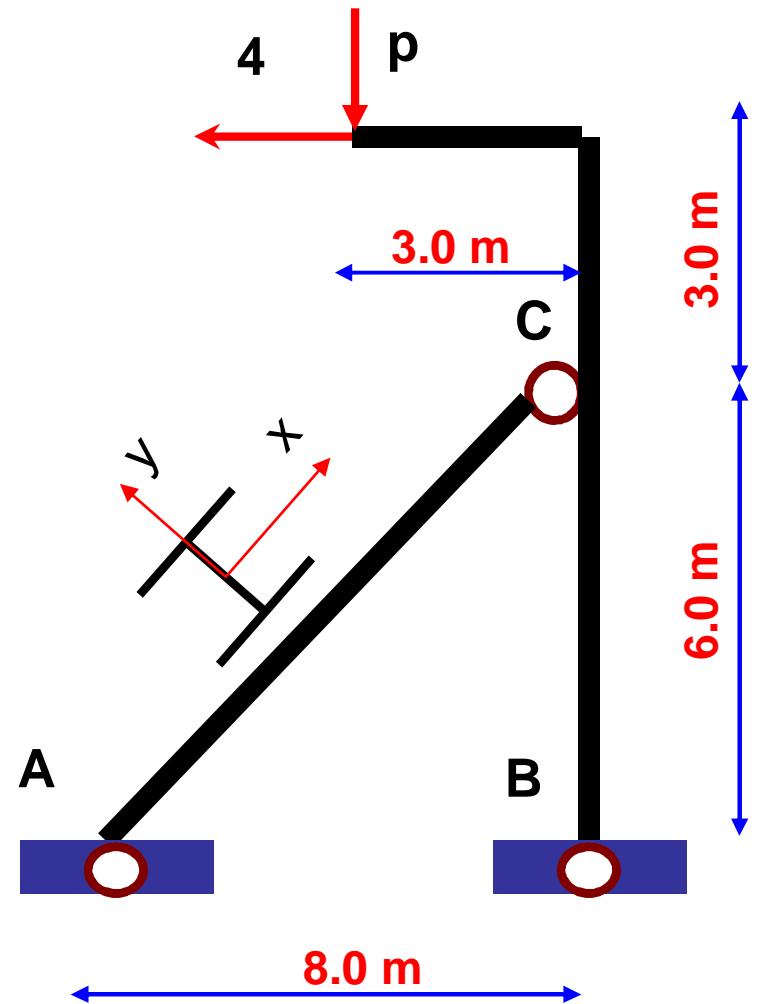
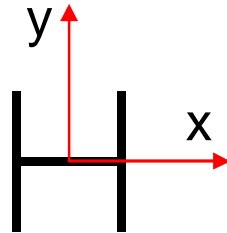
$$I_x = 4670$$

$$I_y = 13900$$

$$E = 2000$$

$$A = 254$$

$$st \ 44$$



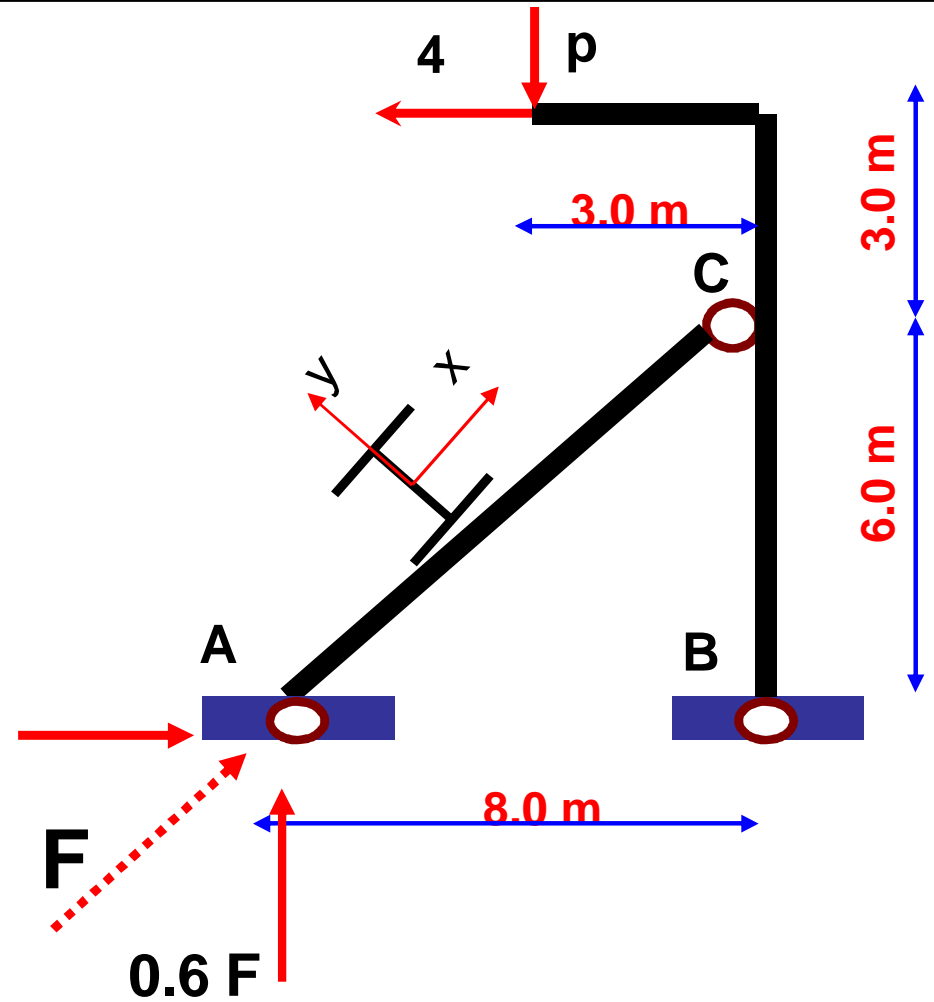
Required in column AB:

- (1) max safe load P.
- (2) critical load P_{cr} and factor of safety.
- (3) derive the formula of Buckling length

$$\Sigma M_B = 0.0$$

$$0.6 F * 8 = 4 * 9 + 3P$$

$$F = 0.625P + 7.5$$



$$(N = 0.625 P + 7.5)$$

$$\text{For st44 ---- } \sigma_{pc} = 1.3 \text{ t/cm}^2$$

العمود A-C مثبت بـ hinge على الارض (hinged – hinged) ---- $L_b = 1000 \text{ cm}$

خواص القطاع

$$A = 254$$

$$I_y = 13900$$

$$x = 16$$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} = \sigma_{pc}$$

$$(Lb = 1000 \text{ cm})$$

$$i = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{4670}{254}} = 4.28 \text{ cm}$$

$$\lambda = \frac{Lb}{i} = \frac{1000}{4.28} = 233 < 96$$

$$P_{cr} = \frac{\pi^2 E I}{Lb^2} = \frac{\pi^2 * 2000 * 4670}{(1000)^2} = 92 \text{ ton}$$

$$\sigma_{pb} = \frac{6000}{233^2} = 0.11 \text{ t / cm}^2$$

$$\sigma_{\max} = \frac{7.5 + 0.625P}{254} * \frac{1.3}{0.11} = 1.3$$

$$>>>> P_{\max} = 32 \text{ ton}$$

$$FoS = \frac{92}{32} = 2.81$$

Example six

Final 2008

given

for IBE32

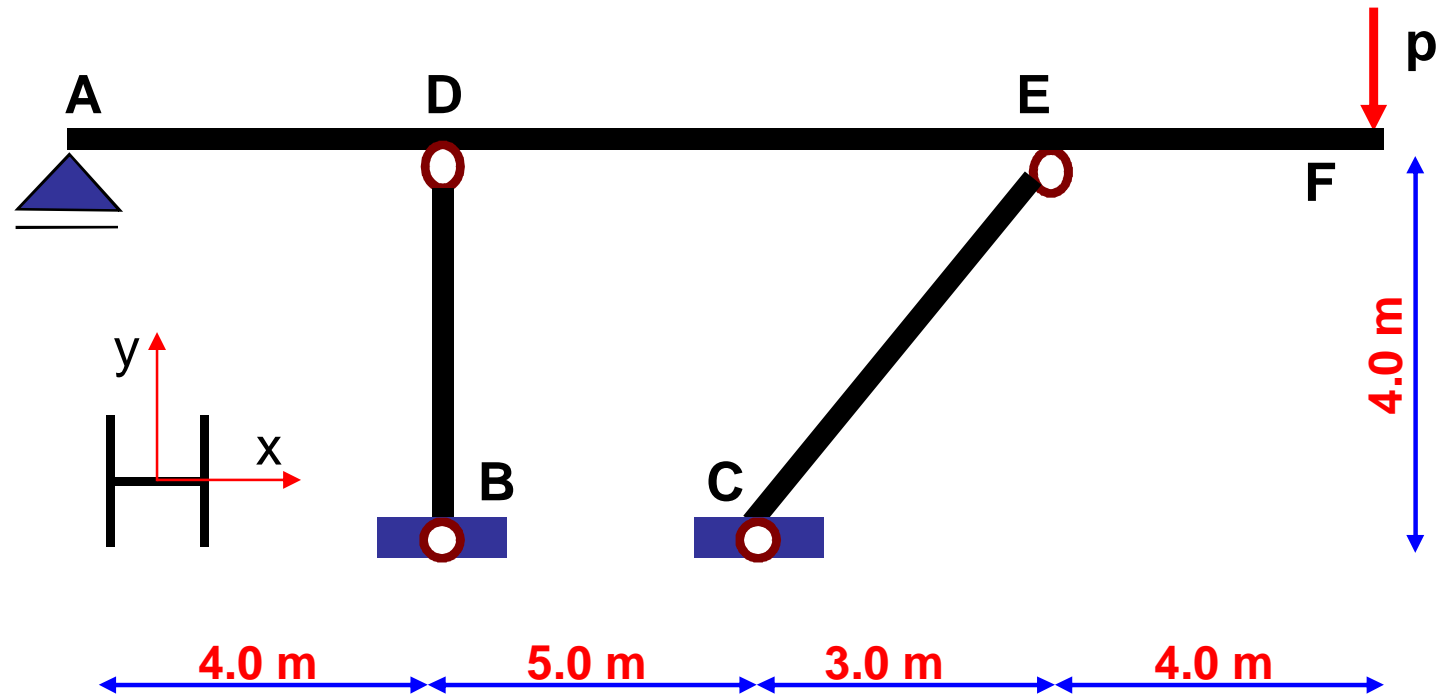
$$I_x = 4670$$

$$I_y = 13900$$

$$E = 2000$$

$$A = 254$$

$$st \ 44$$

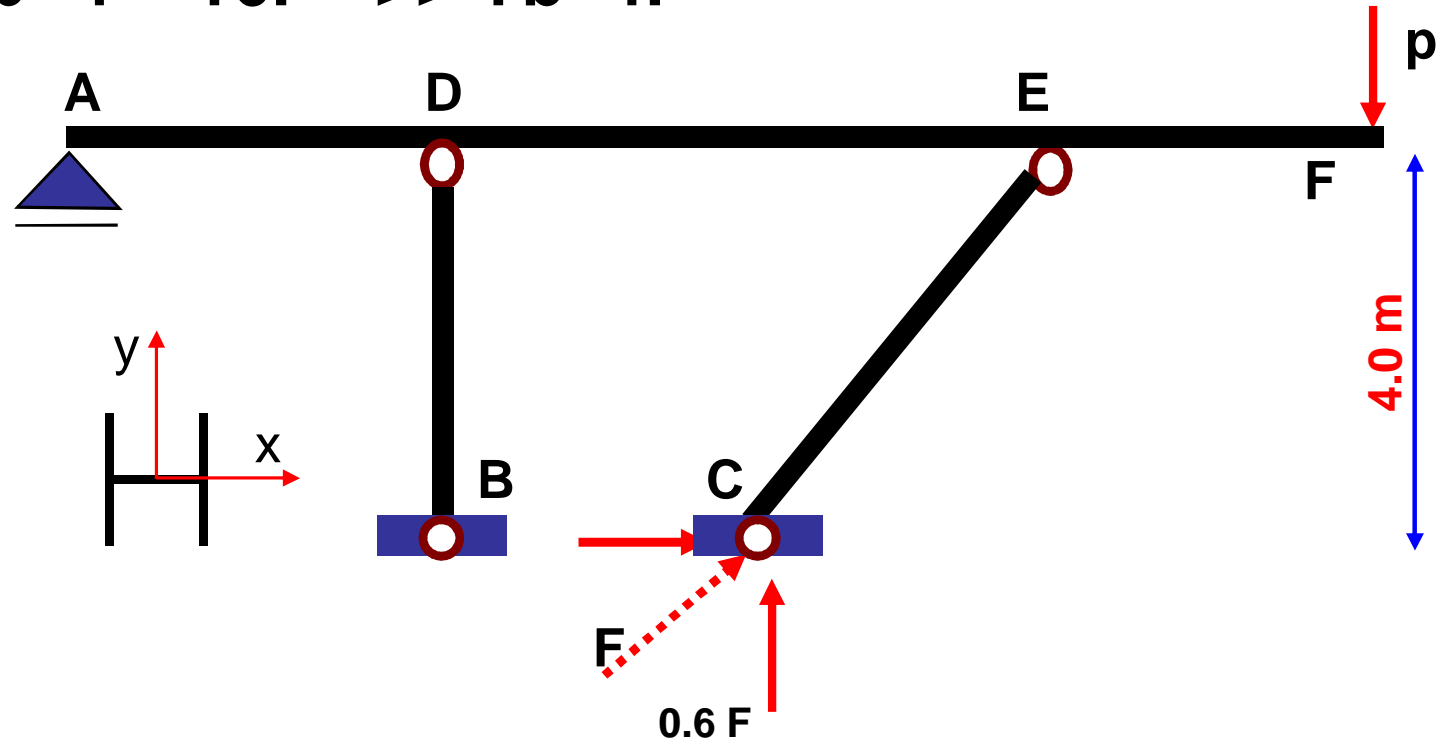


Required in column BD:

- (1) max safe load P.
- (2) critical load P_{cr} and factor of safety.
- (3) derive the formula of Buckling length

Force in member CE is zero

$\Sigma M_A = 0.0$ $Y_b * 4 = 16P \Rightarrow Y_b = 4P$



العمود B-D مثبت به hinge على الارض (hinged – hinged) ---- $L_b = 400\text{cm}$

خواص القطاع
 $A=254, \quad I_y=13900, \quad x = 16$

$(N = 4P)$

For st44 ---- $\sigma_{pc} = 1.3 \text{ t/cm}^2$

$$\sigma_{\max} = \frac{P}{A} * \frac{\sigma_{pc}}{\sigma_{pb}} = \sigma_{pc}$$

$$(Lb = 400 \text{ cm})$$

$$i = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{4670}{254}} = 4.28 \text{ cm}$$

$$\lambda = \frac{Lb}{i} = \frac{400}{4.28} = 93.4 < 96$$

$$P_{cr} = \frac{\pi^2 E I}{Lb^2} = \frac{\pi^2 * 2000 * 4670}{(400)^2} = 576 \text{ ton}$$

$$\sigma_{pb} = 1.3 - 0.00007 * 93^2 = 0.6945 \text{ t / cm}^2$$

$$\sigma_{\max} = \frac{4P}{254} * \frac{1.3}{0.6945} = 1.3$$

$$>>>> P_{\max} = 44 \text{ ton}$$

$$FoS = \frac{576}{44} = 13$$

Example seven

$$E = 2000$$

st 52

for BF1B No 32

$$A = 171 \text{ cm}^2$$

$$I_x = 32250 \text{ cm}^4$$

$$I_y = 9910 \text{ cm}^4$$

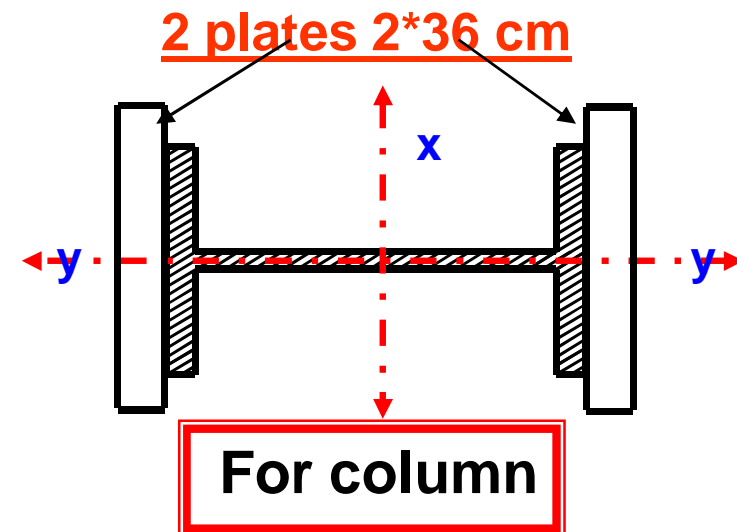
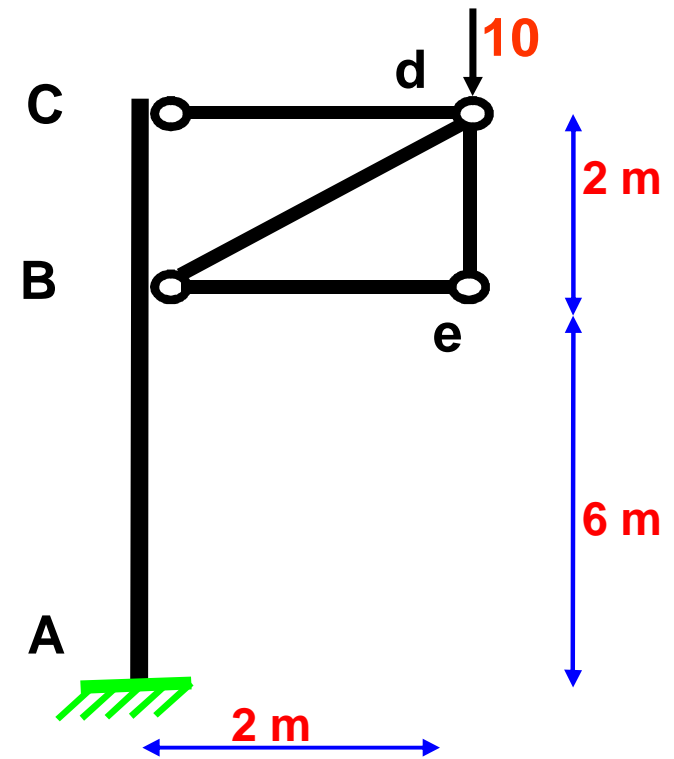
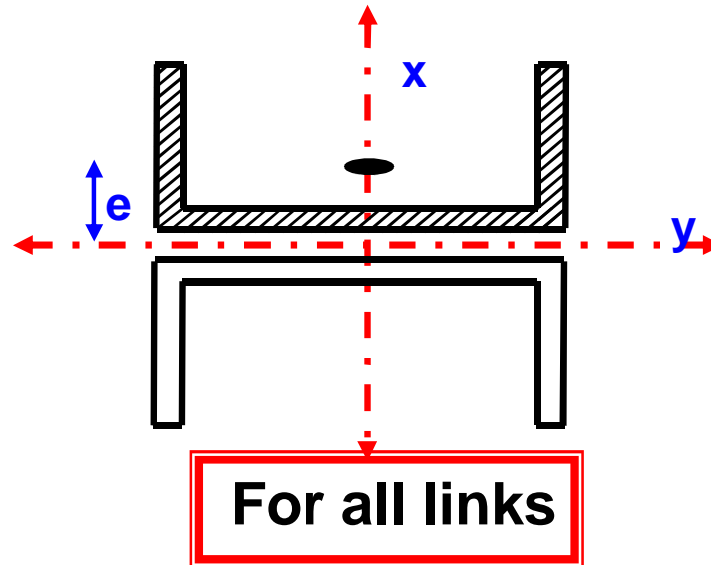
for CH No 10

$$A = 13.5 \text{ cm}^2$$

$$I_x = 206 \text{ cm}^4$$

$$I_y = 29.3 \text{ cm}^4$$

$$e = 1.55 \text{ cm}$$



Required check all members

Column section

$$A = 171 + 2 \cdot (36 \cdot 2) = 315$$

$$I_x = 32250 + 2 \cdot (36 \cdot 2^3 / 12 + 36 \cdot 2 \cdot 17^2) = 73914 \text{ cm}^4$$

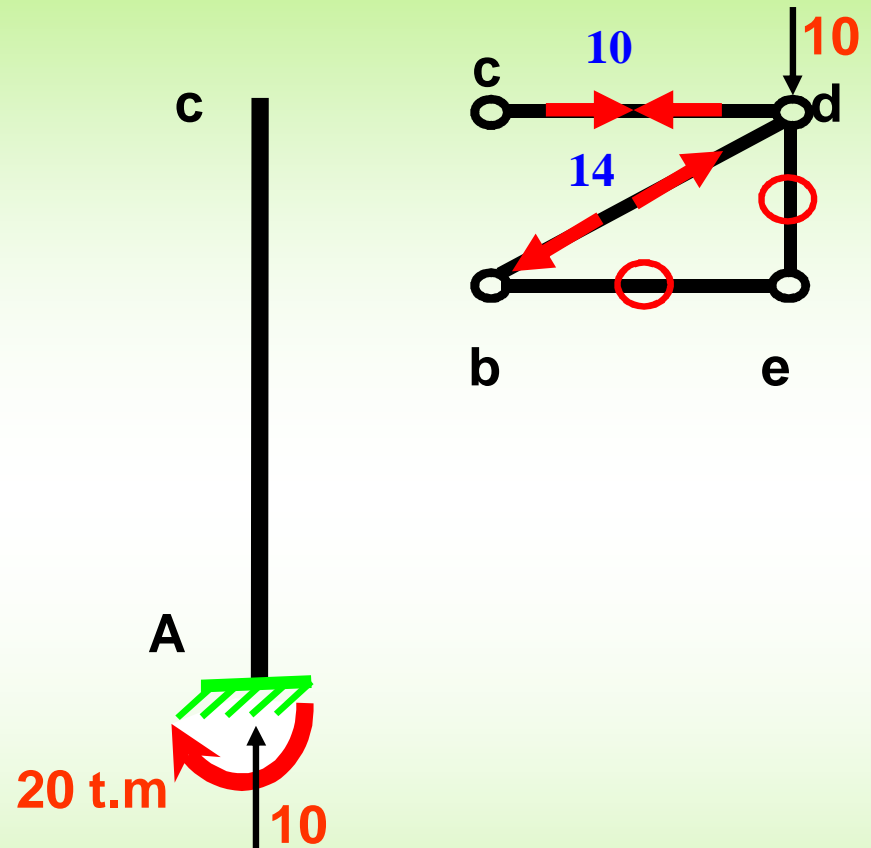
$$I_y = 9910 + 2 \cdot (2 \cdot 36^3 / 12) = 25462 \text{ cm}^4$$

Link section

$$A = 2 \cdot 13.5 = 27$$

$$I_x = 2 \cdot 206 = 412 \text{ cm}^4$$

$$I_y = 2 \cdot (29.3 + 13.5 \cdot 1.55^2) = 123.5 \text{ cm}^4$$



Member (be and ed) --- zero member

Member (cd) --- tension member

Member bd

$$Lb = 283cm$$

$$i \min = \sqrt{\frac{123.5}{27}} = 2.138$$

$$\lambda = \frac{283}{2.138} = 132 > 104$$

$$\sigma_{pb} = \frac{6000}{132^2} = 0.343$$

$$\sigma \max = \frac{14.14}{27} * \frac{1.1}{0.343} = 1.67 > 1.1 \quad \text{unsafe}$$

column ac

$$Lb = 1600 \text{ cm}$$

$$i \text{ min} = \sqrt{\frac{25462}{315}} = 9$$

$$\lambda = \frac{1600}{9} = 178 > 104$$

$$\sigma_{pb} = \frac{6000}{178^2} = 0.1916$$

$$\sigma \text{ max} = \frac{10}{315} * \frac{1.1}{0.1916} + \frac{2000}{73914} * 18 = 0.669 < 1.1 \text{ safe}$$

Effect of Temperature

$$\Delta L = \frac{P.L}{E.A} \dots\dots\dots 1$$

$$\Delta L = \alpha.\Delta t.L \dots\dots\dots 2$$

$$\frac{P.L}{E.A} = \alpha.\Delta t.L$$

$$>> P = \alpha.\Delta t.E.A$$

$$P \leq P_{cr}$$

$$\Delta t \leq \frac{\pi^2 . I_{min}}{\alpha . A . L_b^2}$$

Example one

given

for IBE32

$$I_x = 12510$$

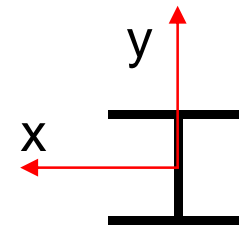
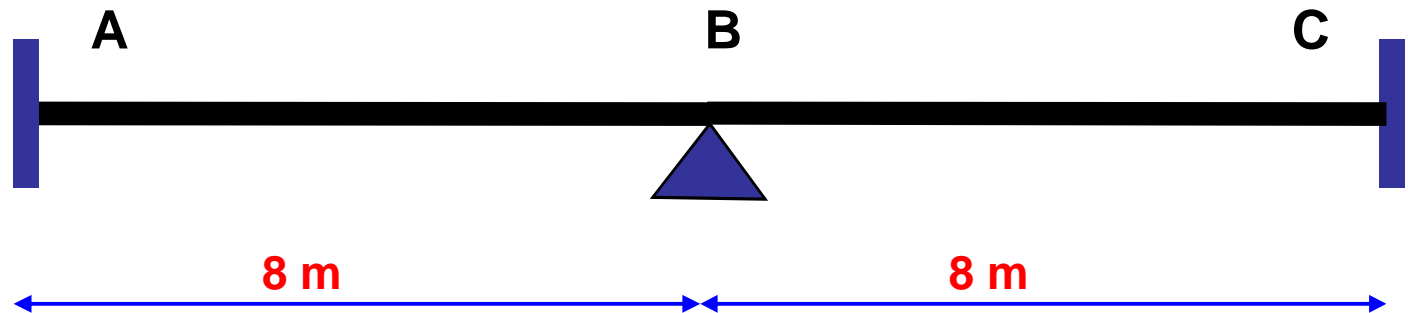
$$I_y = 782$$

$$E = 2100$$

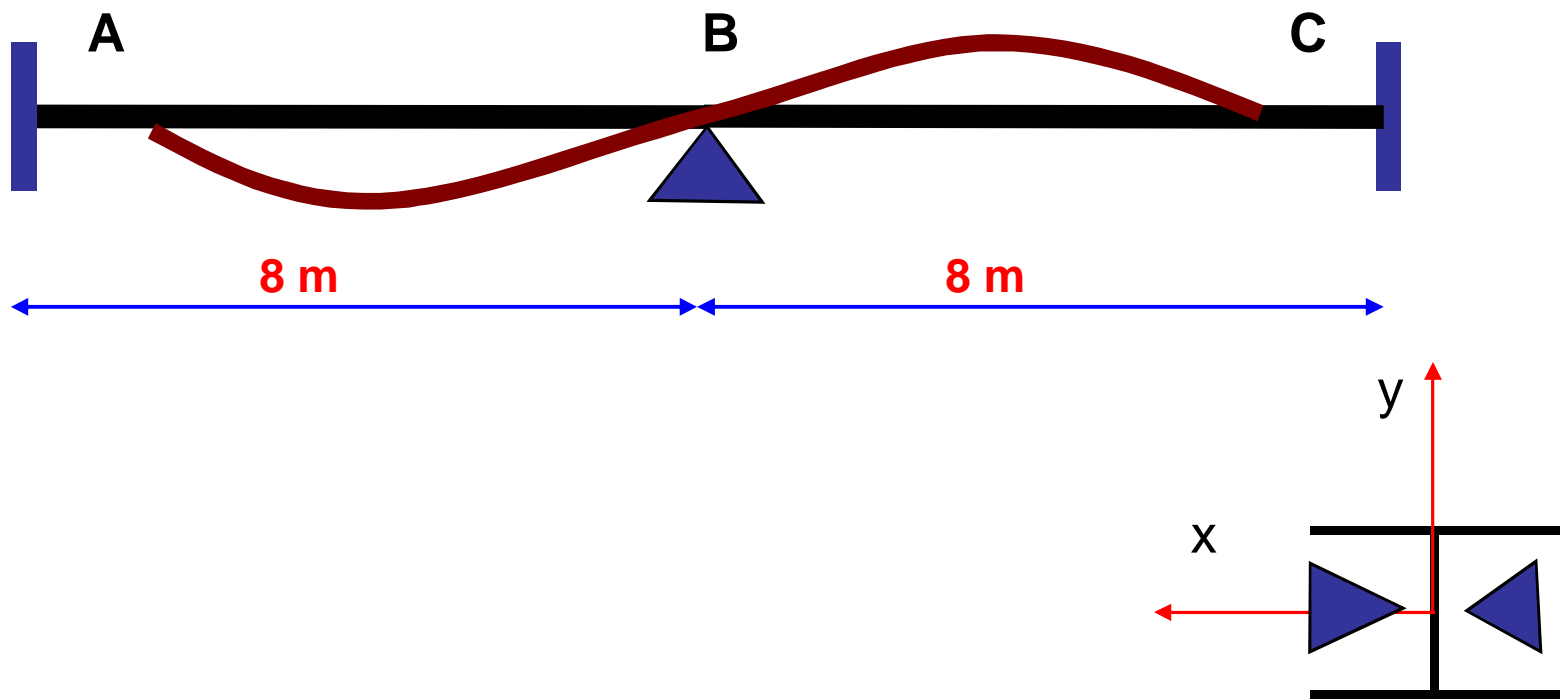
$$A = 77.8$$

st 52

$$\alpha = 1.2 \cdot 10^{-5}$$



Required Δt_{\max}



$$Lb = 0.7 * 800 = 560cm$$

$$\Delta t \leq \frac{\pi^2 * 782}{1.2 * 10^{-5} * 77.8 * 560^2} = 26.36$$

Example two

given

for IBE32

$$I_x = 12510$$

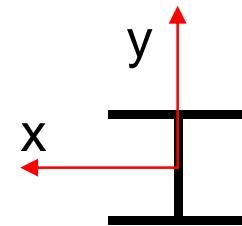
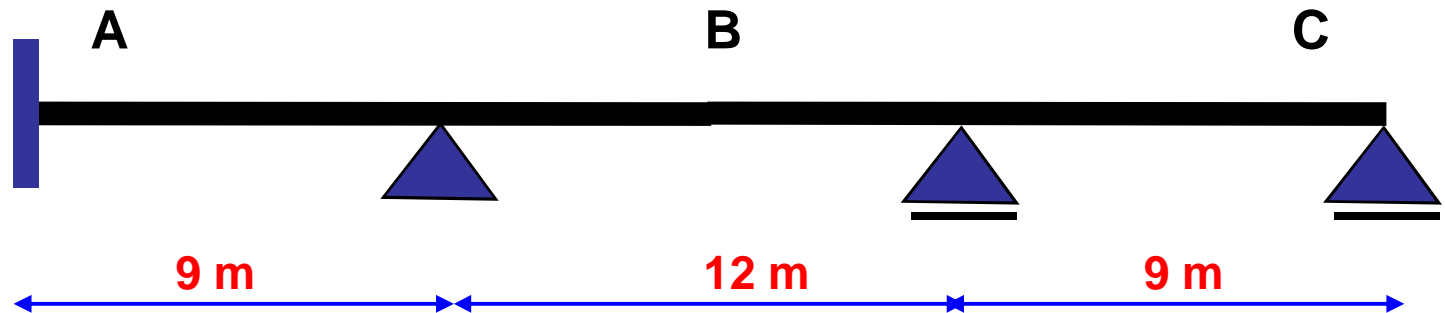
$$I_y = 782$$

