

Higher Secondary - First year

# STATISTICS

Practical  
Book



## 11th standard – Statistics Practical

### Introduction

Statistical tools are important for us in daily life. They are used in the analysis of data pertaining to various activities such as production, consumption, distribution, banking and insurance, trade, transport, etc. Practical work also gives students many opportunities to use their minds to adopt suitable statistical tools and methods on various types of analysis for the given sample data.

### Objectives

- It facilitates comparison with similar data.
- Tabulation of data
- Compares the tabular data with diagrammatic representation of data
- Represents the data in a graph
- Presents the data in suitable diagrams
- Distinguishes diagrammatic and graphical representation of data
- Calculates the mathematical averages and the positional averages
- Computes quartiles, Deciles, Percentiles and interprets
- Measures the spread or dispersion
- Understands the theorems on probability and applies in problems
- Measures the Skewness
- Fittings Binomial and poisson distribution

### Instructions To Students

Students must attend all the practical classes. They must also remember that there is a great degree of co-ordination between theory problems and practical problems.

- ❖ The following are some of the items that they must bring to the Practical Classes.
  - Practical observation note book
  - Practical record
  - Pencil sharpener
  - Eraser
  - A measuring scale
  - Graph sheets
  - Compass and protractor
  - Calculator
- ❖ Come prepared with theory part of the practical subject.
- ❖ They should submit the practical records periodically for correction and evaluation.
- ❖ They must maintain strict discipline and silence in the statistical laboratory.
- ❖ They should write the date and experiment number in their observation note books.

**Students must answer any three out of five questions from the following topics**

1. Formation of Frequency Table
2. Diagrammatic Representation of data
3. Graphical Representation of data
4. Measures of Central Tendency
5. Measures of Dispersion
6. Measures of Skewness
7. Simple problem in probability
8. Probability Mass Function and Probability Density Function
9. Computation of Mean and Variance for Random Variables
10. Fitting Binomial Distribution
11. Fitting Poisson Distribution

## MODEL QUESTION PAPER I

**Marks – 15    Duration: 1 ½ hrs.**

**Answer any three of the following.**

**3 × 5 = 15**

1. Construct a stem and leaf plot for the following and find range, Median and Mode.  
1.13, 0.72, 0.91, 1.44, 1.03, 0.88, 0.99, 0.73, 0.91, 0.98, 1.21, 0.79, 1.14, 1.19, 1.08, 0.94, 1.06, 1.11, 1.01.
2. Administration of a school wished to initiate suitable preventive measures against breakage of equipments in its chemistry laboratory. Information collected about breakage of equipment occurred during the year 2017 in the laboratory are given below.

| Equipments       | Burette | Conical flask | Test tube | Pipette |
|------------------|---------|---------------|-----------|---------|
| No. of Breakages | 45      | 75            | 150       | 30      |

Draw pareto diagram for the above data. Which equipment requires more attention in order to reduce breakages?

3. Find the  $Q_1$ ,  $Q_3$ ,  $D_7$ , of the frequency distribution given below.

| Marks in Statistics | < 10 | 10 - 20 | 20 - 30 | 30 - 40 | 40 - 50 | 50 - 60 | 60 - 70 | > 70 |
|---------------------|------|---------|---------|---------|---------|---------|---------|------|
| Number of students  | 8    | 12      | 20      | 32      | 30      | 28      | 12      | 4    |

4. Given three identical boxes I, II and III containing two coins. In Box I both coins are gold coin, in Box II both are silver coin, in Box III there is one gold and one silver coin. A person chooses a box at random and takes out a coin if the coin is gold what is the probability that the other coin in the box is also gold?
5. A set of 4 coins are tossed 64 times. The number of occurrences of head is tabulated as follows.

| Number of Heads | 0 | 1  | 2  | 3  | 4 |
|-----------------|---|----|----|----|---|
| Number of times | 3 | 15 | 23 | 17 | 6 |

Fit a Binomial distribution for the foresaid data and find the expected frequencies.

## ANSWERS FOR MODEL QUESTION PAPER - I

1. **Aim:** To construct a stem and leaf plot and finding the values of range, Median and Mode.

### Formula:

Range :  $L - S$

Median : Mid value of the data

Mode : Term occurring most frequently

### Calculation:

Write the numbers in ascending order

0.72, 0.73, 0.79, 0.88, 0.91, 0.91, 0.94, 0.98, 0.99, 1.01, 1.03, 1.06, 1.08, 1.11, 1.13, 1.14, 1.19, 1.21, 1.39, 1.44.

| Stem | Leaf          |
|------|---------------|
| 0.7  | 2, 3, 9       |
| 0.8  | 8             |
| 0.9  | 1, 1, 4, 8, 9 |
| 1.0  | 1, 3, 6, 8    |
| 1.1  | 1, 3, 4, 9    |
| 1.2  | 1             |
| 1.3  | 9             |
| 1.4  | 4             |

### Range:

[From the stem and leaf plot, the lowest and highest value can be found out easily]

Highest value (L) = 1.44, Lowest value (S) = 0.72

Range =  $L - S = 1.44 - 0.72 = 0.72$

### Median:

$$\begin{aligned} &= \frac{10^{\text{th}} \text{ item} + 11^{\text{th}} \text{ item}}{2} \\ &= \frac{1.01 + 1.03}{2} = \frac{2.04}{2} = 1.02 \end{aligned}$$

**Mode:** 0.91

### Result:

Range = 0.72

Median = 1.02

Mode = 0.91

2. **Aim:** To draw a pareto diagram.