$$E = 2100$$

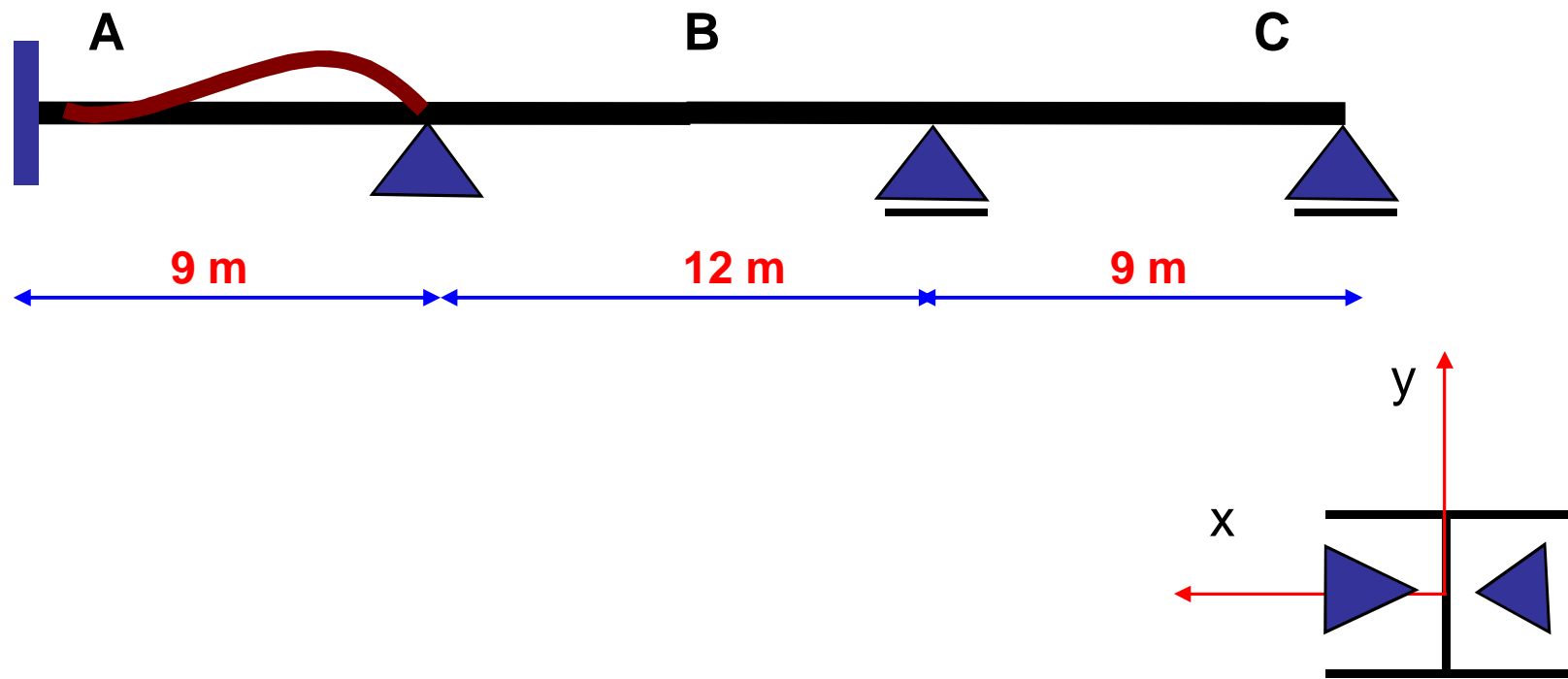
$$A = 77.8$$

st 52

$$\alpha = 1.2 \cdot 10^{-5}$$



Required Δt_{\max}



$$Lb = 0.7 * 900 = 630cm$$

$$\Delta t \leq \frac{\pi^2 * 782}{1.2 * 10^{-5} * 77.8 * 630^2} = 14.8$$

الاحتياجات

$$M = P.y$$

$$y'' = -\frac{Py}{EI} \quad C^2 = \frac{P}{EI}$$

$$y'' + C^2.y = 0.0$$

$$y = c1 . \cos cx + c2 . \sin cx$$

$$\text{at } x = 0 \rightarrow y = 0 \rightarrow c1 = 0.0$$

$$\text{at } x = L \rightarrow y = 0 \rightarrow c2.\sin cL = 0.0$$

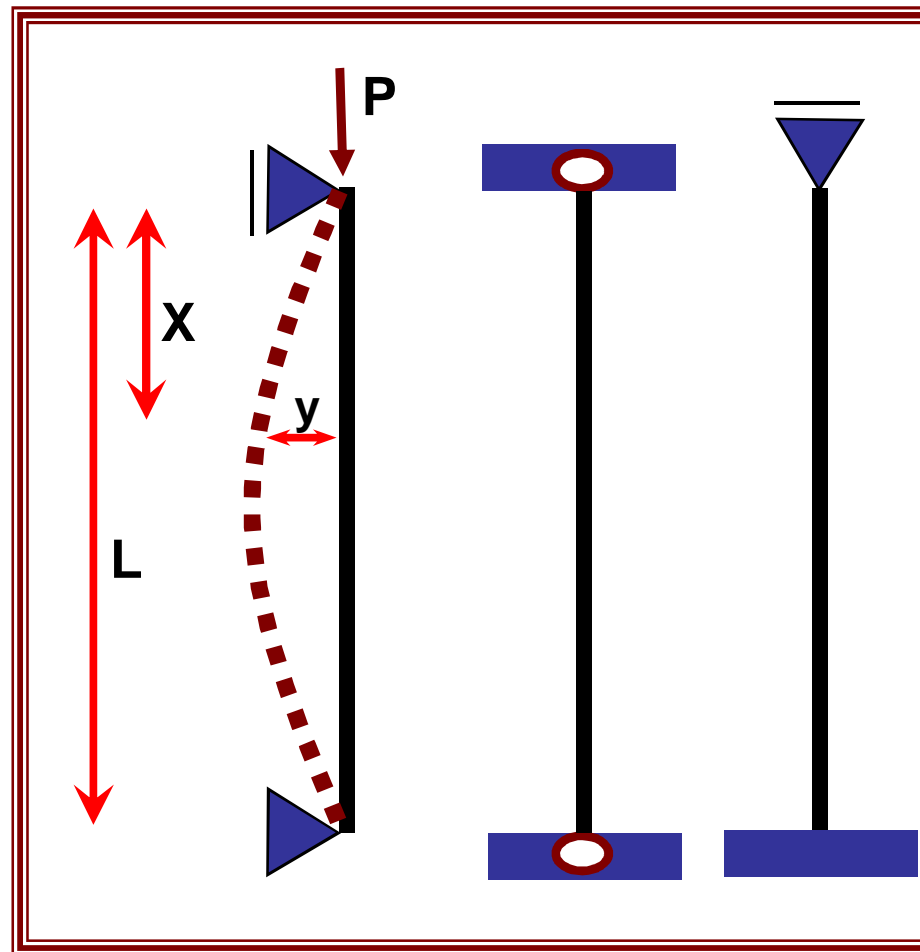
$$c2.\sin cL = 0.0 \gg \sin cL = 0.0$$

$$\gg cL = n * \pi \gg c^2 L^2 = n^2 * \pi^2$$

$$\frac{PL^2}{EI} = n^2 * \pi^2 \gg P_{cr} = n^2 \frac{\pi^2 EI}{L^2}$$

$$P_{cr} = n^2 \frac{\pi^2 EI}{L^2} = n^2 \frac{\pi^2 EI}{Lb^2}$$

$$Lb = L$$



$$M = -P.(\Delta - y)$$

$$y'' = -\frac{-P.(\Delta - y)}{EI} = \frac{P}{EI}\Delta - \frac{P}{EI}y$$

$$y'' + C^2.y - C^2.\Delta = 0.0$$

$$y = c1 . \cos cx + c2 . \sin cx + \Delta$$

$$\text{at } x=0 \rightarrow y=0 \rightarrow c1 = -\Delta$$

$$\text{at } x=0 \rightarrow y'=0 \rightarrow c2 = 0.0$$

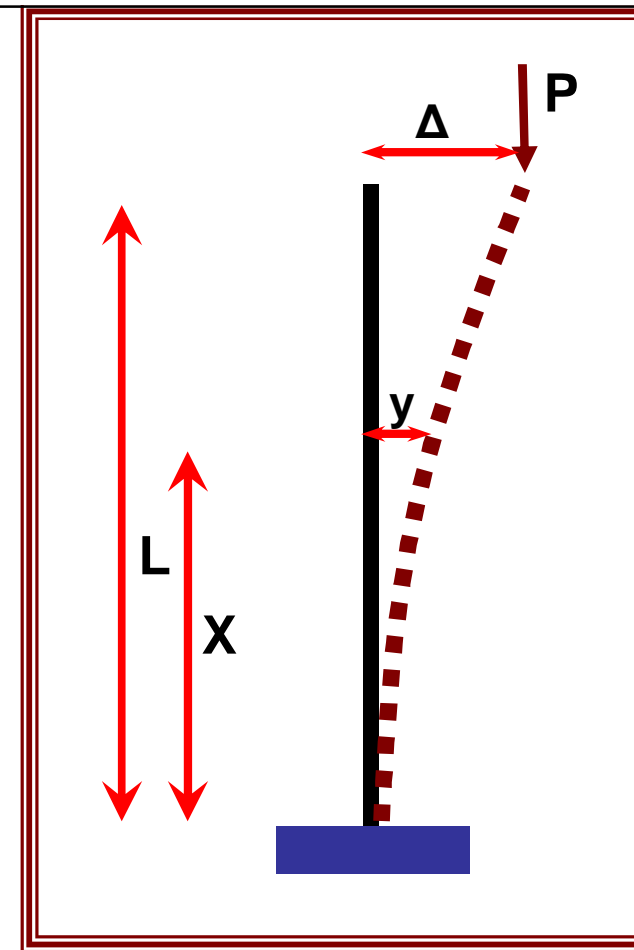
$$\text{at } x=L \rightarrow y=\Delta$$

$$\rightarrow y = -\Delta \cos cL + \Delta = \Delta \rightarrow \cos cL = 0.0$$

$$\rightarrow cL = n * \pi / 2 \gg c^2 L^2 = \frac{n^2 * \pi^2}{4}$$

$$\frac{PL^2}{EI} = n^2 * \pi^2 \gg P_{cr} = n^2 \frac{\pi^2 EI}{(2L)^2} = n^2 \frac{\pi^2 EI}{(Lb)^2}$$

$$C^2 = \frac{P}{EI}$$



$$Lb=2L$$

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Determinacy and without calculation

Eng : Aymman abdo

Determinacy

For Beams and Frames

b >>>> No. members
r >>>> No. of Reaction
J >>>> No. of joints
c >>>> each of intermediate hinge (intermediate hinge - 1) عدد العناصر التي يفصلها

$3b + r$ >>>> مجاهيل

$3j + c$ >>>> معادلات

معادلات < مجاهيل

Unstable

معادلات = مجاهيل

Stable & determinate

معادلات > مجاهيل

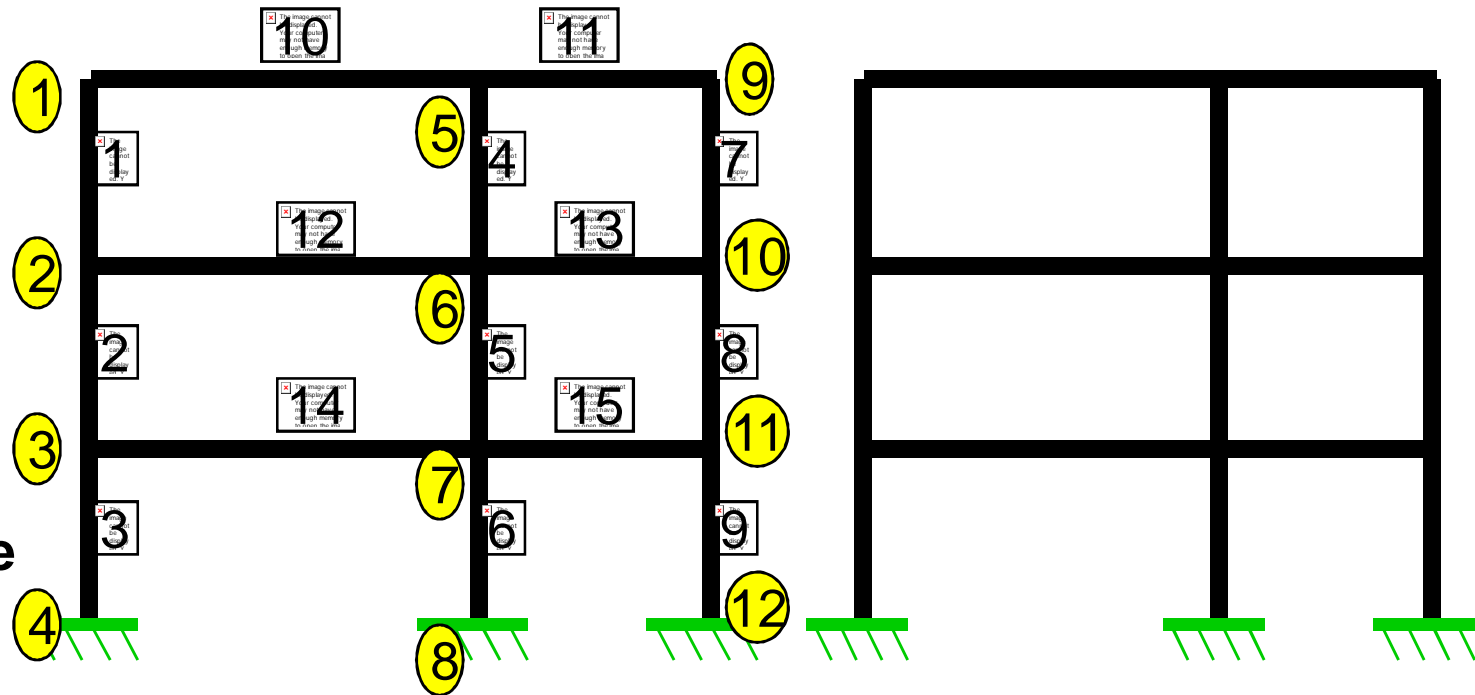
Stable & indeterminate

$b = 15$, $J = 12$
 $c = 0$, $r = 9$

$3b + r = 54$ unknowns

$3j + c = 36$ equations

Unknowns > equations
 > stable & Indeterminate
 > $(54 - 36 = 18$ degree)

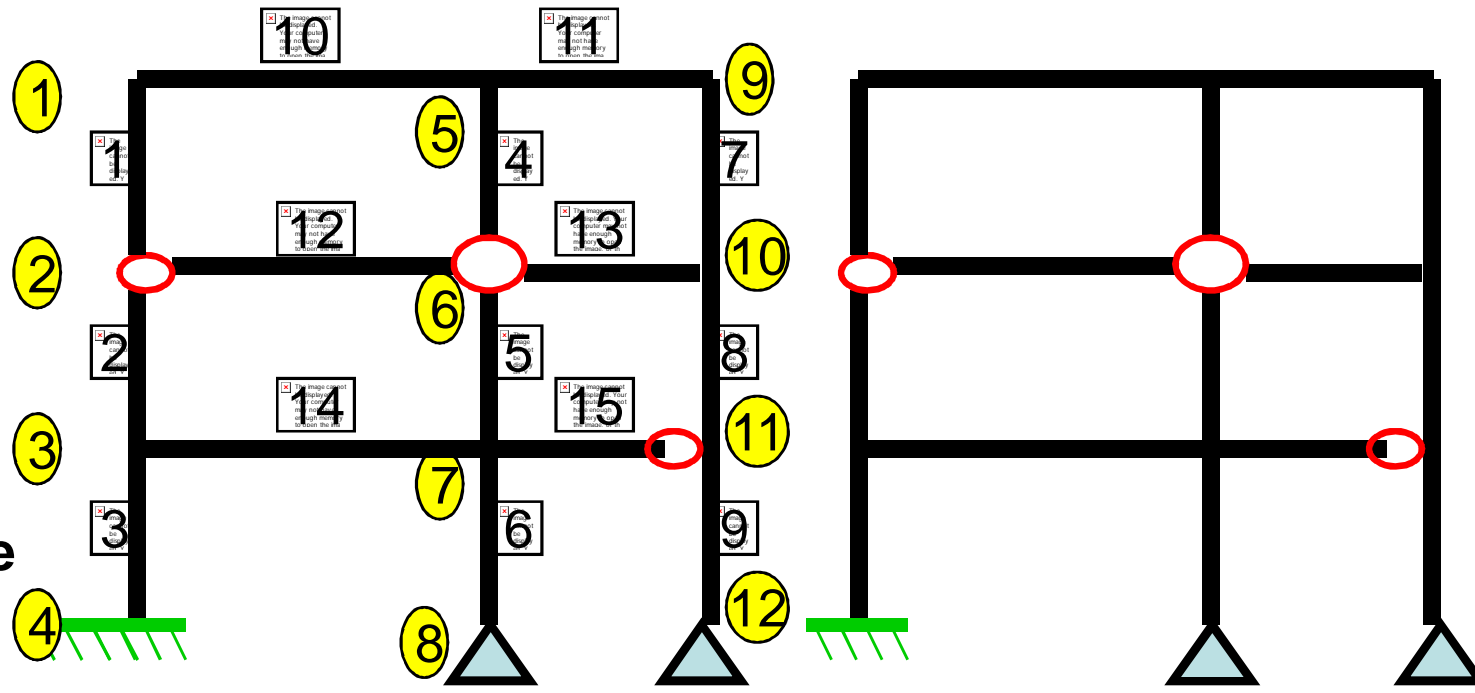


$b = 15$, $J = 12$
 $c = 6$, $r = 5$

$3b + r = 50$ unknowns

$3j + c = 42$ equations

Unknowns > equations
 > stable & Indeterminate
 > $(50 - 42 = 8$ degree)



first

$$b = 16, J = 13$$

$$c = 6, r = 5$$

$$3b + r = 53 \text{ unknowns}$$

$$3j + c = 45 \text{ equations}$$

> stable & Indeterminate

$$> (53 - 45 = 8 \text{ degree})$$

second

$$b = 15, J = 12$$

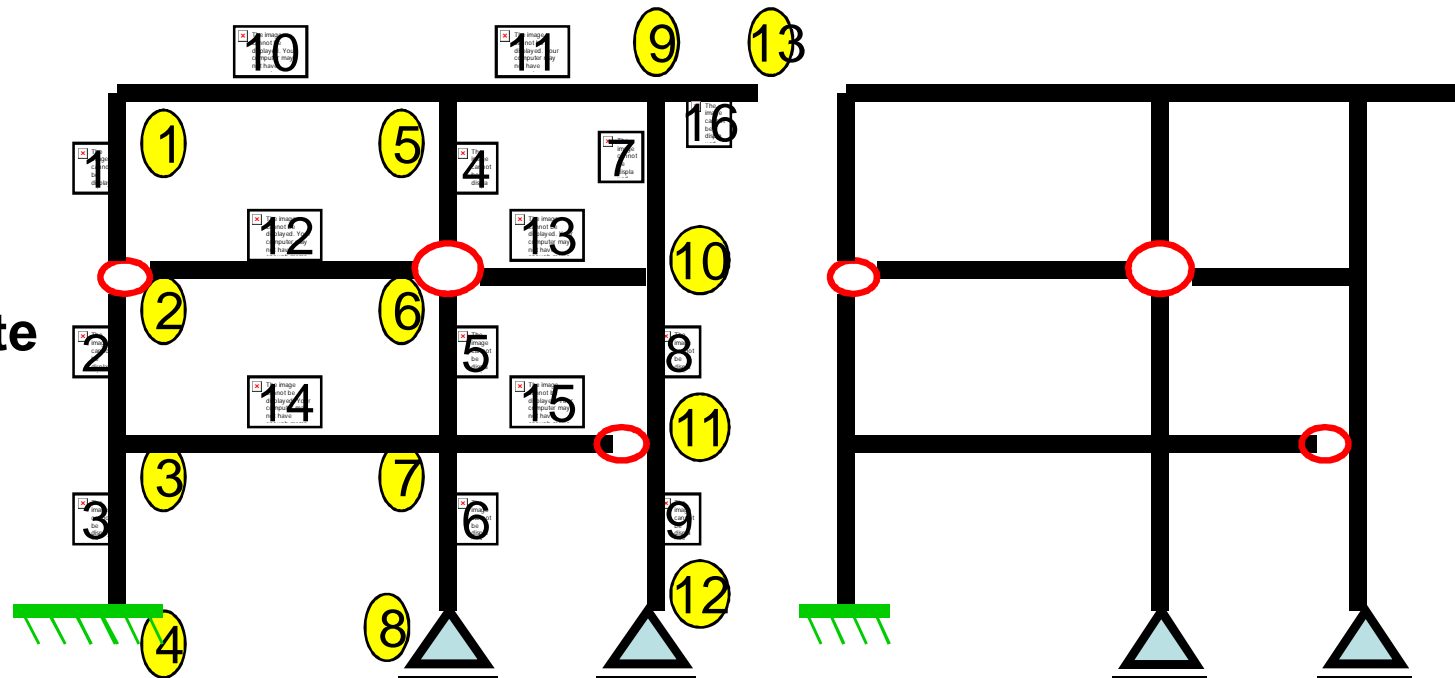
$$c = 6, r = 5$$

$$3b + r = 50 \text{ unknowns}$$

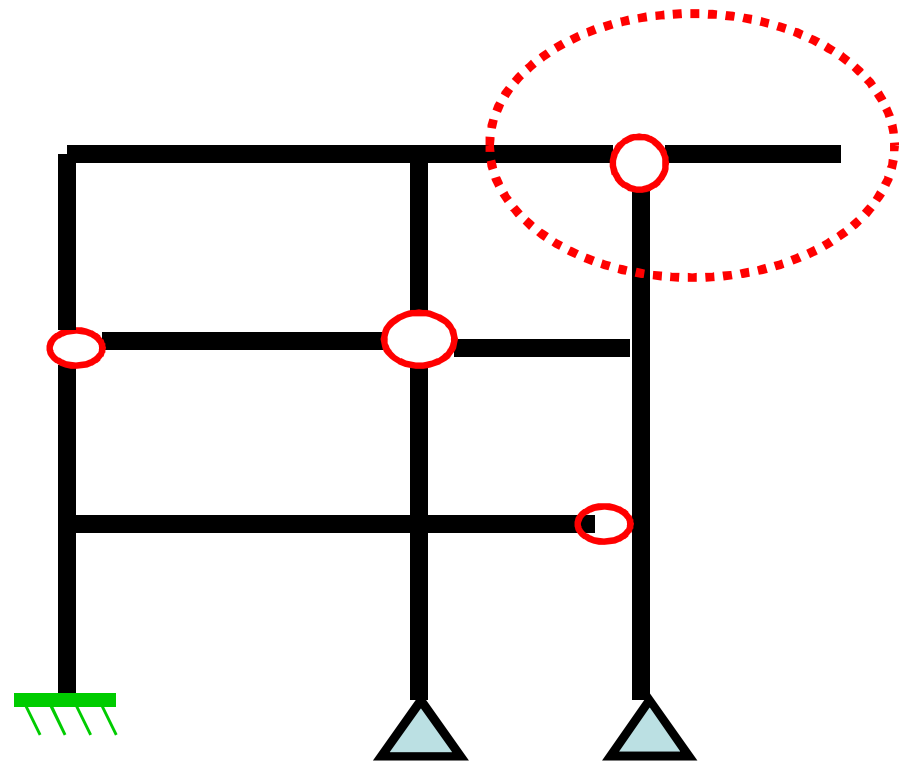
$$3j + c = 42 \text{ equations}$$

> stable & Indeterminate

$$> (50 - 42 = 8 \text{ degree})$$



Unstable



Determinacy

For Trusses

b >>>> No. members
r >>>> No. of Reaction
J >>>> No. of joints

$b + r$ >>>> مجاهيل

$2j$ >>>> معادلات

معادلات < مجاهيل

Unstable

معادلات = مجاهيل

Stable & determinate

معادلات > مجاهيل

Stable & indeterminate

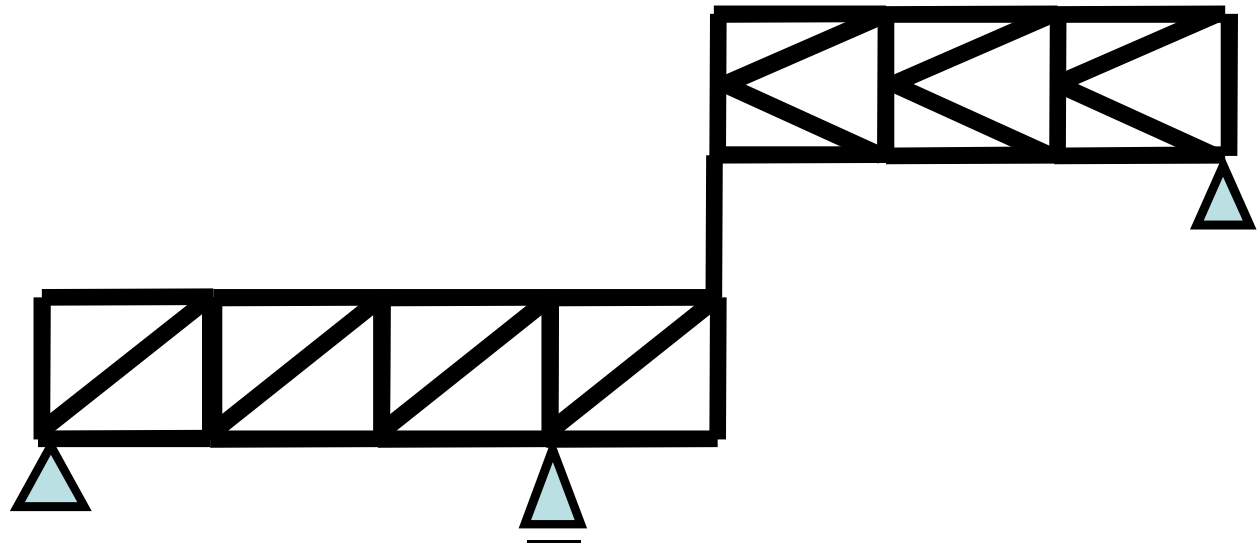
$$b = 37, J = 21$$

$$, r = 5$$

$$b + r = 42 \text{ unknowns}$$

$$2j = 42 \text{ equations}$$

Unknowns = equations
> stable & determinate



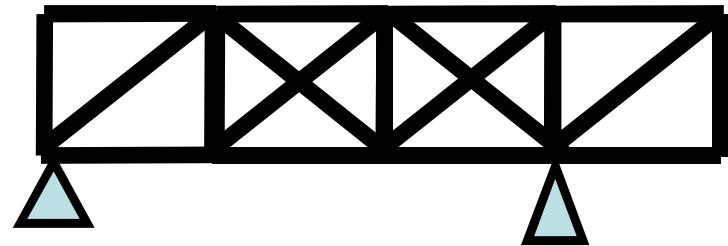
$$b = 19, J = 10$$

$$, r = 4$$

$$b + r = 23 \text{ unknowns}$$

$$2j = 20 \text{ equations}$$

> stable & Indeterminate
> (23 - 20 = 3 degree)

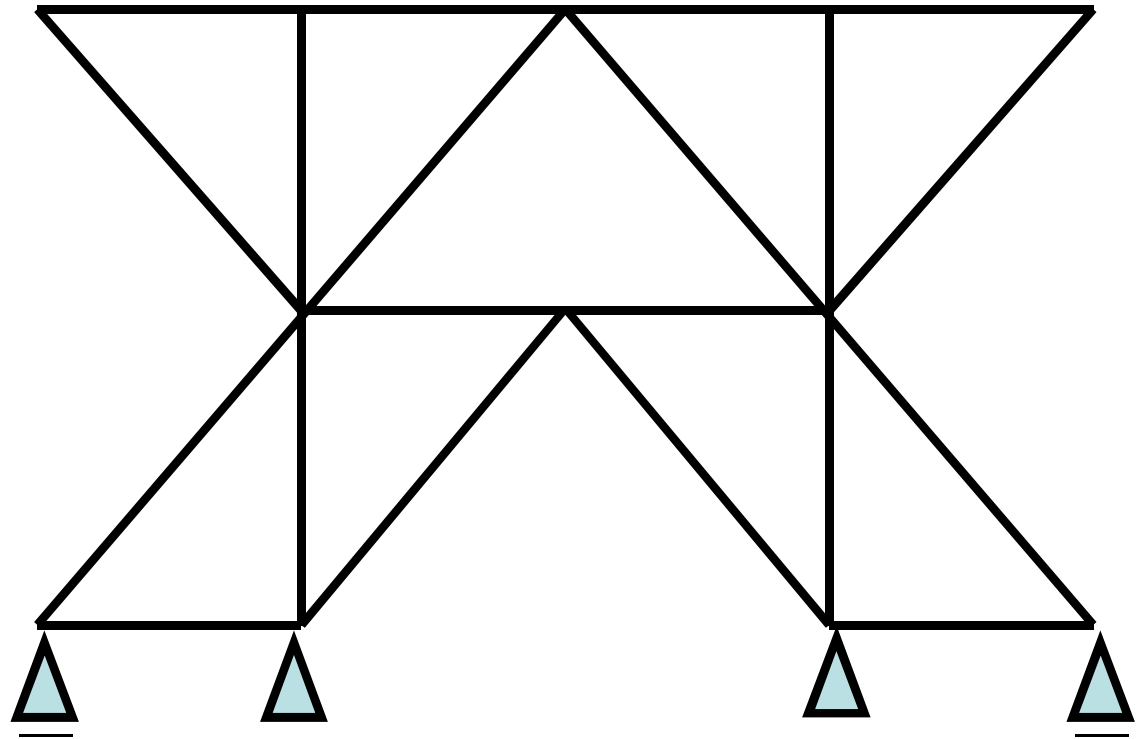


$b = 20$, $J = 12$
 , $r = 6$

$b + r = 26$ unknowns

$2j = 24$ equations

> stable & Indeterminate
> $(26 - 24 = 2 \text{ degree})$

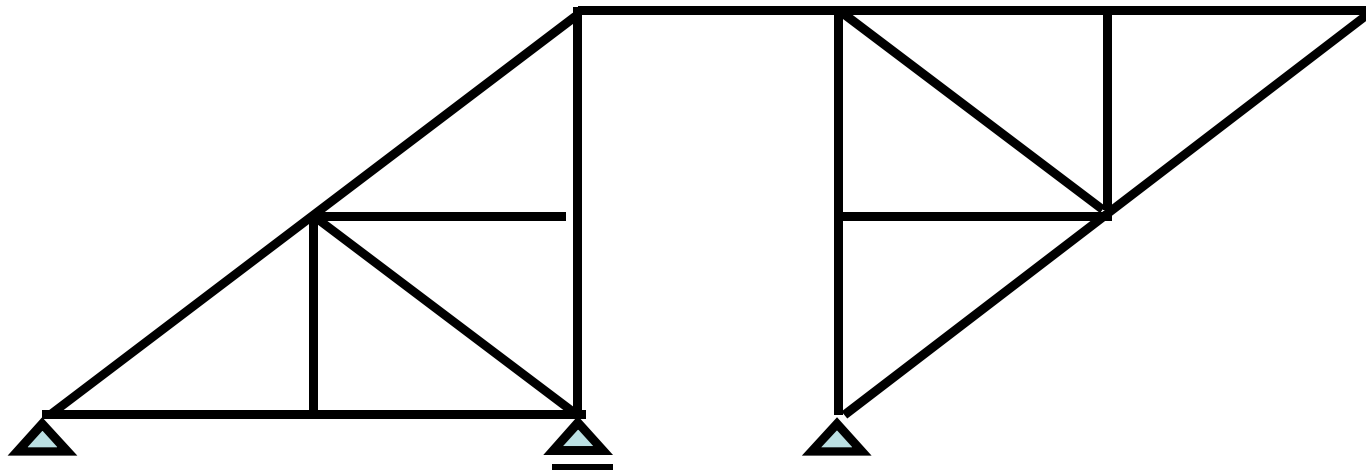


$b = 19$, $J = 12$
 , $r = 5$

$b + r = 24$ unknowns

$2j = 24$ equations

> stable & determinate



بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Without Calculation

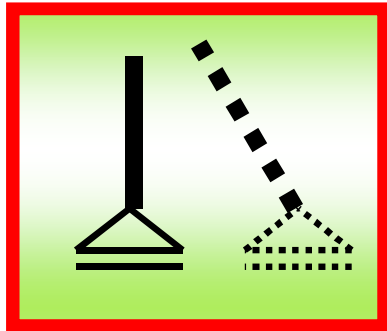
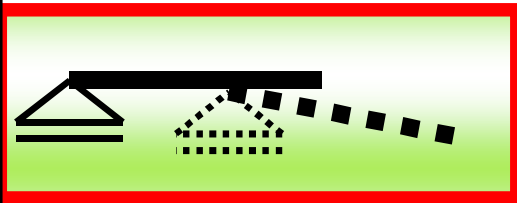
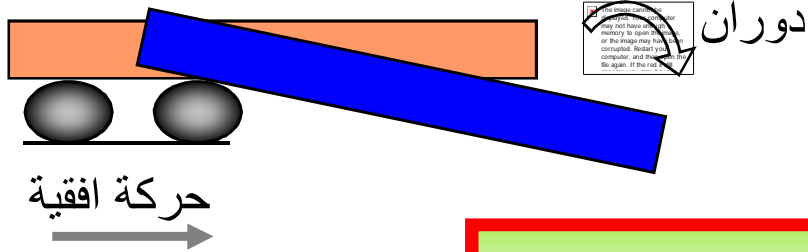
*determinate and Indeterminate
structure*

Eng : Aymman abdo

1- SUPPORTS

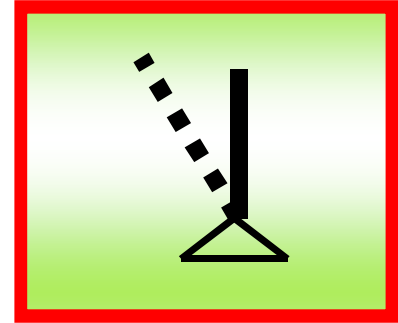
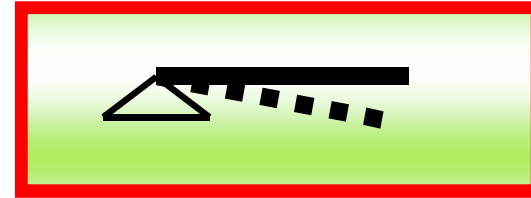
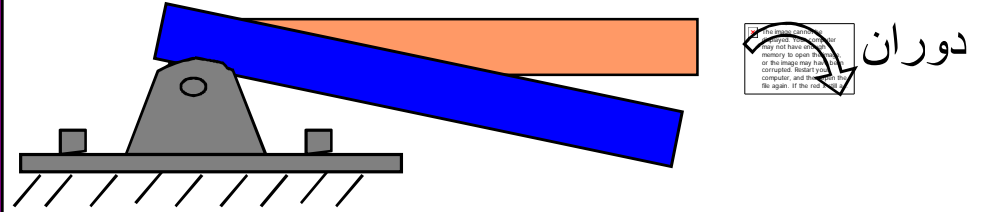
ROLLER SUPPORT

تسمح بالحركة الأفقية والدوران



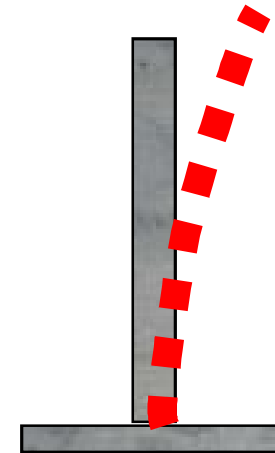
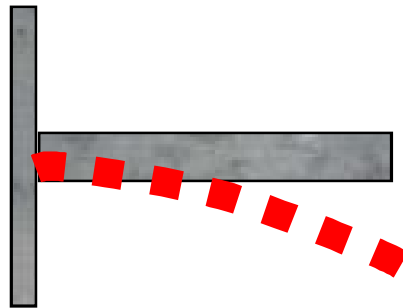
HINGED SUPPORT

تسمح بالدوران فقط

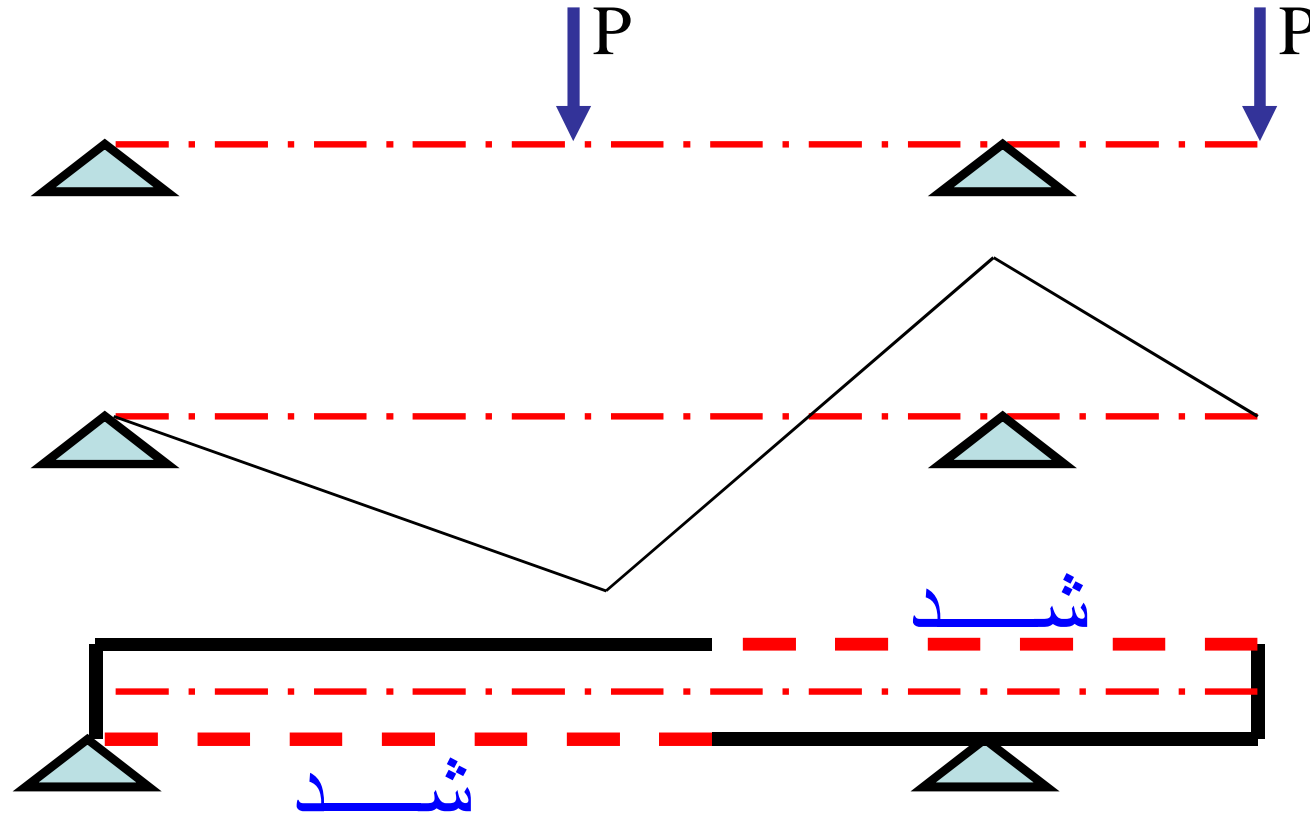


FIXED SUPPORT

لا تسمح بأي حركة



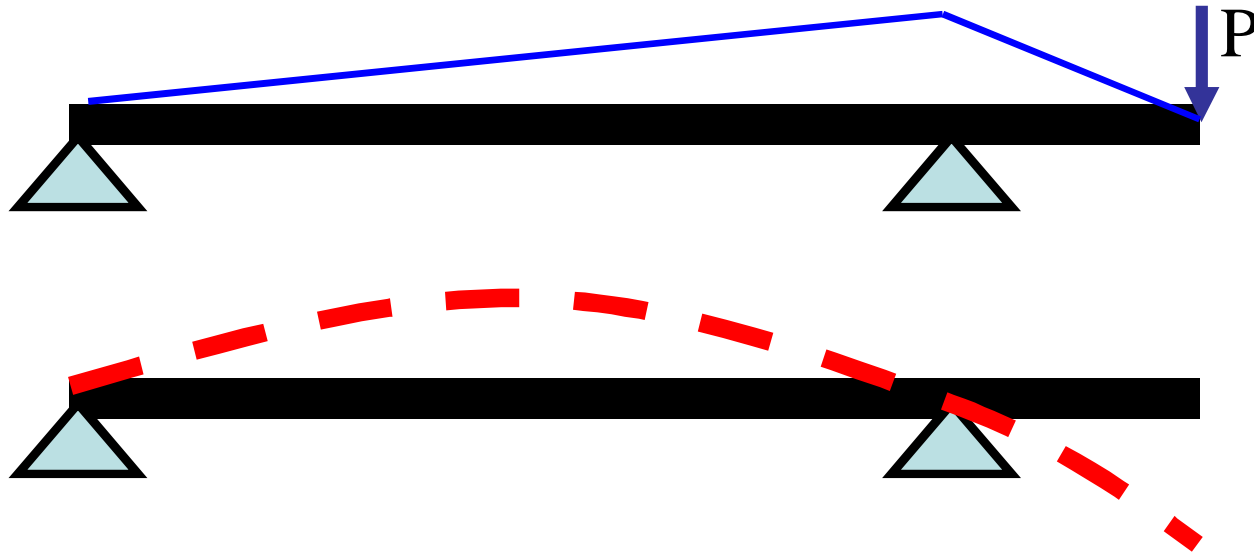
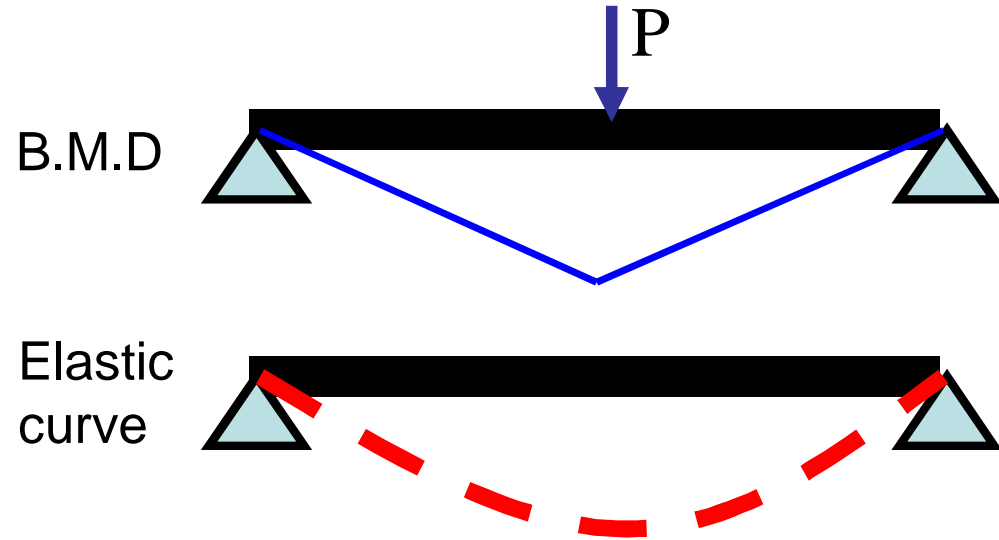
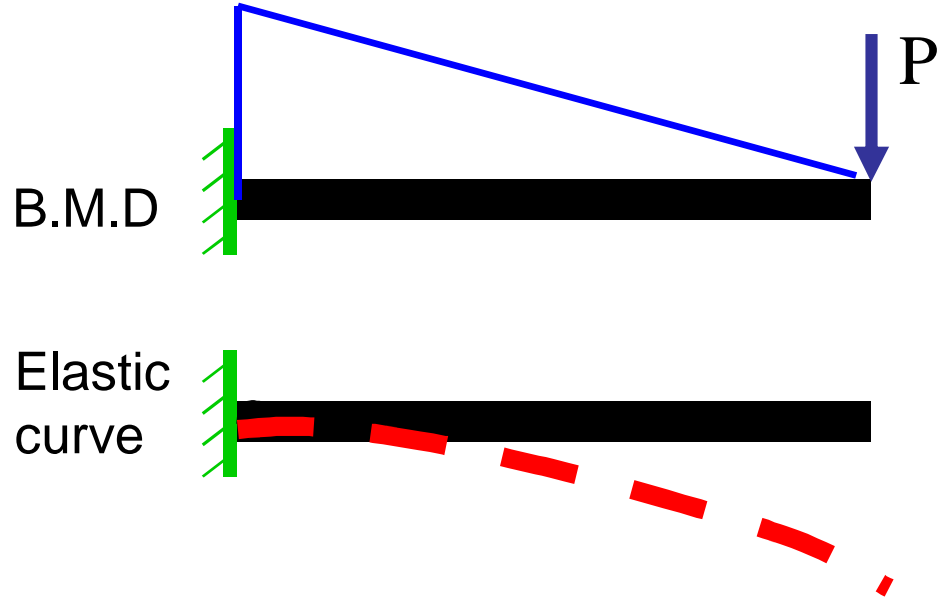
1- يكون العزم فى اتجاه الشد



2- يجب ان تكون الزاويه ثابتة قبل وبعد التشكل

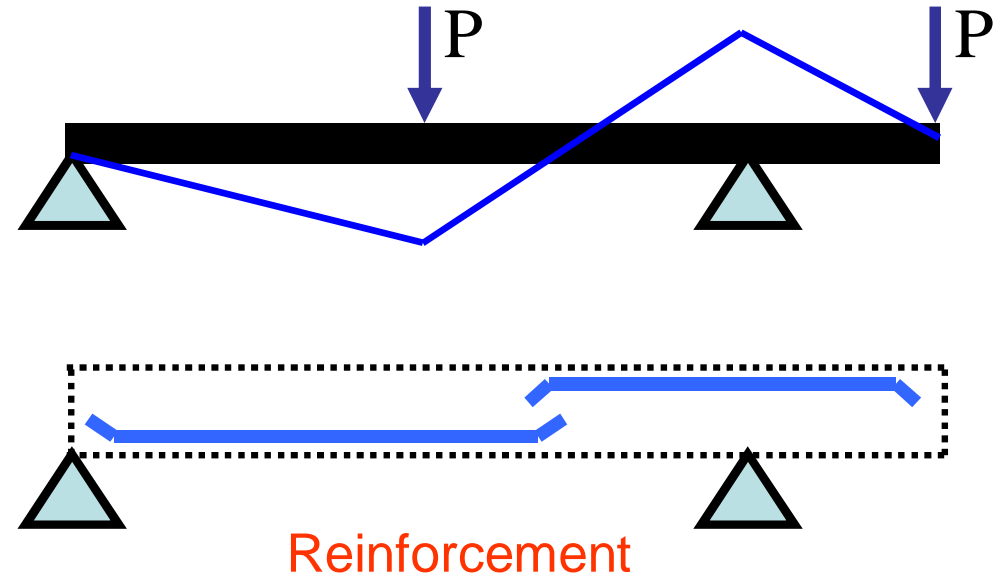
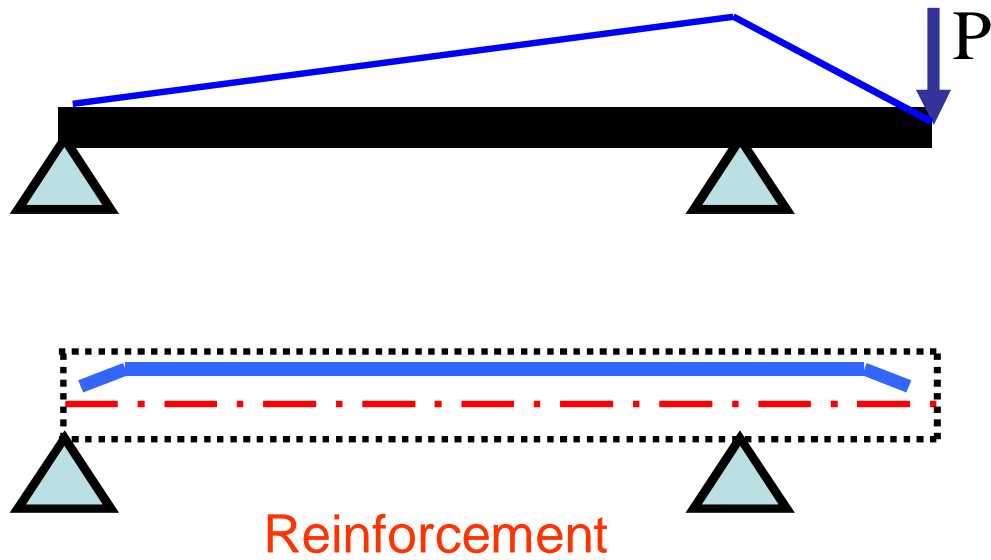
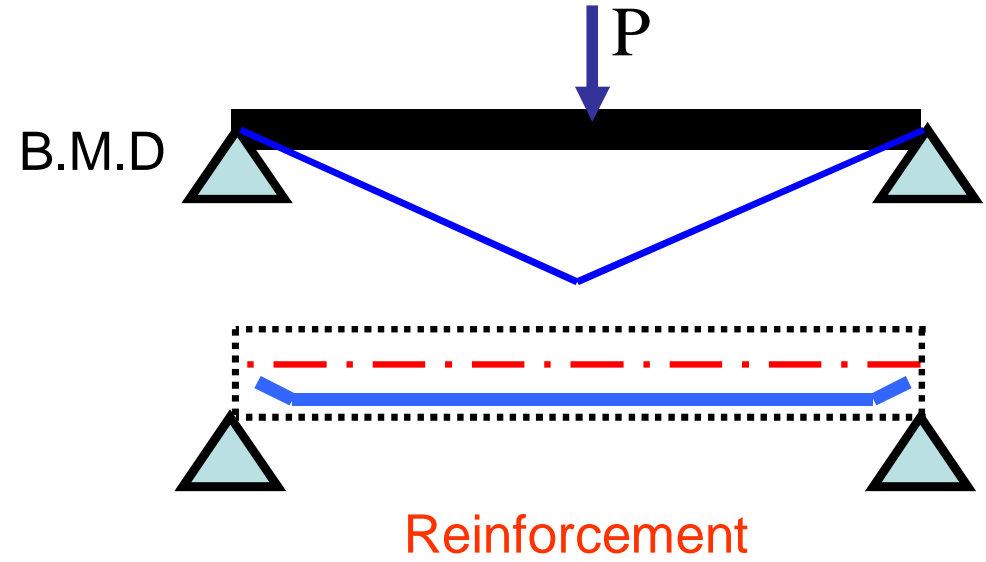
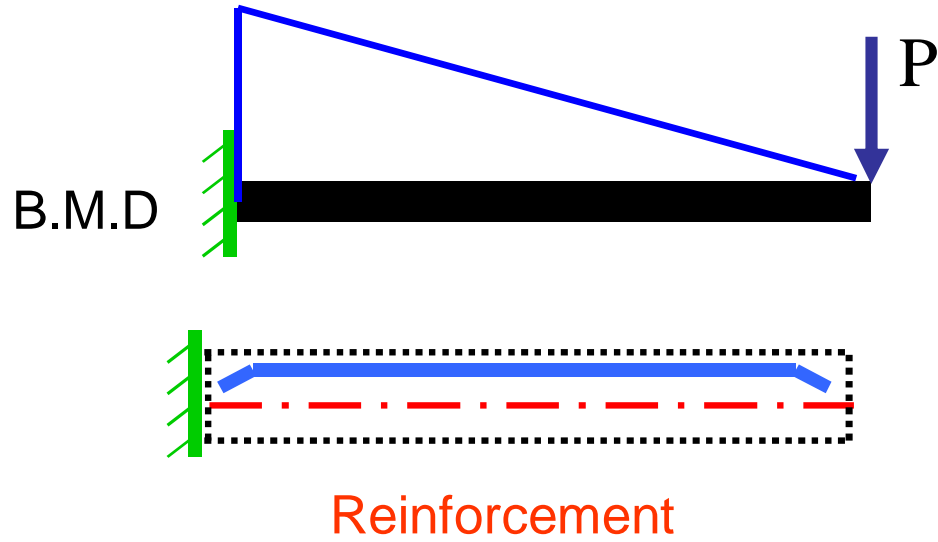


3- يكون الانحناء محدب ناحية العزم



الاجزاء التي لا يوجد عليها عزوم
ترسم في الـ elastic curve
خط مستقيم

4- يكون التسليح ناحية العزم

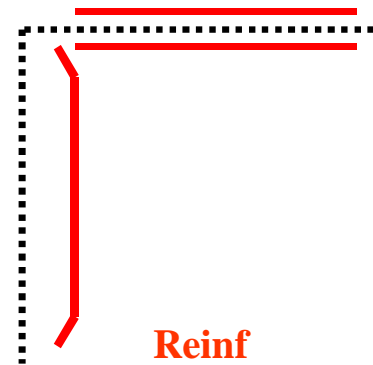
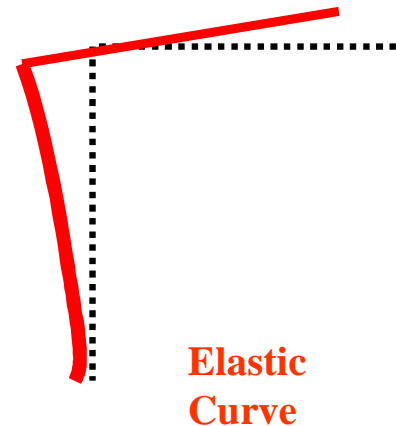
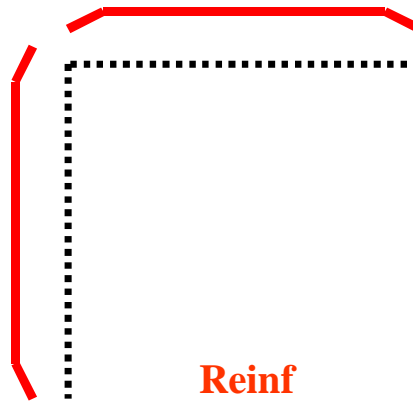
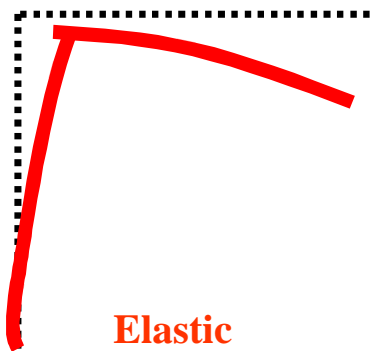
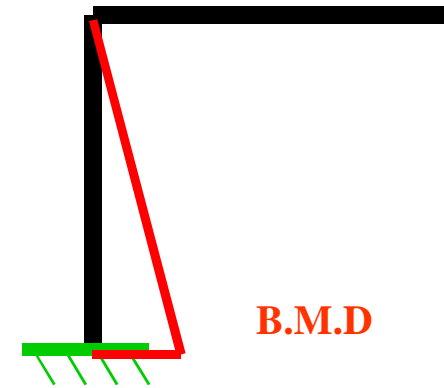
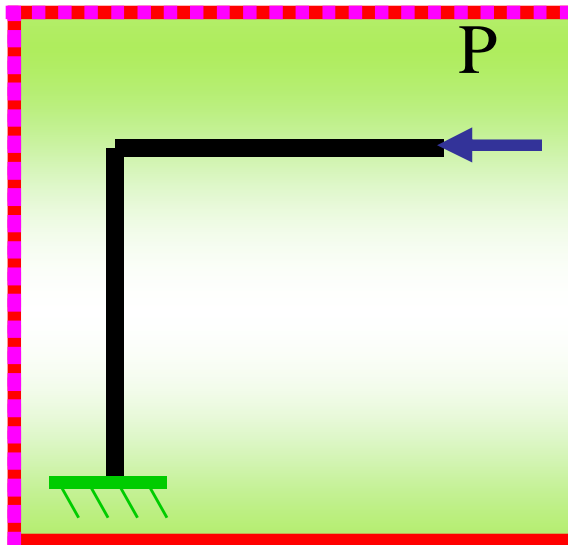
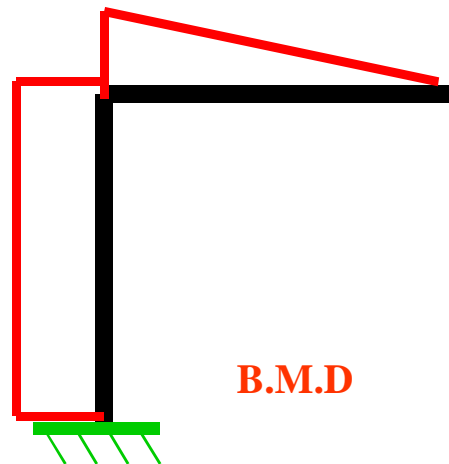
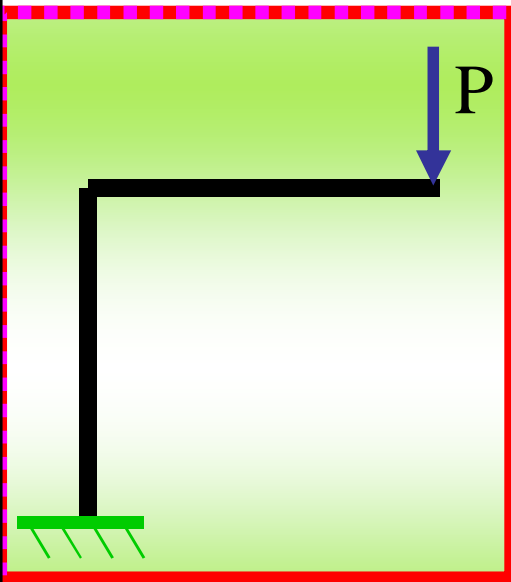