**Given:** Data

**Formula:** To find percentage

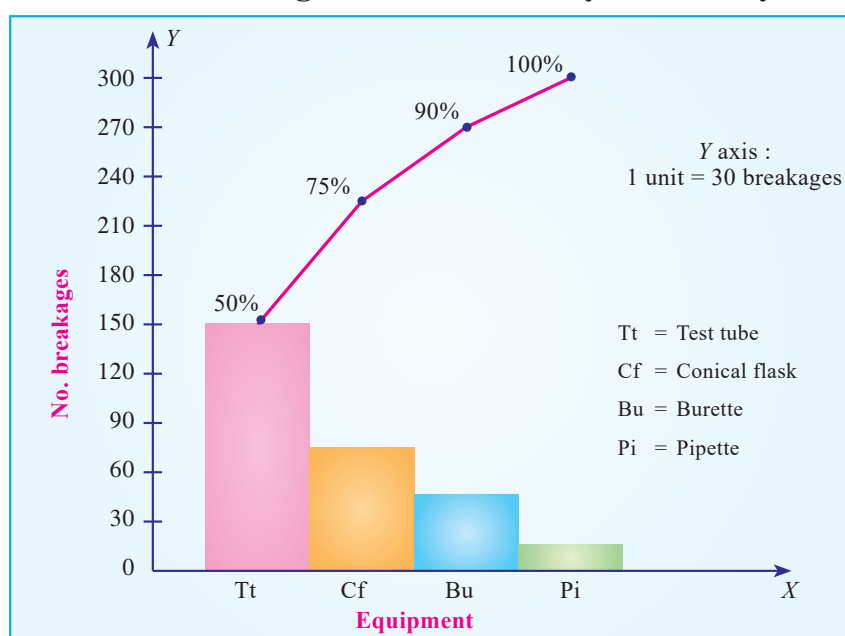
$$\frac{\text{value}}{\text{total}} \times 100$$

**Calculation:**

Arrange the given data in descending order of frequency.

| Equipment     | No. of Breakages (f) | Number of Breakages in percentage | Cumulative percentage |
|---------------|----------------------|-----------------------------------|-----------------------|
| Test tube     | 150                  | $\frac{150}{300} \times 100 = 50$ | 50                    |
| Conical flask | 75                   | $\frac{75}{300} \times 100 = 25$  | 75                    |
| Burette       | 45                   | $\frac{45}{300} \times 100 = 15$  | 90                    |
| Pipette       | 30                   | $\frac{30}{300} \times 100 = 10$  | 100                   |
| Total         | 300                  | 100                               |                       |

**No of breakages in the chemistry laboratory**



50% of breakage is due to test tube and 25% is due to conical flask.

**Result:**

- Pareto diagram is drawn
- The school administration has to focus more attention on reducing the breakages of test tubes and conical flasks.

**3. Aim:** To find  $Q_1$ ,  $Q_3$  and  $D_7$

**Formula:**

$$Q_1 = l + \frac{\frac{N}{4} - m}{f} \times c$$

$$Q_3 = l + \frac{\frac{3N}{4} - m}{f} \times c$$

$$D_7 = l + \frac{\frac{7N}{10} - m}{f} \times c$$

**Calculations:**

To find  $Q_1$  and  $Q_3$

| Marks in Statistics | No. of Students f | Cumulative frequency |
|---------------------|-------------------|----------------------|
| Below 10            | 8                 | 8                    |
| 10 – 20             | 12                | 20 (m)               |
| 20-30 (l)           | 20 f              | 40                   |
| 30 – 40             | 32                | 72                   |
| 40 – 50             | 30                | 102 (m)              |
| 50-60 (l)           | 28 f              | 130                  |
| 60 – 70             | 12                | 142                  |
| Above 70            | 4                 | 146                  |

$$\frac{N}{4} = \frac{146}{4} = 36.5$$

$$Q_1 = l + \frac{\frac{N}{4} - m}{f} \times c$$

$$l = 20, \quad \frac{N}{4} = 36.5, \quad f = 20 \quad m = 20 \quad c = 10$$

$$Q_1 = 20 + \frac{36.5 - 20}{20} \times 10$$

$$= 20 + \frac{16.5}{2} = 20 + 8.25 = 28.25$$

$$3 \frac{N}{4} = 3 \times \frac{146}{4} = 3 \times 36.5 = 109.5$$



$$Q_3 = l + \frac{\frac{3N}{4} - m}{f} \times c$$

$$l = 50, \quad \frac{3N}{4} = 109.5, \quad f = 28 \quad m = 102 \quad c = 10$$

$$Q_3 = 50 + \frac{109.5 - 102}{28} \times 10$$

$$= 50 + \frac{7.5}{28} \times 10$$

$$= 50 + 2.68 = 52.68$$

$$\frac{7N}{10} = 7 \times \frac{146}{10} = 102.2$$

$$D_7 = l + \frac{\frac{7N}{10} - m}{f} \times c$$

$$l = 50, \quad \frac{7N}{10} = 102.2, \quad f = 28 \quad m = 102 \quad c = 10$$

$$D_7 = 50 + \frac{102.2 - 102}{28} \times 10$$

$$= 50 + \frac{0.2}{28} \times 10 = 50 + \frac{20}{28}$$

$$= 50 + 0.71 = 50.71$$

### Result:

$$Q_1 = 28.25, \quad Q_3 = 52.68, \quad D_7 = 50.71$$

### 4. Aim: To find the probability by using Baye's theorem

**Formula:** 
$$P(E_i/A) = \frac{P(E_i) P(A/E_i)}{\sum_{i=1}^3 P(E_i) P(A/E_i)}$$

### Calculation:

$E_1, E_2$  and  $E_3$  be the events that the boxes I, II, III are chosen respectively.

$$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$$

Let A be the event that the gold coin is drawn

$$P(A/E_1) = \frac{2}{2} = 1$$

$$P(A/E_2) = \frac{0}{2} = 0$$

$$P(A/E_3) = \frac{1}{2}$$



$$P(E_1/A) = \frac{\frac{1}{3} \times 1}{\frac{1}{3} \times 1 + \frac{1}{3} \times 0 + \frac{1}{3} \times \frac{1}{2}} = \frac{\frac{1}{3}}{\frac{1}{3} + \frac{1}{6}}$$

$$= \frac{\frac{1}{3}}{\frac{2+1}{6}} = \frac{\frac{1}{3}}{\frac{3}{6}} = \frac{1}{3} \times \frac{6}{3} = \frac{2}{3}$$

**Result:** Probability that the second coin in the box is gold is 2/3.

**5. Aim:** To fit a Binomial distribution

**Formula:**

- (i)  $\bar{x} = \frac{\sum f x}{N}$
- (ii)  $p = \frac{\bar{x}}{n}, q = 1 - p$
- (iii)  $p(x) = n C_x p^x q^{n-x}, x = 0, 1, \dots, n$
- (iv)  $F(0) = N \times p(0)$
- (v)  $F(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} \times F(x)$

**Calculation:**  $n = 4$

| x            | f         | f x        |
|--------------|-----------|------------|
| 0            | 3         | 0          |
| 1            | 15        | 15         |
| 2            | 23        | 46         |
| 3            | 17        | 51         |
| 4            | 6         | 24         |
| <b>Total</b> | <b>64</b> | <b>136</b> |