Determinate Structures

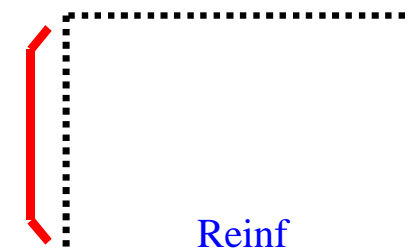
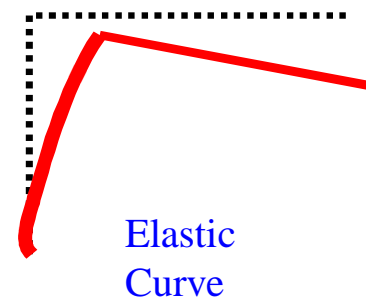
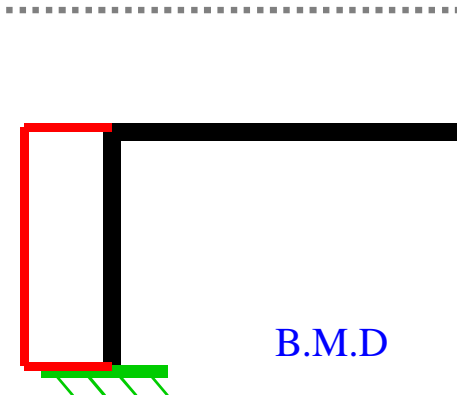
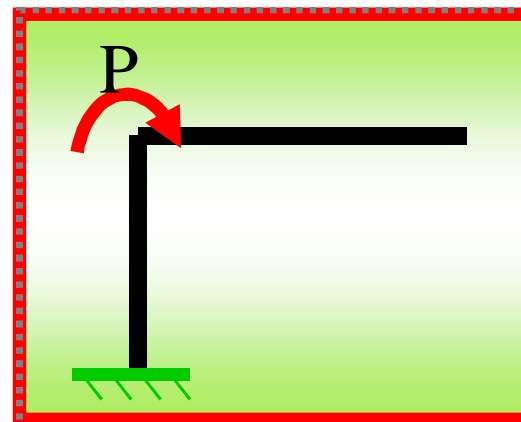
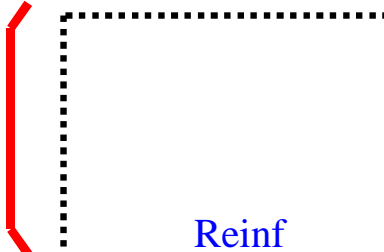
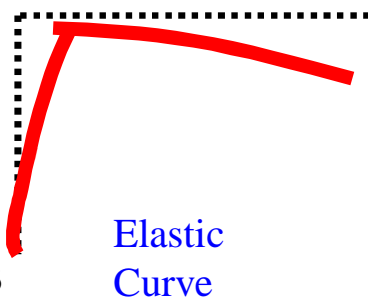
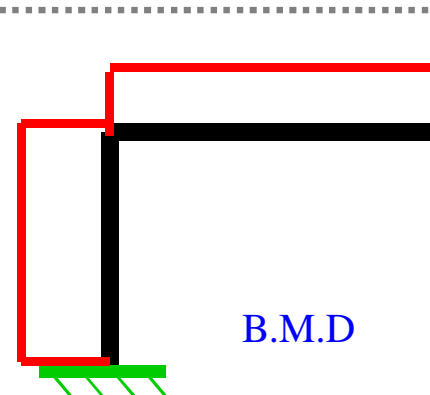
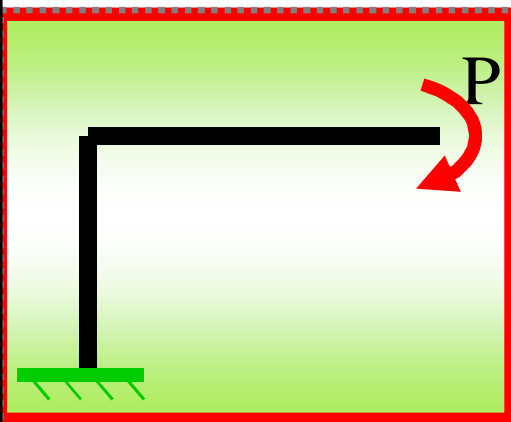
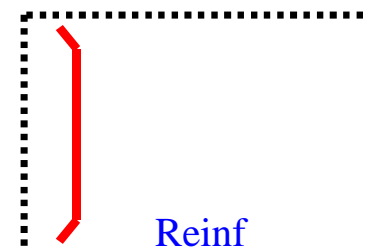
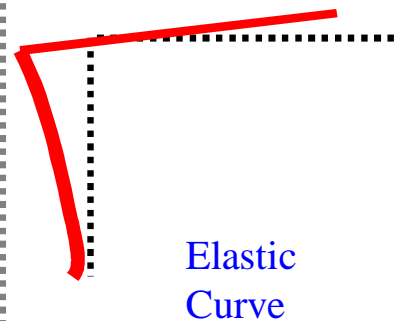
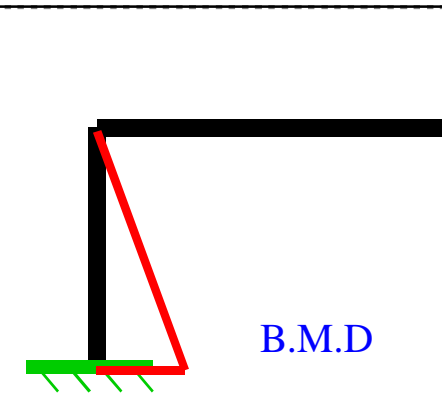
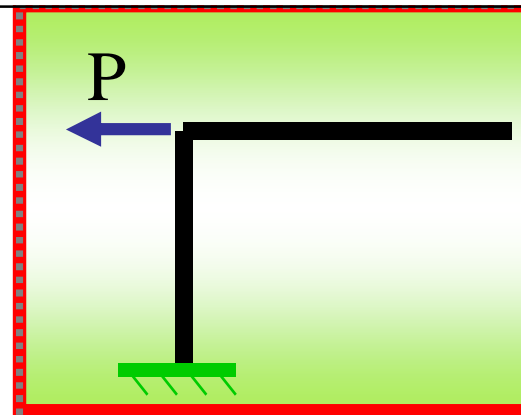
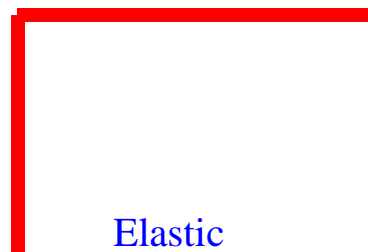
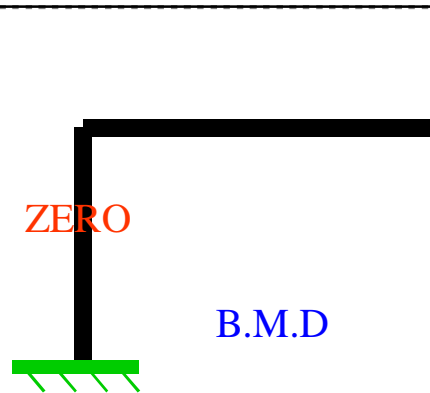
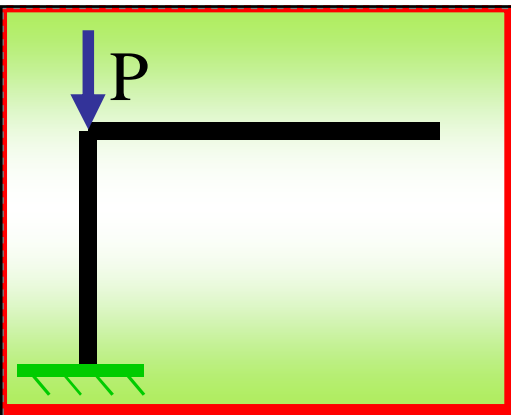
في هذه المنشآت سيكون من السهل توقع اتجاهات الـ reactions

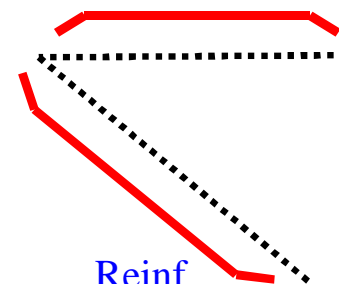
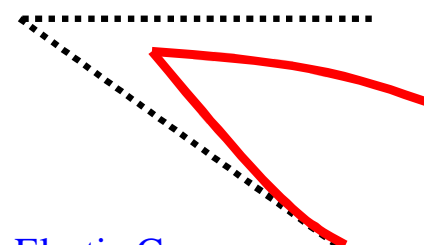
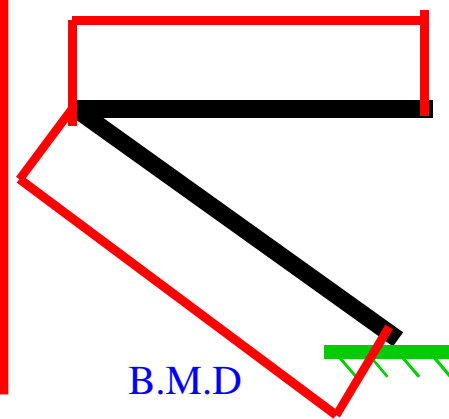
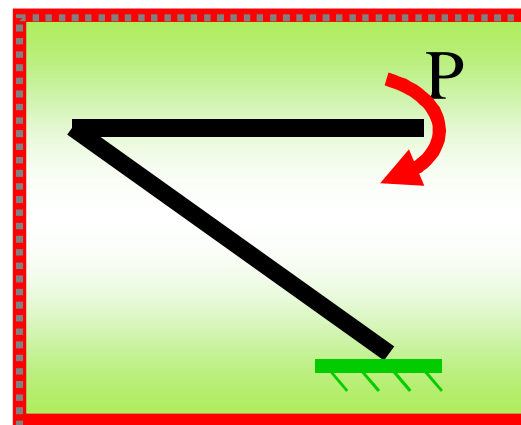
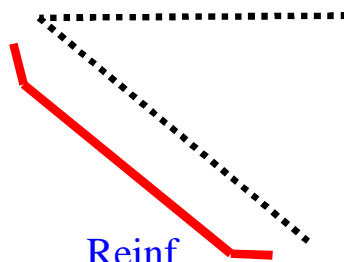
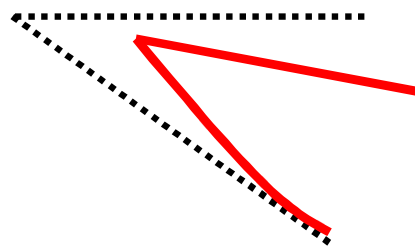
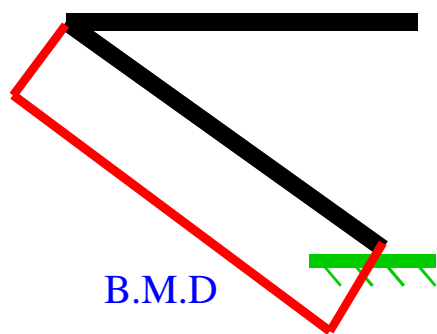
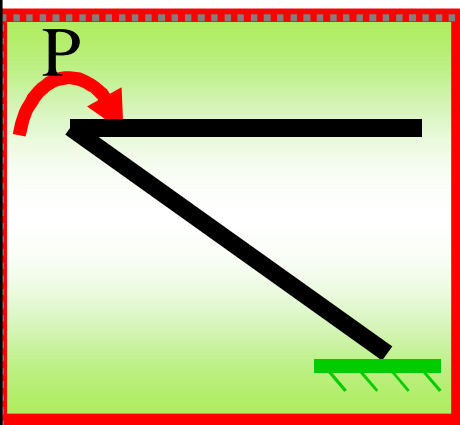
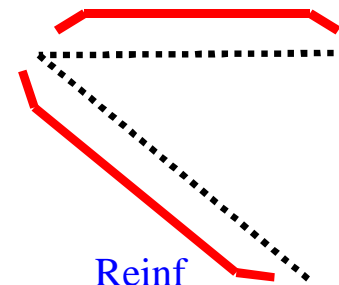
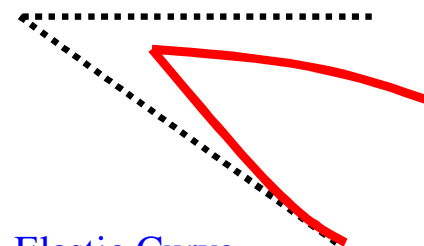
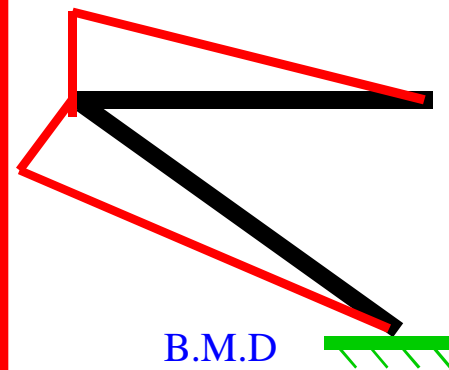
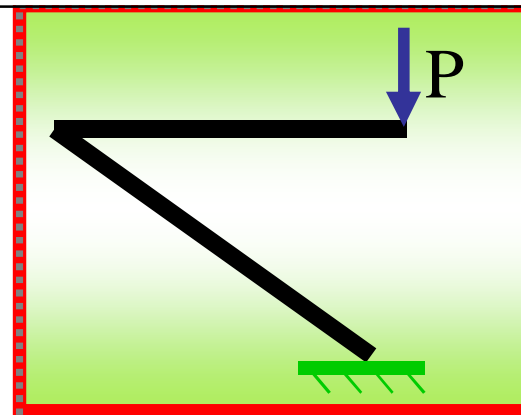
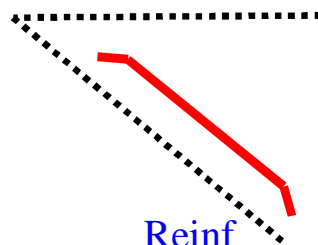
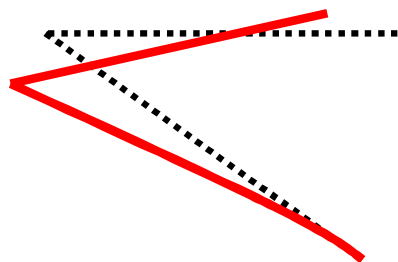
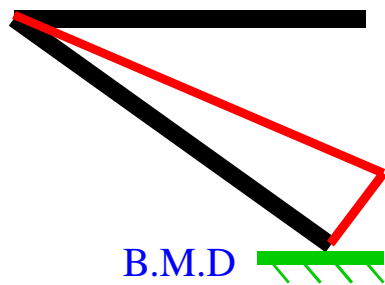
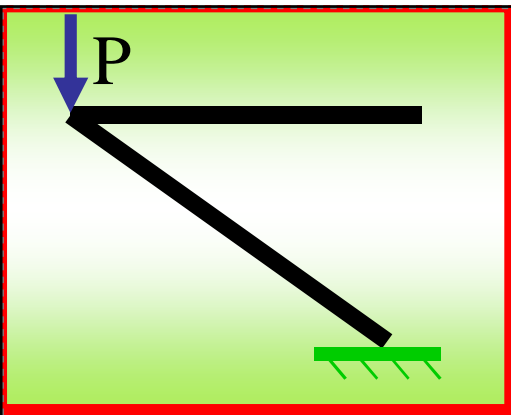
1- رسم العزوم

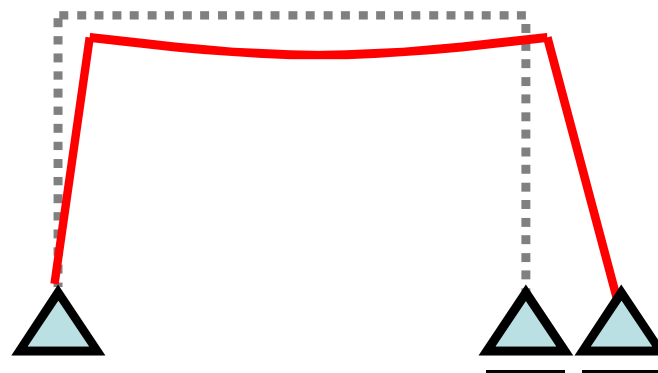
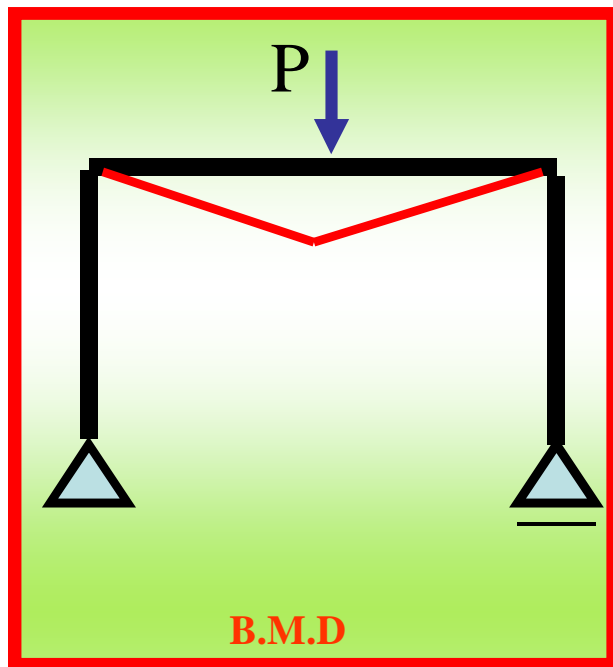
2- يتم تحديد اتجاه التحديد حسب شكل العزم واماكن التسليح



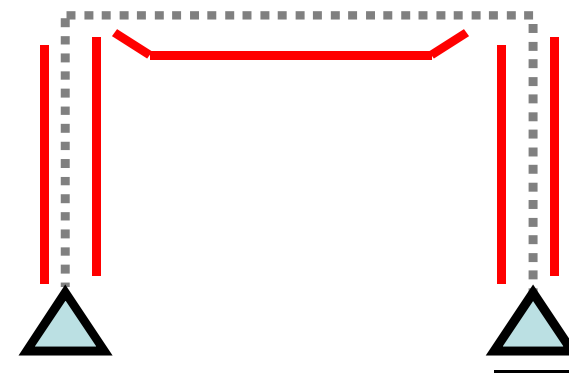
تسليح الضغط او الشد اذا كان هو
الموجود فقط يكون في الاتجاهين



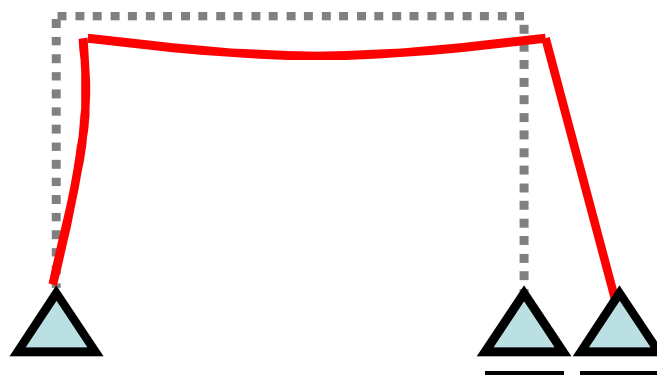
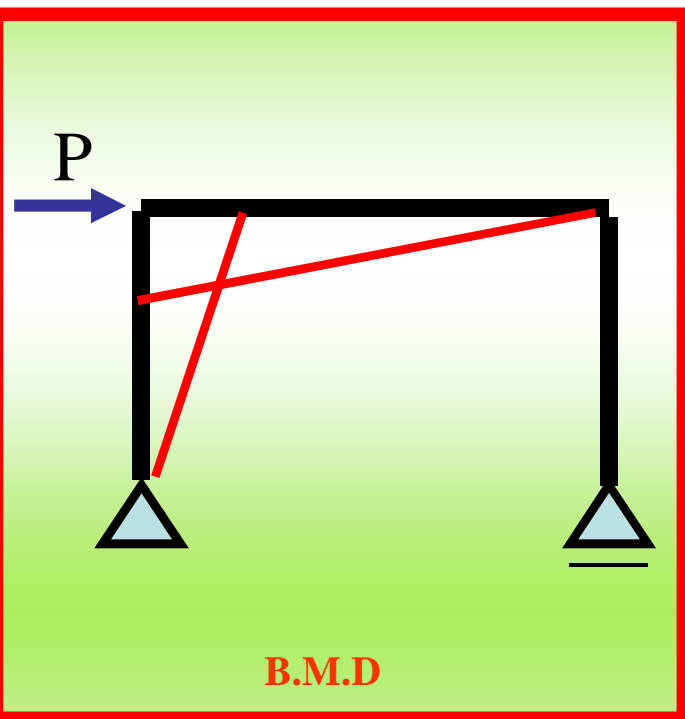




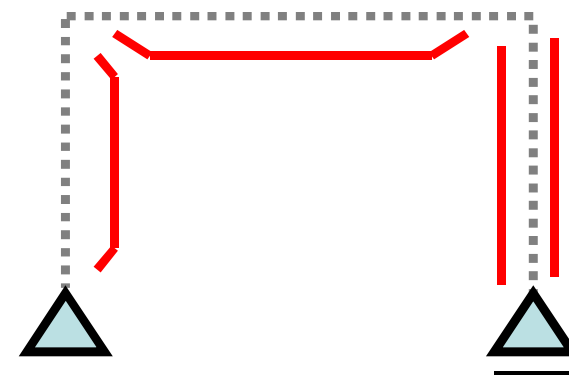
Elastic Curve



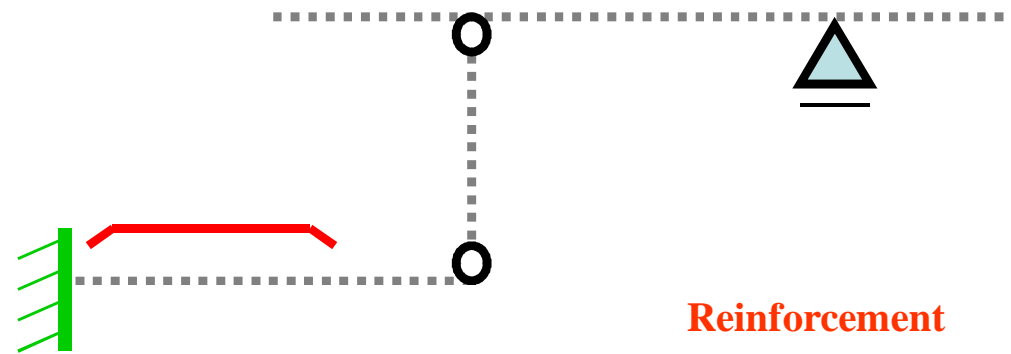
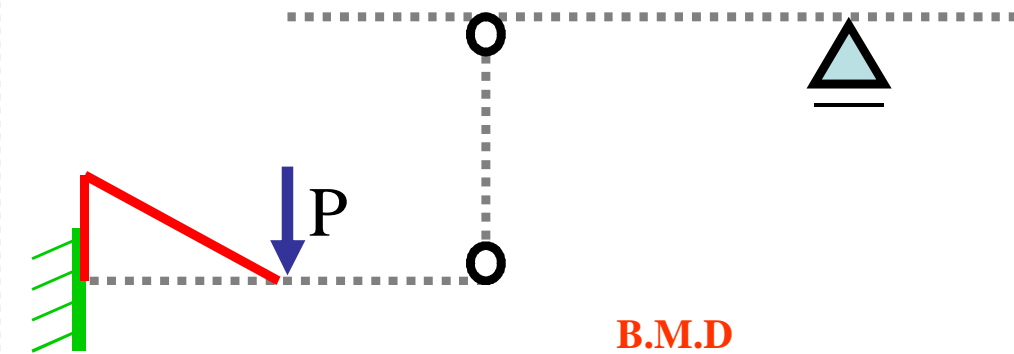
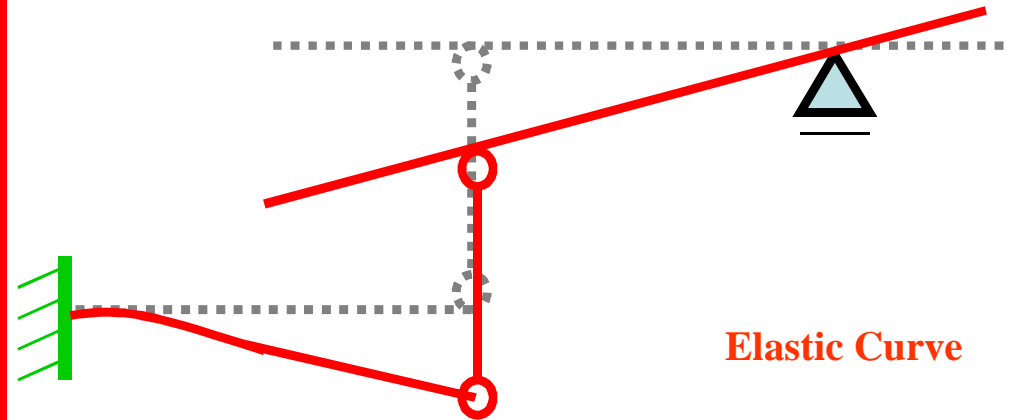
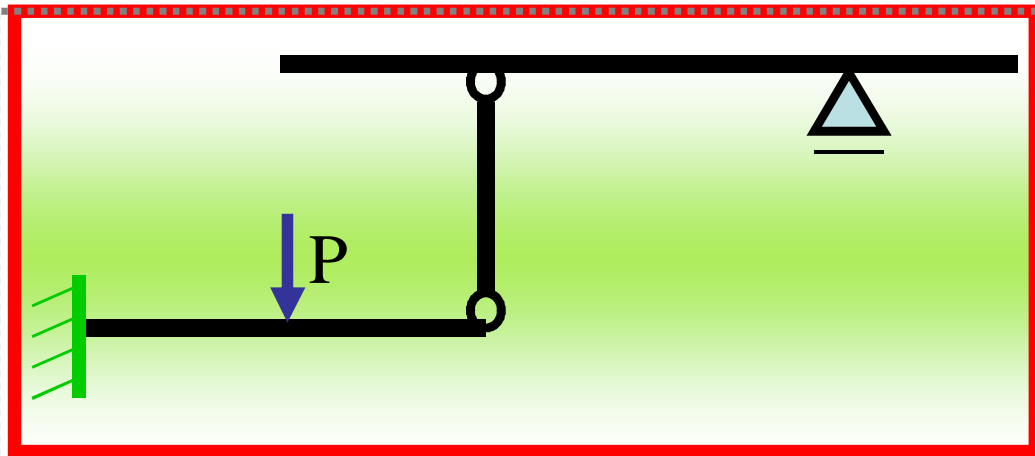
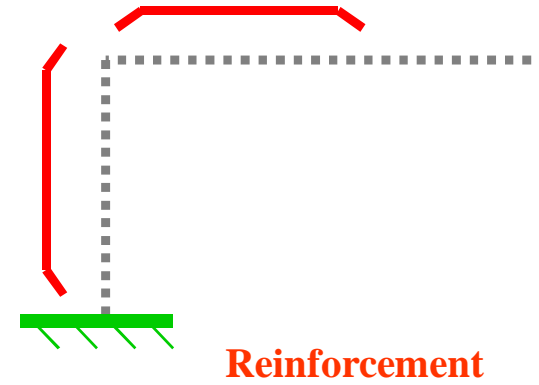
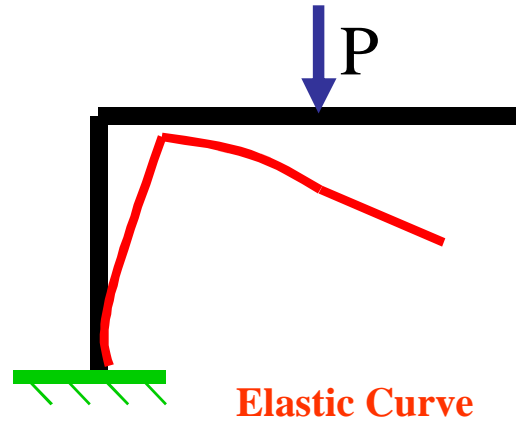
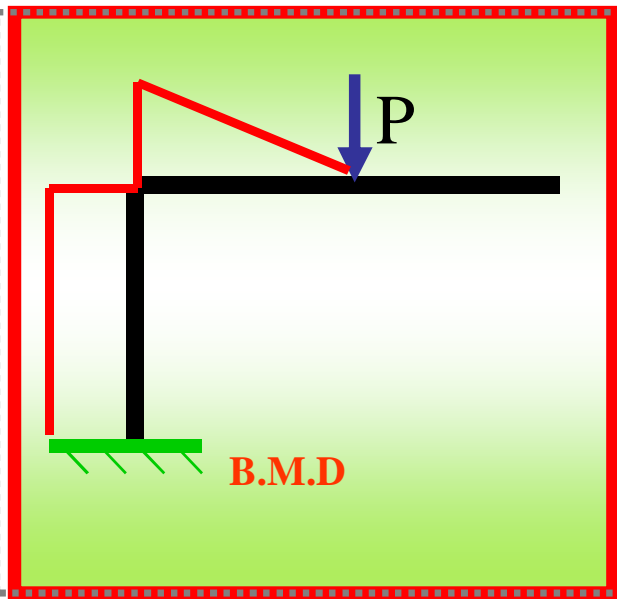
Reinforcement

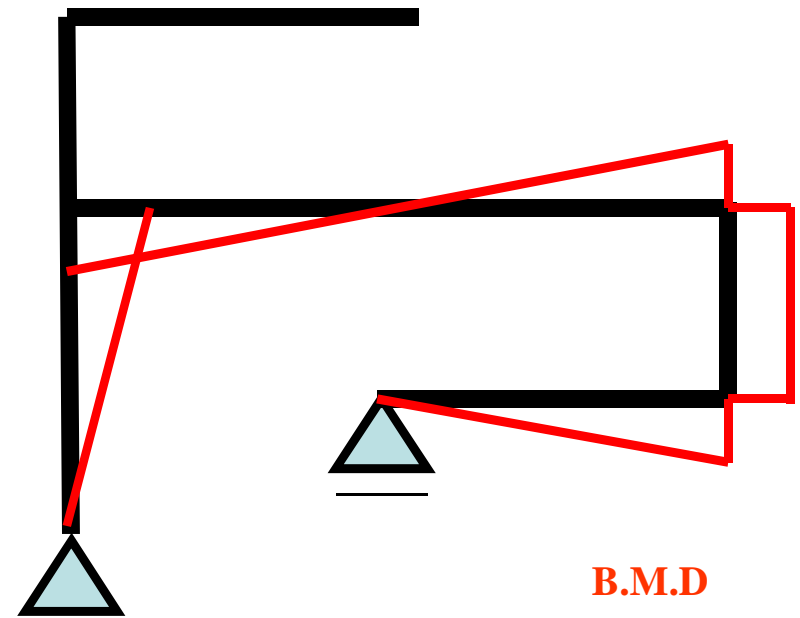
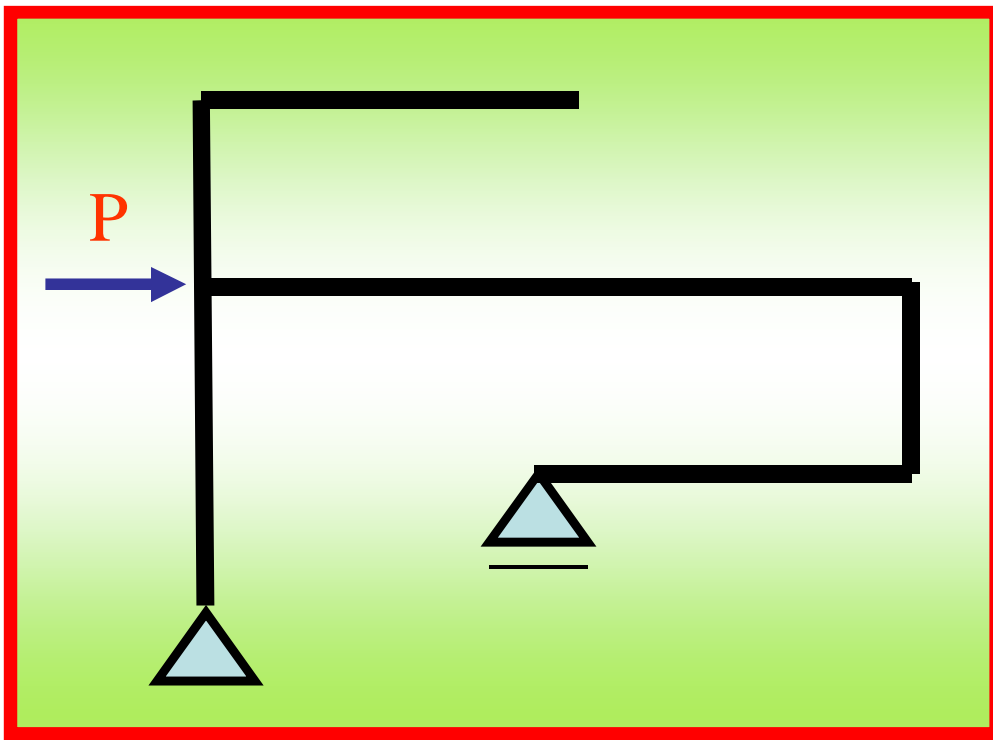


Elastic Curve

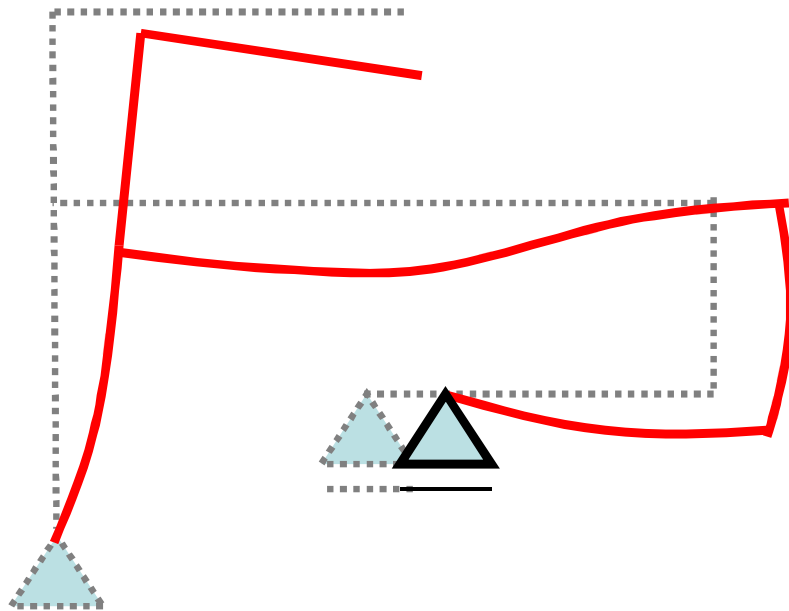


Reinforcement

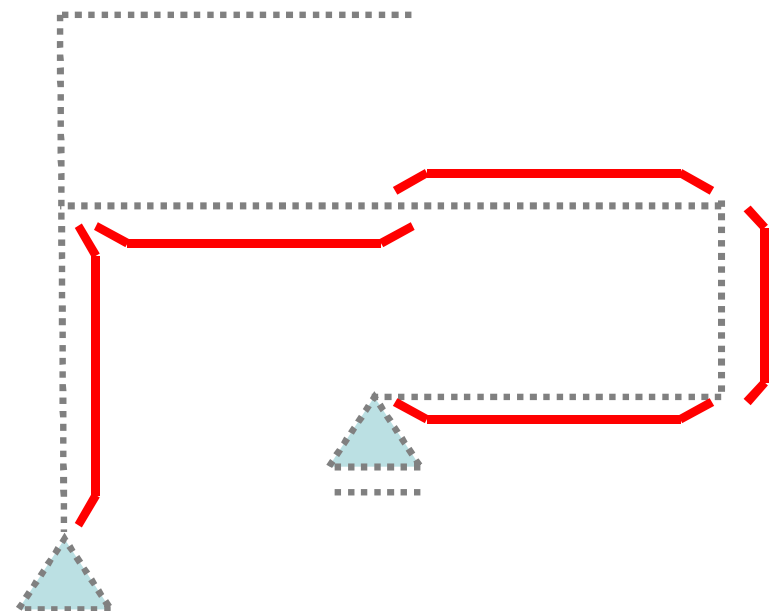




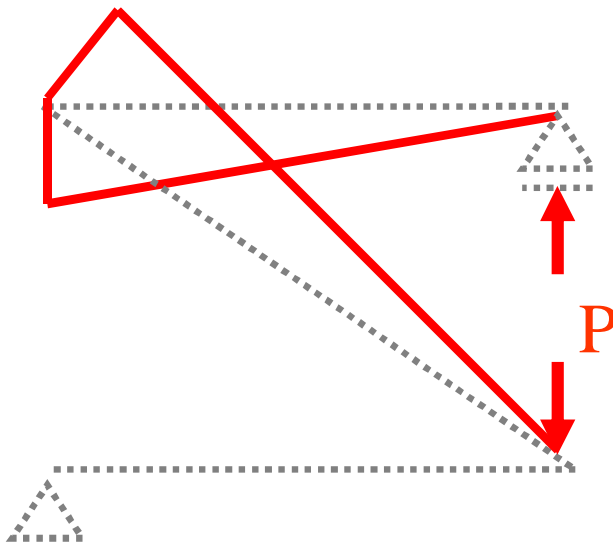
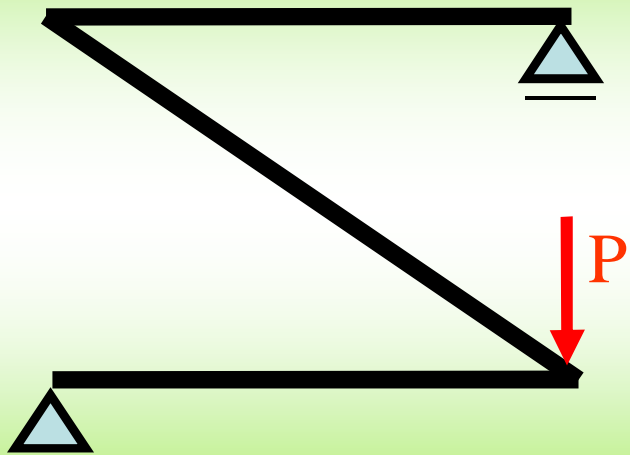
B.M.D



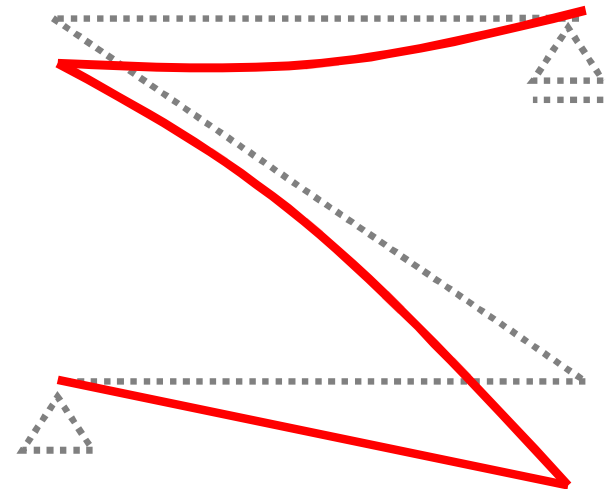
Elastic Curve



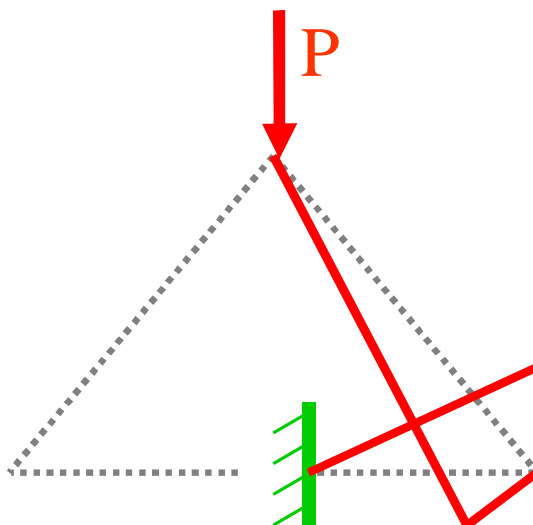
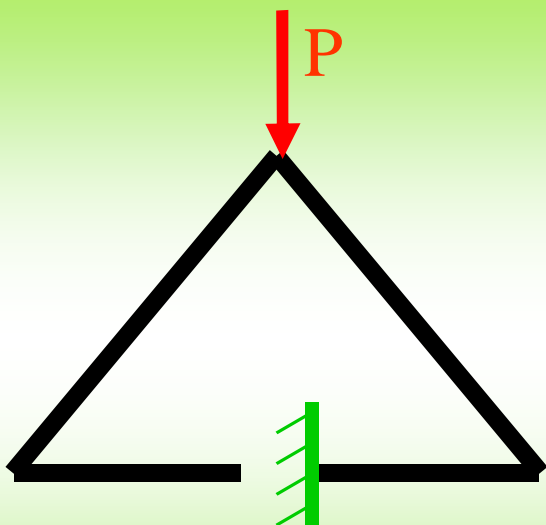
Reinforcement



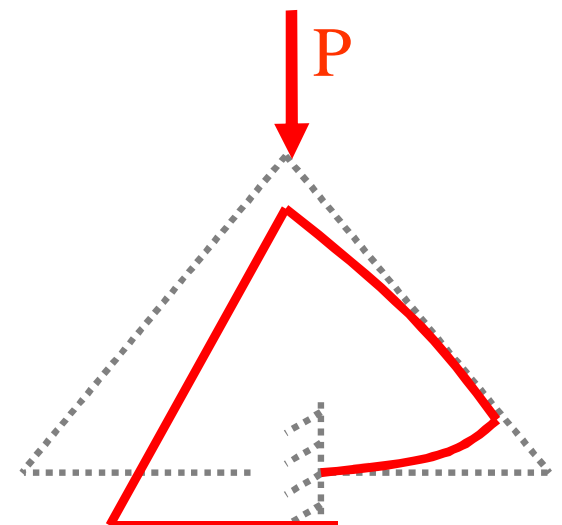
B.M.D



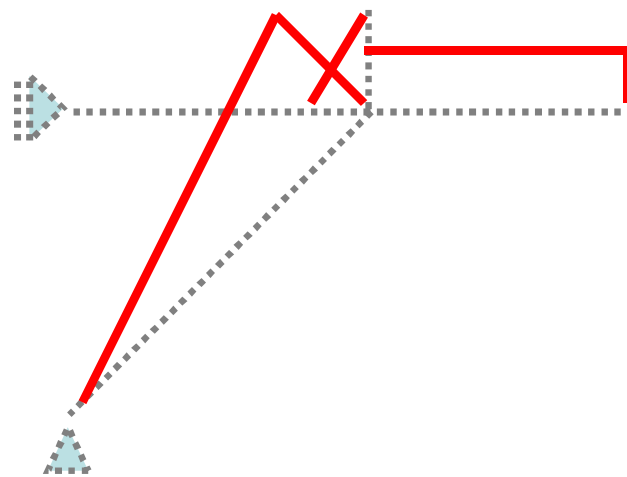
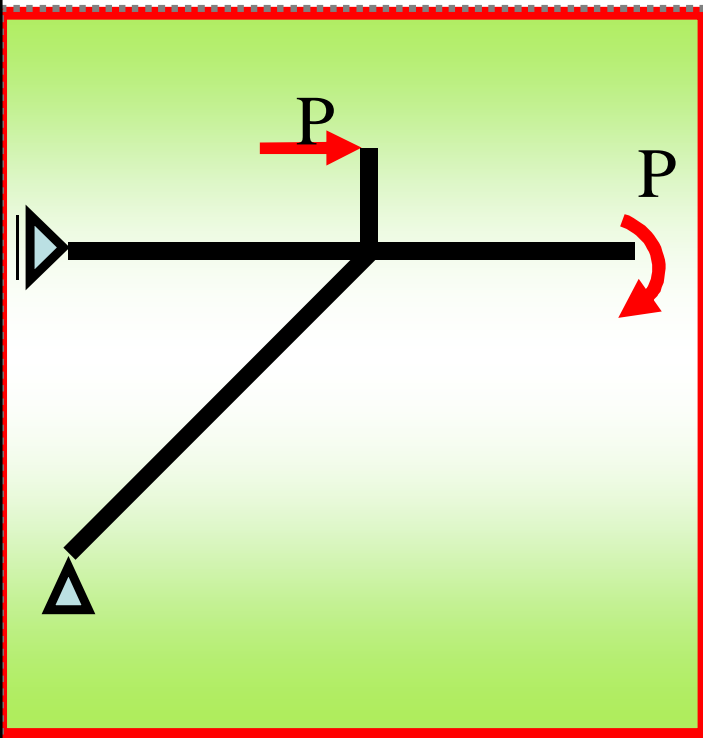
Elastic Curve



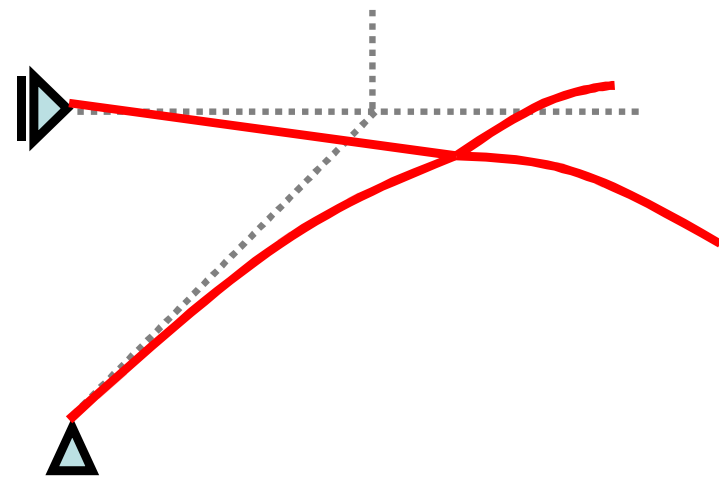
B.M.D



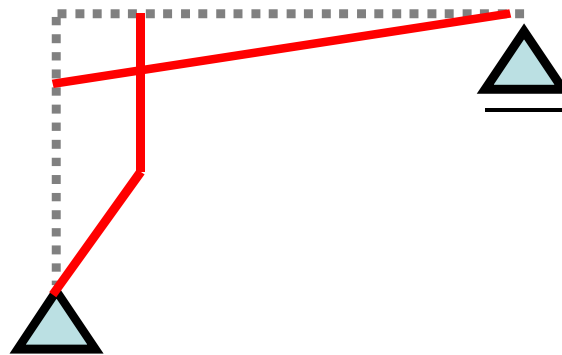
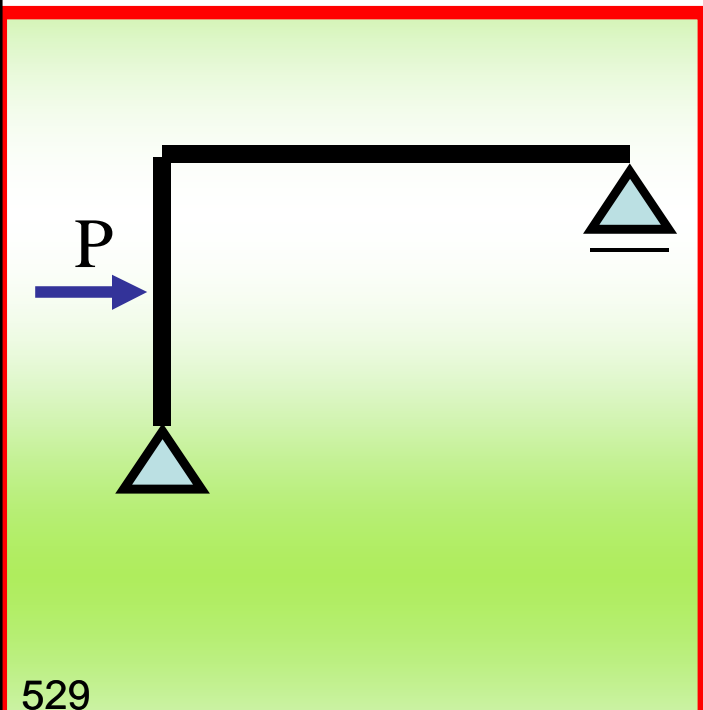
Elastic Curve



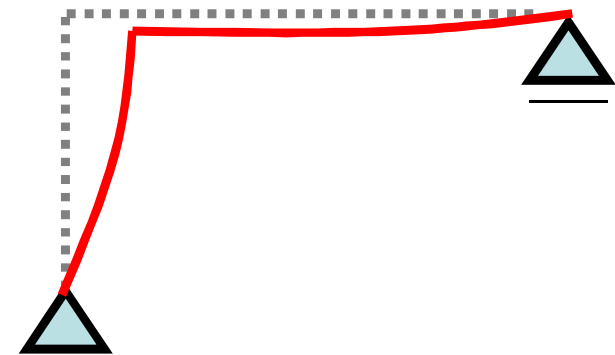
B.M.D



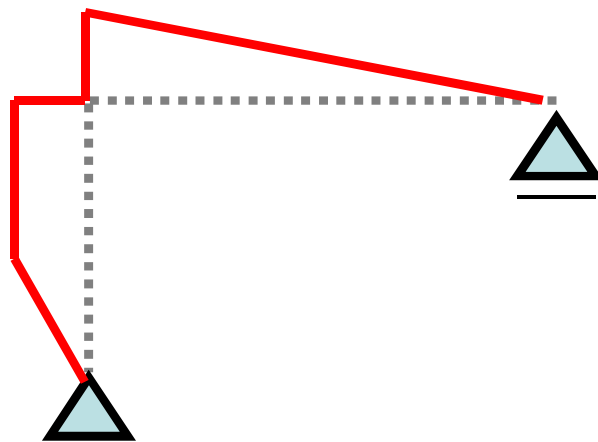
Elastic Curve



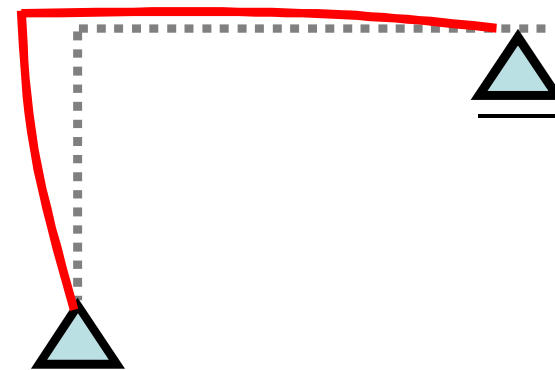
B.M.D



Elastic Curve

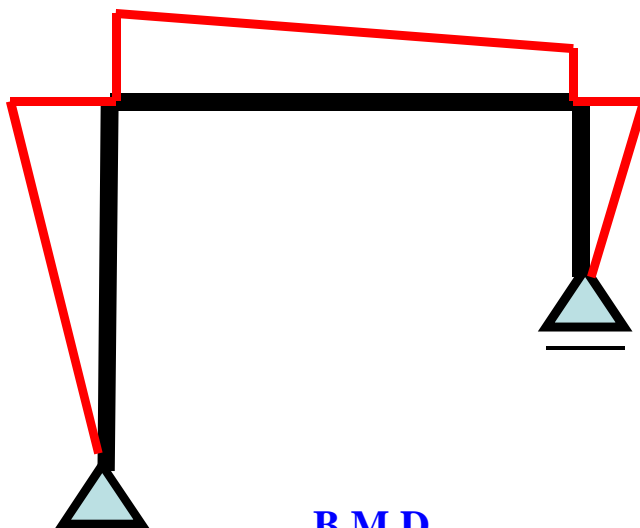


B.M.D

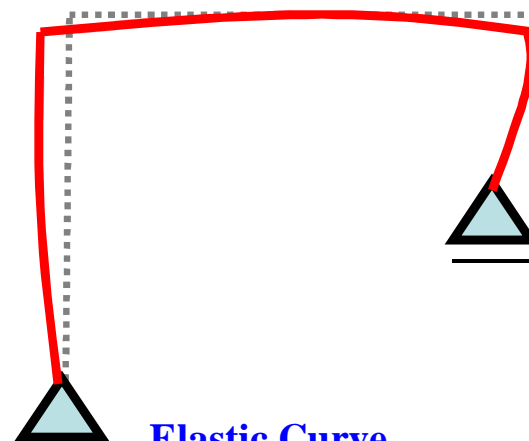


Elastic Curve

530



B.M.D

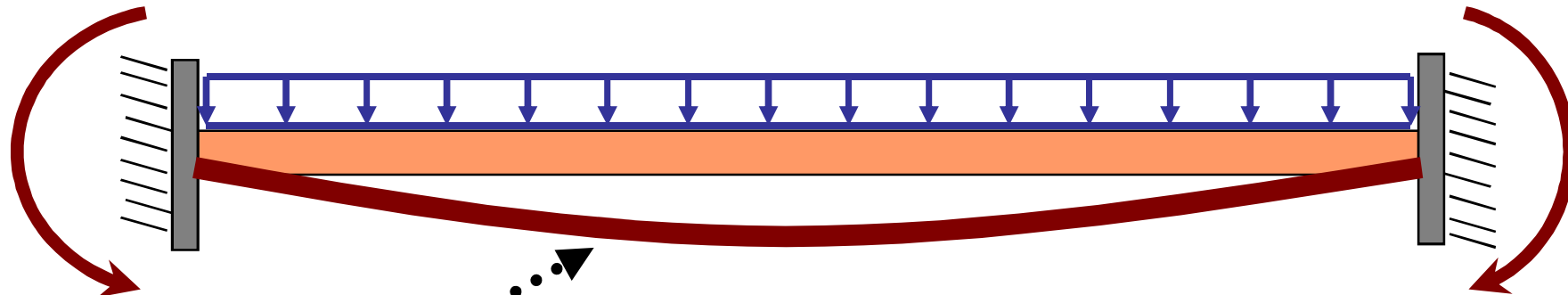


Elastic Curve

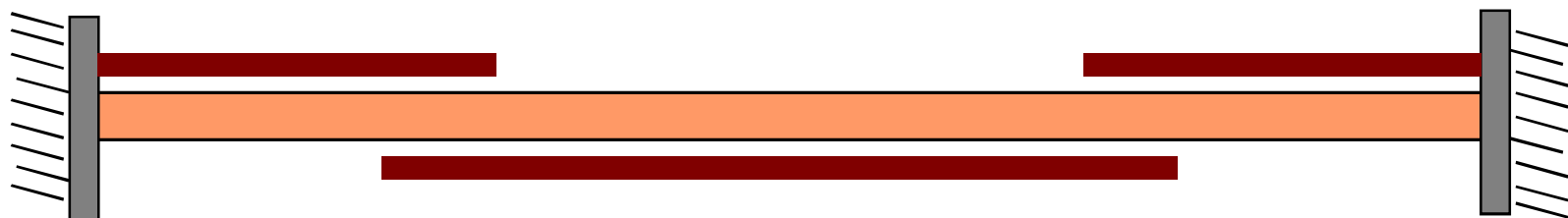
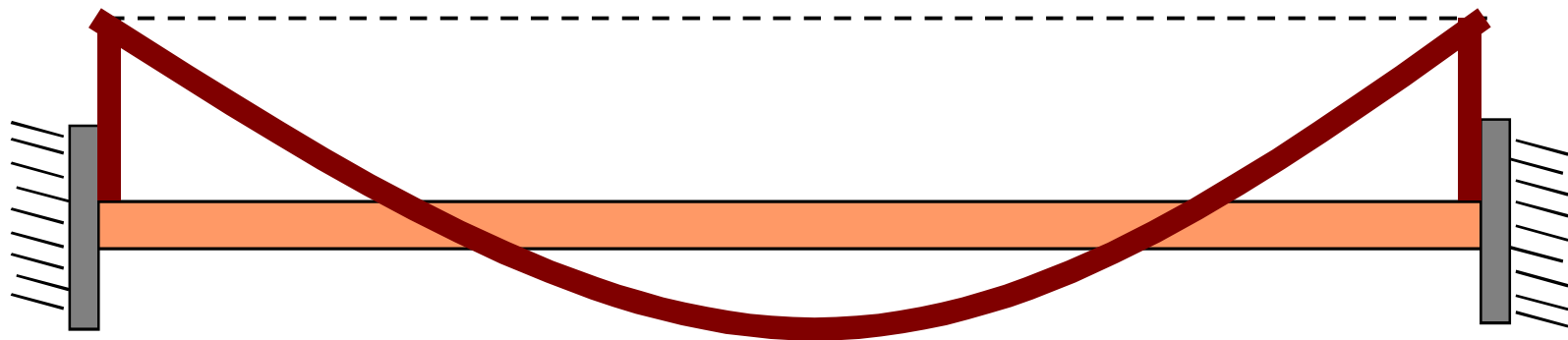
530

يدور العزم بهذا الاتجاه حتى يقوم
بقفل الزاوية عند الـ fixed support

يدور العزم بهذا الاتجاه حتى يقوم
بقفل الزاوية عند الـ fixed support

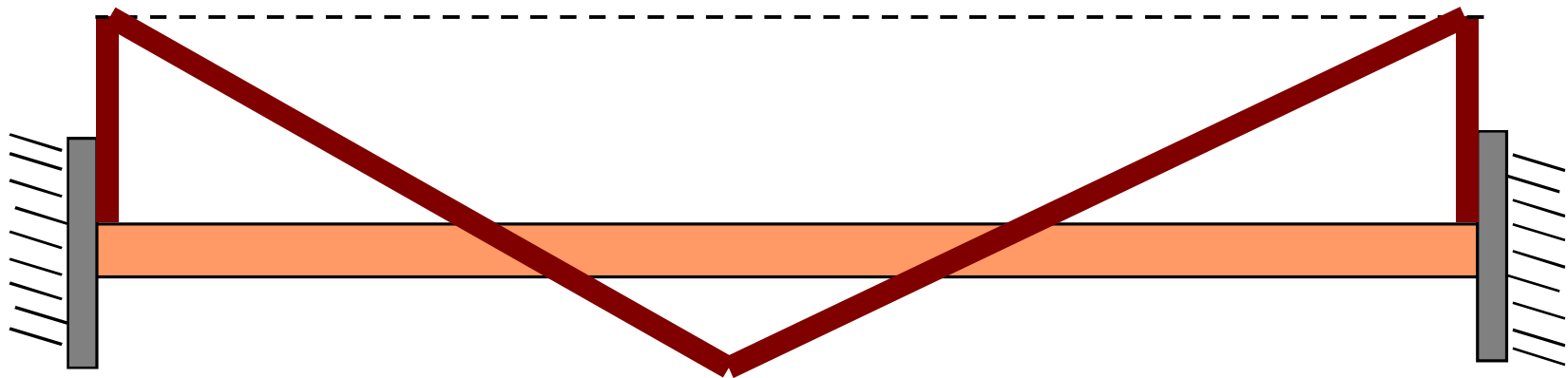
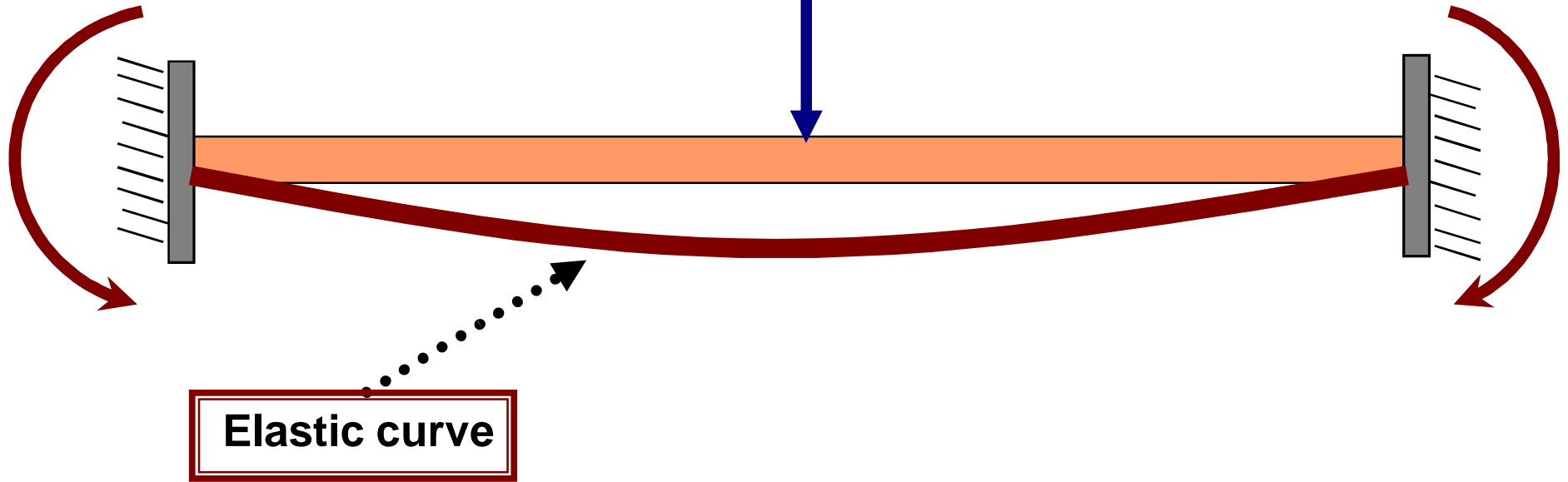


Elastic curve

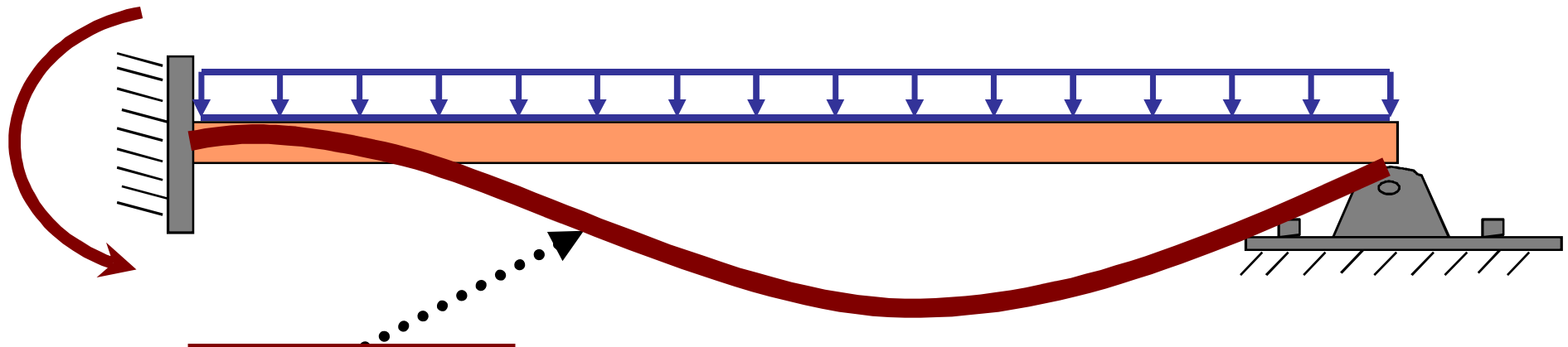


يدور العزم بهذا الاتجاه حتى يقوم
بقفل الزاوية عند الـ fixed support

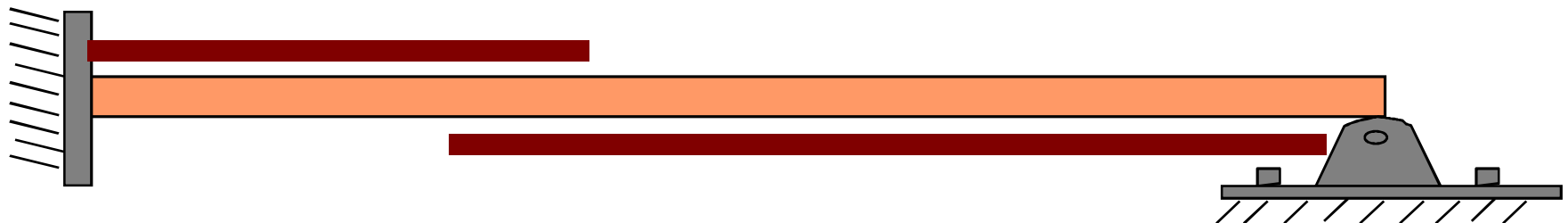
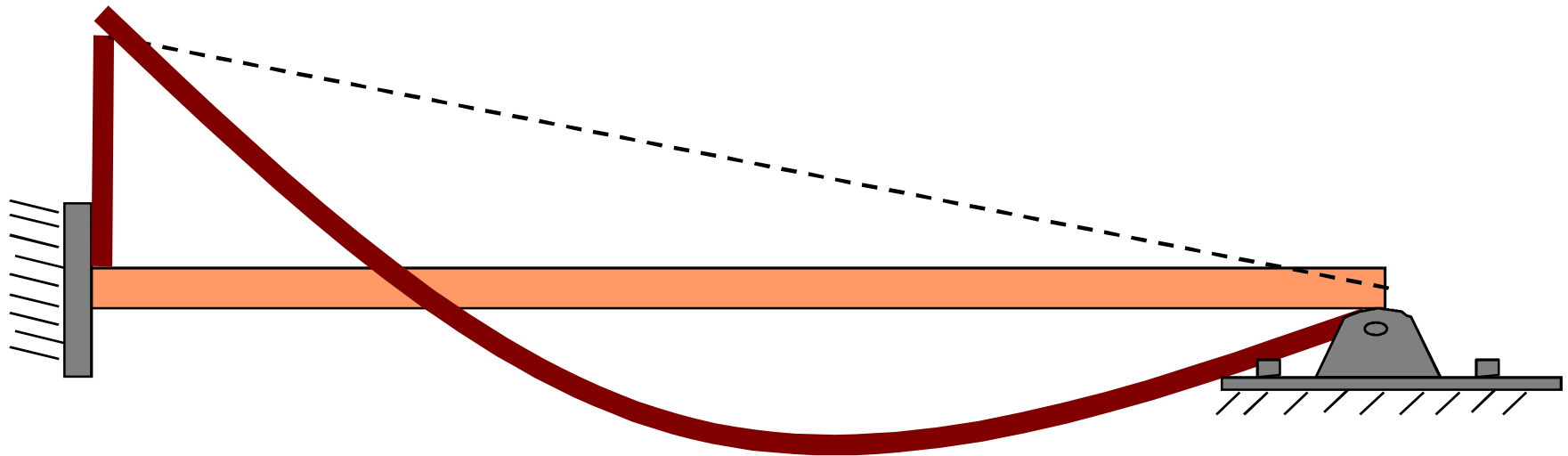
يدور العزم بهذا الاتجاه حتى يقوم
بقفل الزاوية عند الـ fixed support