- (i)  $\bar{x} = \frac{136}{64} = 2.125$
- (ii)  $p = \frac{\bar{x}}{n} = \frac{2.125}{4} = 0.53125 = 0.53$   
 $q = 1 - p = 1 - 0.53 = 0.47$



$$(iii) \quad p(x) = 4c_x (0.53)^x (0.47)^{4-x}, \quad x = 0, 1, \dots, 4$$

$$(iv) \quad F(0) = N \times p(0)$$

$$p(0) = 4c_0 (0.53)^0 (0.47)^{4-0}$$

$$= 1 \times 1 \times (0.47)^4 = 0.05$$

$$F(0) = 64 \times 0.05 = 3.2 \cong 3$$

$$(v) \quad F(x+1) = \frac{n-x}{x+1} \times \frac{p}{q} \times F(x)$$

When  $x = 0$

$$F(0+1) = \frac{4-0}{0+1} \times \frac{0.53}{0.47} \times F(0)$$

$$= \frac{4}{1} \times 1.13 \times 3.2$$

$$F(1) = 14.464 \cong 14$$

When  $x = 1$

$$F(1+1) = \frac{4-1}{1+1} \times \frac{0.53}{0.47} \times F(1)$$

$$= \frac{3}{2} \times 1.13 \times 14.464$$

$$F(2) = 24.54 \cong 25$$

When  $x = 2$

$$F(2+1) = \frac{4-2}{2+1} \times \frac{0.53}{0.47} \times F(2)$$

$$= \frac{2}{3} \times 1.13 \times 24.52$$

$$F(3) = 18.47 \cong 18$$

When  $x = 3$

$$F(3+1) = \frac{4-3}{3+1} \times \frac{0.53}{0.47} \times F(3)$$

$$= \frac{1}{4} \times 1.13 \times 18.47$$

$$F(4) = 5.21 \cong 5$$

**Result:**

(i) The fitted binomial distribution is

$$P(X = x) = 4c_x (0.53)^x (0.47)^{4-x}, \quad x = 0, 1, 2, 3$$

(ii) The Expected frequencies are

| x                    | 0 | 1  | 2  | 3  | 4 |
|----------------------|---|----|----|----|---|
| Observed frequencies | 3 | 15 | 23 | 17 | 6 |
| Expected frequencies | 3 | 14 | 25 | 18 | 5 |

## MODEL QUESTION PAPER II

Marks – 15    Duration: 1 ½ hrs.

Answer any three of the following.

$3 \times 5 = 15$

1. The number of hours spent by a School Student on Various activities on a working day, is given below. Construct a pie chart using the angle measurement.

| Activity        | Sleep | School | Play | Homework | Others |
|-----------------|-------|--------|------|----------|--------|
| Number of hours | 8     | 6      | 3    | 3        | 4      |

Draw a pie chart to represent the above information.

2. The following is the distribution of marks obtained by 109 Students in a Subject in an institution. Find the Geometric mean.

| Marks          | 4 - 8 | 8 - 12 | 12 - 16 | 16 - 20 | 20 - 24 | 24 - 28 | 28 - 32 | 32 - 36 | 36 - 40 |
|----------------|-------|--------|---------|---------|---------|---------|---------|---------|---------|
| No of Students | 6     | 10     | 18      | 30      | 15      | 12      | 10      | 6       | 2       |

3. The wholesale price of a commodity for seven consecutive days in a month is a follows.

| Days                    | 1   | 2   | 3   | 4   | 5   | 6   | 7   |
|-------------------------|-----|-----|-----|-----|-----|-----|-----|
| Commodity/Price/Quintal | 240 | 260 | 270 | 245 | 255 | 286 | 264 |

Find variance and standard deviation

4. A number is selected randomly from the digits 11 through 19. Consider the events.

$$A = \{11, 14, 16, 18, 19\}, B = \{12, 14, 18, 19\}, C = \{13, 15, 18, 19\}$$

Find (i)  $P(A/B)$  (ii)  $P(A/C)$  (iii)  $P(B/C)$  (iv)  $P(B/A)$

5. The p.d.f of a continuous random variable X is given by

$$f(x) = \begin{cases} \frac{x}{2}, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases} \quad \text{find its mean and variance.}$$

## ANSWERS FOR MODEL QUESTION PAPER II

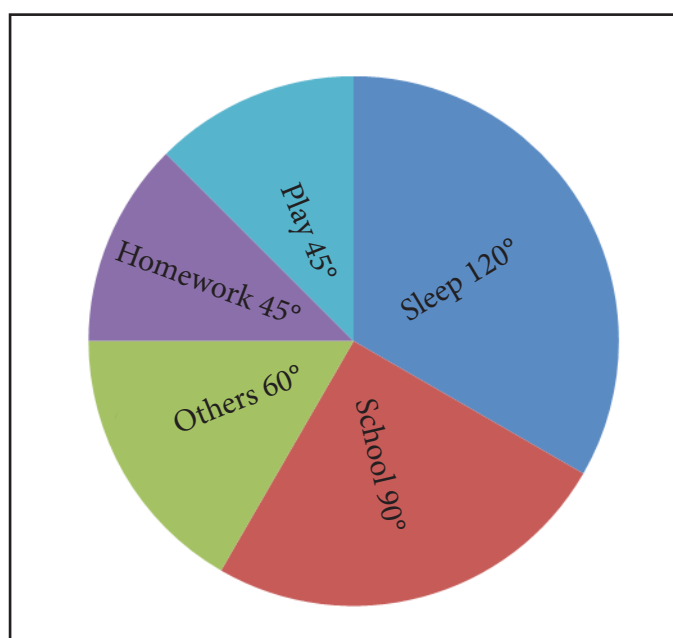
1. **Aim:** Construct a pie chart using the angle measurement

### Formula:

The central angle of a Component is =  $[\text{value of the Component} / \text{Total value}] \times 360^\circ$

### Calculation:

| Activity | Duration in hours | Central angle                               |
|----------|-------------------|---|
| Sleep    | 8                 | $\frac{8}{24} \times 360^\circ = 120^\circ$ |
| School   | 6                 | $\frac{6}{24} \times 360^\circ = 90^\circ$  |
| Play     | 3                 | $\frac{3}{24} \times 360^\circ = 45^\circ$  |
| Homework | 3                 | $\frac{3}{24} \times 360^\circ = 45^\circ$  |
| Others   | 4                 | $\frac{4}{24} \times 360^\circ = 60^\circ$  |
| Total    | 24                | $360^\circ$                                 |