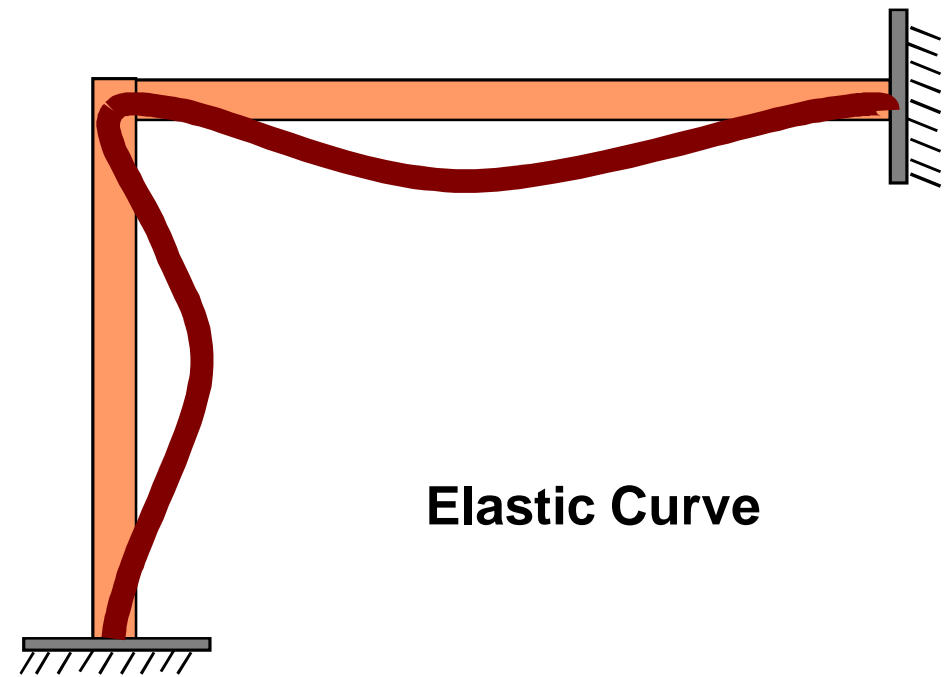
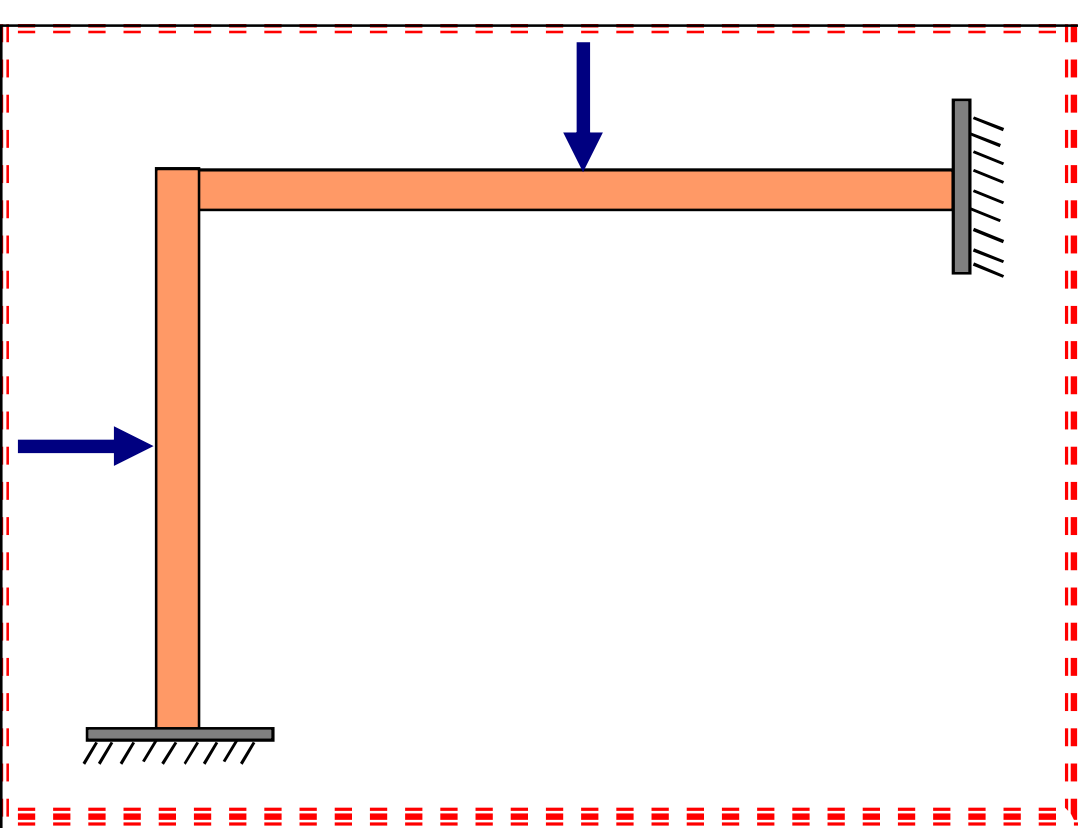


يدور العزم بهذا الاتجاه حتى يقوم
بقفل الزاوية عند الـ fixed support

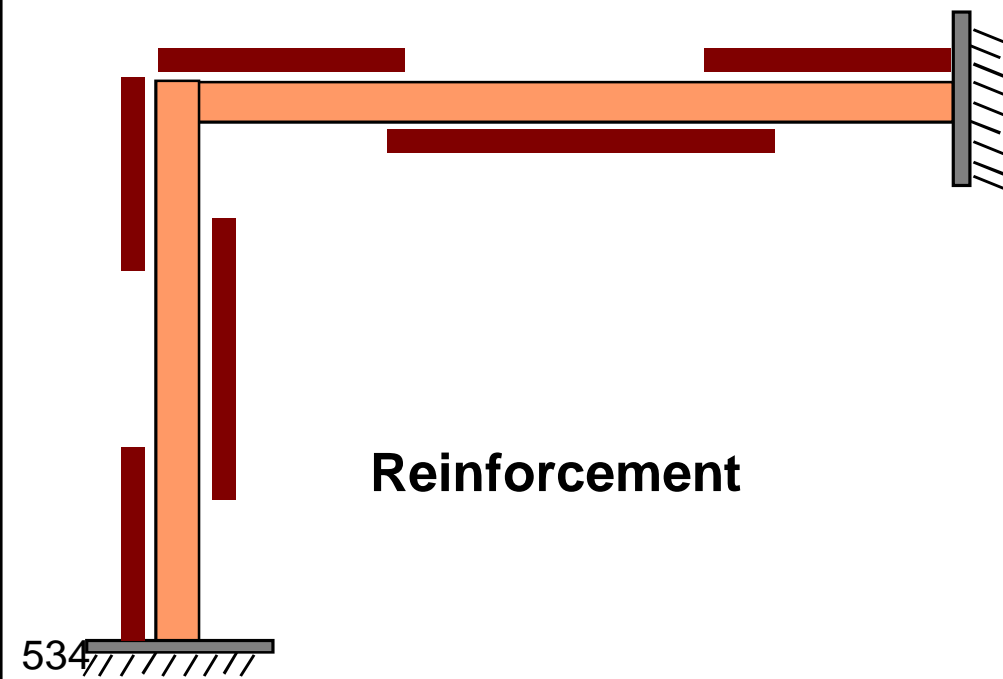


Elastic curve

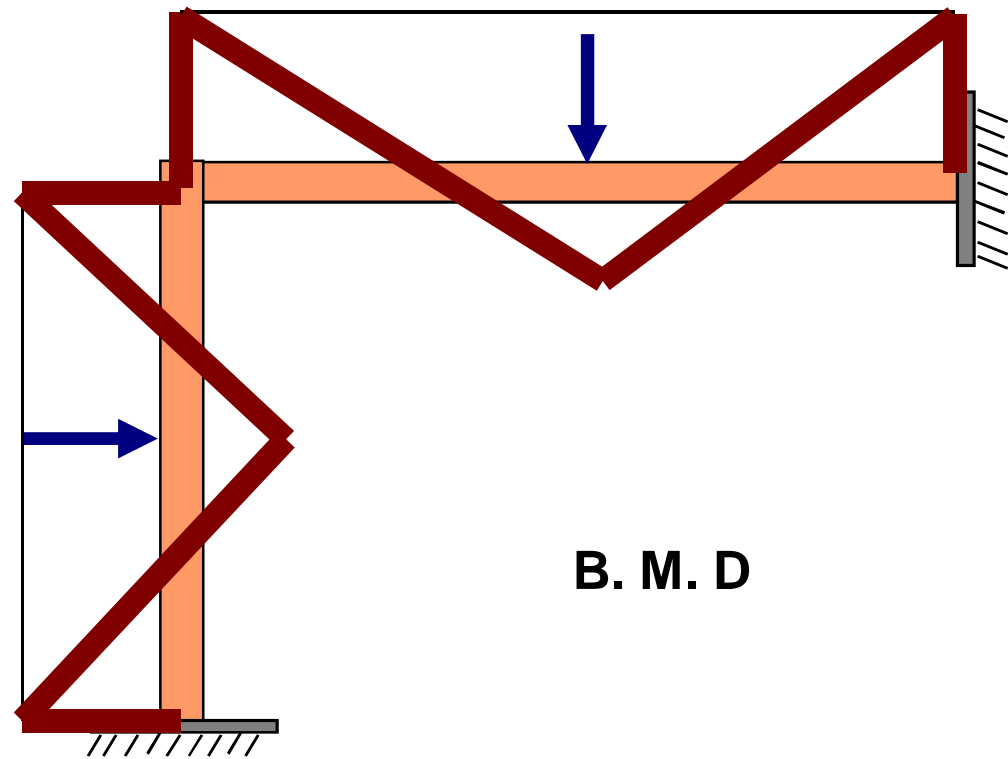




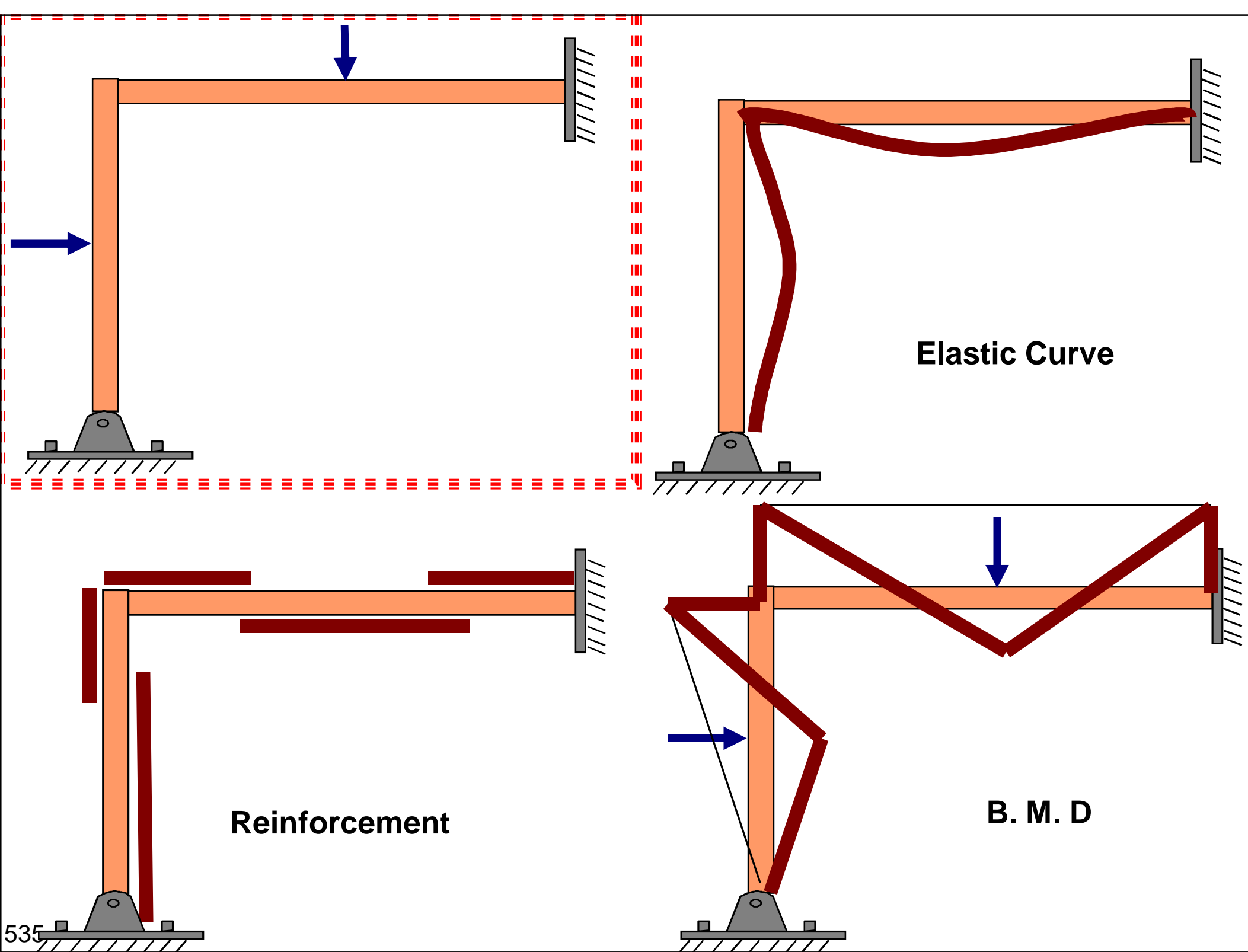
Elastic Curve

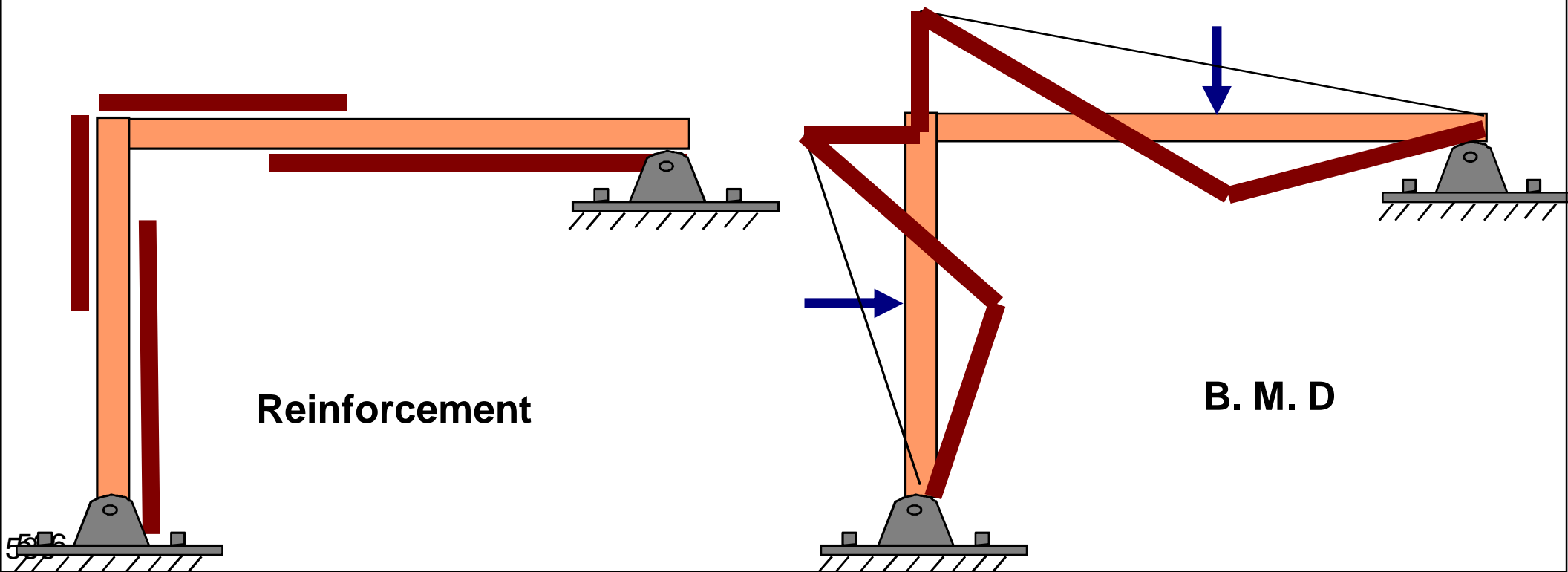
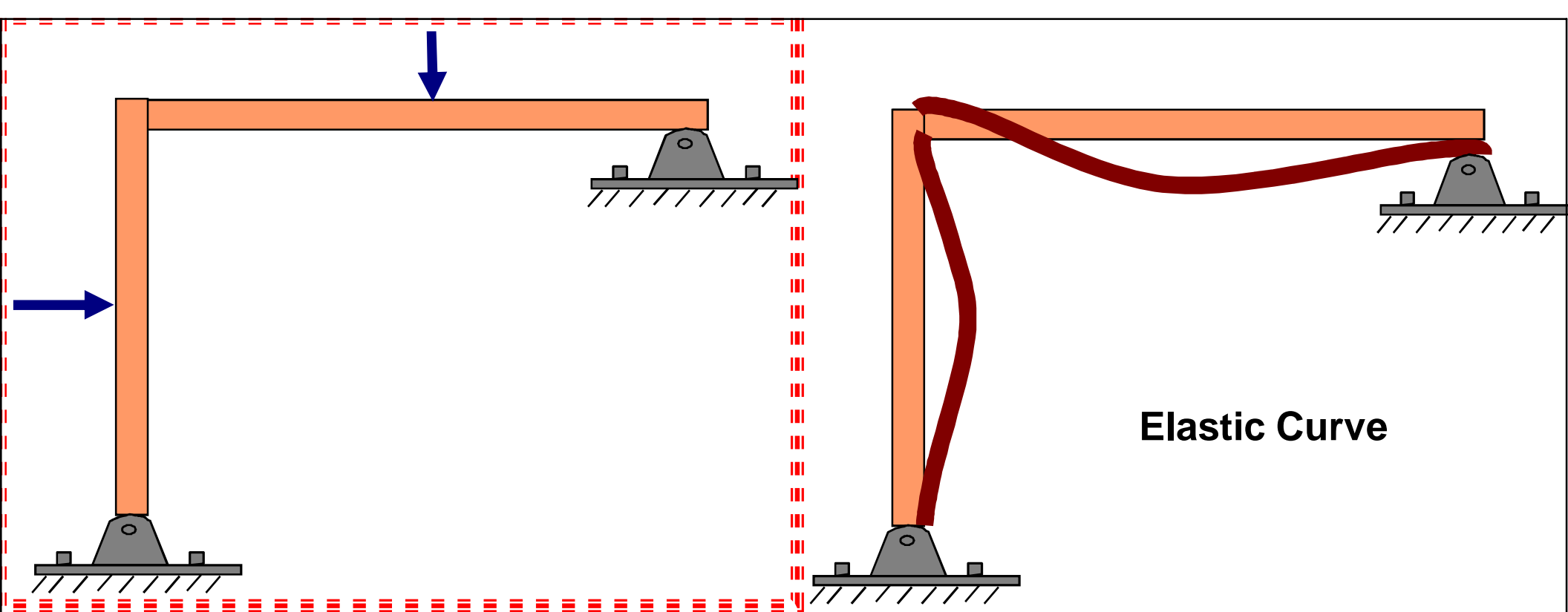


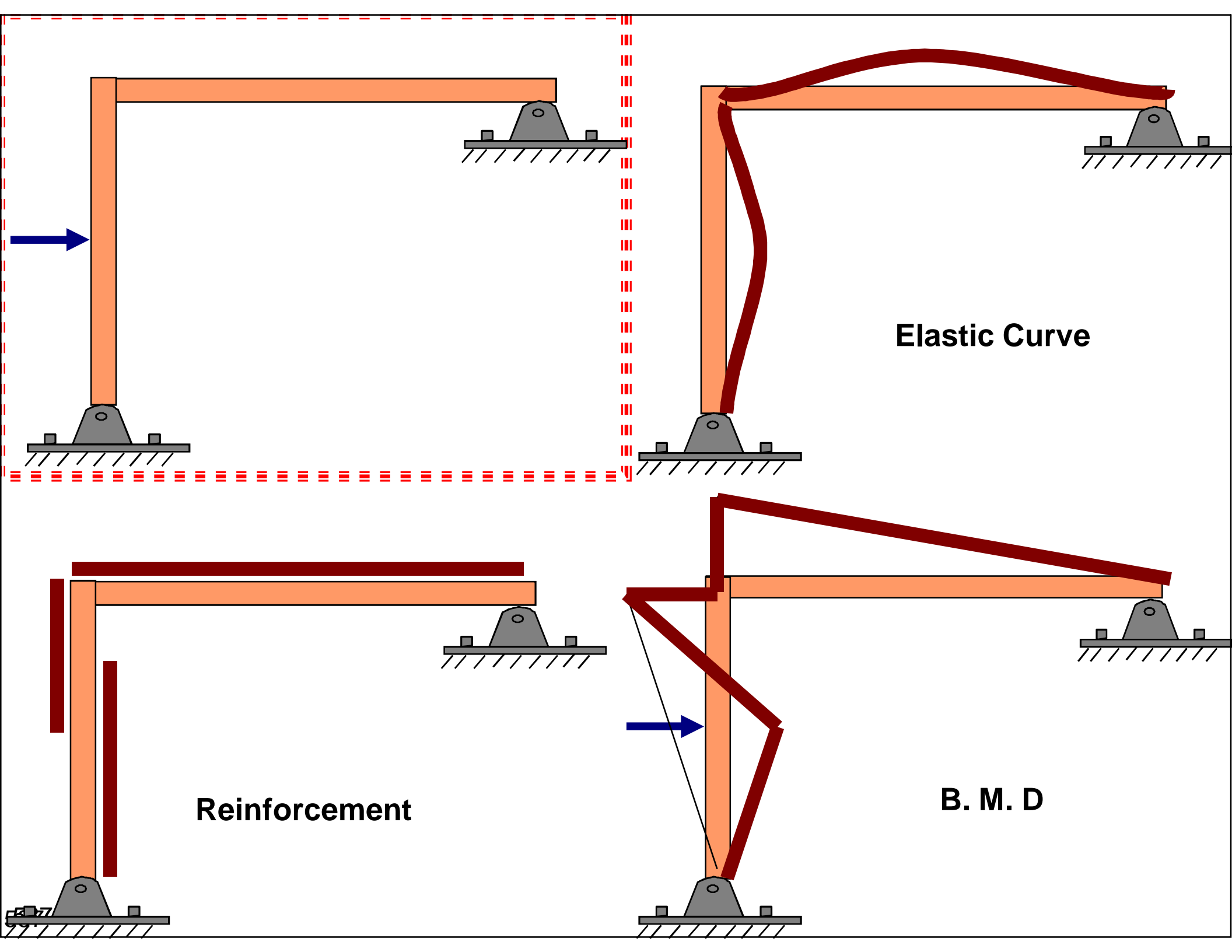
Reinforcement

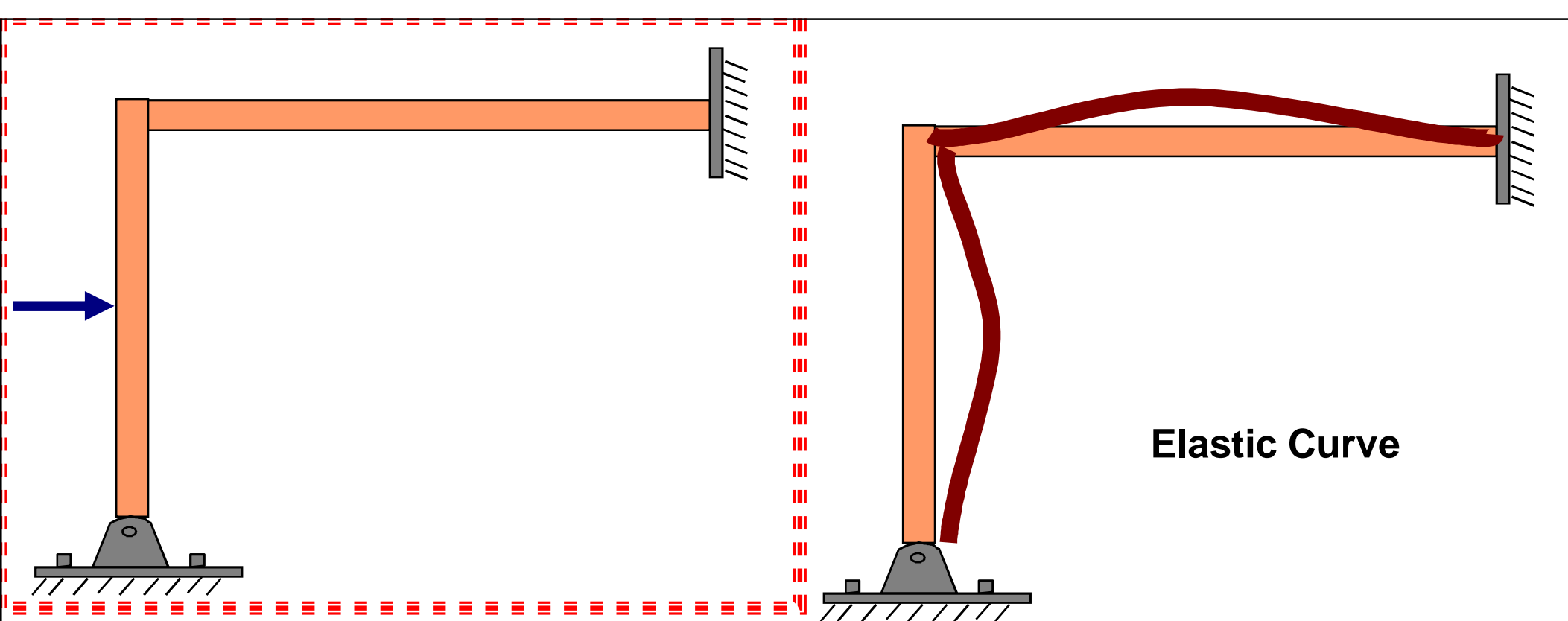


B. M. D

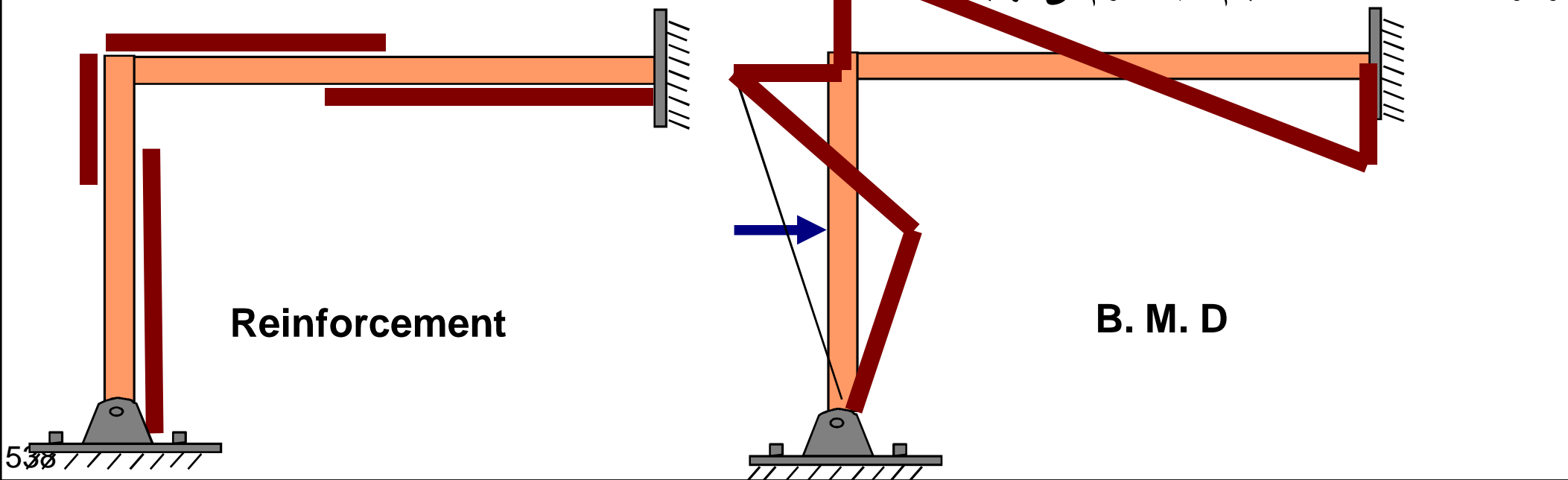


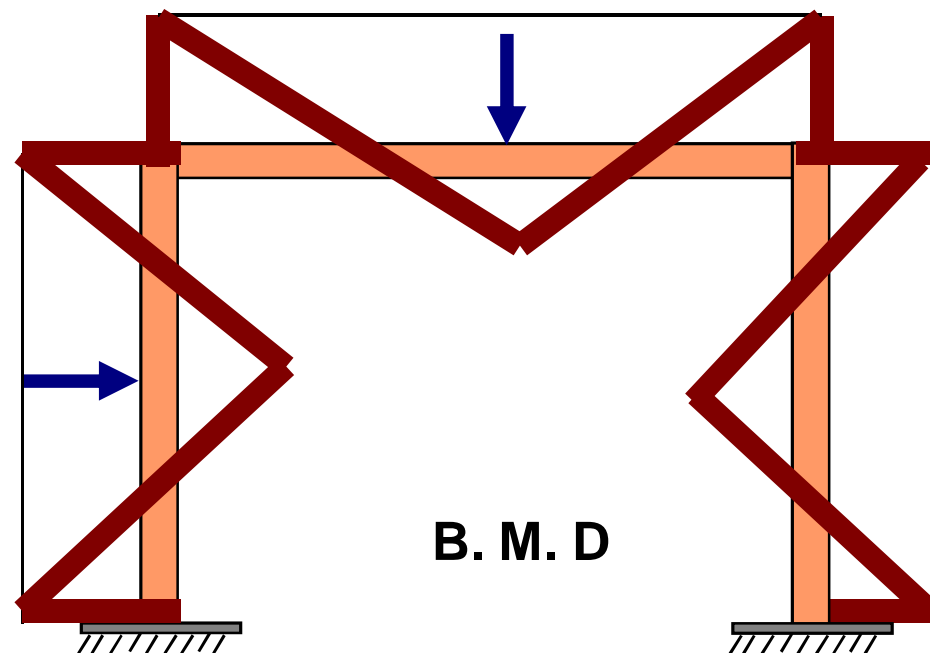
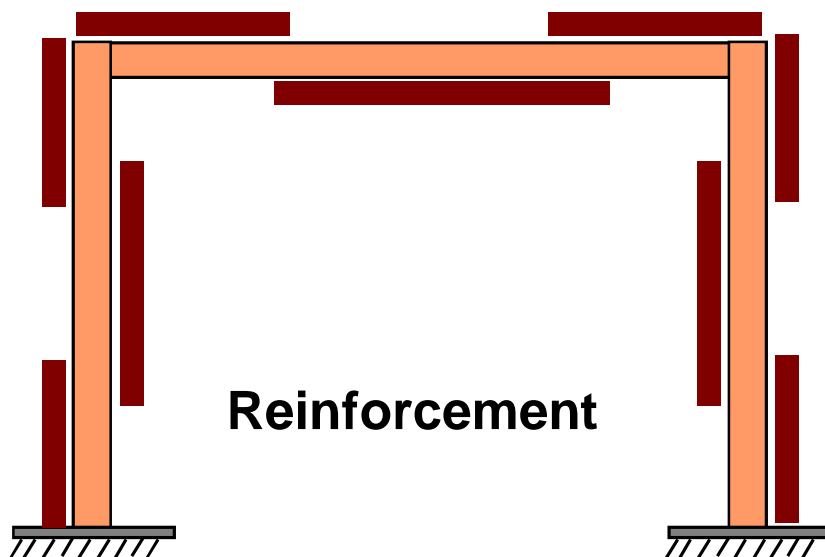
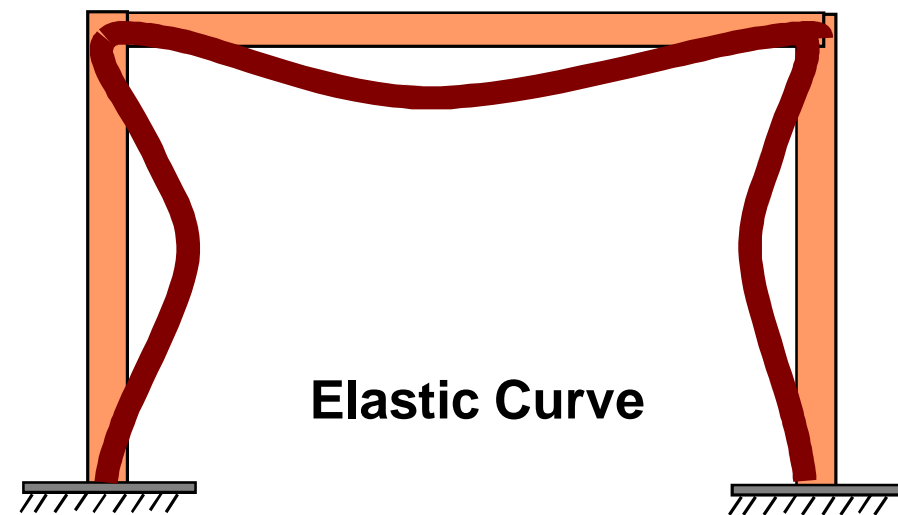
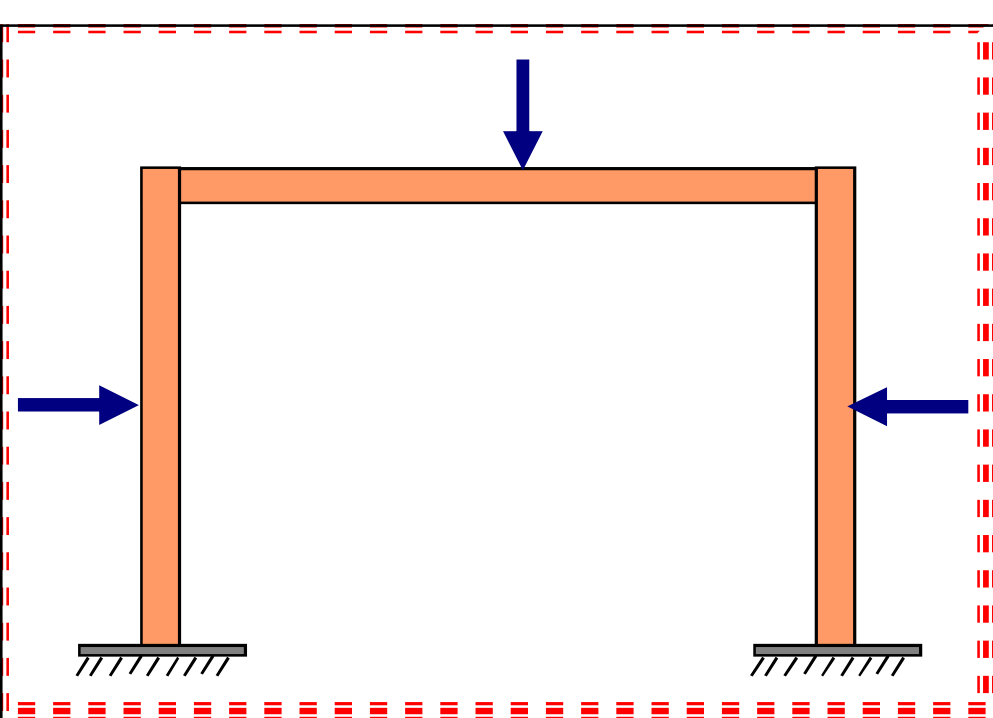






عندما يكون هناك عنصر معلوم العزم في بدايته
وهو fixed - fixed يتم قلب العزم في نهايته

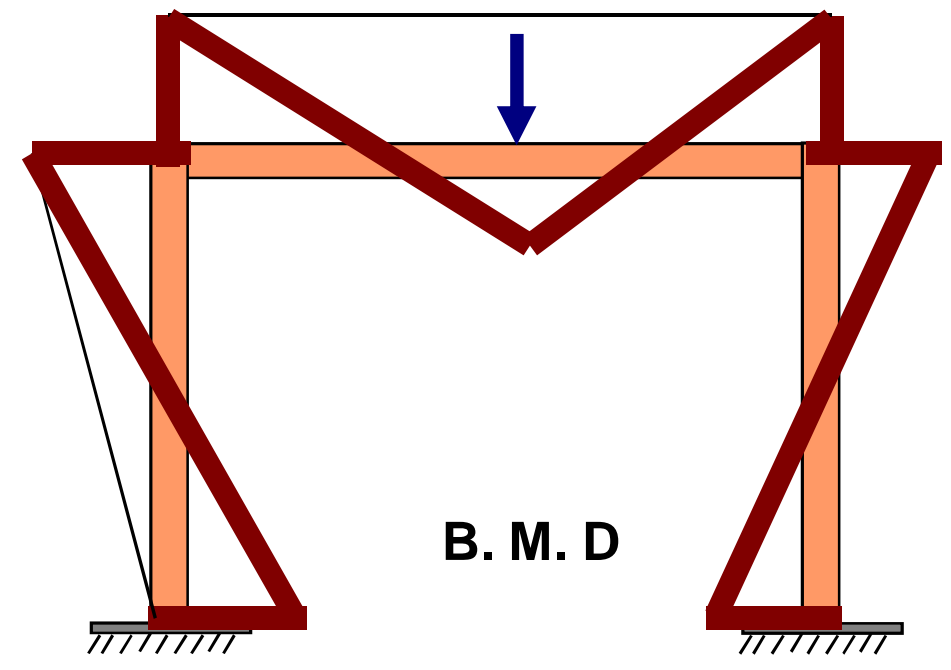
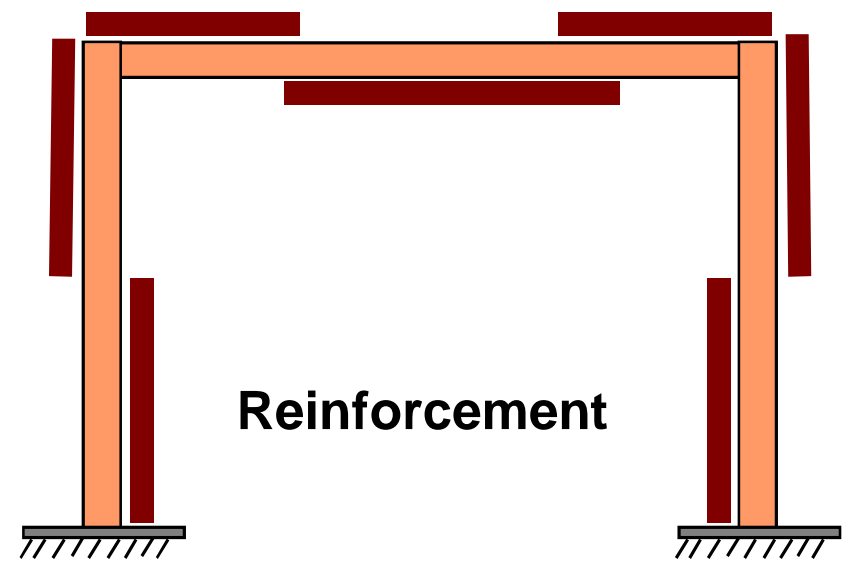
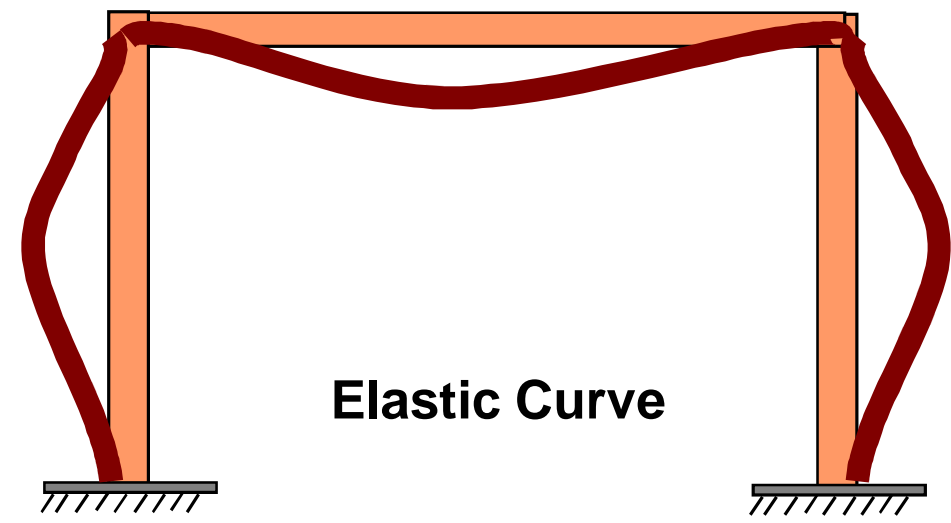
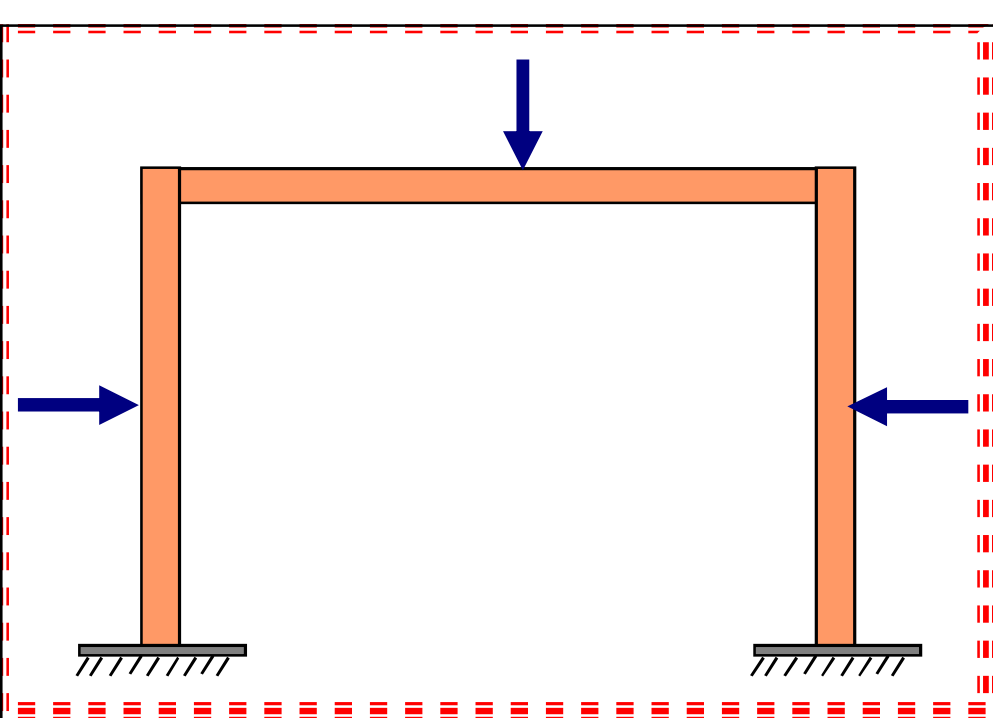


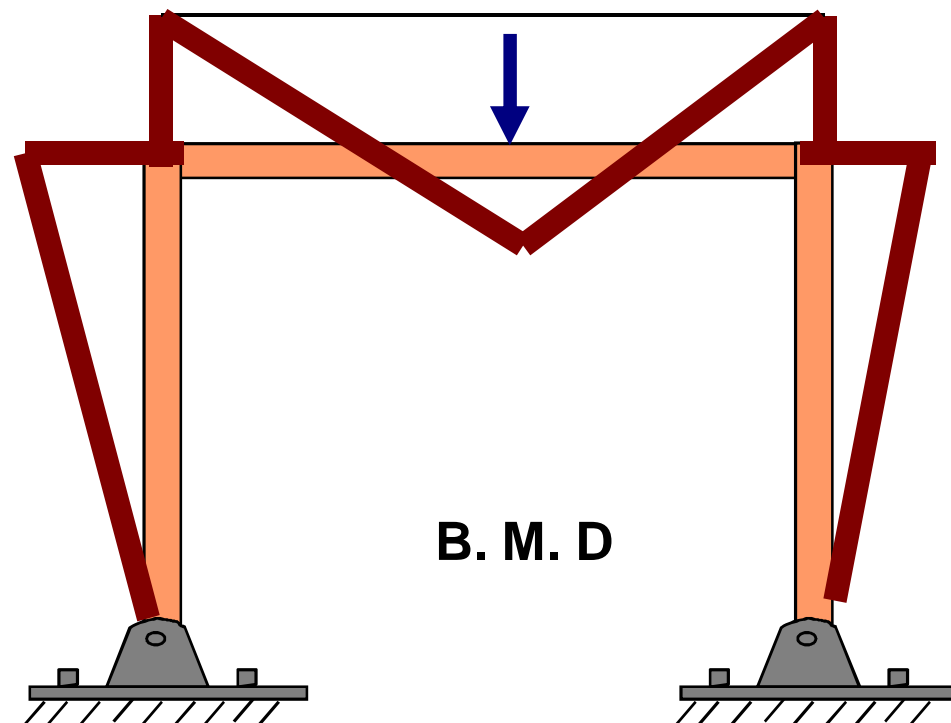
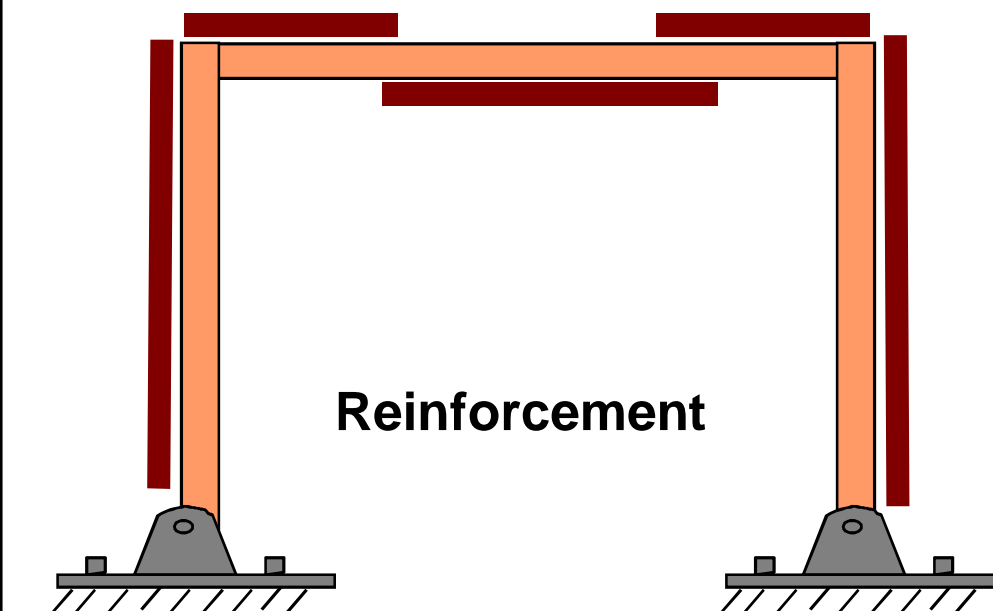
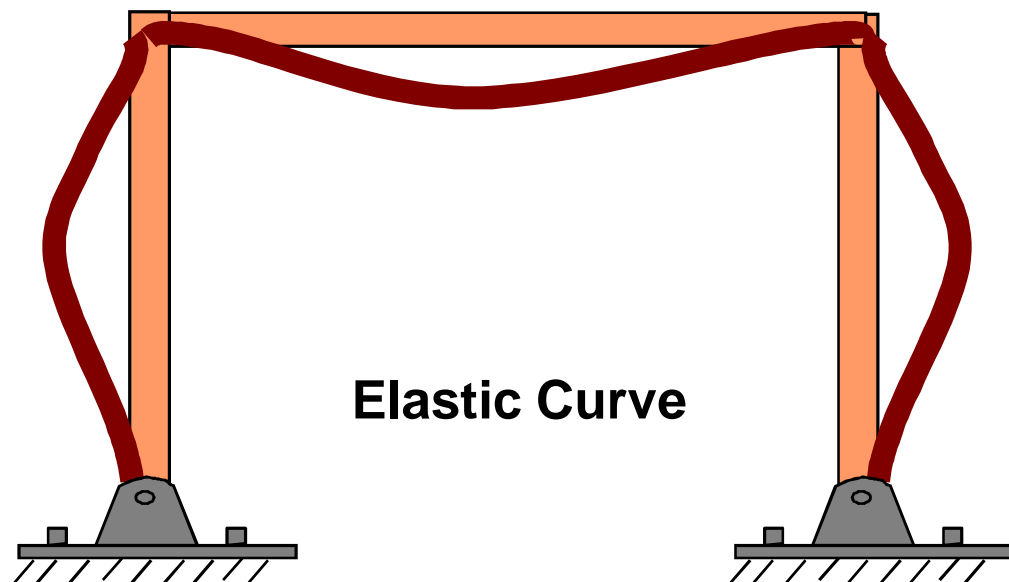
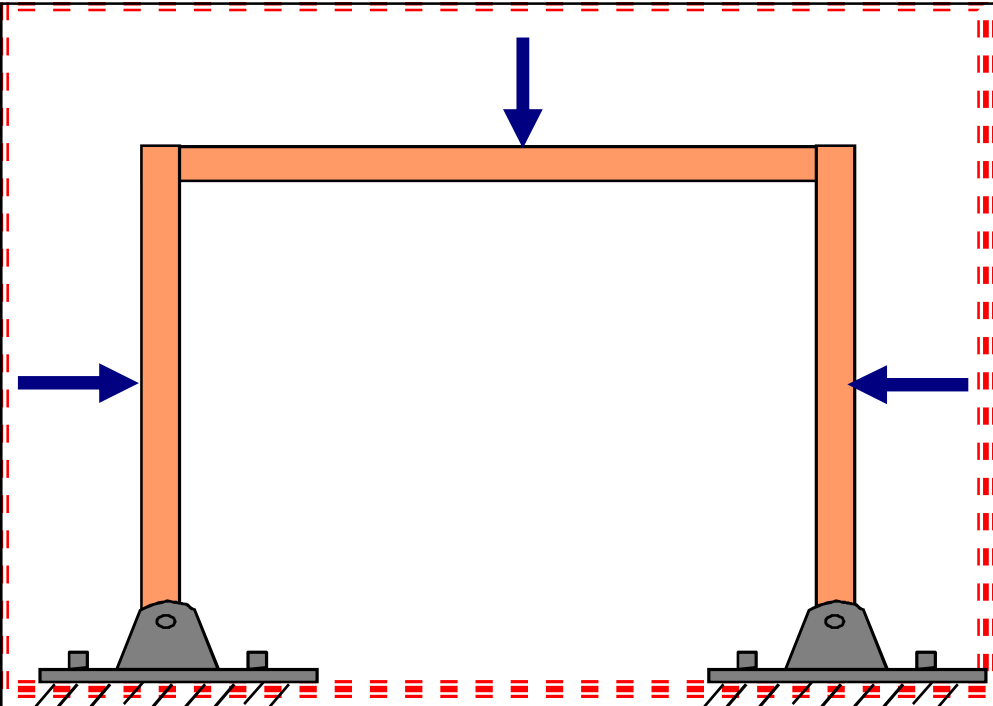


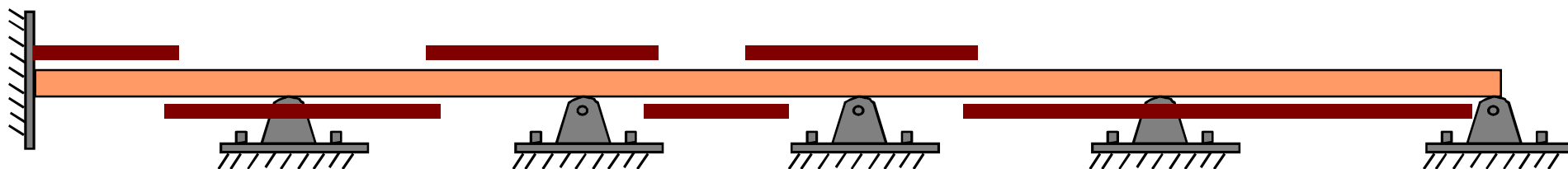
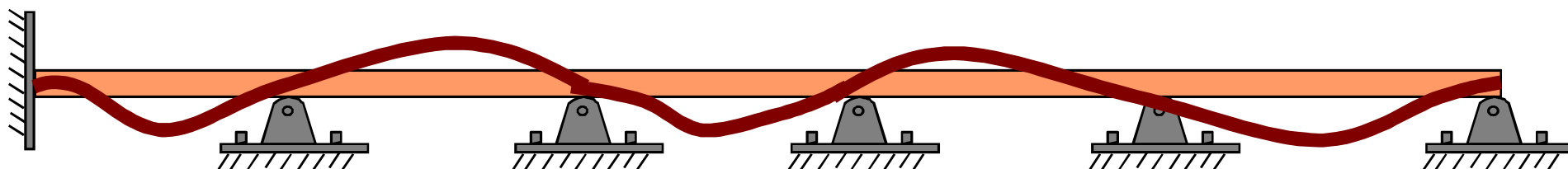
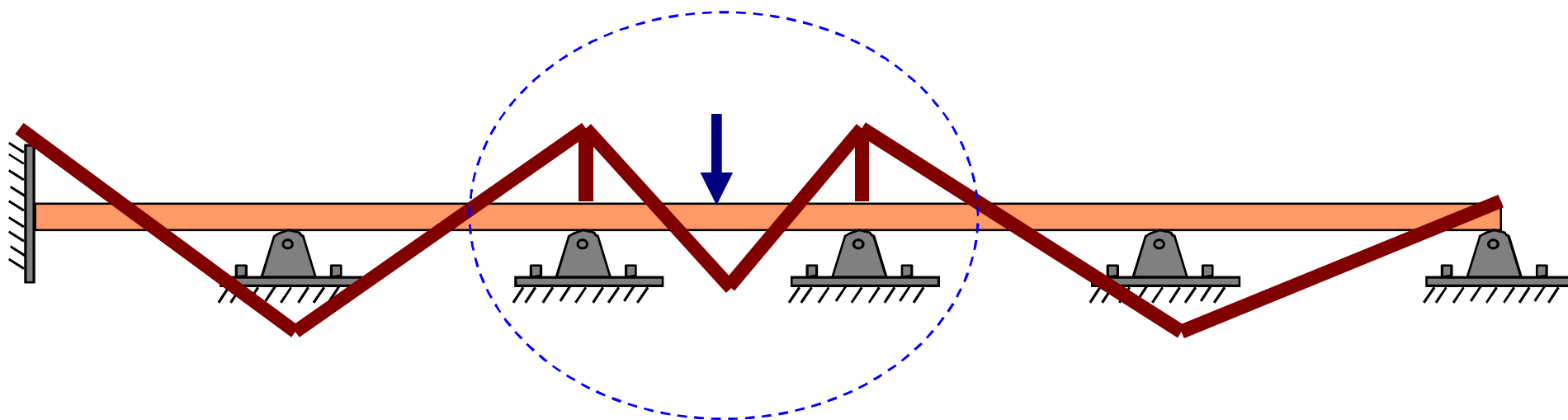
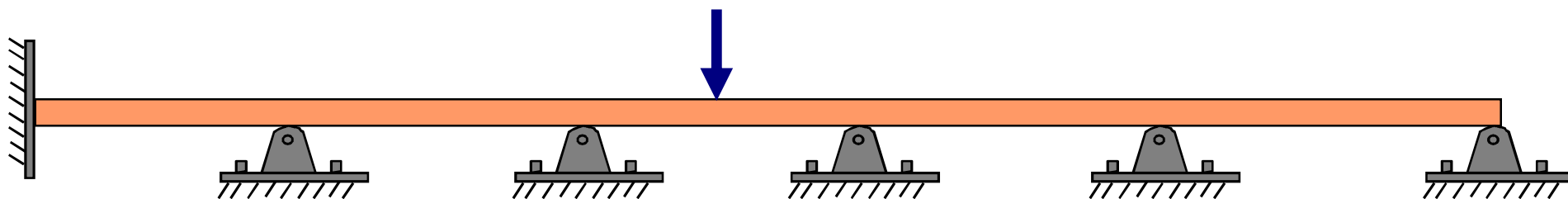
Reinforcement