**2. Aim:** To find the Geometric mean.

**Formula:**

$$G.M = \text{Anti log} \left[ \frac{\sum_{i=1}^n f_i \log x_i}{N} \right]$$

**Calculation:**

| Marks   | Midpoint ( $x_i$ ) | $f_i$ | $\log x_i$ | $f_i \log x_i$ |
|---------|--------------------|-------|------------|----------------|
| 4 – 8   | 6                  | 6     | 0.7782     | 4.6692         |
| 8 – 12  | 10                 | 10    | 1.0000     | 10.0000        |
| 12 – 16 | 14                 | 18    | 1.1461     | 20.6298        |
| 16 – 20 | 15                 | 30    | 1.2553     | 37.6590        |
| 20 – 24 | 22                 | 15    | 1.3424     | 20.1360        |
| 24 – 28 | 26                 | 12    | 1.4150     | 16.800         |
| 28 – 32 | 30                 | 10    | 1.4771     | 14.7710        |
| 32 – 36 | 34                 | 6     | 1.5315     | 9.1890         |
| 36 – 40 | 38                 | 2     | 1.5798     | 3.1596         |
| Total   | N = 109            |       |            | 137.1936       |

$$\begin{aligned} G.M &= \text{Anti log} \left[ \frac{\sum_{i=1}^n f_i \log x_i}{N} \right] \\ &= \text{Anti log} \left[ \frac{137.1936}{109} \right] = \text{Anti log} [1.2587] \\ G.M &= 18.14 \end{aligned}$$

**Result:** Geometric mean marks of 109 students in a subject is 18.14

**3. Aim:** To find the variance and standard deviation

**Formula:**

$$\text{Variance} = \sigma^2 = \frac{\sum d^2}{n} - \left( \frac{\sum d}{n} \right)^2$$

$$\text{Standard deviation } \sigma = \sqrt{\text{variance}}$$

## Calculation:

| Observations | $d = x - A$ | $d^2$ |
|--------------|-------------|-------|
| 240          | -15         | 225   |
| 260          | 5           | 25    |
| 270          | 15          | 225   |
| 245          | -10         | 100   |
| 255          | 0           | 0     |
| 286          | 31          | 961   |
| 264          | 9           | 81    |
|              | 35          | 1617  |

$$\text{Variance} = \sigma^2 = \frac{\sum d^2}{n} - \left( \frac{\sum d}{n} \right)^2$$

$$= \frac{1617}{7} - \left( \frac{35}{7} \right)^2$$

$$= 231 - 5^2$$

$$= 231 - 25$$

$$= 206$$

$$\text{Standard deviation } \sigma = \sqrt{\text{variance}}$$

$$= \sqrt{206}$$

$$= 14.35$$

## Result:

Variance = 206

Standard deviation = 14.35

## 4. Aim: To find the conditional probability

## Formula:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/C) = \frac{P(A \cap C)}{P(C)}$$

$$P(B/C) = \frac{P(B \cap C)}{P(C)}$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

### Calculation:

$$A = \{11, 14, 16, 18, 19\}, B = \{12, 14, 18, 19\}, C = \{13, 15, 18, 19\}$$

$$A \cap B = \{14, 18, 19\} \quad A \cap C = \{18, 19\} = B \cap C$$

$$P(A) = \frac{5}{9} \quad P(B) = \frac{4}{9}$$

$$P(A \cap B) = \frac{3}{9} \quad P(A \cap C) = \frac{2}{9} = P(B \cap C)$$

The probability for the occurrence of A given that B has occurred is

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{3/9}{4/9}$$

$$P(A/B) = 3/4$$

The probability for the occurrence A given that C has occurred is

$$P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{2/9}{4/9}$$

$$P(A/C) = 2/4$$

Similarly the Conditional Probability of B given C

$$P(B/C) = \frac{P(B \cap C)}{P(C)} = \frac{2/9}{4/9}$$

$$P(B/C) = 1/2$$

The Conditional probability of B given A is

$$P(B/A) = \frac{P(B \cap A)}{P(A)} = \frac{3/9}{5/9}$$

$$P(B/A) = 3/5$$

**Result:**  $P(A/B) = 3/4$        $P(A/C) = 2/4$        $P(B/C) = 1/2$        $P(B/A) = 3/5$

**5. Aim:** To find mean and variance

### Formula:

$$E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$\text{Variance}(X) = E(X^2) - [E(X)]^2$$

### Calculation:

$$\begin{aligned} E(X) &= \int_0^2 x \left( \frac{x}{2} \right) dx \\ &= \frac{1}{2} \int_0^2 x^2 dx \\ &= \frac{1}{2} \left[ \frac{x^3}{3} \right]_0^2 \\ &= \frac{1}{2} \left[ \frac{8}{3} - 0 \right] \\ &= \frac{4}{3} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_0^2 x^2 \left( \frac{x}{2} \right) dx \\ &= \frac{1}{2} \int_0^2 x^3 dx \\ &= \frac{1}{2} \left[ \frac{x^4}{4} \right]_0^2 \\ &= \frac{1}{2} \times \frac{16}{4} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \text{Variance } (X) &= E(X^2) - [E(X)]^2 \\ &= 2 - \left( \frac{4}{3} \right)^2 \\ &= 2 - \frac{16}{9} \\ &= \frac{2}{9} \end{aligned}$$

### Result:

$$\text{Mean} = E(X) = \frac{4}{3}$$

$$\text{Variance } (X) = 2/9$$

## MODEL QUESTION PAPER III

### STATISTICS PRACTICAL

Marks – 15 Duration: 1 ½ hrs.

Answer any three of the following.

3 × 5 = 15

1. Construct a bi-variate frequency distribution table for the following data of twenty students.

|                     |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Marks in Economics  | 15 | 12 | 17 | 20 | 23 | 14 | 20 | 18 | 15 | 21 | 10 | 16 | 22 | 18 | 16 | 15 | 17 | 19 | 15 | 20 |
| Marks in Statistics | 20 | 21 | 22 | 21 | 23 | 20 | 22 | 21 | 24 | 23 | 22 | 24 | 22 | 23 | 20 | 23 | 20 | 22 | 24 | 23 |

2. Find the measures of central tendencies for the following data.

|                 |       |       |       |       |       |
|-----------------|-------|-------|-------|-------|-------|
| Wages (₹)       | 60-70 | 50-60 | 40-50 | 30-40 | 20-30 |
| No of labourers | 5     | 10    | 20    | 5     | 3     |

3. Draw Box - Whisker plot for the following

3, 5, 10, 22, 11, 20, 12, 19, 17, 17, 16

4. Two coins are tossed one by one. First throw is considered as X and second throw is considered as Y. The joint probability distribution of X and Y is given by

|       |      |      |
|-------|------|------|
| $X/Y$ | 1    | 0    |
| 1     | 0.25 | 0.25 |
| 0     | 0.25 | 0.25 |

Verify  $E(XY) = E(X)E(Y)$

5. The following mistakes per page were observed in a book

|                               |     |    |    |   |   |
|-------------------------------|-----|----|----|---|---|
| Number of mistakes (per page) | 0   | 1  | 2  | 3 | 4 |
| Number of pages               | 211 | 90 | 19 | 5 | 0 |

Fit a poisson distribution and estimate the expected frequencies.



## ANSWERS MODEL QUESTION PAPER III

### 1. Aim:

To construct a bi-variate frequency distribution.

### Calculation

Let X denote marks in Economics

Y denote marks in Statistics

| Eco/Stat | 10 - 12  | 12 - 14  | 14 - 16   | 16 - 18   | 18 - 20    | 20 - 22    | 22 - 24  |
|----------|----------|----------|-----------|-----------|------------|------------|----------|
| 18 - 20  |          |          |           |           |            |            |          |
| 20 - 22  |          | <i>l</i> | <i>ll</i> | <i>ll</i> | <i>l</i>   | <i>l</i>   |          |
| 22 - 24  | <i>l</i> |          | <i>l</i>  | <i>l</i>  | <i>lll</i> | <i>lll</i> | <i>l</i> |
| 24 - 26  |          |          | <i>ll</i> | <i>l</i>  |            |            |          |

Frequency Distribution Table

| X/Y          | 10 - 12  | 12 - 14  | 14 - 16  | 16 - 18  | 18 - 20  | 20 - 22  | 22 - 24  | Total     |
|--------------|----------|----------|----------|----------|----------|----------|----------|-----------|
| 18 - 20      |          |          |          |          |          |          |          |           |
| 20 - 22      |          | 1        | 2        | 2        | 1        | 1        |          | 7         |
| 22 - 24      | 1        |          | 1        | 1        | 3        | 3        | 1        | 10        |
| 24 - 26      |          |          | 2        | 1        |          |          |          | 3         |
| <b>Total</b> | <b>1</b> | <b>1</b> | <b>5</b> | <b>4</b> | <b>4</b> | <b>4</b> | <b>1</b> | <b>20</b> |

### 2. Aim:

To find the measures of central tendencies

### Formula:

$$\bar{x} = \frac{\sum_{i=1}^n f_i x_i}{N}, \quad x_i \text{ is the midpoint of the class interval.}$$

$$\text{Median} = l + \frac{\frac{N}{2} - m}{f} \times C$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times C$$

### Calculation

| Wages   | No of labourers (f) | Mid value $x$ | $f \times x$                        |
|---------|---------------------|---------------|-------------------------------------|
| 20 - 30 | 5                   | 25            | 125                                 |
| 30 - 40 | 10                  | 35            | 350                                 |
| 40 - 50 | 20                  | 45            | 900                                 |
| 50 - 60 | 5                   | 55            | 275                                 |
| 60 - 70 | 3                   | 65            | 195                                 |
|         | <b>N = 43</b>       |               | <b><math>\sum f x = 1845</math></b> |

$$\text{A.M} \quad \bar{x} = \frac{\sum f x}{N} = \frac{1845}{43} = 42.9$$

## Median

| Wages   | No of labourers<br>(f) | Cumulative<br>frequency |
|---------|------------------------|-------------------------|
| 20 – 30 | 5                      | 5                       |
| 30 – 40 | 10                     | 15                      |
| 40 – 50 | 20                     | 35                      |
| 50 – 60 | 5                      | 40                      |
| 60 – 70 | 3                      | 43                      |
|         | N = 43                 |                         |

$$\frac{N}{2} = \frac{43}{2} = 21.5$$

$$f = 20, m = 15, l = 40, \frac{N}{2} = 21.5, C = 10$$

$$\begin{aligned} \text{Median} &= l + \frac{\frac{N}{2} - m}{f} \times C \\ &= 40 + \frac{21.5 - 15}{20} \times 10 \\ &= 40 + \frac{6.5}{20} \times 10 \\ &= 40 + \frac{65}{20} \\ &= 40 + 3.25 = 43.25 \end{aligned}$$

## Mode

| Wages   | No of labourers<br>(f) |
|---------|------------------------|
| 20 – 30 | 5                      |
| 30 – 40 | 10                     |
| 40 – 50 | 20                     |
| 50 – 60 | 5                      |
| 60 – 70 | 3                      |

$$f_1 = 20, f_0 = 10, l = 40, f_2 = 5, c = 10$$

$$\text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 40 + \frac{20-10}{2 \times 20-10-5} \times 10$$

$$= 40 + \frac{10}{40-15} \times 10$$

$$= 40 + \frac{100}{25}$$

$$= 40 + 4 = 44$$

### Result :

Measures of Central tendencies

Arithmetic mean  $\bar{x} = 42.9$

Median = 43.25

Mode = 44

### 3. Aim: Draw box- whisker plot.

#### Formula

$$Q_1 = \left( \frac{n+1}{4} \right)^{th} \text{ item}$$

$$Q_2 = \left( \frac{n+1}{2} \right)^{th} \text{ item}$$

$$Q_3 = 3 \left( \frac{n+1}{4} \right)^{th} \text{ item}$$

### Calculation :

First arrange in the ascending order

3,5,10,11,12,16,17,17,19,20,22

$$n=11 \quad Q_1 = \left( \frac{n+1}{4} \right)^{th} \text{ item}$$

$$Q_1 = \left( \frac{11+1}{4} \right)^{th} \text{ item}$$

$$= \left( \frac{12}{4} \right)^{th} \text{ item}$$

$$= 3^{rd} \text{ item}$$

$$Q_1 = 10$$



$$\begin{aligned}Q_2 &= \left(\frac{n+1}{2}\right)^{th} \text{ item} \\&= \left(\frac{11+1}{2}\right)^{th} \text{ item} \\&= \left(\frac{12}{2}\right)^{th} \text{ item} \\&= 6^{th} \text{ item} \\Q_2 &= 16\end{aligned}$$

$$\begin{aligned}Q_3 &= 3\left(\frac{n+1}{4}\right)^{th} \text{ item} \\&= 3\left(\frac{11+1}{4}\right)^{th} \text{ item} \\&= (3 \times 3)^{th} \text{ item} \\&= 9^{th} \text{ item} \\Q_3 &= 19.\end{aligned}$$