Elastic Curve

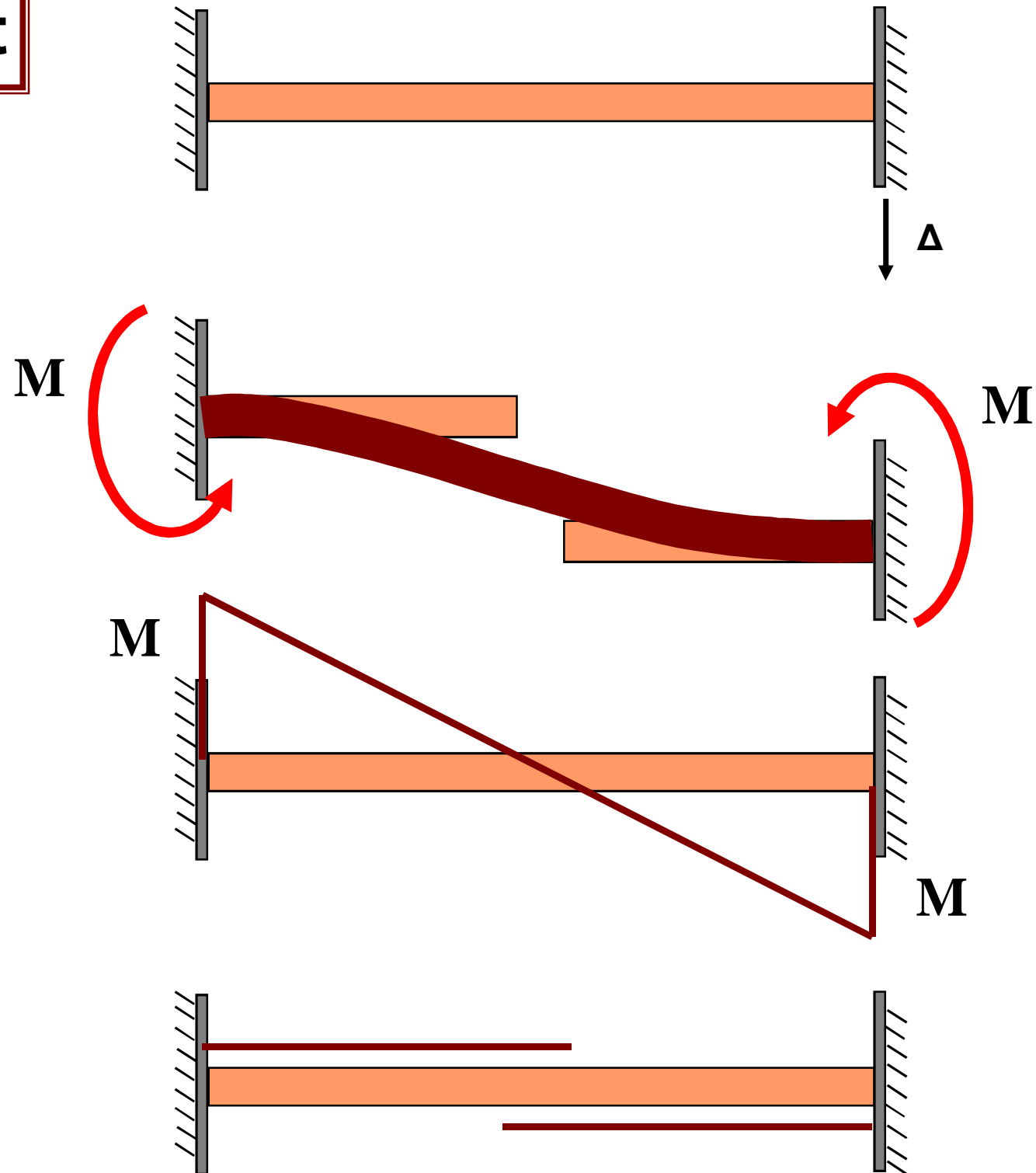
B. M. D

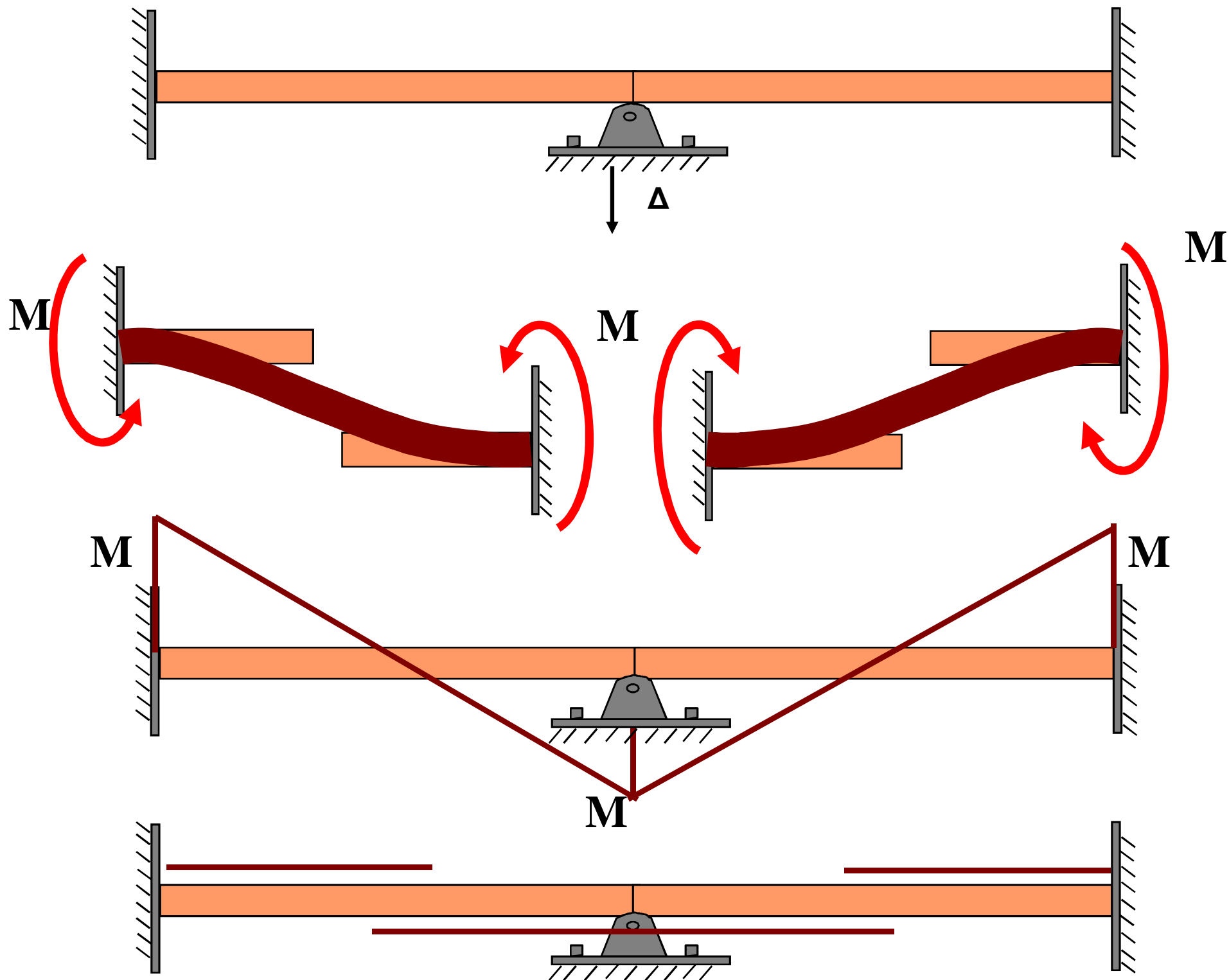


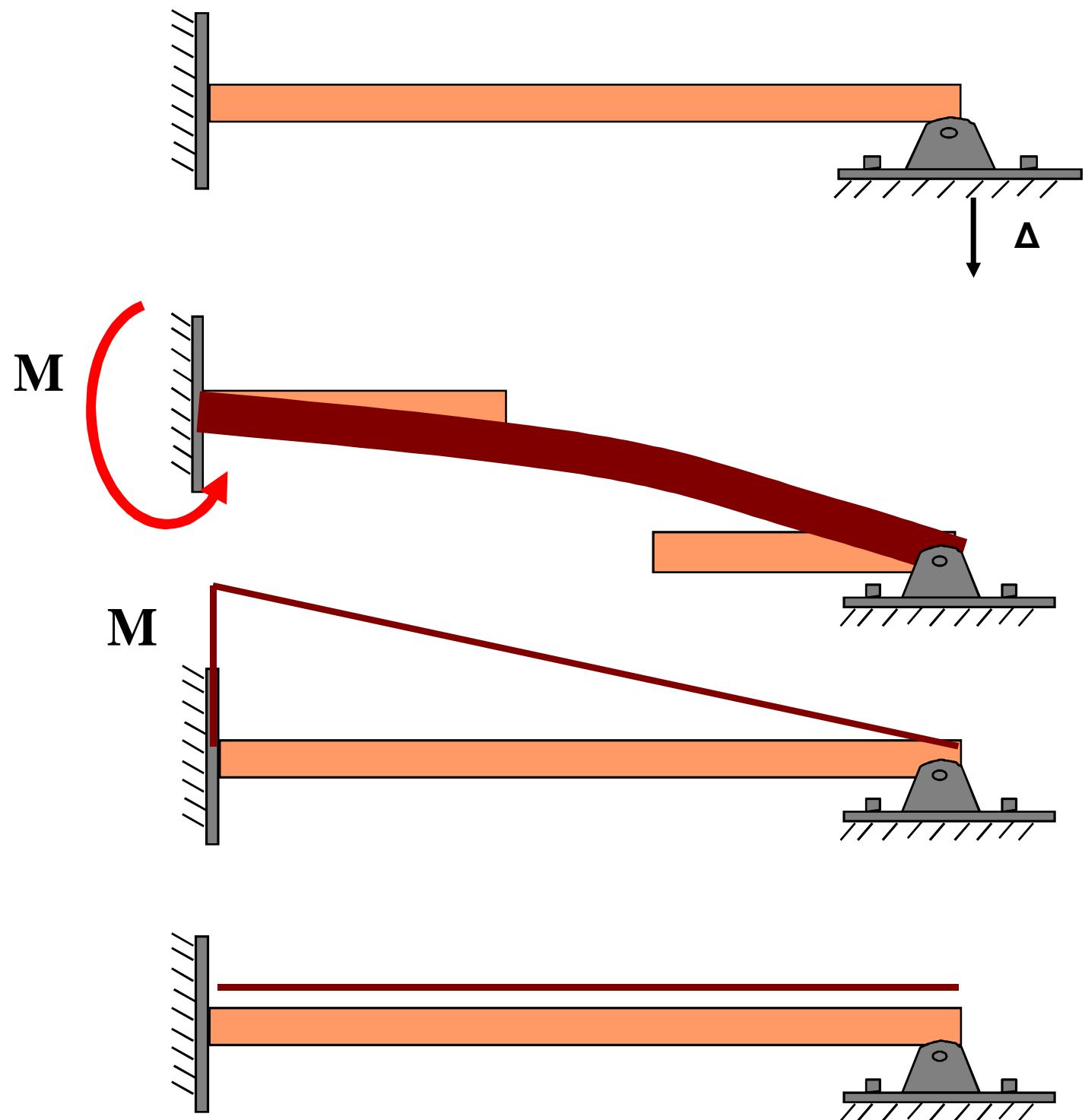


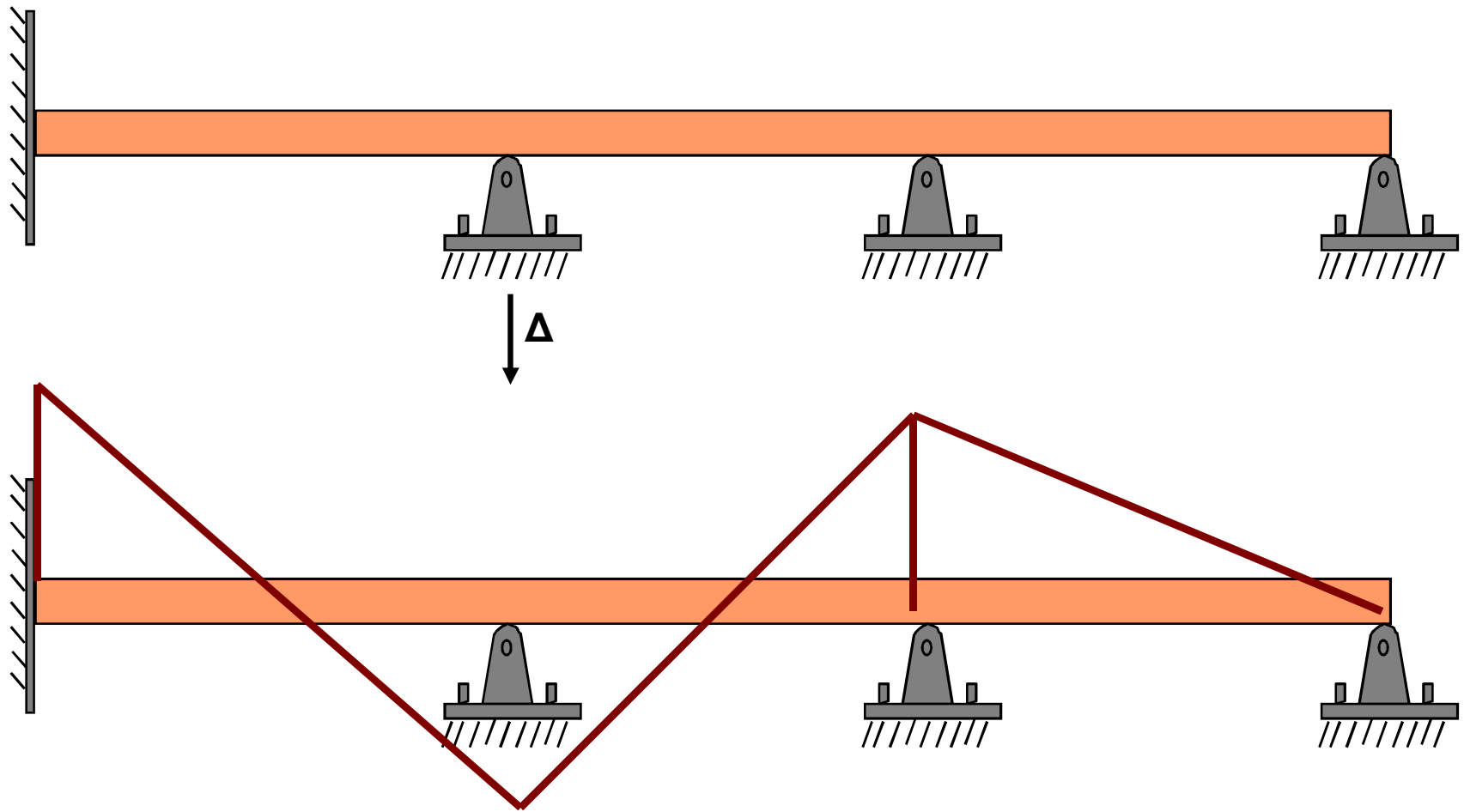


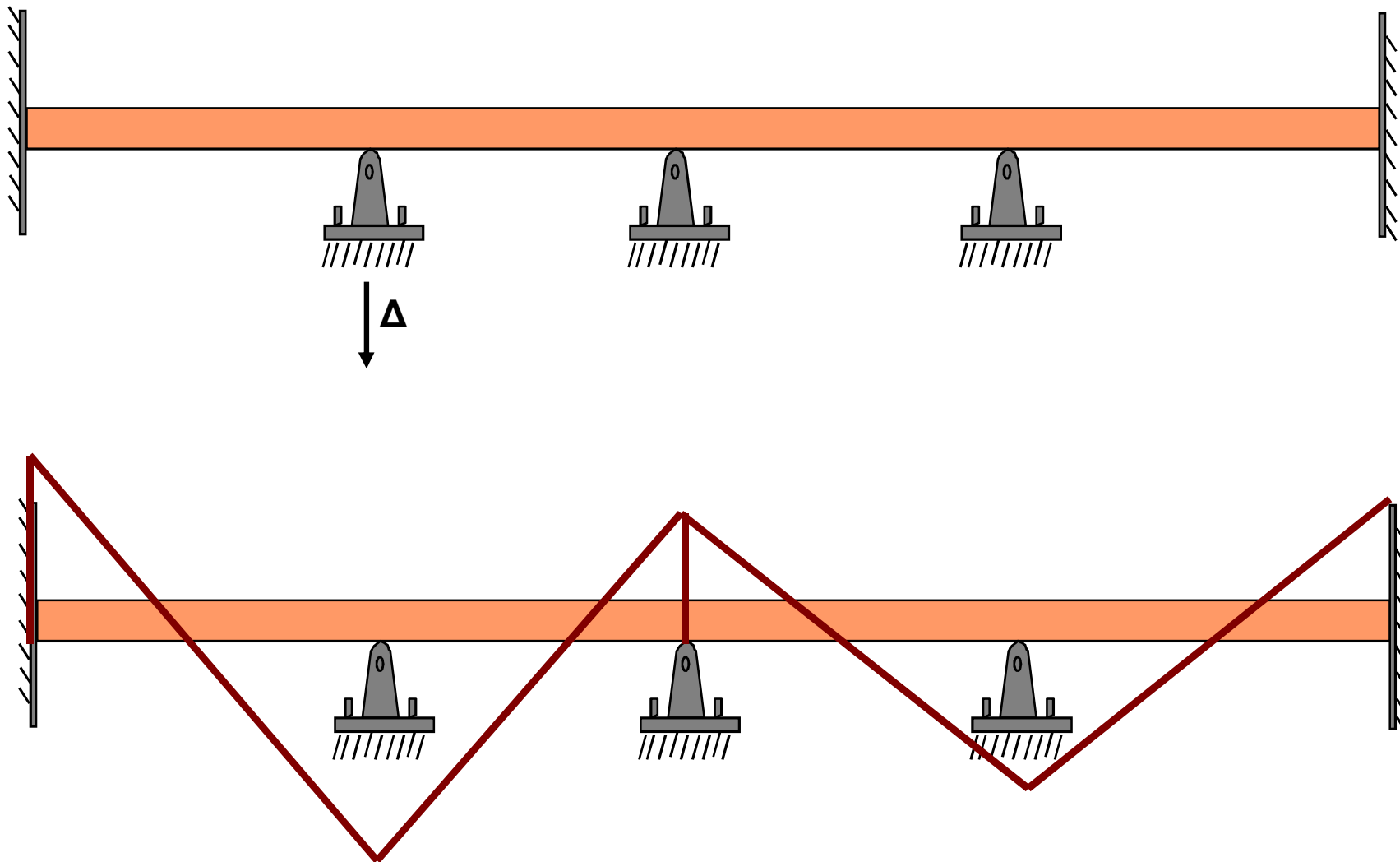
Settlement







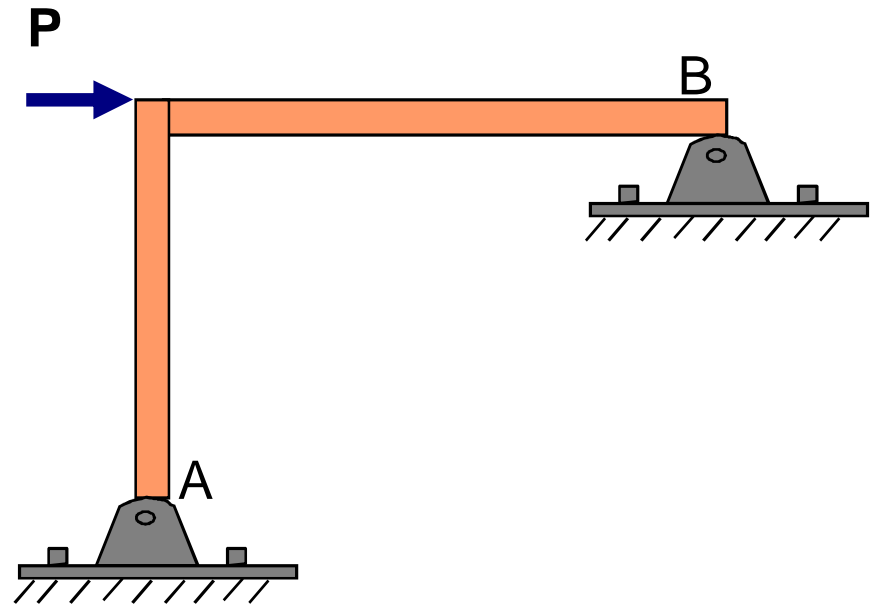
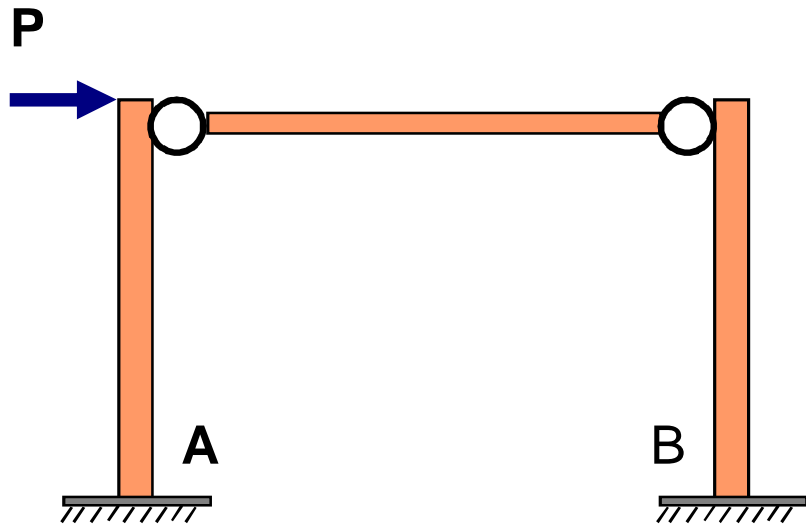




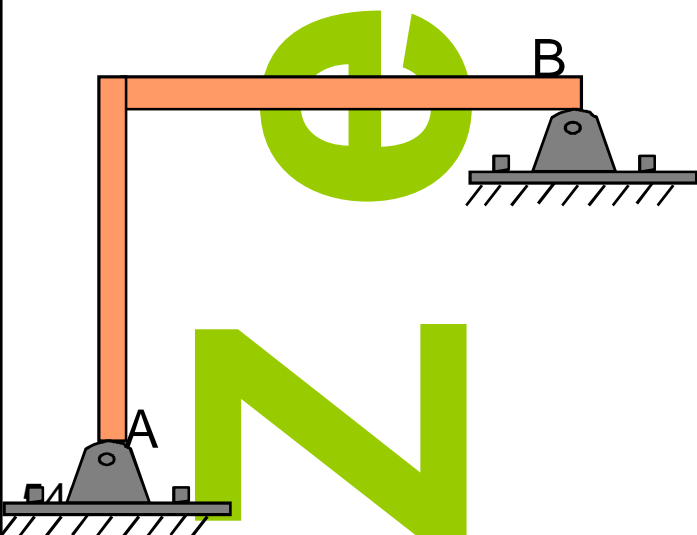
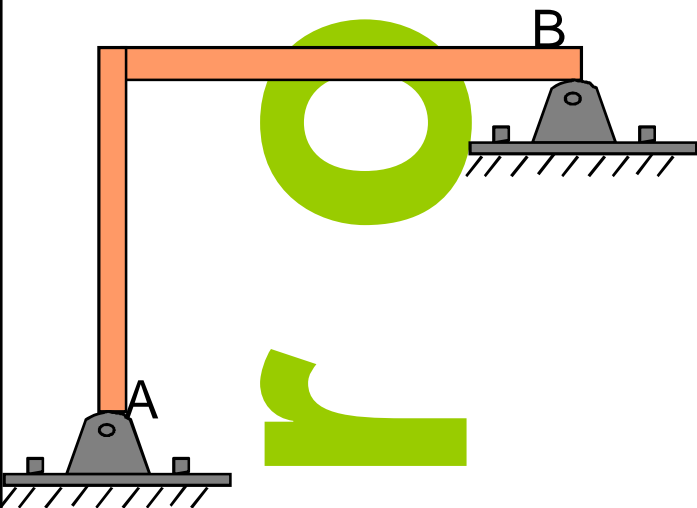
Final 2009

For the following two shapes draw B.M.D, reinforcement and elastic curve due to:

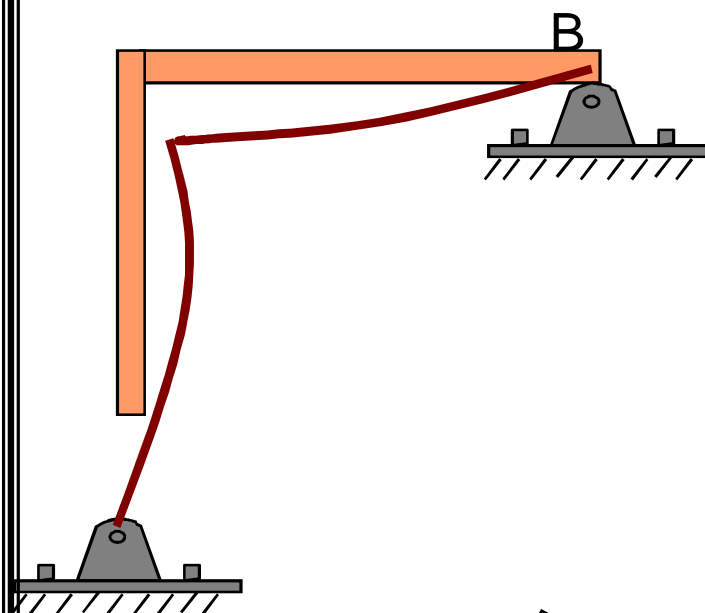
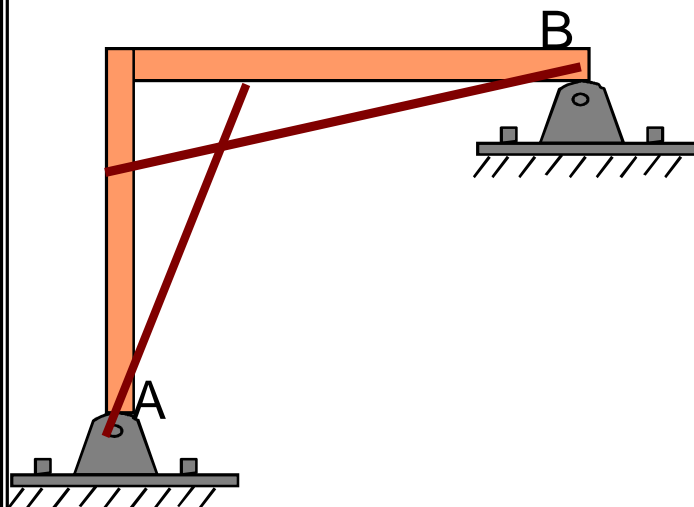
- (a) given loads.
- (b) settlement at A.
- (c) uniform rise in temperature.



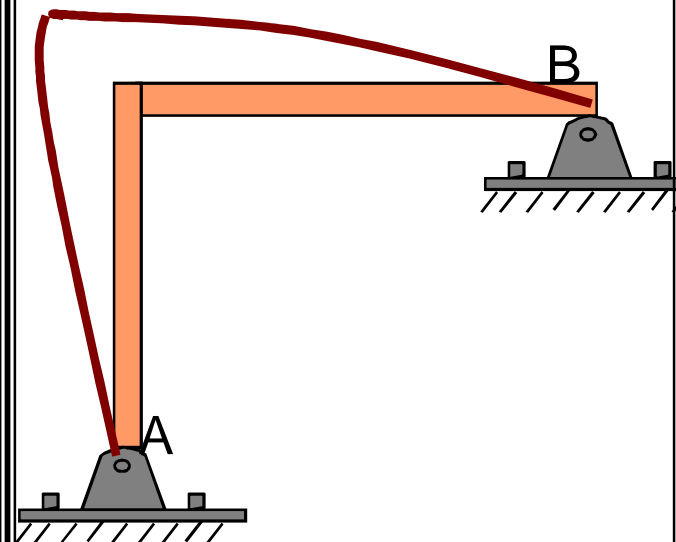
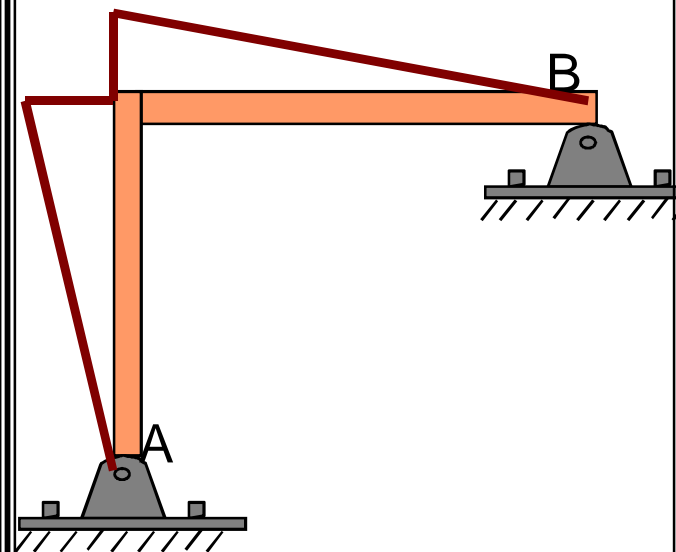
Given loads



Settlement at A

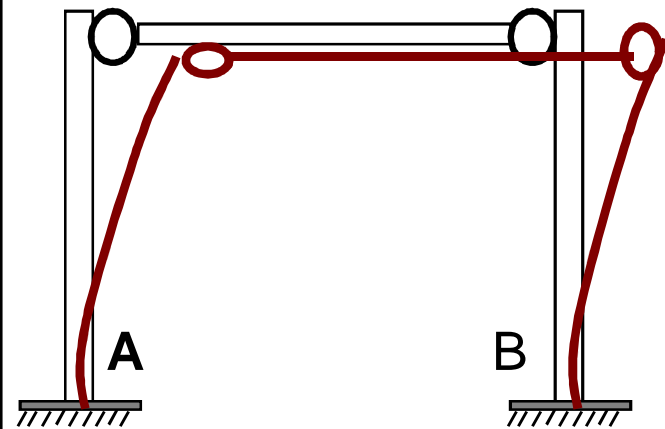
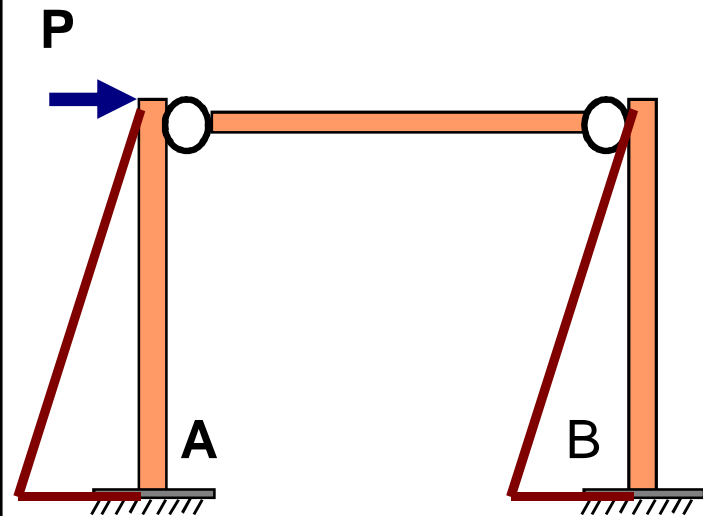


Uniform temperature

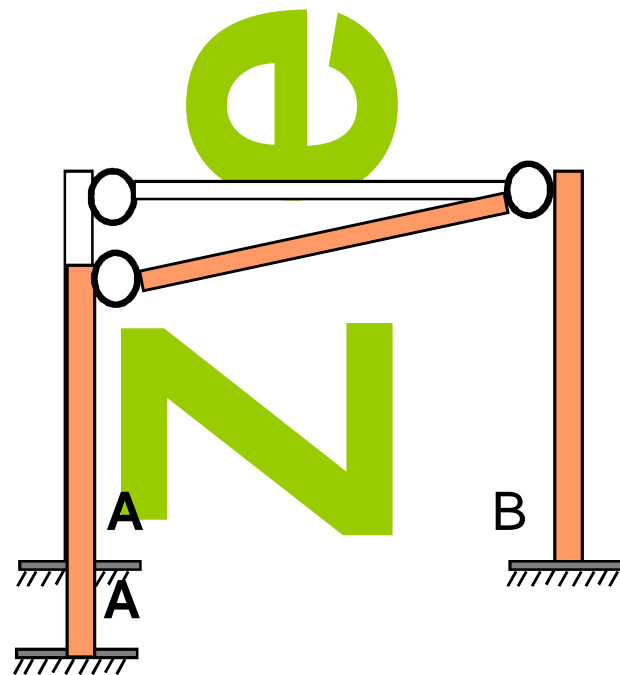
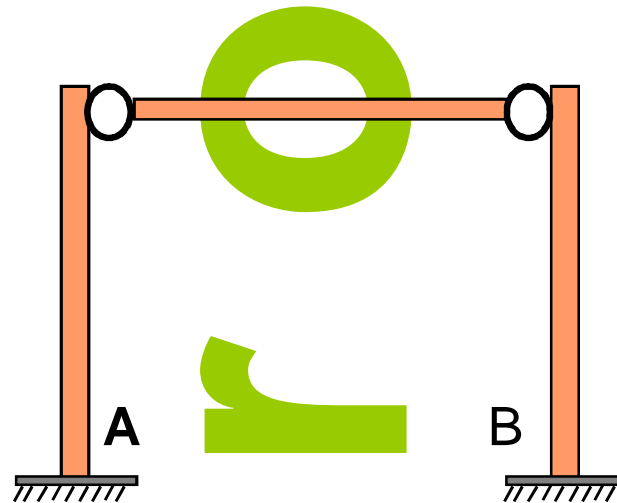


التسليح في اتجاه العزم

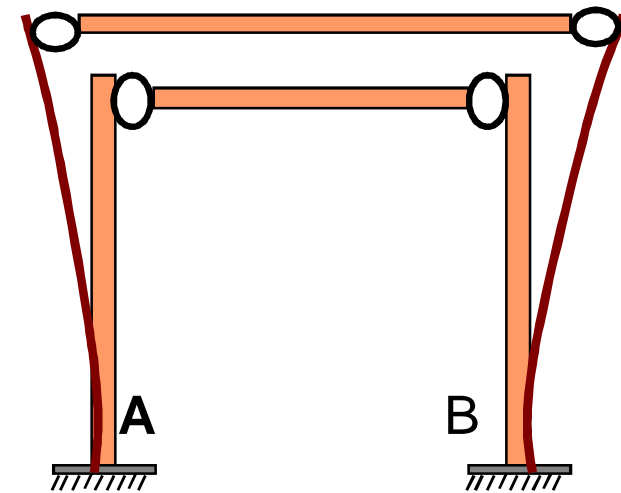
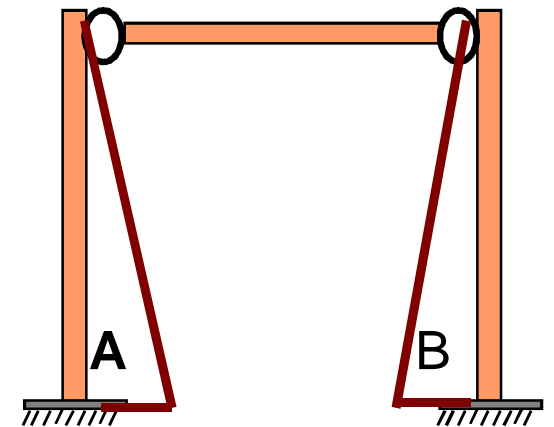
Given loads



Settlement at A



Uniform temperature



الحمد لله الذى هدانا للاسلام
ولولا الله لما اهتدينا

الحمد لله الذى وفقنا لهذا العمل
واتمنى من الله ان ينفعنا به بعد الممات

واتمنى من الله ان يوفق كل المهندسين لما يحبه الله ويرضاه

للطباعة يمكن الطباعة كل اربعة شرائح في صفحة واحدة