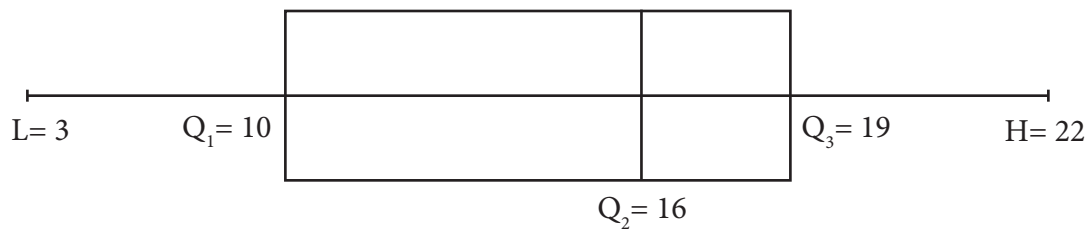
Minimum Value = 3 and Maximum Value = 22

$$L = 3$$

$$Q_1 = 10 \quad Q_2 = 16 \quad Q_3 = 19 \quad H = 22$$

$$L=3$$

$$Q_1=10 \quad Q_2=16 \quad Q_3=19 \quad H=22$$



## Result:

Box- Whisker plot is drawn using Five number summary.

#### 4. Aim

To Verify  $E(xy) = E(x) E(y)$

#### Formula

$$E(xy) = \sum_j \sum_i x_i y_j P_{ij}(x, y)$$

$$E(x) = \sum_i x_i P_i$$

$$E(y) = \sum_j y_j p_j$$

| X/Y   | 1    | 0    | Total |
|-------|------|------|-------|
| 1     | 0.25 | 0.25 | 0.5   |
| 0     | 0.25 | 0.25 | 0.5   |
| Total | 0.5  | 0.5  | 1     |

#### Calculation:

A random variable  $x, y$  can take the values 0 and 1

$$E(xy) = \sum_i \sum_j x_i y_j P(x_i, y_j)$$

$$= 1 \times 0.25 + 0 + 0.25 + 0 + 0.25 + 0 \times 0.25$$

$$= 0.25$$

$$E(x) = \sum_i x_i p_i$$

$$= 1 \times 0.5 + 0 + 0 - 5$$

$$= 0.5$$

$$E(y) = \sum_j y_j p_j$$

$$= 1 \times 0.5 + 0 \times 0.5 = 0.5$$

$$E(xy) \times E(y) = 0.5 \times 0.5 = .25$$

$$E(xy) = E(x) E(y)$$

#### Result:

$\therefore E(xy) = E(x) E(y)$  is verified

#### 5. Aim

To fit a Poisson distribution and estimate the expected frequencies

#### Formula

$$\text{i) } \bar{x} = \frac{\sum fx}{\sum f}$$

$$\text{ii) } \lambda = \bar{x}$$

$$\text{iii) } P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\text{iv) } P(0) = \frac{e^{-\lambda} \lambda^0}{0!} = e^{-\lambda} \quad \text{v) } F(0) = N \times P(0) \quad \text{vi) } F(x+1) = \frac{\lambda}{x+1} \times F(x)$$

| $x$          | $f$        | $fx$       |
|--------------|------------|------------|
| 0            | 211        | 0          |
| 1            | 90         | 90         |
| 2            | 19         | 38         |
| 3            | 5          | 15         |
| 4            | 0          | 0          |
| <b>Total</b> | <b>325</b> | <b>143</b> |

$$\text{i) Mean } \bar{x} = \frac{\sum fx}{\sum f} = \frac{143}{325} = 0.44 \quad \text{ii) } \lambda = \bar{x} = 0.44$$

$$\text{iii) } P(x=x) = \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-0.44} \times (0.44)^x}{x!}$$

$$\text{iv) } P(0) = \frac{e^{-0.44} \times (0.44)^0}{0!} = e^{-0.44} = 0.6440.$$

$$\text{v) } F(0) = N \times P(0) = 325 \times 0.6440 = 209.43.$$

$$\text{vi) } F(x+1) = \frac{\lambda}{x+1} \times F(x)$$

$$F(1) = F(0+1) = \frac{0.44}{0+1} \times 209.43 = 92.15$$

$$F(2) = F(1+1) = \frac{0.44}{1+1} \times 92.15 = 20.27$$

$$F(3) = F(2+1) = \frac{0.44}{2+1} \times 20.27 = 2.972$$

$$F(4) = F(3+1) = \frac{0.44}{3+1} \times 2.97 = 0.33$$

## Result:

Fitted Poisson distribution is

$$P(x=x) = \frac{e^{-0.44} \times 0.44^x}{x!}, x = 0, 1, 2, \dots$$

Expected Frequencies are given below

| $x$                | 0   | 1  | 2  | 3 | 4 | <b>Total</b> |
|--------------------|-----|----|----|---|---|--------------|
| Observed frequency | 211 | 90 | 19 | 5 | 0 | <b>325</b>   |
| Expected frequency | 210 | 92 | 20 | 3 | 0 | <b>325</b>   |



## MODEL QUESTION PAPER IV

### STATISTICS PRACTICAL

Marks – 15 Duration: 1 ½ hrs.

Answer any three of the following.

$3 \times 5 = 15$

1. Draw more than ogive curve for the following data showing the marks secured by the students of class XI in a school.

| Marks | No of students |
|-------|----------------|
| 0-10  | 2              |
| 10-20 | 4              |
| 20-30 | 9              |
| 30-40 | 10             |
| 40-50 | 8              |
| 50-60 | 5              |
| 60-70 | 3              |
| 70-80 | 2              |

- a) Estimate the total number of students who secured marks more than 33.
- b) Find the median of the data
2. (i) From the known data, mean = 7.35, mode = 8 and variance = 1.69 then find the Karl – Pearson coefficient of skewness.
- (ii) If  $Q_1 = 40$ ,  $Q_2 = 50$ ,  $Q_3 = 60$  find Bowley's Coefficient of skewness.
- 3) A Question Paper contains section A with 5 Questions and section B with 7 Questions. A Student is required to attempt 8 Questions in all, Selecting at least 3 from each section. In how many ways can a student select the Questions?
- 4) A discrete random variable has the following distribution function.

| $x$    | 0 | 1  | 2  | 3  | 4  | 5   | 6   | 7   | 8   |
|--------|---|----|----|----|----|-----|-----|-----|-----|
| $P(x)$ | a | 3a | 5a | 7a | 9a | 11a | 13a | 15a | 17a |

Find (i) a (ii)  $P(x < 3)$  (iii)  $P(x \geq 5)$  (iv)  $P(3 < x < 7)$

5. Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and the standard deviation 5. What Percentage of students scored (i) more than 60 marks
- (ii) less than 56 marks (iii) between 45 and 65 marks.

## ANSWERS MODEL QUESTION PAPER IV

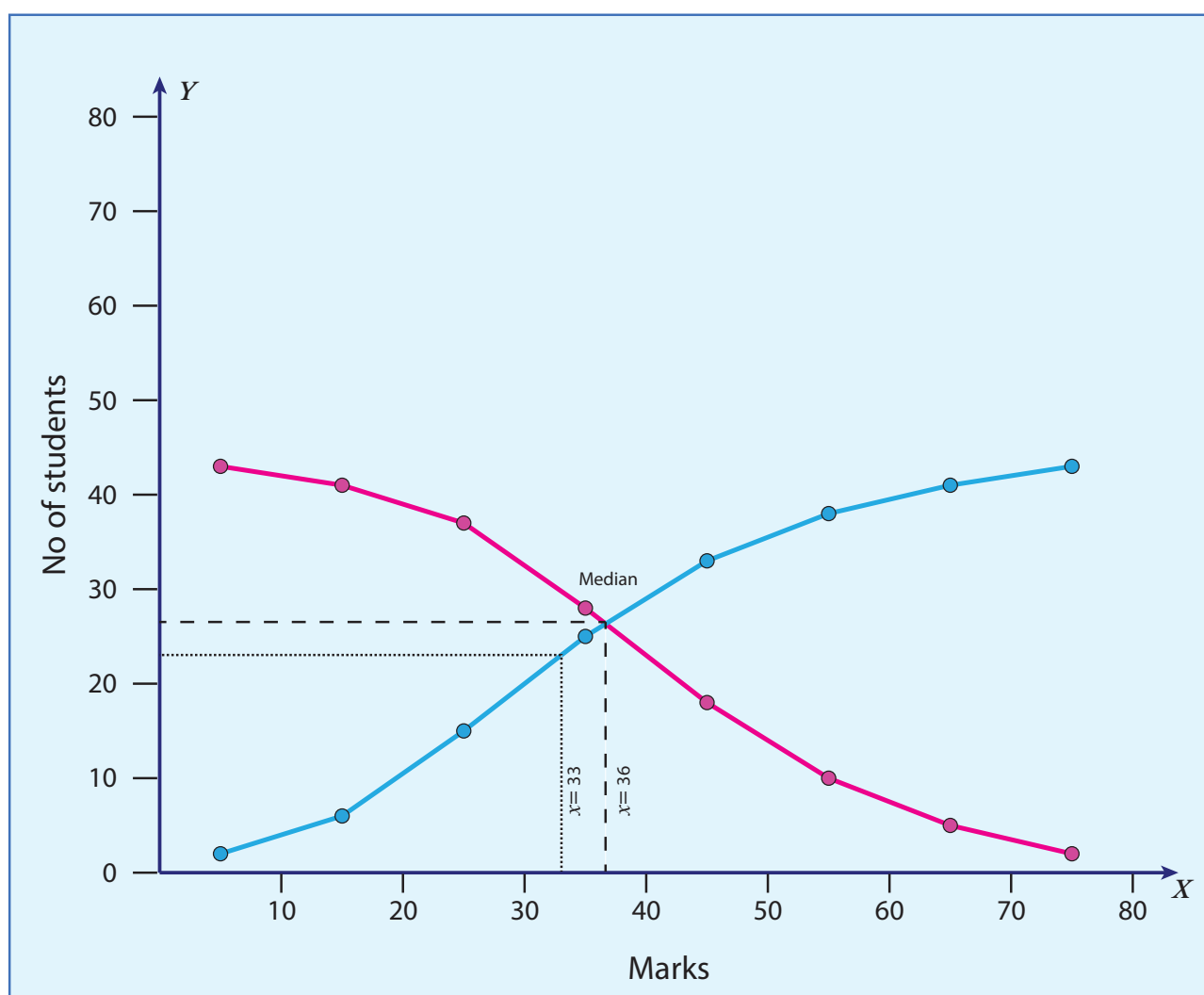
### 1. Aim

To find the median of the data by using ogive curve

### Calculation.

More then ogive

| More than ogive |                | Less than ogive    |                |
|-----------------|----------------|--------------------|----------------|
| Marks less than | No of students | Marks greater than | No of students |
| 10              | 2              | 0                  | 43             |
| 20              | 6              | 10                 | 41             |
| 30              | 15             | 20                 | 37             |
| 40              | 25             | 30                 | 28             |
| 50              | 33             | 40                 | 18             |
| 60              | 38             | 50                 | 10             |
| 70              | 41             | 60                 | 5              |
| 80              | 43             | 70                 | 2              |



### Result:

- The total number of students who secured marks more than 33=26 students
- Median =36



## 2. Aim

To find the coefficient of skewness

Given data (i) mean 7.35, Mode =8, variance 1.69

(ii)  $Q_1 = 40$   $Q_2 = 50$   $Q_3 = 60$

### Formula:

(i) Karl – Pearson coefficient of skewness

$$\frac{\text{Mean} - \text{mode}}{\text{Standard deviation}}$$

$$(i) \text{ Bowler's Coefficient of skewness} = \frac{Q_3 + Q_1 - 2Q_2}{Q_3 - Q_1}$$

### Calculation:

(i) Karl Pearson coefficient of skewness

$$\begin{aligned} \text{Standard deviation} &= \sqrt{\text{variance}} \\ &= \sqrt{1.69} = 1.3 \end{aligned}$$

$$\begin{aligned} \text{Coeff. of skewness} &= \frac{7.35 - 8}{1.3} \\ &= \frac{-0.65}{1.3} = -0.5 \end{aligned}$$

ii) Bowley's coefficient of skewness

$$\begin{aligned} &= \frac{40 + 60 - 2 \times 50}{60 - 40} \\ &= \frac{100 - 100}{20} = 0 \end{aligned}$$

Given distribution is symmetric

### Result:

(i) Karl Pearson Coefficient of skewness = -0.5

(ii) Bowley's coefficient of skewness = 0 and hence the distribution is symmetric.

### 3. Aim

To find the number of ways selecting the question

Given data

| Section | No. of questions |
|---------|------------------|
| A       | 5                |
| B       | 7                |

### Calculation:

To select at least 3 questions from each section to take totally 8 questions.

| Section | Number of Selections |   |   |
|---------|----------------------|---|---|
| A (5)   | 3                    | 4 | 3 |
| B(7)    | 5                    | 4 | 3 |

Combinations

$${}^5C_3 \times {}^7C_5 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} \times \frac{7 \times 6}{2 \times 1} = 10 \times 21 = 210$$

$${}^5C_4 \times {}^7C_4 = {}^5C_1 \times {}^7C_3 = 5 \times \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 175$$

$${}^5C_5 \times {}^7C_3 = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35$$

Total number of selection is =210+ 175+35=420

### Result:

Number of ways selection 8 questions from section A and B is 420.

### 4. Aim

To find a and their probabilities

### Calculation

Since  $P(x = x)$  is probability mass function  $\sum P(x = x) = 1$

$$P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3) + P(x = 4) + P(x = 5) + P(x = 6) + P(x = 7) + P(x = 8) = 1$$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$81a = 1$$

$$a = \frac{1}{81}$$

$$\begin{aligned} \text{(ii) } P(x < 3) &= P(x = 0) + P(x = 1) + P(x = 2) \\ &= a + 3a + 5a \end{aligned}$$

$$= \frac{9}{81}$$

$$\begin{aligned} \text{(iii)} \quad P(x \geq 5) &= P(x=5) + P(x=6) + P(x=7) + P(x=8) \\ &= 11a + 13a + 15a + 17a \\ &= \frac{56}{81} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad P(3 < x < 7) &= P(x=4) + P(x=5) + P(x=6) \\ &= 9a + 11a + 13a \\ &= \frac{33}{81} \end{aligned}$$

## Result

$$\begin{aligned} \text{(i)} \quad a &= \frac{1}{81} & \text{(ii)} \quad P(x < 3) &= \frac{9}{81} & \text{(iii)} \quad P(x \geq 5) &= \frac{56}{81} \\ \text{(iv)} \quad P(3 < x < 7) &= \frac{33}{81} \end{aligned}$$

**5. Aim :** To find the percentage of students scoring more than / less than particular marks using normal distribution

Given data

$$\mu = 60, \sigma = 5$$

## Formula

$$\text{(i)} \quad \lambda = \frac{x - \mu}{\sigma}$$

(ii) Standard normal table

## Calculation:

(i) more than 60 marks

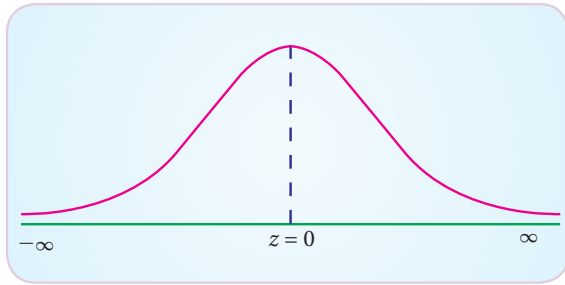
$$P(x > 60) = P(z > \frac{60 - 60}{5})$$

$$= P(z > 0)$$

$$= P(0 < z < \infty)$$

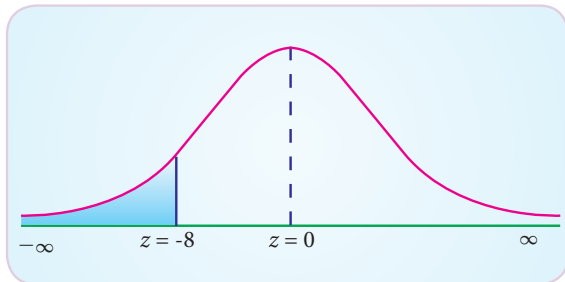
$$= 0.5000$$

Percentage of students scoring more than 60 marks =  $0.5 \times 100 = 50\%$



$$\begin{aligned} \text{(ii) Less than 56 marks} &= P(x < 56) \\ &= P\left(z < \frac{56-60}{5}\right) \\ &= P(z < -0.8) \\ &= P(-\infty < z < 0) - P(-0.8 < z < 0) \\ &= 0.5000 - P(0 < z < 0.8) \\ &= 0.5 - 0.2881 \\ &= 0.2119 \end{aligned}$$

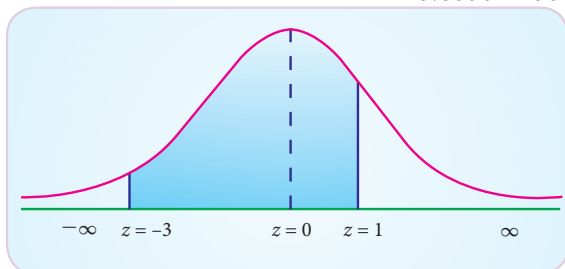
Percentage of students scored less than 56 marks  
 $= 0.2119 \times 100 = 21.19\%$



(ii) Between 45 and 65 marks

$$\begin{aligned} P(45 < x < 65) &= P\left(\frac{45-60}{5} < z < \frac{65-60}{5}\right) \\ &= P(-3 < z < 1) \\ &= P(-3 < z < 0) + P(0 < z < 1) \\ &= 0.4986 + 0.3413 \\ &= 0.8399 \end{aligned}$$

Percentage of students scored marks between 45 and 65 is  
 $= 0.8399 \times 100 = 83.99\%$



## Result

- (i) Percentage of students scored more than 60 marks = 50%
- (ii) Percentage of students scored less than 56 marks = 21.19%
- (iii) Percentage of students scored marks between 45 and 65 is 83.99%